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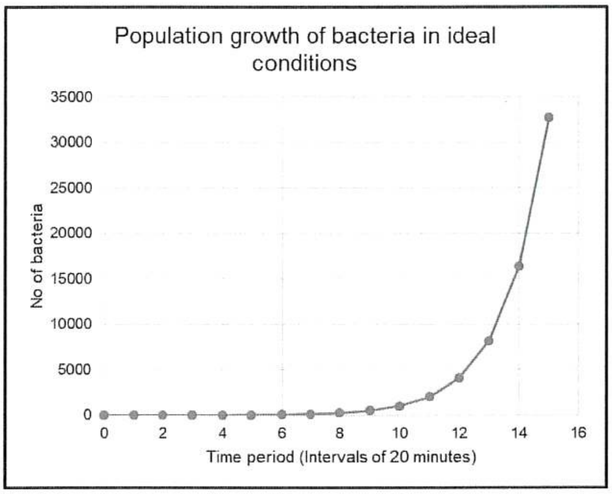
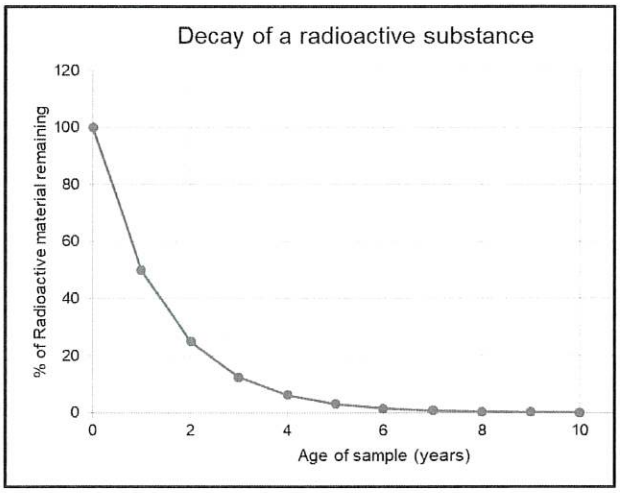
Task 1:

Indices

Did you know?

Indices are also referred to as exponents





e.g. $2^3 = 8$

3 is the 'exponent'

$2^3 = 2 \times 2 \times 2$

It tells us how many times a number is multiplied by itself

This is where exponential graphs come from!

Indices 1

Simplify the following

1. $x^3 \times x^8 =$

2. $\frac{9^8}{9} =$

3. $(2^3)^5 =$

4. $\frac{4^4 \times 4}{(4^2)^3} =$

5. $16^{\frac{1}{2}} =$

6. What is the reciprocal of 16

7. What is 4^{-3}

8. What is $\left(\frac{2}{5}\right)^{-1}$

Indices 2

Simplify the following

1. $t^5 \times t^4 =$

2. $\frac{8^7}{8^2} =$

3. $(3^4)^2 =$

4. $\frac{5^7 \times 5}{(5^3)^3} =$

5. $(8)^{\frac{1}{3}} =$

6. $y^0 =$

7. What is $4^{-3} =$

8. What is $\left(\frac{2}{3}\right)^{-2} =$

Roots and Indices Maze

Can you find the way from one side of the table to the other?

- Begin in the highlighted box
- Move vertically or horizontally one box at a time ....no diagonal moves allowed
- You may only land on boxes which are equivalent in value to the highlighted one

$2^6 \times 2^3$	$3^2 \times 2^3$	$(\sqrt{16})^2$	$(2^3)^3$	$8^3 \div 8$	$4^4 \times 4^{-3}$	$(\sqrt[3]{8})^4$	8×4^2
$\sqrt{8}^3$	$(2^3)^2$	$8^7 \times 8^{-5}$	4^3	$2^{-2} \times 2^7$	64^0	$2^5 \times 2^3$	$4^7 \div 2^3$
$(\sqrt{64})^3$	8^2	$2^2 \times 2^3$	$2^3 \times 2^3$	$(2^3)^3$	$(\sqrt[3]{8})^6$	$4^6 \times 4^{-3}$	$2^2 \times 4^2$
2^6	$(\sqrt{64})^2$	$4^6 \times 4^{-2}$	$(\sqrt{16})^3$	$(2^2)^4$	$8^3 \div 2^3$	$2^{-3} \times 2^7$	$(2^2)^4$
3^5	$2^6 \times 2^1$	8^3	$4^5 \div 2^4$	$(-4)^{-3}$	$(2^2)^3$	$(\sqrt{8})^3$	$4^6 \div 2^6$
$4^3 \times 4^{-3}$	$(2^5)^1$	$(\sqrt[3]{64})^2$	$2^3 \times 8$	$2^{-1} \times 2^7$	$(\frac{1}{4})^{-3}$	16^2	64

Hint : What is the value of 2^6

Matching Pairs

Match the expressions in Column A with their equivalent expression in Column B

A	B
$\left(\frac{9}{16}\right)^{\frac{1}{2}}$	$\frac{3}{2}$
$(4)^{\frac{3}{2}}$	8
$(-5)^{-2}$	$\frac{1}{16}$
$(16)^{-\frac{3}{2}}$	$\frac{1}{4}$
$(2)^{-3}$	$\frac{3}{4}$
$(64)^{-\frac{1}{3}}$	$\frac{1}{25}$
$\left(\frac{4}{9}\right)^{-\frac{1}{2}}$	$\frac{1}{8}$
4^{-2}	$\frac{1}{64}$

Indices Challenge

Where does it belong?

