YEAR 7 – REASONING WITH NUMBER

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Developing number sense

What do I need to be able to do?

By the end of this unit you should be able to:

- Know and use mental addition/ subtraction
- Know and use mental multiplication/ division
- Know and use mental arithmetic for decimals
- Know and use mental arithmetic for fractions
- Use factors to simplify calculations
- Use estimation to check mental calculations
- Use number facts
- Use algebraic facts

Keywords

Commutative: changing the order of the operations does not change the result
Associative: when you add or multiply you can do so regardless of how the numbers are grouped
Dividend: the number being divided
Divisor: the number we divide by
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign
Equation: a mathematical statement that two things are equal
Quotient: the result of a division

Mental methods for addition/ subtraction

Addition is commutative

6 + 3 = 3 + 6

The order of addition does not change the result

Subtraction the order has to stay the same

360 – 147 = 360 – 100 – 40 – 7

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction

Mental methods for multiplication/ division

Multiplication is commutative

2 × 4 = 4 × 2

The order of multiplication does not change the result

Partitioning can help multiplication

24 x 6 = 20 x 6 + 4 x 6
= 120 + 24
= 144

Division is not associative

Chunking the division can help 4000 ÷ 25
"How many 25's in 100" then how many chunks of that in 4000

Mental methods for decimals

Multiplying by a decimal < 1 will make the original value smaller   e.g x 0.1 = ÷ 10

Methods for multiplication 1.2 x 0.03

12 × 3 = 36
1.2 × 3 = 3.6
1.2 × 0.3 = 0.36
1.2 × 0.03 = 0.036

12 × 3 = 36
÷ 10   ÷ 100   ÷ 1000
1.2 × 0.03 = 0.036

Methods for addition 2.3+2.4

2 + 2 = 4
0.3 + 0.4 = 0.7
4 + 0.7 = 4.7

Methods for division 1.5 ÷ 0.05

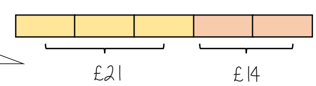
Multiply by powers of 10 until the divisor becomes an integer

1.5 ÷ 0.05
x100   x100
150 ÷ 5 = 30

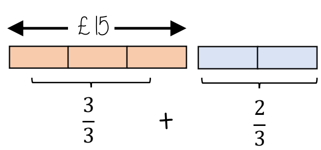
Mental methods for fractions

Use bar models where possible

I've spent $\frac{2}{5}$ of my money I have £21 left



How much did they have to begin with?



What is $\frac{5}{3}$ of £15?

Using factors to simplify calculations

30 x 16

10 x 3 x 4 x 4
2 x 5 x 3 x 2 x 2 x 2 x 2
10 x 3 x 2 x 8
16 x 10 x 3

Multiplication is commutative
Factors can be multiplied in any order

Estimation

Estimations are useful – especially when using fractions and decimals to check if your solution is possible.

Most estimations round to 1 significant figure

Estimations are useful – especially when using fractions and decimals to check if your solution is possible.

210 + 899 < 1200

This is true because even if both numbers were rounded up, they would reach 300 + 900.
The correct estimation would be
200 + 900 = 1100.

Number facts

Use 124 x 5 = 620

For multiplication, each value that is multiplied or divided by powers of 10 needs to happen to the result

620÷ 124 = 50

For division you must consider the impact of the divisor becoming smaller or bigger.
Smaller – the answer will be bigger
(It is being shared into less parts)
Bigger – the answer will be smaller
(It is being shared into more parts)

Algebraic facts

a + b = 5

2a + 2b = 10 (Everything x 2)

0.1a + 0.1b = 0.5 (Everything ÷ 10)

a + b + 2 = 7 (Add 2 to the total)

The unknown quantity isn't changing but the variables change what is done to give the result

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Sets and probability

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify and represent sets
- Interpret and create Venn diagrams
- Understand and use the intersection of sets
- Understand and use the union of sets
- Generate sample spaces for single events
- Calculate the probability of a single event
- Understand and use the probability scale

