YEAR 10 – USING NUMBER...

Non-calculator methods

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What do I need to be able to do?

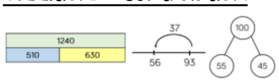
By the end of this unit you should be able to:

- Use mental/written methods for the four number operations
- Use four operations for fractions
- Write exact answers
- Round to decimal places and significant figures
- Estimate solutions
- Understand limits of accuracy
- Understand financial maths

Keywords

Truncate: to shorten, to shorten a number (no rounding), to shorten a shape (remove a part of the shape)
Round: making a number simpler, but keeping its place value close the what it originally was
Credit: money that goes into a bank account
Debit: money that leaves a bank account
Profit: the amount of money after income - costs
Tax: money that the government collects based on income, sales and other activities.
Balance: The amount of money in a bank account
Overestimate: Rounding up – gives a solution higher than the actual value
Underestimate: Rounding down – gives a solution lower than the actual value

Addition/ Subtraction



Modelling methods for addition/ subtraction

- Bar models
- Number lines
- Part/ Whole diagrams

Addition is commutative

6 + 3 = 3 + 6

The order of addition does not change the result

Subtraction the order has to stay the same

360 – 147 = 360 – 100 – 40 – 7

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction
- Show your relationships by writing fact families

Formal written methods

	H	T	O
	1	8	7
+	5	4	2

	H	T	O
	4	2	7
–	2	4	9

Remember the place value of each column. You may need to move 10 ones to the ones column to be able to subtract

Decimals have the same methods remember to align the place value

Division methods

Short division

3584 ÷ 7 = 512

$$7\overline{)3\,{}^{3}5\,8\,{}^{1}4}\quad 5\;1\;2$$

Complex division

÷ 24 = ÷ 6 ÷ 4

Break up the divisor using factors

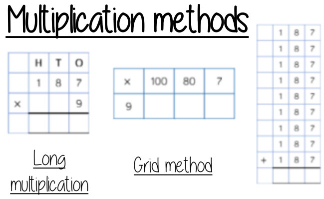
Division with decimals

The placeholder in division methods is essential – the decimal lines up on the dividend and the quotient

24 ÷ 0.02 ⟶ 24 ÷ 0.2 ⟶ 240 ÷ 2

All give the same solution as represent the same proportion.
Multiply the values in proportion until the divisor becomes an integer

Multiplication methods



Long multiplication (column)

Grid method

Repeated addition

Less effective method especially for bigger multiplication

Multiplication with decimals

Perform multiplications as integers
eg 0.2 x 0.3 ⟶ 2 x 3

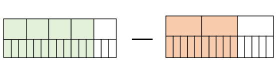
Make **adjustments** to your answer to match the question: 0.2 x 10 = 2
0.3 x 10 = 3

Therefore 6 ÷ 100 = <u>0.06</u>

Four operations with fractions

Addition and Subtraction

$$\frac{4}{5} - \frac{2}{3}$$

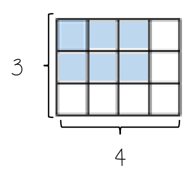


$$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$$

Multiplication

$$\frac{3}{4} \times \frac{2}{3}$$

$$= \frac{6}{12} = \frac{1}{2}$$



Division

$$\frac{2}{5} \div \frac{3}{4}$$

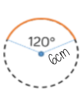
$$\frac{2}{5} \times \frac{4}{3}$$

Multiplying by a reciprocal gives the same outcome

$$= \frac{8}{15}$$

Exact Values

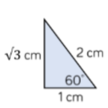
Leave in terms of π



$$= \frac{120}{360} \times 36\pi$$

$$= \frac{1}{3} \times 36\pi = \underline{12\pi}$$

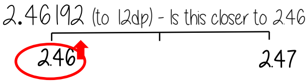
Leave as a surd



$$\text{Tan}30 = \frac{1}{\sqrt{3}}$$

Rounding

2.46192 (to 2dp) - Is this closer to 2.46 or 2.47



2.46 | 192

This shows the number is closer to 2.46

Significant Figures

370 to 1 significant figure is 400
37 to 1 significant figure is 40
3.7 to 1 significant figure is 4
0.37 to 1 significant figure is 0.4
0.00000037 to 1 significant figure is 0.0000004

SF: Round to the first nonzero number

Estimation

Round to 1 significant figure to estimate

21.4 x 3.1 ≈ 20 x 3 ≈ 60

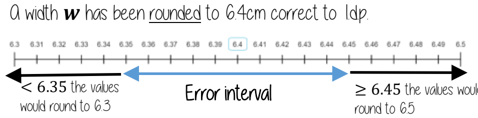
The equal sign changes to show it is an estimation

This is an **underestimate** because both values were rounded down

It is good to check all calculations with an estimate in all aspects of maths – it helps you identify calculation errors.

