Using Graphs

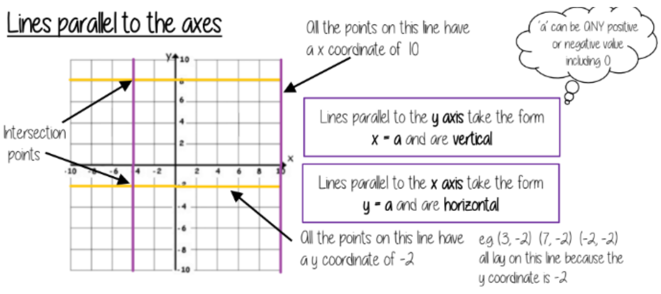
What do I need to do...

- Reflect shapes in given lines
- Construct and interpret conversion graphs and other real-life straight line graphs
- Construct and interpret distance/time graphs
- Construct and interpret speed/time graphs
- Construct and interpret piece-wise graphs
- Recognise and interpret graphs that illustrate direct and inverse proportion

Key Words

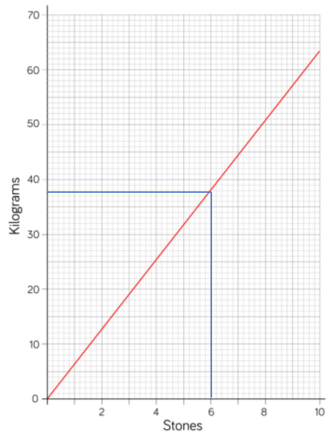
Quadratic, Parabola, Curve, Substitute, Equation, Vertical, Horizontal, Estimate, Cube, Cubic, Asymptote, Infinity, Reciprocal, Tends towards, Gradient, y-intercept, Coordinate, Roots, Solution, Meets, Exponential, Growth, Decay, Rapid, Radius, Diameter, Pythagoras' Theorem, Origin, Simplify, Tangent, Curve, Equidistant

Horizontal and vertical lines

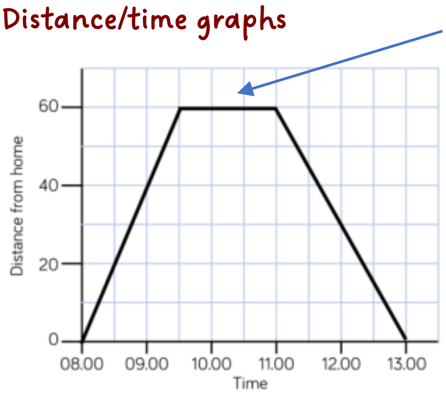


Conversion graphs

We can use the conversion graph to change between stones and kilograms. The blue line shows us that 6 stones would be approximately 38kg.



Distance/time graphs

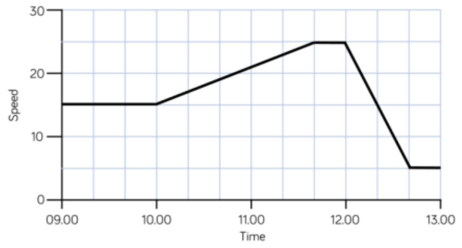


stationary

The distance-time graph shows Dora's visit to a friend and back. We can use the formula speed = distance ÷ time to calculate Dora's speed.

Speed/time graphs

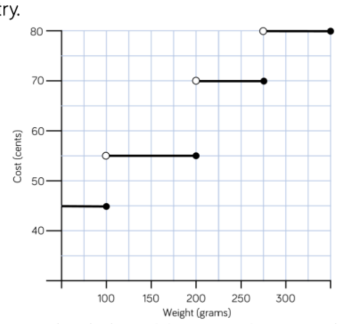
The graph shows the speed of a boat over a four hour period.



The gradient represents the change in speed (acceleration). A negative gradient shows slowing down (deceleration).

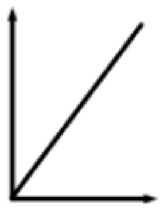
Piece-wise graphs

The graph shows the cost, in cents, of posting letters of different weights in a country.

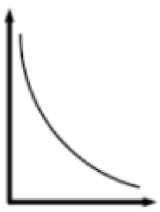


Letters up to and including 100 g cost 40 c to send.

Direct and inverse proportion

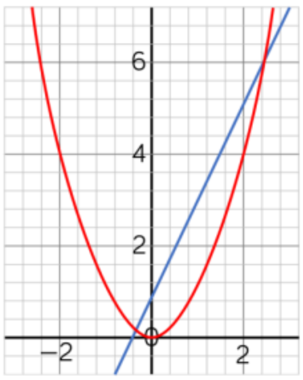


direct proportion



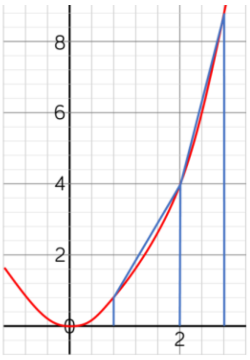
inverse proportion

Approximate solutions



The diagram shows the graphs $y = x^2$ and $y = 2x + 1$. We can estimate solutions by looking at the points of intersection.

Estimate area under a curve



To estimate the area under a curve, we need the formula below to calculate the area of trapzia.

Area of a trapezium $= \frac{1}{2}(a + b)h$

Non Linear Graphs

What do I need to do...

- Plot/read from quadratic, cubic and reciprocal graphs
- Recognise graph shapes
- Identify and interpret roots and intercepts of quadratics
- Understand and use exponential graphs (HIGHER)
- Find and use the equation of a circle (HIGHER)
- Find the equation of the tangent to any curve (HIGHER)

Key Words

Quadratic, Parabola, Curve, Substitute, Equation, Vertical, Horizontal, Estimate, Cube, Cubic, Asymptote, Infinity, Reciprocal, Tends towards, Gradient, y-intercept, Coordinate, Roots, Solution, Meets, Exponential, Growth, Decay, Rapid, Radius, Diameter, Pythagoras' Theorem, Origin, Simplify, Tangent, Curve, Equidistant

Table of values

Complete the table for $y = x^2 - 2x + 2$

x	−3	−2	−1	0	1	2	3	4
y	17				1			10

Use a calculator to substitute the values into

$x^2 - 2x + 2$

Enter negatives using the bracket keys...

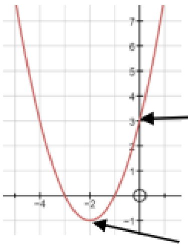
$(-2)^2 - 2 \times (-2) + 2$

Quadratic graphs

$y = x^2 + 4x + 3$

If x^2 is the highest power in your equation then you have a quadratic graph

It will have a parabola shape



Intersection with the y axis

Turning point

Cubic graphs

$y = x^3 + 2x^2 - 2x + 1$

If x^3 is the highest power in your equation then you have a cubic graph

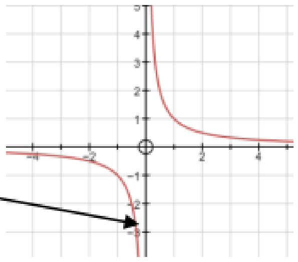
Reciprocal graphs

$y = \frac{1}{x}$

Reciprocal graphs never touch the y axis

This is because x cannot be 0

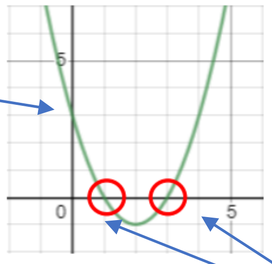
This is an asymptote



Roots and intercepts

$y = x^2 - 4x + 3$

y-intercept

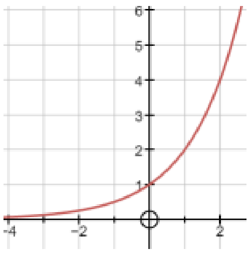


Roots

Exponential graphs

$y = 2^x$

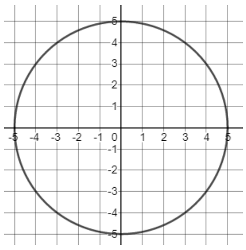
Exponential graphs have a power of x



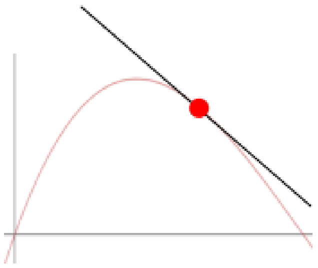
Equation of a circle

The equation of a circle with centre (0,0) is written in the format $x^2 + y^2 = r^2$.

In the equation $x^2 + y^2 = 25$, the circle has a radius of 5cm.



Equation of a tangent to a curve



We find the equation of a tangent in the same way we find the equation of a line, using y=mx+c.