Keywords

Set: collection of things
Element: each item in a set is called an element
Intersection: the overlapping part of a Venn diagram (AND ∩)
Union: two ellipses that join (OR ∪)
Mutually Exclusive: events that do not occur at the same time
Probability: likelihood of an event happening
Bias: a built-in error that makes all values wrong (unequal) by a certain amount, e.g. a weighted dice
Fair: there is zero bias, and all outcomes have an equal likelihood
Random: something happens by chance and is unable to be predicted

Identify and represent sets

The **universal set** has this symbol ξ – this means EVERYTHING in the Venn diagram is in this set

A set is a collection of things – you write sets inside curly brackets { }

ξ = {the numbers between 1 and 50 inclusive}

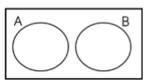
My sets can include every number between 1 and 50 including those numbers

A = {Square numbers}

A = {1, 4, 9, 16, 25, 36, 49}

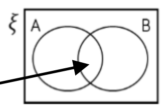
All the numbers in set A are square number and between 1 and 50

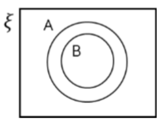
Interpret and create Venn diagrams



Mutually exclusive sets
The two sets have nothing in common
No overlap

Union of sets
The two sets have some elements in common – they are placed in the intersection



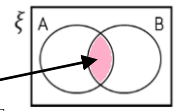


Subset
All of set B is also in Set A so the ellipse fits inside the set

The box
Around the outside of every Venn diagram will be a box. If an element is not part of any set it is placed outside an ellipse but inside the box

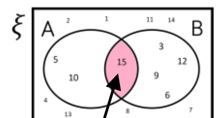
Intersection of sets

Elements in the intersection are in set A AND set B



The notation for this is $A \cap B$

ξ = {the numbers between 1 and 15 inclusive}
A = {Multiples of 5} B = {Multiples of 3}

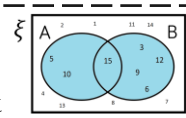


The element in $A \cap B$ is 15

In this example there is only one number that is both a multiple of 3 and a multiple of 5 between 1 and 15

Union of sets

Elements in the union could be in set A OR set B

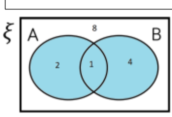


The notation for this is $A \cup B$

ξ = {the numbers between 1 and 15 inclusive}
A = {Multiples of 5} B = {Multiples of 3}

The elements in $A \cup B$ are 5, 10, 15, 3, 9, 6, 12

There are 7 elements that are either a multiple of 5 OR a multiple of 3 between 1 and 15



This Venn shows the **number of elements** in each set

Sample space – for single events

A sample space for rolling a six-sided dice is S = {1, 2, 3, 4, 5, 6}

A sample space for this spinner is S = {Pink, Blue, Yellow}

You only need to write each element once in a sample space diagram

- A Sample space represents a possible outcome from an event
- They can be interpreted in a variety of ways because they do not tell you the probability

Probability of a single event

Probability = $\frac{\text{number of times event happens}}{\text{total number of possible outcomes}}$

P(Blue) = $\frac{4}{10}$ ← There are 4 blue sectors; There are 10 sectors overall

$= \frac{2}{5}$

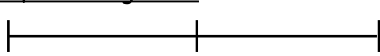
Probability notation P(event)

Probability can be a fraction, decimal or percentage value

$\frac{4}{10} = \frac{40}{100} = 0.40 = 40\%$

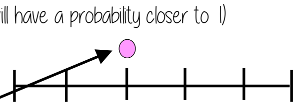
Probability is always a value between 0 and 1

The probability scale



Impossible: 0 or 0%
Even chance: 0.5, $\frac{1}{2}$ or 50%
Certain: 1 or 100%

The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)



There are 2 pink and 2 yellow balls, so they have the same probability

There are 5 possible outcomes
So 5 intervals on this scale, each interval value is $\frac{1}{5}$

Sum of probabilities

Probability is always a value between 0 and 1

The probability of getting a blue ball is $\frac{1}{5}$
∴The probability of NOT getting a blue ball is $\frac{4}{5}$
The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

P(white chocolate) = 1 – 0.15 – 0.35
= 0.5

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Prime numbers and Proof

What do I need to be able to do?