Five numbers are arranged below in order from least to greatest

$$x, \quad x^3, \quad x^4, \quad x^2, \quad x^0$$

- Where does $-x^{-1}$ belong in the list above?

Hints

- The numbers are arranged in order ($x < x^3 < x^4 < x^2 < x^0$)
- When is a cubed number greater than a squared number?
- Are there any of the terms that you know the value of ?
- Draw a number line and try some values in the expressions – what happens?

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Task 2: **Surds**

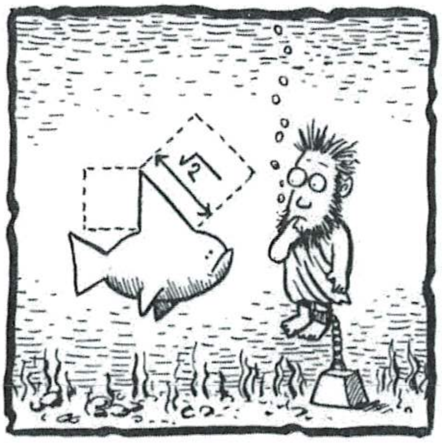
Did you know?

Maths can be murderous!

You will have heard of Pythagoras and his theorem but have you heard of Hippasus who was one of his followers?

Pythagoreans preached that all numbers could be expressed as the ratio of integers – i.e. fractions.

Hippasus is sometimes credited with the discovery of the existence of irrational numbers – proving it for $\sqrt{2}$. Following which, he was drowned at sea!



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Surds 1

1. Simplify $\sqrt{a} + 2\sqrt{a} + 5\sqrt{a}$

2. Simplify $\sqrt{2} \times \sqrt{6}$

3. Simplify fully $(4\sqrt{3})^2$

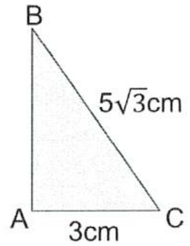
4. Write $\sqrt{45} + \sqrt{20}$ in the form $k\sqrt{5}$

5. Calculate $\frac{\sqrt{54}}{\sqrt{6}}$

6. Rationalise the denominator of $\frac{4}{\sqrt{3}}$

7. A rectangle has an area of $8\sqrt{15}$ cm^2 and a length of $2\sqrt{3}$ cm.
   Find the width of the rectangle

8. Find the length AB



Surds 2

1. Simplify $\sqrt{d} + 6\sqrt{d} - 3\sqrt{d}$

2. Simplify $2\sqrt{b} \times 4\sqrt{3}$

3. Simplify fully $(4\sqrt{5})^2$

4. Write $\sqrt{75} + \sqrt{48} - 2\sqrt{12}$ in the form $k\sqrt{3}$

5. Simplify $\frac{\sqrt{125} - 2\sqrt{20}}{\sqrt{5}}$

6. Rationalise the denominator of $\frac{2\sqrt{2}}{\sqrt{5}}$

7. Evaluate $\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{6}}$

8. A triangle has base of $3\sqrt{2}$ and a perpendicular height of $5\sqrt{8}$

   Calculate the area of the triangle

Another Brick in the Wall

Complete the empty boxes in the pyramid.

Each box is the sum of the two boxes directly below it.

$31\sqrt{3} + 18\sqrt{5}$

$\sqrt{75}$	$2\sqrt{3}$	$\sqrt{245}$	$-\sqrt{180}$	$2\sqrt{243}$

Hint: You may need to simplify some of the surds in the bottom row to get started.

True or False

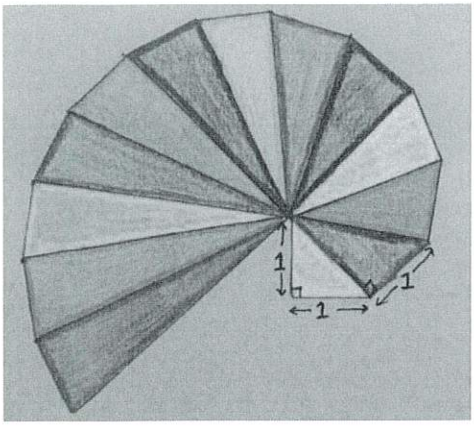
Decide if each of the following expressions is *True* or *False*

1. $\sqrt{9} + \sqrt{4} = \sqrt{13}$
2. $\sqrt{a} \times \sqrt{b} = \sqrt{c}$
3. $\sqrt{(8)^2} = 8$
4. $10\sqrt{2} = \sqrt{8}$
5. $\frac{\sqrt{12}\times\sqrt{3}}{\sqrt{9}} = 2$
6. $\sqrt{2}^3 = 2\sqrt{2}$
7. $\sqrt{ab}^2 = ab$
8. $2\sqrt{100} = \sqrt{200}$

Are there some statements that are 'Sometimes true' but not 'Always true'? Explain why.

Surds Challenge

The Wheel of Theodorus



The diagram shows a spiral made up of right angled triangles.

The shortest side of each triangle measures 1 unit long.

Can you see how it is constructed?

Find the length of the hypotenuse of the first few triangles.

What do you notice ?

Which triangle would have a side length of 3?

What other questions might you want to ask about the diagram?

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Task 3: # Further Factorising

? Did you know? ?

Substitute $x = 9$ into the following two expressions

$$\boldsymbol{x^2 + 3x + 2}$$

What do you notice?

$$(9)^2 + 3(9) + 2 = 81 + 27 + 2 = 110$$

$$\boldsymbol{(x + 2)(x + 1)}$$

$$(9 + 2)(9 + 1) = 11 \times 10 = 110$$

Both give the same answer as the expressions are equivalent

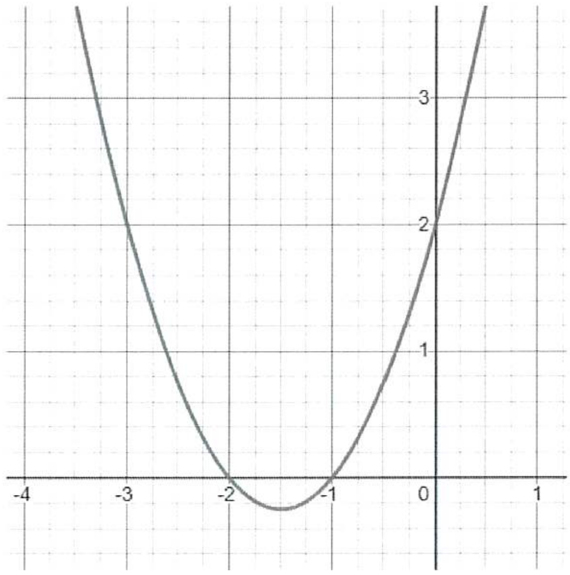
One of the expressions was a lot easier to evaluate! Why?

Which is best?

$$x^2 + 3x + 2 \quad \text{or} \quad (x + 2)(x + 1)$$

expanded form — factorised form

$$y = x^2 + 3x + 2 \qquad y = (x + 2)(x + 1)$$



Factorising is a key skill for both sketching graphs and solving equations, both of which will be covered later.

Sometimes it is more helpful to factorise an expression, other times better to be expand it, depending on the context.

Further Factorising 1

Factorise the following fully:

1. $x^2 + 5x - 6$
2. $x^2 + 13x - 30$
3. $y^2 - 13y + 30$
4. $t^2 + 2t - 15$
5. $k^2 - 2k - 24$
6. $p^2 - 10p + 21$
7. $x^2 - 16x$
8. $3x(2x - 1) + 4(1 - 2x)$

