

YEAR 7 — REASONING WITH NUMBER

Prime numbers and Proof

What do I need to be able to do?

By the end of this unit you should be able to:

- Find and use multiples
- Identify factors of numbers and expressions
- Recognise and identify prime numbers
- Recognise square and triangular numbers
- Find common factors including HCF
- Find common multiples including LCM

Keywords

Multiples: found by multiplying any number by positive integers
Factor: integers that multiply together to get another number.
Prime: an integer with only 2 factors
Conjecture: a statement that might be true (based on reasoning) but is not proven
Counterexample: a special type of example that disproves a statement
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
HCF: highest common factor (biggest factor two or more numbers share)
LCM: lowest common multiple (the first time the times table of two or more numbers match)

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3

3, 6, 9, 12, 15...

$3x, 6x, 9x \dots$

This list continues and doesn't end

Non example of a multiple

45 is not a multiple of 3 because it is 3×15

Not an integer

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Factors

Arrays can help represent factors

Factors of 10: 1, 2, 5, 10

10×1 or 1×10

5×2 or 2×5

Factors and expressions

$6x \times 1$ OR $6 \times x$

$2x \times 3$

$3x \times 2$

The number itself is always a factor

Prime numbers

- Integer
- Only has 2 factors
- and itself

The first prime number

The only even prime number

2

Learn or how-to quick recall...

2, 3, 5, 7, 11, 13, 17, 19, 23, 29...

Square and triangular numbers

Square numbers

Representations are useful to understand a square number n^2

1, 4, 9, 16, 25, 36, 49, 64 ...

odd, even, odd

Triangular numbers

Representations are useful — an extra counter is added to each new row

Add two consecutive triangular numbers and get a square number

1, 3, 6, 10, 15, 21, 28, 36, 45...

Common factors and HCF

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF — Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

Common factors (factors of both numbers): 1, 2, 3, 6

HCF = 6

6 is the biggest factor they share

Common multiples and LCM

Common multiples are multiples two or more numbers share

LCM — Lowest common multiple

LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54

12: 12, 24, 36, 48, 60

LCM = 36

The first time their multiples match

Comparing fractions

Compare fractions using a LCM denominator

$\frac{3}{5}$ and $\frac{7}{10}$

$\frac{6}{10}$ and $\frac{7}{10}$

Product of prime factors

Multiplication part-whole models

30 = 2 x 15 = 2 x 3 x 5

30 = 3 x 10 = 3 x 2 x 5

30 = 5 x 6 = 5 x 2 x 3

All three prime factor trees represent the same decomposition

Multiplication is commutative

$30 = 2 \times 3 \times 5$

Multiplication of prime factors

Using prime factors for predictions

e.g. 60: 30×2 or $2 \times 3 \times 5 \times 2$

150: 30×5 or $2 \times 3 \times 5 \times 5$

Conjectures and counterexamples

Conjecture

1, 2, 4, ...

The numbers in the sequence are doubling each time.

A pattern that is noticed for many cases

Counterexamples

This sequence isn't doubling it is adding 2 each time

Only one counterexample is needed to disprove a conjecture