Limits of accuracy

A width w has been <u>rounded</u> to 6.4cm correct to 1dp.

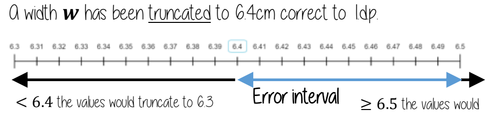


The error interval

$6.35 \leq w < 6.45$

Any value within these limits would round to 6.4 to 1dp

A width w has been <u>truncated</u> to 6.4cm correct to 1dp.



$6.4 \leq w < 6.5$

Any value within these limits would <u>truncate</u> to 6.4 to 1dp

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Indices & Roots

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify square and cube numbers
- Calculate higher powers and roots
- Understand powers of 10 and standard form
- Know the addition and subtraction rule for indices
- Understand power zero and negative indices
- Calculate with numbers in standard form

Keywords

Standard (index) Form: A system of writing very big or very small numbers
Commutative: an operation is commutative if changing the order does not change the result.
Base: The number that gets multiplied by a power
Power: The exponent – or the number that tells you how many times to use the number in multiplication
Exponent: The power – or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Negative: A value below zero.
Coefficient: The number used to multiply a variable

Square and cube numbers

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Square numbers

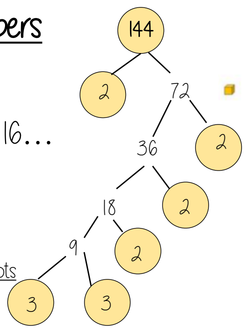
1, 4, 9, 16...

144 = 2 x 2 x 2 x 2 x 3 x 3

2 x 2 x 3 x 2 x 2 x 3

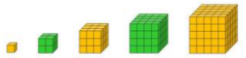
12 x 12

Prime factors can find square roots



$\sqrt{144} = 12$

Cube numbers

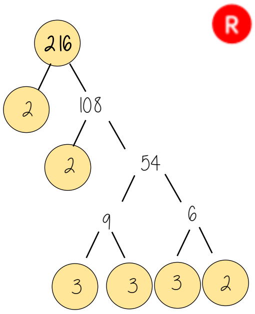


1, 8, 27, 64, 125...

216 = 2 x 2 x 2 x 3 x 3 x 3

2 x 3 x 2 x 3 x 2 x 3

6 x 6 x 6



$\sqrt[3]{216} = 6$

Higher powers and roots

x^n ← n – power (number of times multiplied by itself)

x – the base number.

$\sqrt[n]{x}$ ← Finding the nth root of any value

Other mental strategies for square roots

$\sqrt{810000} = \sqrt{81} \times \sqrt{10000}$

$= 9 \times 100$

$= 900$

Standard form

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$A \times 10^n$

Any number between 1 and less than 10 → A

Any integer → n

0.001
$1 \times \frac{1}{1000}$
1×10^{-3}

10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^1	10^0	10^{-1}	10^{-2}	10^{-3}
10	1	0.1	0.01	0.001

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Example

3.2×10^4
$= 3.2 \times 10 \times 10 \times 10 \times 10$
$= 32000$

Non-example

0.8×10^4

$5.3 \times 10^{0.7}$

Numbers in standard form with negative powers will be less than 1

$3.2 \times 10^{-4} = 3.2 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = 0.00032$

Addition/ Subtraction Laws

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

Zero and negative indices

$x^0 = 1$

Any number divided by itself = 1

$\frac{a^6}{a^6} = a^6 \div a^6$

$= a^{6-6} = a^0 = 1$

Negative indices do not indicate negative solutions

$2^2 = 4$
$2^1 = 2$
$2^0 = 1$
$2^{-1} = \frac{1}{2}$
$2^{-2} = \frac{1}{4}$

Looking at the sequence can help to understand negative powers

Powers of powers

$(x^a)^b = x^{ab}$

$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$

The same base and power is repeated. Use the addition law for indices

$(2^3)^4 = 2^{12}$ ← $a \times b = 3 \times 4 = 12$

NOTICE the difference

$(2x^3)^4 = 2x^3 \times 2x^3 \times 2x^3 \times 2x^3$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$(2x^3)^4 = 16x^{12}$

Standard form calculations

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Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard from at the end

$6 \times 10^5 + 8 \times 10^5$

Method 1

= 600000 + 800000
= 1400000
= 1.4×10^5

Method 2

= $(6 + 8) \times 10^5$
= 14×10^5
= $1.4 \times 10^1 \times 10^5$
= 1.4×10^6

This is not the final answer → (14×10^5)