Gradients and Lines

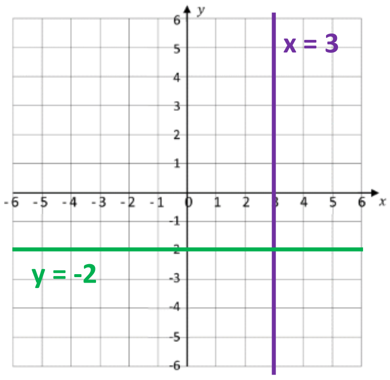
What do I need to do...

- Find equations of lines parallel to the axes
- Plot straight line graphs
- Be able to interpret $y = mx + c$
- Calculate the gradient
- Find the y-intercept
- Solve simultaneous equations graphically
- Find the equation of perpendicular lines

Key Words

Parallel, horizontal, vertical, straight line, axis, equation, graph, intercept, linear, table of values, gradient, y-intercept, scale, point, coordinates, substitute, satisfies, above, below, simultaneous, interception, solutions, product, reciprocal, negative reciprocal.

Equations of lines parallel to the axes



Completing a table of values

Complete the table of values for $y = 2x + 3$.

x → × 2 → + 3 → y

x	-2	-1	0	1	2	3
y	-1	1	3	5	7	9

Interpret y = mx + c

The equation of a line is $y = mx + c$.

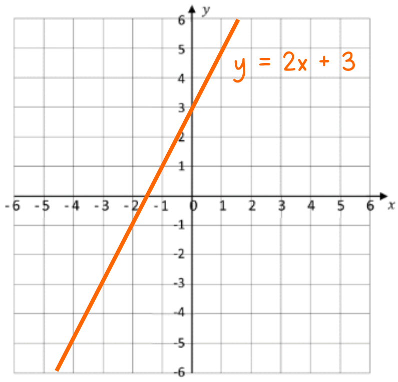
$y = mx + c$

m → gradient

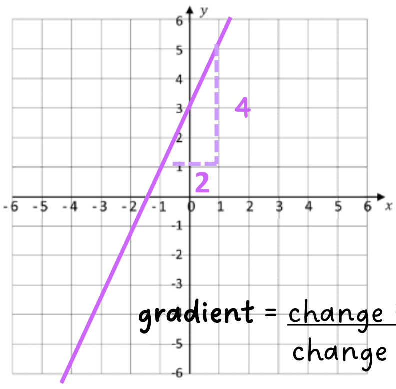
c → y-intercept

Plot straight line graphs

Use the points from your table of values to generate coordinates, e.g. (3, 9).
Plot these points and join with a straight line.

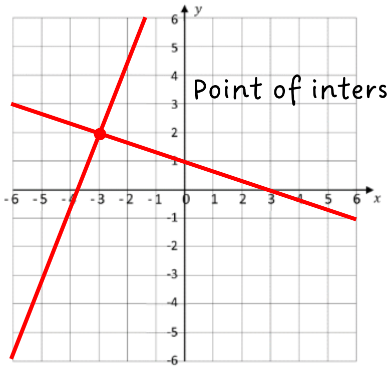


Finding the gradient



gradient = $\frac{\text{change in y-axis}}{\text{change in x-axis}} = \frac{4}{2} = 2$

Solve simultaneous equations graphically



(x, y)

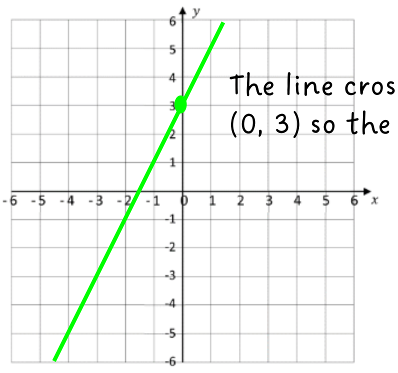
Point of intersection (-3, 2)

Solutions

$x = -3$

$y = 2$

Finding the y-intercept



The line crosses the y-axis at (0, 3) so the y-intercept is 3.

Perpendicular lines (HIGHER ONLY)

Perpendicular lines are defined as two lines that intersect at right angles. To find the gradient of a perpendicular line we need to know the negative reciprocal of the gradient.

The negative reciprocal of 5 would be -1/5.
The negative reciprocal of -1/4 would be 4.
The negative reciprocal of 2/3 would be -3/2.