Further Factorising 2

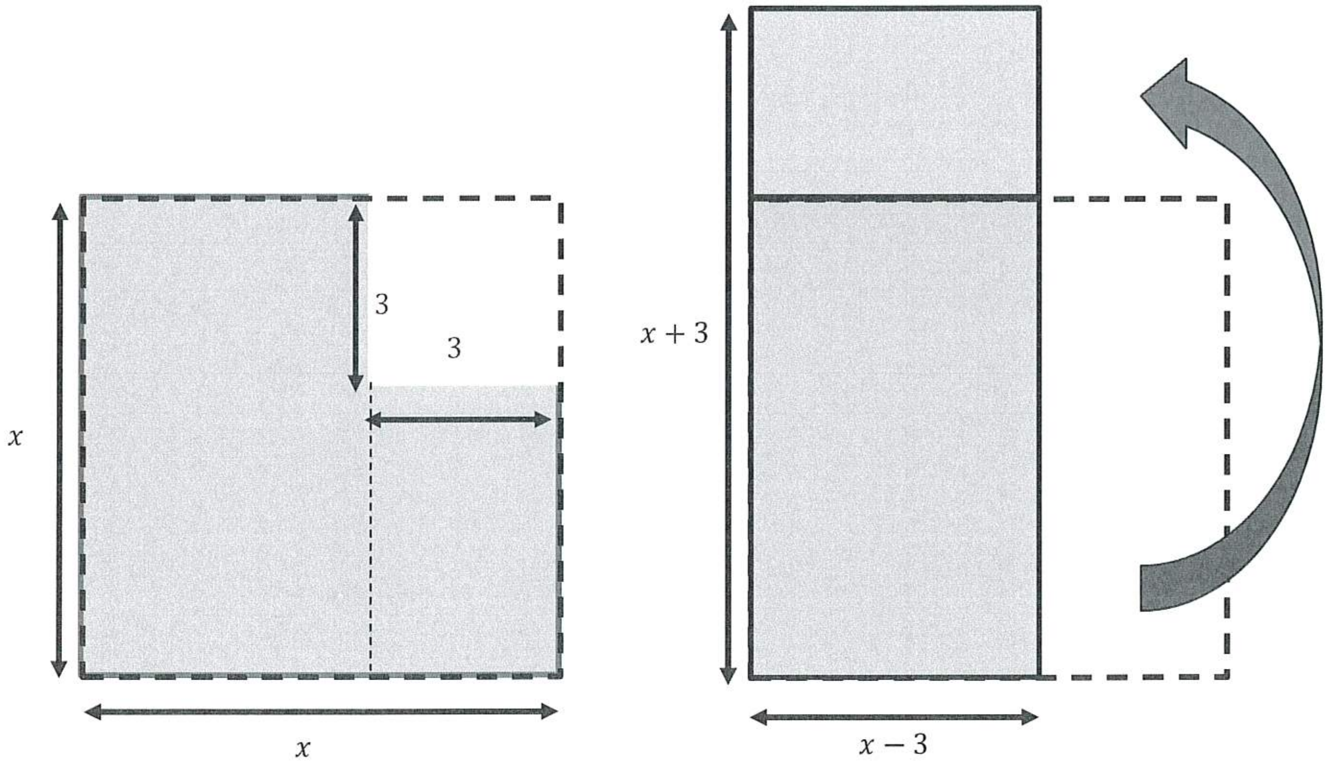
Factorise the following fully:

1. $x^2 + 6x - 7$
2. $y^2 + y - 12$
3. $y^2 - 11y + 28$
4. $t^2 + 7t - 18$
5. $k^2 + 9k + 20$
6. $x^2 + x - 56$
7. $p^2 - 25p$
8. $x^2(3x - 4) + (4 - 3x)$

Difference of Two Squares

A special case for factorising is the difference of two squares.

Expressions such as $x^2 - 3^2$, where the coefficient of x is zero.



$$x^2 - 3^2 = (x-3)(x+3)$$

Try factorising these expressions using the difference of two squares

1. $x^2 - 6^2$
2. $y^2 - 144$
3. $x^2 - y^2$
4. $4t^2 - 81$
5. $x^2 - 5$

$$ax^2 + bx + c$$

So far we have been factorising quadratic expressions where $a = 1$. For example, $x^2 - 2x - 15$

Time to try some trickier quadratics!

Have a go at this one...

Factorise $\quad \mathbf{6x^2 + 19x + 10}$

If you got $6x^2 + 19x + 10 = (3x + 2)(2x + 5)$ Well done! ★

Feeling confident? You can try the Trickier Quadratics questions below

There are many methods for factorising quadratics where $a > 1$

You might want to refresh your memory on the method that you learnt at school if you are going to tackle the following questions.

Trickier Quadratics

1. $\mathbf{3x^2 - 10x - 8}$
2. $\mathbf{2x^2 - 7x + 6}$
3. $\mathbf{4y^2 + 20y + 9}$
4. $\mathbf{6x^2 - 13x - 8}$
5. $\mathbf{20x^2 + x - 12}$

Further Factorising Problems

These expressions are slightly different to the previous ones but can still be factorised.

1. $2t^2 - 32$
2. $x^3 - 7x^2 + 12x$
3. $x^4 - x^2 - 2$
4. $y^4 - 625$

Factorising Challenge

Without a calculator

What is the value of each of the following?
calculators not allowed

$$9^2 - 1^2$$

$$99^2 - 1^2$$

$$999^2 - 1^2$$

Still without a calculator

Without using a calculator, find the value of

$$\frac{122 \times (122^2 + 4 \times 123)}{124} - \frac{124 \times (124^2 - 4 \times 123)}{122}$$

Top and Bottom

Simplify

$$\frac{x^2 - 3x - 10}{x^2 + 7x + 10}$$

Some possible hints!

Without a calculator Hint	Still without a calculator Hint	Top and Bottom Hint
▪ Can you factorise $9^2 - 1^2$? ▪ How does this help?	▪ Replace 123 by n and 122 by n−1 ▪ Now go on to factorise	▪ Factorise the numerator then the denominator ▪ What do you notice?

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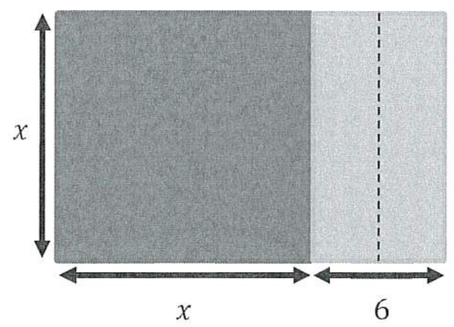
Task 4: Completing the Square

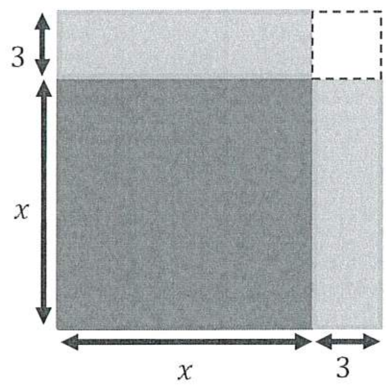
Did you know?

These are different forms of the same algebraic expression

$$x^2 + 6x \quad = \quad x(x + 6) \quad = \quad (x + 3)^2 - 9$$

expanded form	factorised form	completed square form





Do the diagrams help you see why this is called **Completing the square**?

Completing the square 1

Write these expressions in the form $(x+a)^2+b$

1. x^2+4x
2. x^2+4x+5
3. y^2-8y
4. y^2-8y+7
5. $x^2-12x+41$
6. $k^2+10k-2$
7. y^2+3y+1
8. p^2-2p+1

Completing the square 2

Write these expressions in the form $(x+a)^2+b$

1. x^2+10x
2. $x^2+10x+30$
3. y^2-2y
4. y^2-2y+3
5. $x^2-8x+25$
6. $k^2+14k-1$
7. y^2+5y+6
8. t^2+6t+9

Completing The Square Challenge

Different Forms

It is important to be able to convert expressions between the different forms:

expanded form　　　factorised form　　　completed square form

In this problem there are 4 sets of three equivalent expressions, however, some expressions are missing. Match the sets and find the 3 missing expressions.

$a^2 - 2a - 8$		$a^2 - 8a + 15$
	$a^2 + 2a - 15$	$(a + 2)(a + 4)$
$(a + 1)^2 - 16$	$(a - 3)(a - 5)$	
$(a + 5)(a - 3)$	$(a - 1)^2 - 9$	$(a + 3)^2 - 1$

Extra Puzzle 1

What is the value of

$$\frac{\dfrac{(5^2 - 3^2)}{5 + 3} + \dfrac{(4^2 - 2^2)}{4 + 2} + \dfrac{(3^2 - 1^2)}{3 + 1}}{2} \ ?$$

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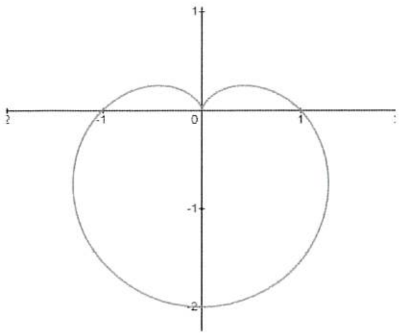
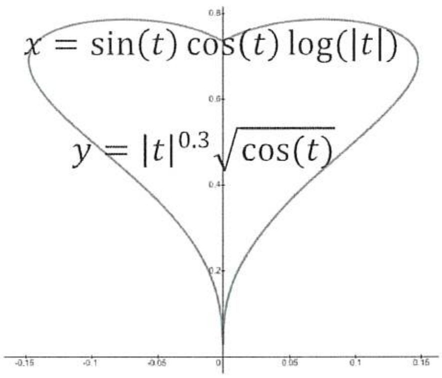
Task 5: Rearranging Factorising

Did you know?

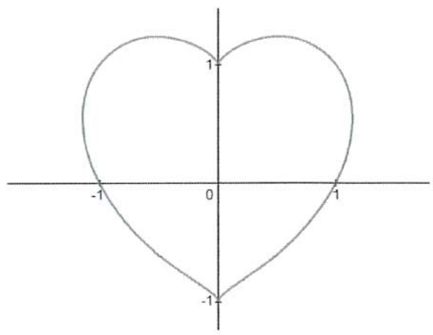
- Being able to express equations in different forms gives us different information
- Later we'll be looking at information needed to sketch graphs
- If you continue your maths studies to A Level Further Maths, you will draw graphs such as these

$r = 1 - sin\theta$





$(x^2 + y^2 - 1)^3 - x^2y^3 = 0$



Futher Factorising 1

1. The equation of a line is given as
    i. $3y + 4x - 2 = 0$.
    b. What is the gradient of the line?

2. A rectangle has area A, length y and width $x - 2$. Write an expression for the length of the rectangle, y, in terms of A and x

3. Make x the subject of:
    a. $ax - y = z + bx$

4. The equation of a line is given as
    i. $5(\mathrm{b} - \mathrm{p}) = 2(\mathrm{b} + 3)$

5. John says the first step to rearranging
    a. $\frac{x-a}{f} = 3g$ is to add $a\ to\ 3g$. Is he right? Explain your answer.

6. Make a the subject of
    a. $5(a - t) = 3(a + x)$

7. Make x the subject of
    a. $ay + x = 4x + xb$

8. Make x the subject of
    a. $2\pi\sqrt{x + t} = 4$

Further Factorising 2

1. Make y the subject of
   $xy + 6 = 7 - ky$

2. Find an expression for the area of a rectangle with length, $(y - x)$ and width, $(x - 2)$

3. Rewrite your expression in Q2 to have y expressed in terms of A and x

4. Make y the subject of
   $\frac{4}{y} + 1 = 2x$

5. Displacement can be expressed as
    i. $s = ut + \frac{1}{2}at^2$

   Express a in terms of s, u and t

6. Make y the subject of $\sqrt{by^2 - x} = D$

7. The area of a trapezium has formula
    i. $A = \frac{1}{2}\left(\frac{a+b}{h}\right)$

   Express h in terms of A, a and b

8. Make t the subject $b(t + a) = x(t + b)$

Equivalent quadratics

Sort the expressions below in to 4 sets of 4 equivalent expressions

$x^2 - 25$	$2x^2 - 2$
$(x+5)(x+6) - x - 55$	$(x+5)(x-5)$
$2(x^2 - 1)$	$(x+5)^2 - 10x - 50$
$2(x+3)(x-1)$	$2(x+1)(x-1)$
$(x+5)^2 - 50$	$2(x+2)^2 - 4x - 14$
$2x^2 + 4x - 6$	$(x+5)(x-5) + 10x$
$2(x+1)^2 - 8$	$(x-5)(x+6) - x + 5$
$x^2 + 10x - 25$	$2(x+1)^2 - 4(x+1)$

Rearrange Challenge

The Quadratic Formula

We've all used the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- But where does it come from?
- Can you prove why the quadratic formula works?

Rearrange these steps in order to prove the quadratic formula

$$ax^2 + bx + c = 0 \longrightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$\left(x + \frac{b}{2a}\right) = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$	$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} = -\frac{c}{a}$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	$ax^2 + bx = -c$
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	$\left(x + \frac{b}{2a}\right) = \pm\sqrt{\left(\frac{b^2 - 4ac}{4a^2}\right)}$	$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

Match the steps below with the algebra above for a slightly easier version

Step 1: Subtract c from both sides

Step 2: Divide both sides by a

Step 3: Complete the square on the left hand side

Step 4: Add $\frac{b^2}{4a^2}$ to both sides

Step 5: Make the right hand side into a single expression

Step 6: Take the square root of both sides

Step 7: Simplify the denominator on the right hand side

Step 8: Subtract $\frac{b}{2a}$ from both sides

Step 9: You now have the quadratic formula!

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Task 6: Solving Quadratics

Did you know?

I have picked two numbers that multiply to make zero.

What can you say about my numbers?

At least one of them must be zero

This is useful when using factorising to solve equations.

If $a \times b = 0$, then either $a = 0$ or $b = 0$ (or both!)

Historically zero wasn't accepted as a number until fairly recently!

Solving with Quadratics 1

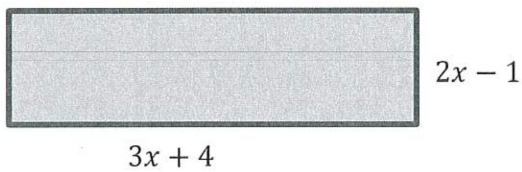
Solve the following

1. $x^2 = 16$
2. $x^2 - 16x = 0$
3. $(x+1)(2x-3) = 0$
4. $x^2 - 3x + 2 = 0$
5. $(2x-5)(4x+3) = 0$
6. $3x^2 + 14x - 5 = 0$
7. $(x+3)^2 = 25$
8. $\frac{3}{x} + \frac{4}{x-1} = 10$