By the end of this unit you should be able to:

- Find and use multiples
- Identify factors of numbers and expressions
- Recognise and identify prime numbers
- Recognise square and triangular numbers
- Find common factors including HCF
- Find common multiples including LCM

Keywords

Multiples: found by multiplying any number by positive integers
Factor: integers that multiply together to get another number.
Prime: an integer with only 2 factors.
Conjecture: a statement that might be true (based on reasoning) but is not proven.
Counterexample: a special type of example that disproves a statement.
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
HCF: highest common factor (biggest factor two or more numbers share)
LCM: lowest common multiple (the first time the times table of two or more numbers match)

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3

3, 6, 9, 12, 15…

$3x, 6x, 9x$ …

This list continues and doesn't end

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Non example of a multiple

4.5 is not a multiple of 3 because it is 3 x 1.5

Not an integer

Factors

Arrays can help represent factors

5 x 2 or 2 x 5

Factors of 10
1, 2, 5, 10

10 x 1 or 1 x 10

The number itself is always a factor

Factors and expressions

| x | x | x | x | x | x |

$6x \times 1$ OR $6 \times x$

Factors of $6x$
6, x, 1, $6x$, $2x$, 3, $3x$, 2

x	x
x	x
x	x

$2x \times 3$

| x | x | x |
| x | x | x |

$3x \times 2$

Prime numbers

- Integer
- Only has 2 factors
- and itself

2

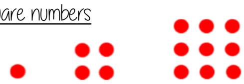
The first prime number
The only even prime number

Learn or how-to quick recall…

2, 3, 5, 7, 11, 13, 17, 19, 23, 29…

Square and triangular numbers

Square numbers



Representations are useful to understand a square number n^2

1, 4, 9, 16, 25, 36, 49, 64 …

Triangular numbers

Representations are useful – an extra counter is added to each new row

Add two consecutive triangular numbers and get a square number

1, 3, 6, 10, 15, 21, 28, 36, 45…

Common factors and HCF

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

18 — 1, 2, 3, 6, 9, 18

30 — 1, 2, 3, 5, 6, 10, 15, 30

Common factors (factors of both numbers)
1, 2, 3, 6

HCF = 6

6 is the biggest factor they share

Common multiples and LCM

Common multiples are multiples two or more numbers share

LCM – Lowest common multiple

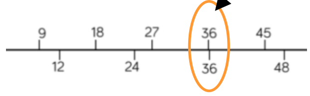
LCM of 9 and 12

9 — 9, 18, 27, 36, 45, 54

12 — 12, 24, 36, 48, 60

LCM = 36

The first time their multiples match



Comparing fractions

$\frac{3}{5}$ and $\frac{7}{10}$ ← Compare fractions using a LCM denominator → $\frac{6}{10}$ and $\frac{7}{10}$

Conjectures and counterexamples

Conjecture

1, 2, 4….
The numbers in the sequence are doubling each time.

A pattern that is noticed for many cases

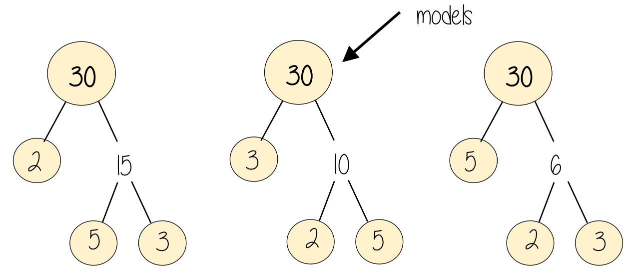
Counterexamples

This sequence isn't doubling it is adding 2 each time

Only **one** counterexample is needed to disprove a conjecture

Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

Multiplication is commutative

30 = 2 x 3 x 5 ← Multiplication of prime factors

Using prime factors for predictions

eg 60 — 30 x 2 — 2 x 3 x 5 **x 2**
150 — 30 x 5 — 2 x 3 x 5 **x 5**