Manipulating Expressions

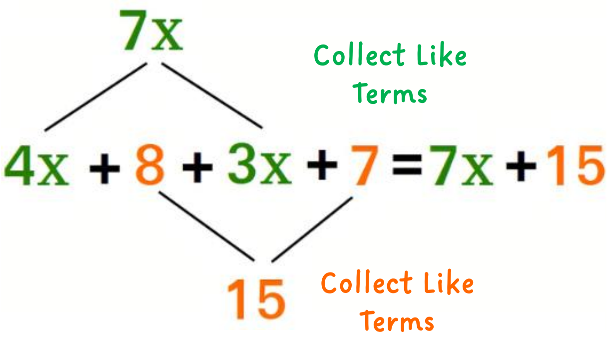
What do I need to do...

- Simplify algebraic expressions
- Use Identities
- Add/Subtract algebraic fractions
- Multiply/Divide algebraic fractions
- Form and solve equations & Inequalities with fractions

Key Words

Expression Term Simplify Coefficient Power Numerator Denominator Like/Unlike Variable Identity Equivalent LCM Difference Sum Invert Product Quotient Reciprocal Cancel Factor Factorise Equation Inequality Solve Solution Quadratic Integer Multiple Odd Even Prove Counterexample

Simplify algebraic expressions



Use Identities

Identity I
$(a + b)^2 = a^2 + 2ab + b^2$

Identity II
$(a - b)^2 = a^2 - 2ab + b^2$

Identity III
$a^2 - b^2 = (a + b)(a - b)$

Identity IV
$(x + a)(x + b) = x^2 + (a + b)x + ab$

Adding algebraic fractions

Ex1 Work out: $\frac{x}{2} + \frac{x}{5}$

Solution

$$\frac{x}{2} + \frac{x}{5} = \frac{5x}{10} + \frac{2x}{10} = \frac{5x + 2x}{10} = \frac{7x}{10}$$

Ex2 Work out: $\frac{x+1}{3} + \frac{x}{4}$

Solution

$$\frac{x+1}{3} + \frac{x}{4} = \frac{4(x+1)}{12} + \frac{3x}{12} = \frac{4x + 4 + 3x}{12} = \frac{7x + 4}{12}$$

Find the LCM of the denominators, then use equivalent fractions

Subtracting algebraic fractions

Ex3 Work out: $\frac{x}{2} - \frac{x-2}{7}$

Solution

$$\frac{x}{2} - \frac{x-2}{7} = \frac{7x}{14} - \frac{2(x-2)}{14} = \frac{7x - 2(x-2)}{14} = \frac{5x + 4}{14}$$

These follow the same method for adding & subtracting numerical fractions!

Multiplying & dividing algebraic fractions

To **multiply algebraic fractions**, we multiply the **numerators** together, and multiply the **denominators** together.

Example

$$\frac{3x^3}{a} \times \frac{5x}{2b} = \frac{3x^3 \times 5x}{a \times 2b} = \frac{15x^4}{2ab}$$

Here we use the multiplication law of indices to multiply the numerators.

To **divide algebraic fractions**, we first write the **reciprocal** of the **dividing fraction** and then multiply the **numerators** and multiply the **denominators**.

Example

$$\frac{4b}{3} \div \frac{7a}{b} = \frac{4b}{3} \times \frac{b}{7a} = \frac{4b \times b}{3 \times 7a} = \frac{4b^2}{21a}$$

To find the reciprocal, flip the fraction.

Solve equations

To solve equations, we use inverse operations
The inverse of add is subtract and the inverse of multiply is divide

Balancing method

$8a - 5 = 11$
+5 +5
$8a = 16$
÷8 ÷8
$a = 2$

Function machine method

$8a - 5 = 11$
a → ×8 → −5 → 11
2 ← ÷8 ← +5 ← 11
$a = 2$

Balancing method

$10 + 6y = 34$
−10 −10
$6y = 24$
÷6 ÷6
$y = 4$

Function machine method

$10 + 6y = 34$
y → ×6 → +10 → 34
4 ← ÷6 ← −10 ← 34
$y = 4$

Balancing method

$\frac{x}{12} - 5 = 4$
+5 +5
×12 $\frac{x}{12} = 9$ ×12
$x = 108$

Function machine method

$\frac{x}{12} - 5 = 4$
x → ÷12 → −5 → 4
108 ← ×12 ← +5 ← 4
$x = 108$

Solving inequalities

Solving inequalities is similar to solving equations, but where an equation has one unique solution, an inequality has a range of solutions.

To **solve an inequality** we calculate the values that an unknown variable can be in that inequality.

Example

$2x + 1 < 9$
$2x < 8$
$x < 4$

Multiplying or dividing by a negative number **changes the direction of the inequality.**

Example

$1 - 2x < 9$
$-2x < 8$
$x > -4$

Parts of an expression