Multiplication and division

$\frac{1.5 \times 10^5}{0.3 \times 10^3}$

Division questions can look like this

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$1.5 \div 0.3 \times 10^5 \div 10^3$

$= 5 \times 10^2$

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

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Types of number & sequences

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand factors and multiples
- Express numbers as a product of primes
- Find the HCF and LCM
- Describe and continue sequences
- Explore sequences
- Find the nth term of a linear sequence

Keywords

Factor: numbers we multiply together to make another number
Multiple: the result of multiplying a number by an integer.
HCF: highest common factor. The biggest factor that numbers share.
LCM: lowest common multiple. The first multiple numbers share.
Arithmetic: a sequence where the difference between the terms is constant
Geometric: a sequence where each term is found by multiplying the previous one by a fixed nonzero number
Sequence: items or numbers put in a pre-decided order

Multiples

The "times table" of a given number (R)

All the numbers in this lists below are multiples of 3.

3, 6, 9, 12, 15…

This list continues and doesn't end

$3x$, $6x$, $9x$ …

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Non example of a multiple

4.5 is not a multiple of 3 because it is 3 x 1.5 ← Not an integer

Factors

(R)

Arrays can help represent factors

5 x 2 or 2 x 5

Factors of 10
1, 2, 5, 10

10 x 1 or 1 x 10

The number itself is always a factor

Factors and expressions

x x x x x x

$6x \times 1$ OR $6 \times x$

Factors of $6x$
6, x, 1, $6x$, $2x$, 3, $3x$, 2

x x / x x / x x } $2x \times 3$

x x x / x x x } $3x \times 2$

Prime numbers

- Integer
- Only has 2 factors
- and itself

2 — The first prime number. The only even prime number

Learn or how-to quick recall…

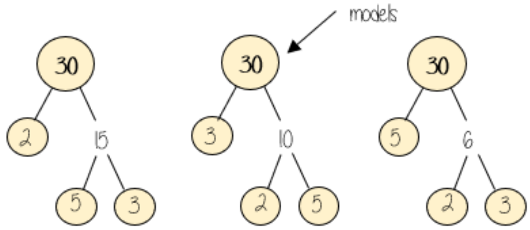
2, 3, 5, 7, 11, 13, 17, 19, 23, 29…

(R)

Product of prime factors

(R)

Multiplication part-whole models



All three prime factor trees represent the same decomposition

30 = 2 x 3 x 5 ← Multiplication of prime factors

Using prime factors for predictions

eg **60**	30 x 2	2 x 3 x 5 **x 2**
150	30 x 5	2 x 3 x 5 **x 5**

Finding the HCF and LCM

(R)

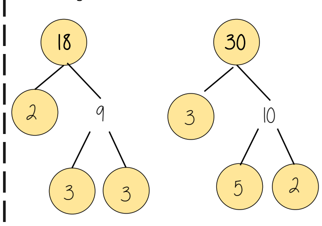
HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, (6), 9, 18

30: 1, 2, 3, 5, (6), 10, 15, 30

6 is the biggest factor they share — HCF = 6



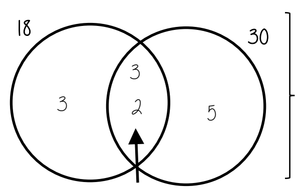
LCM – Lowest common multiple

LCM of 18 and 30

18: 18, 36, 54, 72, (90)

30: 30, 60, (90)

The first time their multiples match — LCM = 90



LCM = 90

HCF = 6

Arithmetic/ Geometric sequences

Arithmetic Sequences change by a common difference. This is found by addition or subtraction between terms

Geometric Sequences change by a common ratio. This is found my multiplication/ division between terms.

Term to term rule – how you get from one term (number in the sequence) to the next term.

Position to term rule – take the rule and substitute in a position to find a term. Eg. Multiply the position number by 3 and then add 2

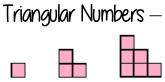
Other sequences

Fibonacci Sequence

1, 1, 2, 3, 5, 8 …

Each term is the sum of the previous two terms

Triangular Numbers – look at the formation



1, 3, 6, 10, 15 …

Square Numbers – look at the formation

1, 4, 9, 16…

Sequences are the repetition of a patten

Finding the nth term

(R)

This is the 4 times table → 4, 8, 12, 16, 20…

4n

↓ ↓ ↓

7, 11, 15, 19, 22

This has the same constant difference – but is 3 more than the original sequence

4n + 3

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence