St Bernadette's Catholic Primary School, Wombourne

How we teach calculations:

Calculation Policy for Mathematics

October 2014

Achieve, Believe, Care

About our Calculation Policy

The following calculation policy has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school. Please note that early learning in number and calculation in Reception follows the 'Development Matters' EYFS document, and this calculation policy is designed to build on progressively from the content and methods established in the Early Years Foundation Stage.

Age stage expectations

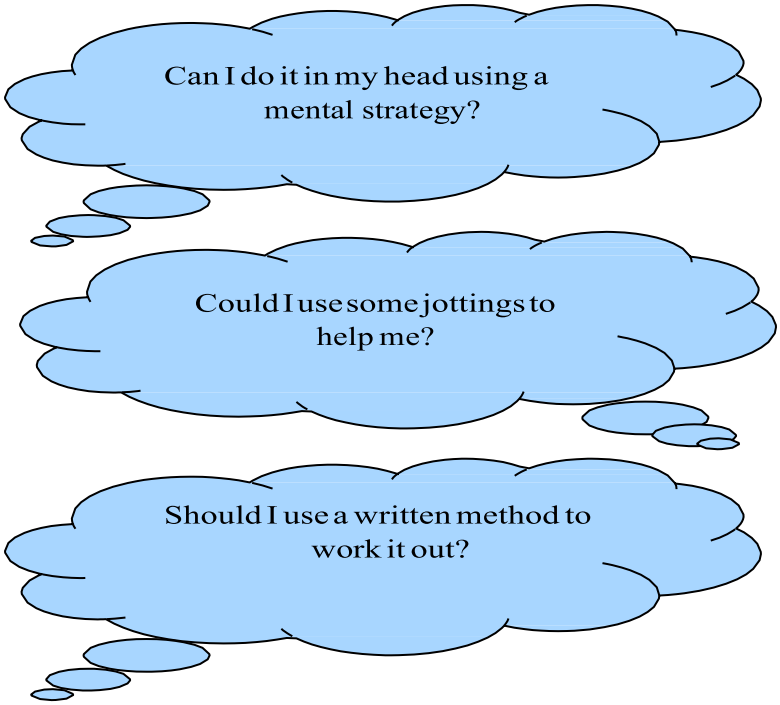
The calculation policy is organised according to age stage expectations as set out in the National Curriculum 2014, **however it is vital that pupils are taught according to the stage that they are currently working at,** being moved onto the next level as soon as they are ready, or working at a lower stage until they are secure enough to move on.

Providing a context for calculation:

It is important that any type of calculation is given a real life context or problem solving approach to help build children"s understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons.

Choosing a calculation method:

Children need to be taught and encouraged to use the following processes in deciding what approach they will take to a calculation, to ensure they select the most appropriate method for the numbers involved:

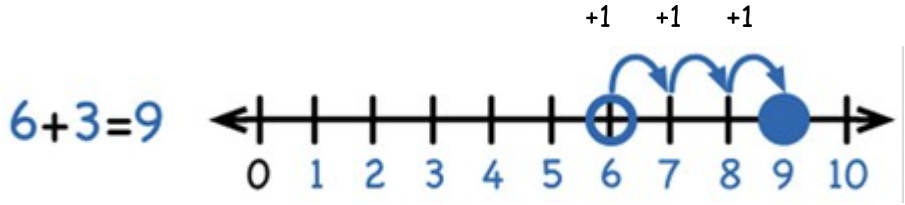


To work out a tricky calculation:
Approximate,
Calculate,
Check it mate!

Addition

Stage 1 Add with numbers up to 20

Use numbered number lines to add, by counting on in ones. Encourage children to start with the **larger** number and count on.



Children should:

Have access to a wide range of counting equipment, everyday objects, number tracks and number lines, and be shown numbers in different con-texts.

Read and write the addition (+) and equals (=) signs within number sentences.

Interpret addition number sentences and solve missing box problems, using con-crete objects and number line addition to solve them:
8 + 3 = ☐ 15 + 4 = ☐ 5 + 3 + 1 = ☐ ☐ + ☐ = 6

This builds on from prior learning of adding by combining two sets of objects into one group (5 cubes and 3 cubes) in Early Years.

Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.

8 + 5

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line

Key skills for addition at Stage 1:

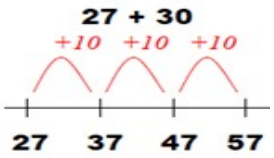
- Read and write numbers to 100 in numerals, incl. 1-20 in words
- Recall bonds to 10 and 20, and addition facts within 20
- Count to and across 100
- Count in multiples of 1 2, 5 and 10
- Solve simple 1-step problems involving addition, using objects, number lines and pictorial representations.

Addition

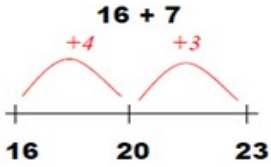
Stage 2 Add with 2-digit numbers

Developing mental fluency with addition and place value involving 2-digit numbers, then establish more formal methods.

Add 2-digit numbers and tens:

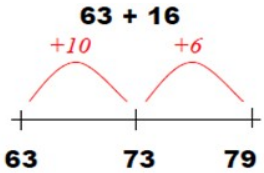


Add 2-digit numbers and units:



Use empty number lines, concrete equipment, hundred squares etc. to build confidence and fluency in mental addition skills.

Add pairs of 2-digit numbers, moving to the partitioned column method when secure



23 + 34 =

	20	+	3
+	30	+	4
	50	+	7
		=	57

STEP 1: Only provide examples that do **NOT** cross the tens boundary until they are secure with the method itself.

STEP 2: Once a child can add a multiple of ten to a 2-digit number mentally (e.g.80+11), they are ready for adding pairs of 2-digit numbers that DO cross the tens boundary

50	+	8	
40	+	3	
90	+	11	
	=	101	

STEP 3: Children who are confident and accurate with this stage should move onto the expanded addition methods with 2 and 3-digit numbers (see Y3).