Solving with Quadratics 2

Solve the following

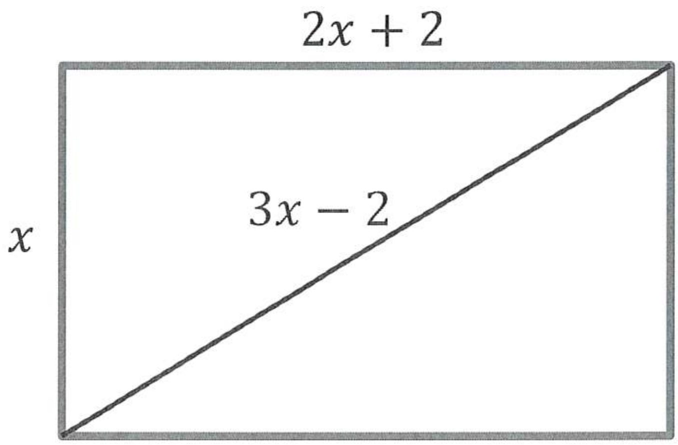
1. $x^2 - 4x - 12 = 0$
2. $x^2 - x = 6$
3. $2x^2 - 11x + 12 = 0$
4. $6x^2 + x - 12 = 0$
5. $3 + 2x - x^2 = 0$
6. $x^2 - 4x - 1 = 0$
7. $\frac{8}{x+2} - \frac{14}{x-3} = 9$
8. The area of this rectangle is $30m^2$



a) Show that $6x^2 + 5x - 34 = 0$
b) Find any possible values for x

Quadthagoras

Find the length, width and diagonal of this rectangle



Up in the air!

An object is launched from a cliff that is $58.8m$ high.

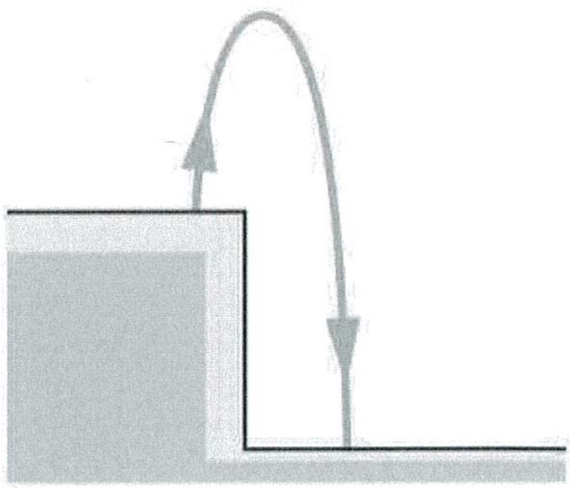
The speed of the object is 19.6 metres per second (m/s).

The equation for the object's height *h* above the ground at time *t* seconds after launch is

$$h = -4.9t2 + 19.6t + 58.8$$

where *h* is in metres.

- When does the object strike the ground?



Quadratics Challenge

Which Way?

In the skills check you saw how we can solve quadratic equations by factorising or completing the square.

We can also use the quadratic formula. For a quadratic $ax^2 + bx + c = 0$ the solutions are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Try solving $x^2 + 4x - 21 = 0$ using each of the three methods.

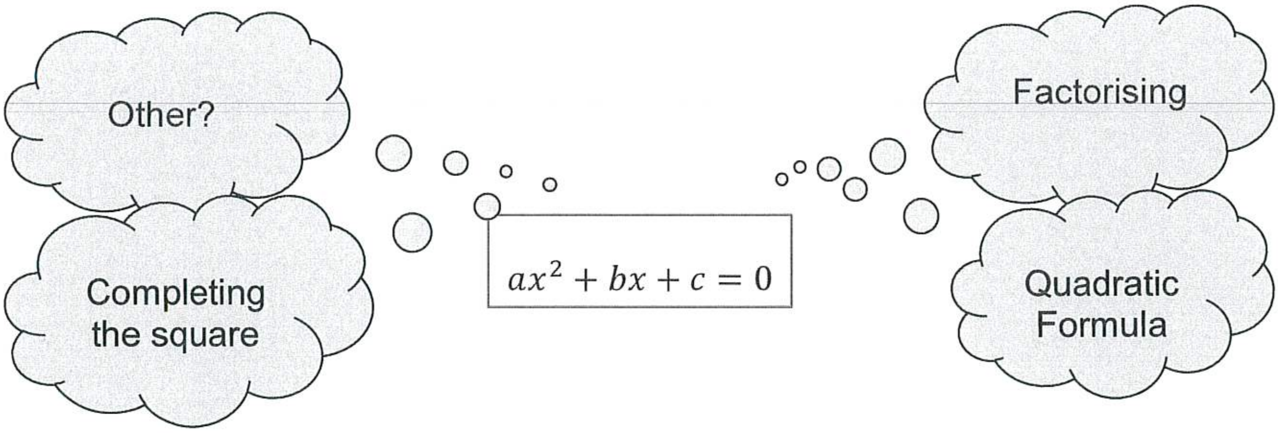
Try solving $3x^2 + 4x - 2 = 0$ using each of the three methods.

Which Way Now?

There is not always one best way to solve a quadratic.

Some methods are better than others for different equations

How can you spot which is the right method for each equation?

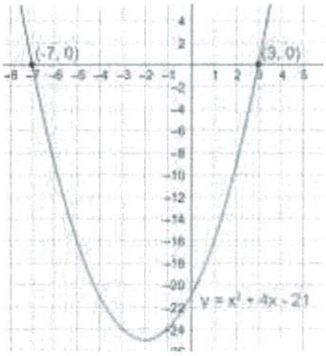


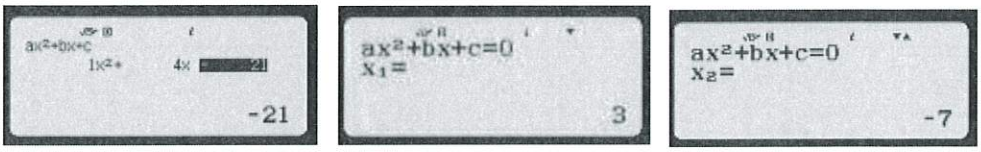
https://undergroundmathematics.org/quadratics/quad-solving-sorter is a really good activity for improving your skills in sorting quadratic equations. You or your teacher may be able to print the cards out to help.

Another Way?

And of course, there are the methods of solving using graphs and/or your calculator

$$x^2 + 4x - 21 = 0$$





Using Graphs

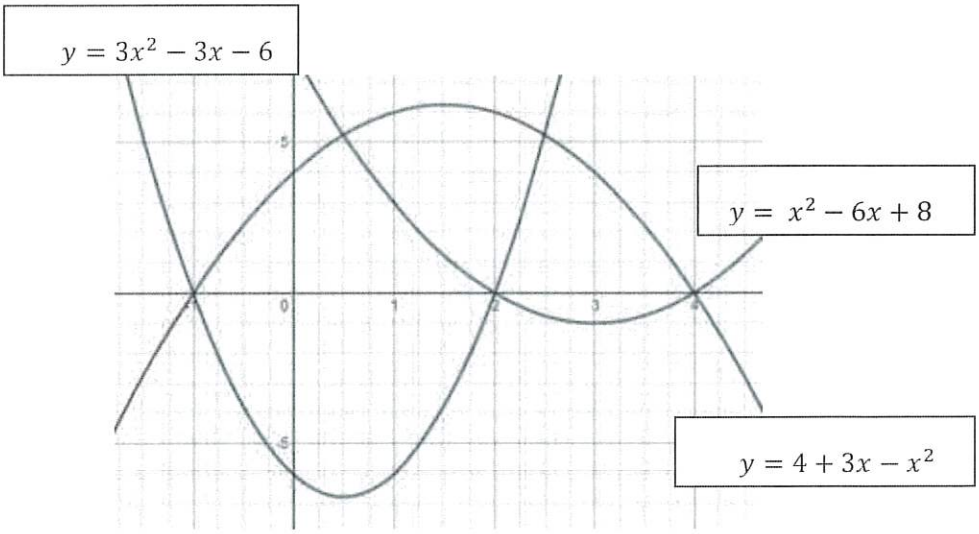
Use the graphs to solve

$$4 + 3x - x^2 = 0$$

$$x^2 - 6x + 8 = 0$$

$$3x^2 - 3x - 6 = 0$$

$$4 + 3x - x^2 = 4$$



Simultaneously

Solve these pairs of equations

1. $y = x^2 + 6x - 9$

$y = 3x + 1$

2. $y = x^2 + 2x + 2$

$y - 4x = 1$

3. A rectangle has length $(a + b)$ and width $3a$.

The area is $60cm^2$ and perimeter is $32\ cm$.

Calculate, algebraically, the values of a and b.

4. In how many places does the line $y = 2x + 2$ intersect the circle $(x + 2)^2 + y^2 = 25$?

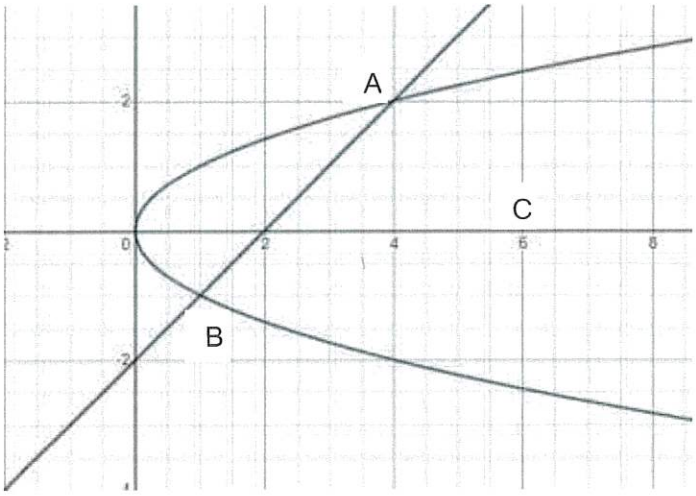
What are the co-ordinates of these intersections?

The diagram shows the graphs of

$y^2 = x$ and $y = x - 2$

The graphs cross at the points A and B.

The point C has co-ordinates (6,0)



- Without the use of a calculator, find the exact area of triangle ABC