

YEAR 10 — SIMILARITY...

Trigonometry

What do I need to be able to do?

By the end of this unit you should be able to:

- Work fluently with hypotenuse, opposite and adjacent sides
- Use the tan, sine and cosine ratio to find missing side lengths
- Use the tan, sine and cosine ratio to find missing angles
- Calculate sides using Pythagoras' Theorem

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)

Scale Factor: the multiplier of enlargement

Constant: a value that remains the same

Cosine ratio: the ratio of the length of the adjacent side to that of the hypotenuse. The sine of the complement

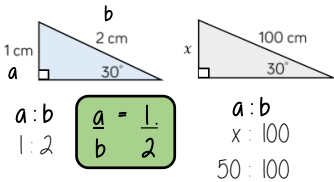
Sine ratio: the ratio of the length of the opposite side to that of the hypotenuse.

Tangent ratio: the ratio of the length of the opposite side to that of the adjacent side.

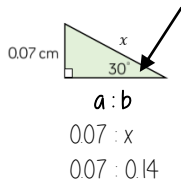
Inverse: function that has the opposite effect.

Hypotenuse: longest side of a right-angled triangle. It is the side opposite the right-angle.

Ratio in right-angled triangles

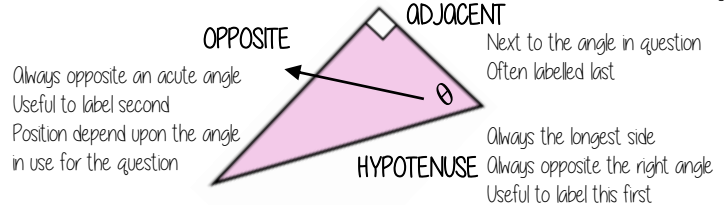


When the angle is the same the ratio of sides a and b will also remain the same



Hypotenuse, adjacent and opposite

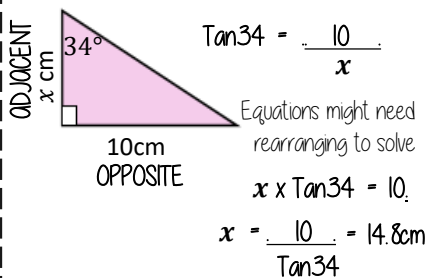
ONLY right-angled triangles are labelled in this way



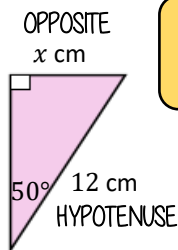
Tangent ratio: side lengths

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Substitute the values into the tangent formula



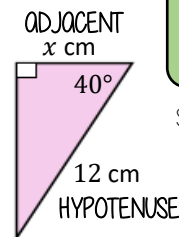
Sin and Cos ratio: side lengths



$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$$

NOTE

The $\sin(x)$ ratio is the same as the $\cos(90-x)$ ratio



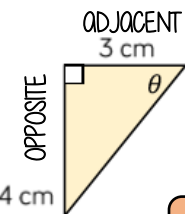
$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$$

Substitute the values into the ratio formula

Equations might need rearranging to solve

Sin, Cos, Tan: Angles

Inverse trigonometric functions



Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$$\theta = \tan^{-1} \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\theta = \sin^{-1} \frac{\text{opposite side}}{\text{hypotenuse side}}$$

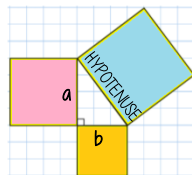
$$\theta = \cos^{-1} \frac{\text{adjacent side}}{\text{hypotenuse side}}$$

$\tan \theta = \frac{4}{3}$
 $\theta = \tan^{-1} \frac{4}{3}$
 $\theta = 36.9^\circ$

Pythagoras theorem



$$\text{Hypotenuse}^2 = a^2 + b^2$$



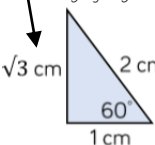
This is commutative — the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras

- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

Key angles

This side could be calculated using Pythagoras



$$\tan 30 = \frac{1}{\sqrt{3}}$$

$$\tan 60 = \sqrt{3}$$

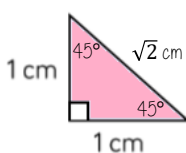
$$\cos 30 = \frac{\sqrt{3}}{2}$$

$$\cos 60 = \frac{1}{2}$$

$$\sin 30 = \frac{1}{2}$$

$$\sin 60 = \frac{\sqrt{3}}{2}$$

Because trig ratios remain the same for similar shapes you can generalise from the following statements



$$\tan 45 = 1$$

$$\cos 45 = \frac{1}{\sqrt{2}}$$

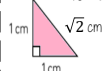
$$\sin 45 = \frac{1}{\sqrt{2}}$$

Key angles 0° and 90°

$$\tan 0 = 0$$

$$\tan 90$$

This value cannot be defined — it is impossible as you cannot have two 90° angles in a triangle



$$\sin 0 = 0$$

$$\sin 90 = 1$$

$$\cos 0 = 1$$

$$\cos 90 = 0$$