To support understanding, pupils may physically make and carry out the calculation with Dienes Base 10 apparatus or place value counters, then compare their practical version to the written form, to help them to build an understanding of it.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary

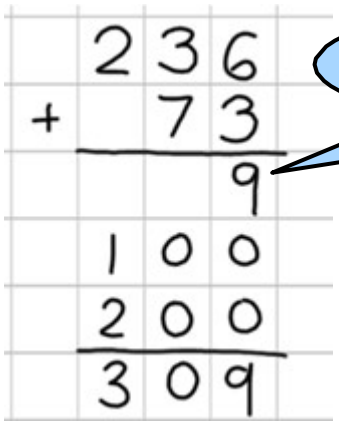
Key skills for addition at Stage 2:

- Add a 2-digit number and ones (e.g. 27 + 6)
- Add a 2-digit number and tens (e.g. 23 + 40)
- Add pairs of 2-digit numbers (e.g. 35 + 47)
- Add three single-digit numbers (e.g. 5 + 9 + 7)
- Show that adding can be done in any order (the commutative law).
- Recall bonds to 20 and bonds of tens to 100 (30 + 70 etc.)
- Count in steps of 2, 3 and 5 and count in tens from any number.
- Understand the place value of 2-digit numbers (tens and ones)
- Compare and order numbers to 100 using < > and = signs.
- Read and write numbers to at least 100 in numerals and words.
- Solve problems with addition, using concrete objects, pictorial representations, involving numbers, quantities and measures, and applying mental and written methods.

Addition

Stage 3 Add numbers with up to 3 digits

Introduce the **expanded column addition** method:



Add the **units** first, in preparation for the compact method

In order to carry out this method of addition:

- Children need to recognise the value of the hundreds, tens and units without recording the partitioning.

Move to the compact **column addition** method, with 'carrying':

Add **units** first.

'Carry' numbers **underneath** the bottom

```
 236
+ 73
 309
  1
```

Children who are very secure and confident with 3-digit expanded column addition should be moved onto the **compact column addition** method, being introduced to "carrying" for the first time. Compare the expanded method to the compact column method to develop an understanding of the process and the reduced number of steps involved.

Remind children the actual value is **'three tens** add **seven tens'**, not 'three add seven, which equals **ten** tens.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, **hundreds boundary, increase, vertical, carry', expanded, compact**

Key skills for addition at Stage 3:

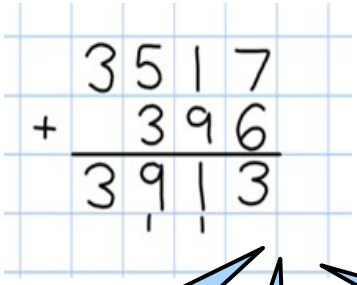
- Read and write numbers to 1000 in numerals and words.
- Add 2-digit numbers mentally, incl. those exceeding 100.
- **Add a three-digit number and ones mentally (175 + 8)**
- **Add a three-digit number and tens mentally (249 + 50)**
- **Add a three-digit number and hundreds mentally (381 + 400)**
- Estimate answers to calculations, using inverse to check answers.
- Solve problems, including missing number problems, using number facts, place value, and more complex addition.
- Recognise place value of each digit in 3-digit numbers (hundreds, tens, ones.)
- Continue to practise a wide range of mental addition strategies, ie. number bonds, adding the nearest multiple of 10, 100, 100 and adjusting, using near doubles, partitioning and recombining.

Addition

Stage 4 Add numbers with up to 4 digits

Move from expanded addition to the compact column method, **adding units first**, and „carrying" numbers **underneath** the calculation. Also include money and measures contexts.

e.g. 3517 + 396 = 3913



Introduce the **compact column addition** method by asking children to add the two given numbers together using the method that they are familiar with (expanded column addition-see Stage 3). Teacher models the compact method with carrying, asking children to discuss similarities and differences and establish how it is carried out.

Add **units** first.

'Carry' numbers **underneath** the bottom

Reinforce correct place value by reminding them the actual value is 5 hundreds add 3 hundreds, **not 5 add 3, for example**

Use and apply this method to money and measurement values.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, „carry", expanded, compact, **thousands, hundreds, digits, inverse**

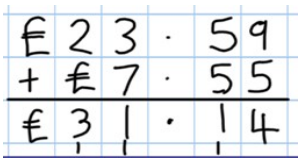
Key skills for addition at Stage 4:

- Select most appropriate method: mental, jottings or written and explain why.
- Recognise the place value of each digit in a four-digit number.
- Round any number to the nearest 10, 100 or 1000.
- Estimate and use inverse operations to check answers.
- Solve 2-step problems in context, deciding which operations and methods to use and why.
- Find 1000 more or less than a given number.
- Continue to practise a wide range of mental addition strategies, ie. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining.
- Add numbers with up to 4 digits using the formal written method of column addition
- Solve 2-step problems in contexts, deciding which operations and methods to use and why.
- Estimate and use inverse operations to check answers to a calculation.

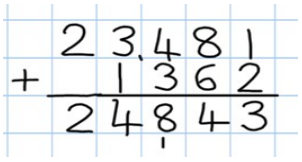
Addition

Stage 5 Add numbers with more than 4 digits

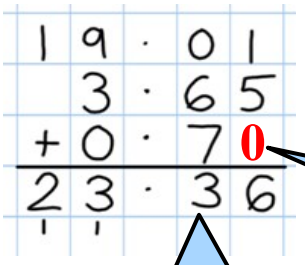
Including money, measures and decimals with different numbers of decimal places.



The decimal point should be aligned in the same way as other place value columns, and must be in the same column in the answer.



Numbers should exceed 4 digits.



Pupils should be able to add more than two values, carefully aligning place value columns

Children should:

- Understand the place value of **tenths and hundredths** and use this to align numbers with different numbers of decimal places.

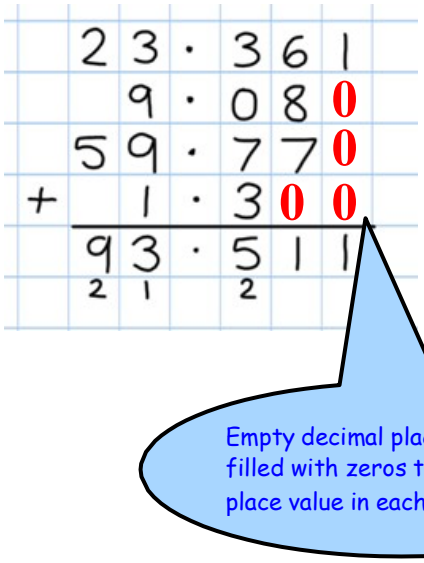
Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, 'carry', expanded, compact, vertical, thousands, hundreds, digits, inverse & **decimal places, decimal point, tenths, hundredths, thousandths**

Key skills for addition at Stage 5:

- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies ie. add the nearest multiple of 10, 100, 100 and adjust; use near doubles, inverse, partitioning and re-combining; using number bonds.
- Use rounding to check answers and accuracy.
- Solve multi-step problems in contexts, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000.
- Add numbers with more than 4 digits using formal written method of columnar addition.

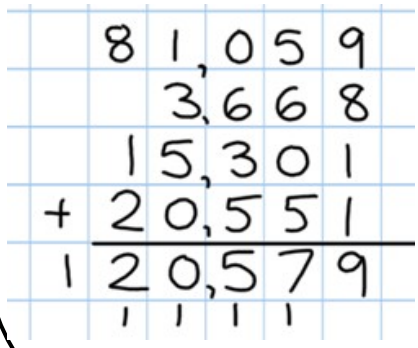
Addition

Stage 6 Add several numbers of increasing complexity



Adding several numbers with different numbers of decimal places (including money and measures):

- Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.
- Zeros could be added into any empty decimal places, to show there is no value to add.



Adding several numbers with more than 4 digits

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, 'carry', expanded, compact, vertical, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths

Key skills for addition at Stage 6:

- Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies.
- Solve multi-step problems in context, deciding which operations and methods to use and why.
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy.
- Pupils understand how to add mentally with larger numbers and calculations of increasing complexity.

Subtraction

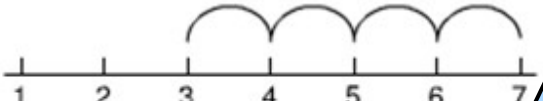
Stage 1 Subtract from numbers up to 20

Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes etc. and in familiar contexts, and are introduced to more formal recording using number lines as below:

Read, write and interpret number sentences with + and = signs

Subtract by taking away

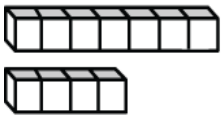
Count back in ones on a numbered number line to take away, with numbers up to 20



$7 - 4 = 3$

Model subtraction using hundred squares and numbered number lines/tracks and practically

This will be introduced practically with the language 'find the distance between' and 'how many more?' in a range of familiar contexts



'Seven is 3 more than four'

'I am 2 years older than my sister'

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_?

Key skills for subtraction at Stage 1:

- Given a number, say **one more or one less**.
- Count to and over 100, **forward and back**, from any number.
- Represent and use **subtraction facts to 20 and within 20**.
- Subtract with **one-digit and two-digit** numbers to 20, including zero.
- Solve one-step problems that involve addition and subtraction, using concrete objects (ie bead string, objects, cubes) and pictures, and missing number problems.
- Read and write numbers from 0 to 20 in numerals and words.

Subtraction

Stage 2 Subtract with 2-digit numbers

Subtract on a number line by counting back, aiming to develop mental subtraction skills.

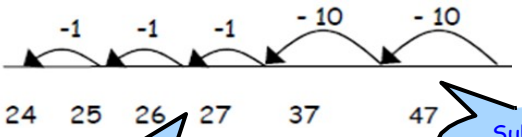
Use Dienes blocks for subtraction calculations too

This strategy will be used for:

- **2-digit numbers subtract units** (by taking away / counting back) e.g. 36—7
- **2-digit numbers subtract tens** (by taking away / counting back) e.g. 48—30
- **Subtracting pairs of 2-digit numbers** (see below)

Subtracting pairs of 2-digit numbers on a number line:

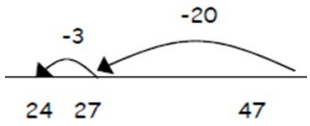
47 - 23 = 24 Partition the second number and subtract it in tens and units, as below:



Then subtract units.

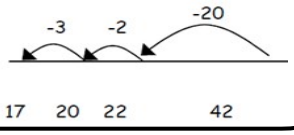
Subtract tens first

Move towards more efficient jumps back, as below:



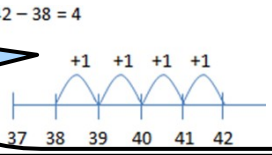
Combine methods with use of a hundred square to reinforce understanding of number value and order

Teaching children to **bridge through ten** can help them to become more efficient, for example 42—25:



Mental strategy - subtract numbers close together by **counting on:**

Start with the smaller number and count on to the largest



Many mental strategies are taught. Children are taught to recognise that when numbers are close together, it is more efficient to **count on** the difference. They need to be clear about the relationship between addition and subtraction.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? **difference, count on, strategy, partition, tens, units**

Key skills for subtraction at Stage 2:

- Recognise the place value of each digit in a two-digit number.
- Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100.
- Subtract using concrete objects, pictorial representations, 100 squares and mentally, including: a two-digit number and ones, a two-digit number and tens, and two two-digit numbers.
- Show that subtraction of one number from another cannot be done in any order.
- Recognise and use inverse relationship between addition and subtraction, using this to check calcula-tions and missing number problems.
- Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representation, and also applying their increasing knowledge of mental and written methods.
- Read and write numbers to at least 100 in numerals and in words.

Subtraction

Stage 3 Subtracting with 2 and 3-digit numbers

Introduce **partitioned column subtraction** method.

STEP 1: introduce this method with examples where **no exchanging** is required.

89 – 35 = 54

80 9

-30 5

50 + 4

When learning to „exchange", explore „partitioning in different ways" so that pupils understand that when you exchange, the **VALUE** is the same ie **72** = 70+2 = 60+12 = 50+22 etc. Emphasise that the **value hasn't changed**, we have just partitioned it in a different way.

STEP 2: introduce „exchanging" through practical subtraction. Make the larger number with Base 10, then subtract 47 from it.

72 - 47

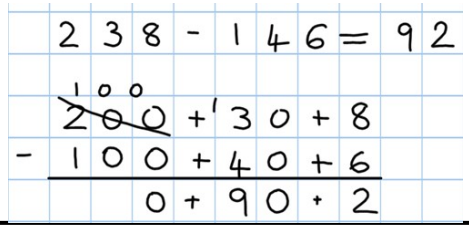
60 ~~70~~ + $^{1}2$

- 40 + 7

20 + 5 = **25**

Before subtracting '7' from the 72 blocks, they will need to exchange a row of 10 for ten units. Then subtract 7, and subtract 4 tens.

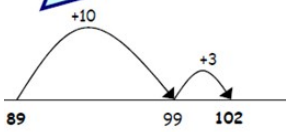
STEP 3: Once pupils are secure with the understanding of "exchanging", they can use the partitioned column method to subtract any 2 and 3-digit



Subtracting money: partition into e.g. £1 + 30p + 8p

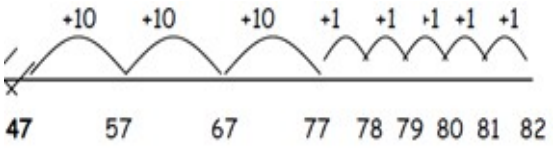
Counting on as a mental strategy for subtraction:

Because counting on in tens is the way we use a 100 square.



Continue to reinforce counting **on** as a strategy for **close-together numbers** (e.g. 121—118), and also for numbers that are "nearly" multiples of 10, 100, 1000 or £s, which make it easier to count on (e.g. 102-89, 131—79, or calculating change from £1 etc.).

- Start at the smaller number and count on in **tens** first, then count on in units to find the rest of the difference:



Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units, **exchange, decrease, hundreds, value, digit**

Key skills for subtraction at Stage 3:

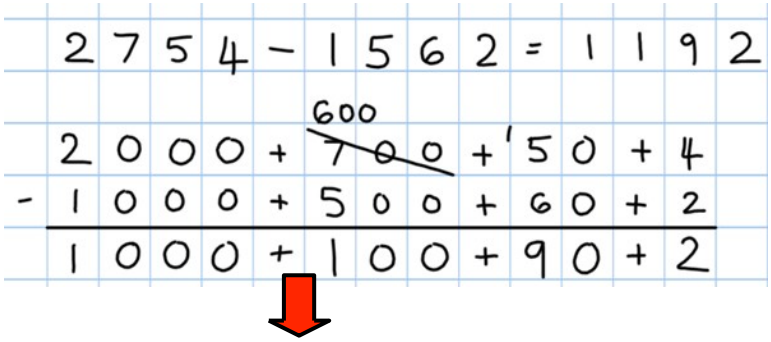
- Subtract mentally a: **3-digit number and ones, 3-digit number and tens, 3-digit number and hundreds.**
- Estimate answers and use inverse operations to check.
- Solve problems, including missing number problems.
- Find 10 or 100 more or less than a given number.
- Recognise the place value of each digit in a 3-digit number .
- Counting up differences as a mental strategy when numbers are close together or near multiples of 10 (see examples above)
- Read and write numbers up to 1000 in numerals and words.
- Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting (e.g. subtracting 19 or 21), and select most appropriate methods to subtract, explaining why.

Approximate, Calculate, Check it mate!

Subtraction

Stage 4 Subtracting with up to 4-digit numbers

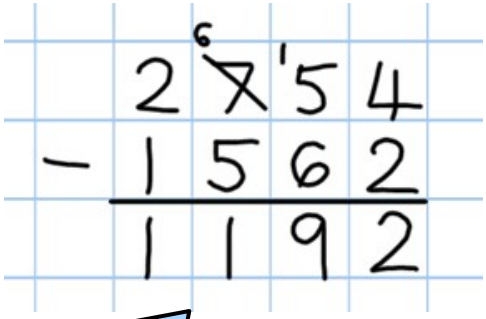
Partitioned column subtraction with 'exchanging' (Decomposition)



As introduced in Y3, but moving towards more complex numbers and values. Use **place value counters** to reinforce 'exchanging'.

Subtracting money: partition into £1 + 30 + 5 for example.

Compact column subtraction



To introduce the compact method, ask children to perform a subtraction calculation with the familiar partitioned column subtraction then display the compact version for the calculation they have done. Ask pupils to consider how it relates to the method they know, what is similar and what is different, to develop an understanding of it

Give plenty of opportunities to apply this to money and measures.

Always encourage children to consider the best method for the numbers involved—mental, counting on, counting back or written method

Mental strategies

A variety of mental strategies must be taught and practised, including counting on to find the difference where numbers are closer together, or where it is easier to

Approximate,

Calculate,

Check it mate!

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, **inverse**

Key skills for subtraction at Stage 4:

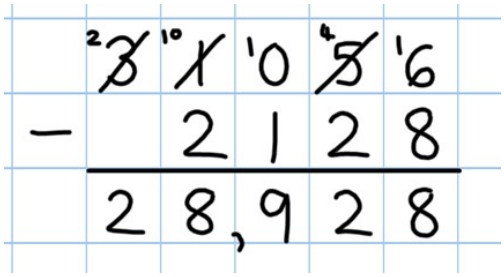
- Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc.
- Children select the most appropriate and efficient methods for given subtraction calculations.
- Estimate and use inverse operations to check answers.
- Solve addition and subtraction 2-step problems, choosing which operations and methods to use and why.
- Solve simple measure and money problems involving fractions and decimals to two decimal places.
- Find 1000 more or less than a given number.
- Count backwards through zero, including negative numbers.
- Recognise place value of each digit in a 4-digit number Round any number to the nearest 10, 100 or 1000
- Solve number and practical problems that involve the above, with increasingly large positive numbers.

Subtraction

Stage 5 Subtract with at least 4-digit numbers

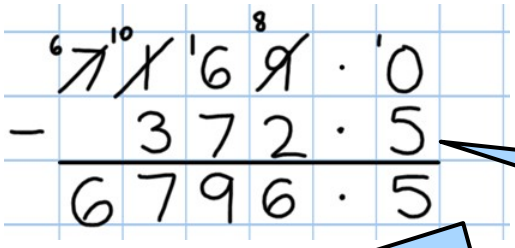
Including money, measures and decimals

Compact column subtraction
(with 'exchanging')



Children who are still not secure with number facts and place value will need to remain on the partitioned column method until ready for the compact method

Subtracting with large numbers



Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.

Add a 'zero' in any empty decimal places to aid understanding of what to subtract in that column.

Create lots of opportunities for subtracting and finding differences with money and measures

Approximate,
Calculate,
Check it mate!

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse, **tenths, hundredths, decimal point, decimal**

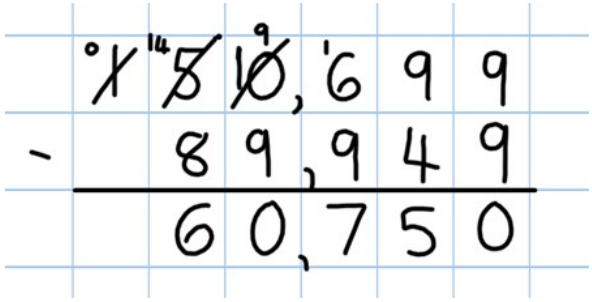
Key skills for subtraction at Stage 5:

- Subtract numbers mentally with increasingly large numbers .
- Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy .
- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Count forwards or backwards in steps of powers of 10 for any given number up to 1 million.
- Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through 0.
- Round any number up to 1 million to the nearest 10, 100, 1000, 10 000 and 100 000.

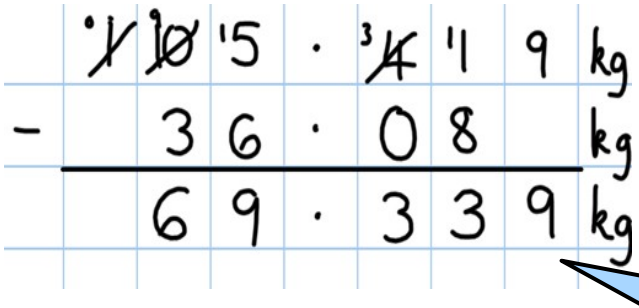
Subtraction

Stage 6

Subtracting with increasingly large and more complex numbers and decimal values.



Using the compact column method to subtract more complex integers



Using the compact column method to subtract money and measures, including decimals with different numbers of decimal places.

Empty decimal places can be filled with **zero** to show the place value in each column.

Pupils should be able to apply their knowledge of a range of mental strategies, mental recall skills, and informal and formal written methods when selecting **the most appropriate method** to work out subtraction problems.

Approximate, Calculate, Check it mate!

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal

Key skills for subtraction at Stage 6:

- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit
- Round any whole number to a required degree of accuracy
- Use negative numbers in context, and calculate intervals across zero.
- Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before choosing how to calculate.

Multiplication

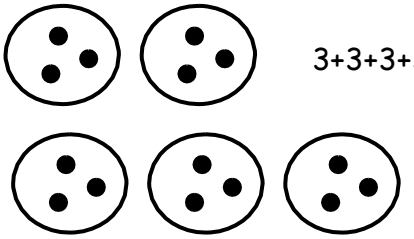
Stage 1

Multiply with concrete objects, arrays and pictorial representations

How many legs will 3 teddies have?

2 + 2 + 2 = 6

There are 3 sweets in one bag. How many sweets are in 5 bags altogether?



3+3+3+3+3 = 15

- Give children experience of counting equal group of objects in 2s, 5s and 10s.
- Present practical problem solving activities involving counting equal sets or groups, as above.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count

Key skills for multiplication at Stage 1:

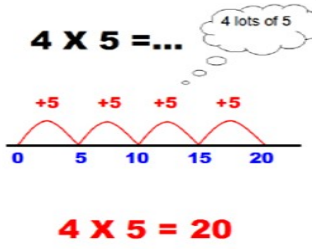
- Count in multiples of 2, 5 and 10.
- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Make connections between arrays, number patterns, and counting in twos, fives and tens.
- Begin to understand doubling using concrete objects and pictorial representations.

Multiplication

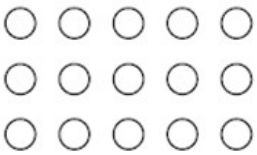
Stage 2 Multiply using arrays and repeated addition (using at least 2s, 5s and 10s)

Use repeated addition on a number line:

- Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using x and = signs.



Use arrays:



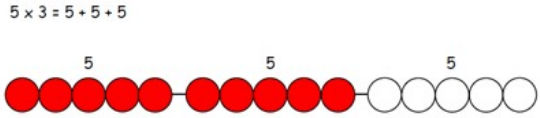
3 x 5 = 15

5 x 3 = 15

5 x 3 = 3 + 3 + 3 + 3 = **15**
3 x 5 = 5 + 5 + 5 = **15**

Use arrays to help teach children to understand the commutative law of multiplication, and give examples such as **3 x**____ = **6**.

Use practical apparatus:



Use mental recall:

- Children should begin to **recall multiplication facts for 2, 5 and 10** times tables through practice in counting and understanding of the operation.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times..

Key skills for multiplication at Stage 2:

- Count in steps of 2, 3 and 5 from zero, and in 10s from any number.
- Recall and use multiplication facts from the **2, 5 and 10** multiplication tables, including recognising odds and evens.
- Write and calculate number statements **using the x and = signs**.
- Show that multiplication can be done in any order (commutative).
- Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods, and multiplication facts.
- Pupils use a variety of language to discuss and describe multiplication.

Multiplication

Stage 3 Multiply 2 digits by a single digit number

Partition the numbers into tens and units. Multiply by the units(least valuable first) then by tens (most valuable)

23x6

2 3	
x 6	
18	6x3
120	6x20
138	

47x7

4 7	
x 7	
28	7x4
280	7x40
308	

We use this method as it partitions like the grid method,but works with units first so follows written methods of subtraction and addition to show a consistent approach. If children do struggle with this method, we use the grid as an intervention method

To do this, children must be able to:
Partition numbers into tens and units
Multiply multiples of ten by a single digit(eg20x6) using knowledge of multiplication facts and place value
Work out multiplication facts not known by repeated addition

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addi- tion, column, row, commutative, sets of, equal groups, times, _times as big as, once, twice, three times..., **partition,multiple, product, tens, units, value**

Key skills for multiplication at Stage 3:

- Recall and use multiplication facts for the **2, 3, 4, 5, 8 and 10** multiplication tables, and multiply multiples of 10.
- Write and calculate number statements using the multiplication tables they know, including **2-digit x single- digit**, drawing upon mental methods, and progressing to reliable written methods.
- Solve multiplication problems, including missing number problems.
- Develop mental strategies using commutativity (e.g. 4 x 12 x 5 = 4 x 5 x 12 = 20 x 12 = 240)
- Solve simple problems in contexts, deciding which operations and methods to use.
- Develop efficient mental methods to solve a range of problems e.g using commutativity (**4 × 12 × 5** = 4 × 5 × 12 = **20 × 12** = 240) and for missing number problems ☐ x 5 = 20, 3 x ☐ = 18, ☐ x ☐ = 32

Multiplication

Stage 4 Multiply 2 and 3 digits by a single digit, using multiplication tables up to 12x12

Partition the numbers into tens and units. Multiply by the units(least valuable first) then by tens (most valuable)

Developing partitioning method:

Eg- 136x5

136
x 5
30 5x6
150 5x30
500 5x100
680

Once children are secure with this method of partitioning, and in carrying for written addition,and only then, move to short multiplication

Children should be able to:
Approximate before they calculate, and make this a regular part of their calculating, going back to check the approximation to check the reasonableness of their answer, e.g
248x8 is approx. 250x10=2500. Record approximation to check against

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times as big as, once, twice, three times... partition, grid method, total, multiple, product, sets of, **inverse**

Key skills for multiplication at Stage 4:

- Count in multiples of 6, 7, 9, 25 and 1000
- Recall multiplication facts for **all multiplication tables up to 12 x 12.**
- Recognise place value of digits in up to 4-digit numbers
- Use place value, known facts and derived facts to multiply mentally, e.g. multiply by 1, 10, 100, by 0, or to multi- ply 3 numbers.
- Use commutativity and other strategies mentally 3 x 6 = 6 x 3 , 2 x 6 x 5 = 10 x 6 , 39x7 = 30 x 7 + 9 x 7.
- Solve problems with increasingly complex multiplication in a range

Multiplication

Stage 5(i) Multiplying up to 4 digits by 1 digit-short multiplication

Introduce by comparing partitioning method to short multiplication method, to see how the steps are related, but notice how there are less steps involved in the contracted method.
Children should be taught to estimate first, eg 78x38-use rounding-80x40=3200, and use approximation to check reasonableness of answer.

236x3		236x3
236		
X6		236
36	6x6	x 6
180	6x30	1416
1200	6x200	2 3
1416		

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, _times as big as, once, twice, three times..., partition, grid method, total, multiple, product, inverse, **square, factor, integer, decimal, short/long multiplication, 'carry'**

Key skills for multiplication at Stage 5:

- Identify multiples and factors, using knowledge of **multiplication tables to 12x12**.
- Solve problems where larger numbers are decomposed into their factors
- Multiply and divide integers and decimals by 10, 100 and 1000
- Recognise and use square and cube numbers and their notation
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately.

Multiplication

Stage 5(ii)- Long multiplication- multiplying up to 6 digits by 2 digit number

Recap on short multiplication with carrying and follow this whilst multiplying by units

24x23

```
  24
 x23
  72
 480
 552
 1
```

we are now multiplying by tens column, so we put zero as place holder in units column

*****Use this as school method as it works with units first as we teach addition and subtraction***** Also show children using tens first as an alternative, answer will be the same

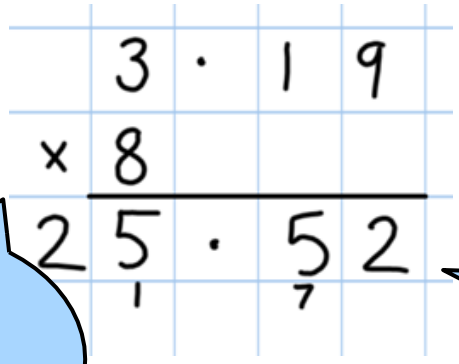
Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, _times as big as, once, twice, three times..., partition, grid method, total, multiple, product, inverse, **square, factor, integer, decimal, short/long multiplication, 'carry'**

Key Skills for multiplicatio

Identify multiples and factors, using knowledge of **multiplication tables to 12x12**.
Solve problems where larger numbers are decomposed into their factors Multiply and divide integers and decimals by 10, 100 and 1000
Recognise and use square and cube numbers and their notation
Solve problems involving combinations of operations, choosing and using calculations and methods appropriately

Multiplication

Stage 6 Short and long multiplication as in Stage 5, and multiply decimals with up to 2d.p by a single digit.



Line up the decimal points in the question and the answer.

Remind children that the single digit belongs in the units column.

This works well for multiplying money (£.p) and other measures.

Children will be able to:

- Use rounding and place value to make approximations before calculating and use these to check answers against.
- Use **short multiplication** (see Y5) to multiply numbers with **more than 4-digits by a single digit**; to multiply money and measures, and to **multiply decimals with up to 2d.p. by a single digit**.
- Use **long multiplication** (see Y5) to multiply numbers with **at least 4 digits by a 2-digit number**.

Approximate,

Calculate,

Check it mate!

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times... partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short / long multiplication, 'carry', **tenths, hundredths, decimal**

Key skills for multiplication at Stage 6:

- Recall multiplication facts for all times tables up to **12 x 12 (as Stage 4 and Stage 5).**
- Multiply multi-digit numbers, up to 4-digit x 2-digit using long multiplication.
- Perform mental calculations with mixed operations and large numbers.
- Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods.
- Estimate answers using round and approximation and determine levels of accuracy.
- Round any integer to a required degree of accuracy.

Division

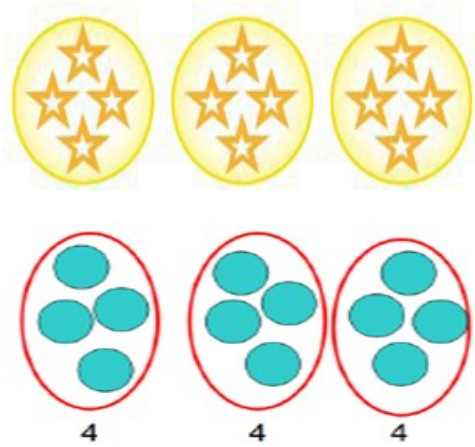
Stage 1 Group and share small quantities

Using objects, diagrams and pictorial representations to solve problems involving **both grouping and sharing**.

How many groups of 4 can be made with 12 stars? = **3**

Grouping:

Sharing:



12 shared between 3 is 4

Example division problem in a familiar context:

There are 6 pupils on this table and there are 18 pieces of fruit to share between us. If we share them equally, how many will we each get?

Can they work it out and give a division statement… ?

"18 shared between 6 people gives you 3 each."

Pupils should :

- use lots of practical apparatus, arrays and picture representations
- Be taught to understand the difference between 'grouping' objects (How many groups of 2 can you make?) and 'sharing' (Share these sweets between 2 people)
- Be able to count in multiples of 2s, 5s and 10s.
- Find **half** of a group of objects by sharing into 2 equal groups.

Key vocabulary: share, share equally, one each, two each…, group, groups of, lots of, array

Key skills for division at Stage 1:

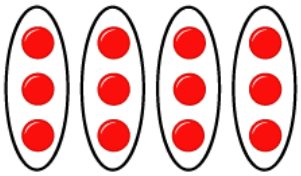
- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher
- Through grouping and sharing small quantities, pupils begin to understand, division, and finding simple frations of objects, numbers and quantities.
- They make connections between arrays, number patterns, and counting in twos, fives and tens.

Division

Stage 2 Group and share, using the ÷ and = sign

Use objects, arrays, diagrams and pictorial representations, and grouping on a number line.

Arrays:



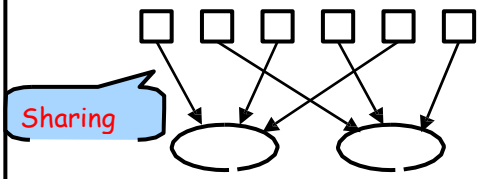
12 ÷ 3 = 4

This represents 12 ÷ 3, posed as how many groups of 3 are in 12?

Pupils should also show that the same array can represent 12 ÷ 4 = 3 if grouped horizontally.

Know and understand sharing and grouping:

6 sweets shared between 2 people, how many do they each get?



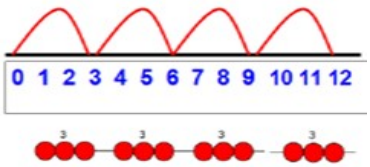
Grouping

There are 6 sweets, how many people can have 2 sweets each?

Children should be taught to recognise whether problems require sharing or grouping.

Grouping using a number line:

Group from zero in equal jumps of the divisor to find out 'how many groups of _ in _ ?'. Pupils could and using a bead string or practical apparatus to work out problems like 'A CD costs £3. How many CDs can I buy with £12?' **This is an important method to develop understanding of division as grouping.**



12 ÷ 3 = 4

Pose **12 ÷ 3** as '**How many groups of 3 are in 12?**'

Key vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, **divide, divided by, divided into, division, grouping, number line, left, left over**

Key skills for division at Stage 2:

- Count in steps of 2, 3, and 5 from 0
- Recall and use multiplication and division facts for the **2, 5 and 10** multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the x, ÷ and = signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Division

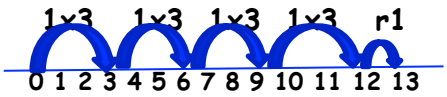
Stage 3 Divide 2-digit numbers by a single digit

(initially where there is no remainder in the final answer,leading to remainders as children improve in confidence)

Children continue to work out unknown division facts by grouping on a number line from zero. They are also now taught the concept of **remainders,** as in the example. This should be introduced practically and with arrays, as well as being translated to a number line. Children should work towards calculating some basic division facts with remainders mentally for the 2s, 3s, 4s, 5s, 8s and 10

GROUPING ON NUMBERLINE

13÷ 4= 4r1



Key vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, **divide, divided by, divided into, division, grouping, number line, left, left over,inverse, carry,remainder,multiple**

Key skills for division at Stage 3:

- Count in steps of 2, 3, and 5 from 0
- Recall and use multiplication and division facts for the **2,3,4,5,8 and 10** multiplication tables. Through doubling, connect 2,4, 8s
- Write and calculate mathematical statements for multiplication and division within the multiplication tables that they know, including for 2 digit numbers x 1 digit numbers using mental methods and jottings.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Develop efficient mental methods, for example, using multiplication and division facts to derive related facts(3x4=12 so 3 x 40=120)
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental meth- ods, and multiplication and division facts, including problems in contexts.

Division

Stage 3 b BASIC LONG DIVISION-CHUNKING (taught before short division to follow whole school partitioning approach)

Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division,

Where **remainders** occur, pupils should express them as fractions, decimals or use rounding, depending upon the problem.

Find out 'How many 36s are in 972?' by subtracting 'chunks' of 36, until zero is reached (or until there is a r
Teach pupils to write a '**useful list**' first at the side that will help th chunks to use, e.g.:

1x36=36
2x36=72
5x36=180
10x36=360
100x36=3600

```
      27
36 ) 972
   - 720      20x
     252
   - 252       7x
       0
```

Answer : 27

Introduce the method in a simple way by limiting the choice of chunks to 'Can we use 10 lots?
Can we use 100 lots?' As children become confident with the process, encourage more efficient chunks
to get to the answer more quickly (e.g. 20x, 5x), and expand

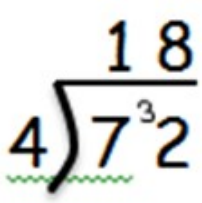
Division

Stage 4 Divide 2 digit numbers by a single digit

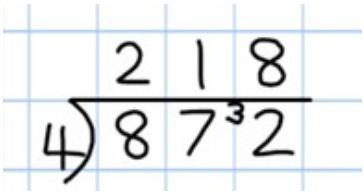
(without remainders initially then leading to remainders)

Continue to develop short division:

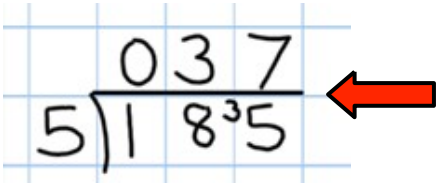
Short division should only be taught once children have secured the skill of calculating 'remainders'.



STEP 1: Pupils must be secure with the process of short division for dividing 2-digit numbers by a single digit (**those that do not result in a final remainder** —see steps in Stage 3), but must understand how to calculate remainders, using this to „carry" remainders within the calculation process (see example).



STEP 2: Pupils move onto dividing numbers with up to **3-digits** by a single digit, however problems and calculations provided should **not result in a final answer with remainder** at this stage. Children who exceed this expectation may progress to Y5 level.



When the answer for the **first column** is zero (1 ÷ 5, as in example), children could initially write a zero above to acknowledge its place, and must always 'carry' the number (1) over to the next digit as a remainder.

Real life contexts need to be used routinely to help pupils gain a full understanding, and the ability to recognise the place of division and how to apply it to problems.

Key vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, **divisible by, factor**

Key skills for division at Stage 4:

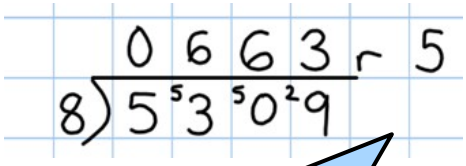
- **Recall multiplication and division facts for all numbers up to 12 x 12.**
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number
- Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example 200 × 3 = 600 so 600 ÷ 3 = 200
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

Division

Stage 5 Divide up to 4 digits numbers by a single digit

(including those with remainders)

Short division, including remainder answers:



Short division with remainders: Now that pupils are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where **pupils consider the meaning of the remainder and how to express it,** ie. as a fraction, a decimal, or as a rounded number or value , depending upon the context of the problem.

The answer to 5309 ÷ 8 could be expressed as **663 and five eighths**, **663 r 5**, as a decimal, or **rounded** as appropriate to the problem involved.

See Stage 6 for how to continue the short division to give a **decimal answer** for children who are confident.

Include **money and measure** contexts.

Approximate,

Calculate,

Check it mate!

If children are confident and accurate:

- Introduce **long division** for pupils who are ready to divide any number by a 2-digit number (e.g. 2678 ÷ 19). This is a Stage 6 expectation.

Key vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor, inverse, **quotient, prime number, prime factors, composite number (non-prime)**

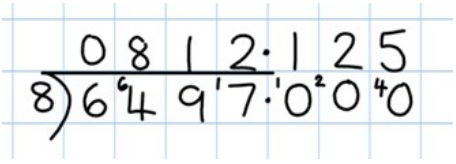
Key skills for division at Stage 5:

- Recall multiplication and division facts for all numbers up to 12 x 12 (as in Y4).
- Multiply and divide numbers mentally, drawing upon known facts.
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two number.
- Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Work out whether a number up to 100 is prime, and recall prime numbers to 19.
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Use multiplication and division as inverses.
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. 98 ÷ 4 = 24 r 2 = 241/2 = 24.5 ≈ 25).
- Solve problems involving combinations of all four operations, including understanding of the equals sign, and including division for scaling by different fractions and problems involving simple rates.

Division

Stage 6 Divide at least 4 digits by both single-digit and 2 -digit numbers (including decimal numbers and quantities)

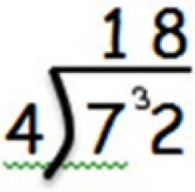
Short division, for dividing by a single digit: e.g. 6497 ÷ 8



Short division with remainders: Pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.

Calculating a decimal remainder: In this example, rather than expressing the remainder as **r 1**, a decimal point is added after the units because there is still a remainder, and the one remainder is carried onto zeros after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy for the problem being solved.

WHILST WORKING WITH SHORT DIVISION METHODS AT UPPER KEY STAGE 2, CHILDREN WILL CONTINUE TO WORK WITH MORE ~~COMPLEX~~ MULTIPLICATION STRATEGIES ,USING FRACTIONS FOR ROUNDING TO DECIMAL REMAINDERS



Key vocabulary: As previously, & **common factor**

Key skills for division at Stage 6:

- Recall and use multiplication and division facts for all numbers to 12 x 12 for more complex calculations
- Use short division where appropriate.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Solve problems involving all 4 operations.
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem. Use written division methods in cases where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specified degrees of accuracy.

Division

Stage 6 Divide at least 4 digits by both single-digit and 2 -digit numbers (including decimal numbers and quantities)

Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division and interpret remainders as whole numbersremainders,fractions or by rounding,as appropriate for the context.

— Long division: 20 divided by 9 gives an answer with an unending sequence of 2s.

$$
\begin{array}{r|l}
 & 2.2222\ldots \\
\hline
9 & 20.0 \\
 & -18 \\
 & \quad 20 \\
 & \quad -18 \\
 & \quad\quad 20 \\
 & \quad\quad -18 \\
 & \quad\quad\quad 20 \\
 & \quad\quad\quad -18 \\
 & \quad\quad\quad\quad 20 \\
 & \quad\quad\quad\quad -18 \\
 & \quad\quad\quad\quad\quad 20
\end{array}
\qquad \Longleftrightarrow \qquad \frac{20}{9} = 2.\overline{2}
$$

This final method is not taught until the children are familiar with and comfortable with all other methods. It is the most complicated method taught as an extension to children performing above expectations as they leave primary school. The method begins by asking

1- How many 9s in 20?
2- 2(2 goes on top). 2x9=18
3- Subtract 18 from 20-answer 2
4- Add decimal point , bring down .0
5- How many 9s in 20?

This method will give an answer as a decimal and often uneneding sequence(introduce term recurring). This method can also be used to enhance knowledge of rounding.