YEAR 9 – REASONING WITH GEOMETRY...

Solving ratio & proportion problems

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with direct proportion
- Use conversion graphs
- Solve problems with inverse proportion
- Solve ratio problems
- Solve 'best buy' problems

Keywords

Proportion: a comparison between two numbers
Ratio: a ratio shows the relative size of two variables
Direct proportion: as one variable is multiplied by a scale factor the other variable is multiplied by the same scale factor.
Inverse proportion:: as one variable is multiplied by a scale factor the other is divided by the same scale factor.

Direct Proportion

As one variable changes the other changes at the same rate.

R

4 cans of pop = £2.40

This is a multiplicative change

4 cans of pop = £2.40 (x 3)
12 cans of pop = £7.20 (x 3)

4 cans of pop = £2.40 (x 0.5)
2 cans of pop = £1.20 (x 0.5)

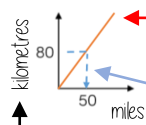
This multiplier is the same
In the same way that this would be for ratio

Sometimes this is easiest if you work out how much one unit is worth first
e.g. 1 can of pop = £0.60

Conversion Graphs

Compare two variables

R



This is always a straight line because as one variable increases so does the other at the same rate

Labelling of both axes is vital

To make conversions between units you need to find the point to compare – then find the associated point by using your graph.
Using a ruler helps for accuracy
Showing your conversion lines help as a "check" for solutions

Inverse Proportion

As one variable is multiplied by a scale factor the other is divided by the same scale factor

Examples of inversely proportional relationships

Time taken to fill a pool and the number of taps running

Time taken to paint a room and the number of workers

T is inversely proportional to G. When T=2 then G=20

÷ 2   × 4

T	1	2	8
G	40	20	5

× 2   ÷ 4

Best Buys

Have a directly proportional relationship

To calculate best buys you need to be able to compare the cost of one unit or units of equal amounts

Shop A
4 cans for £1.20
£1.20 ÷ 4
1 can is £0.30
Or 30p

Shop B
3 cans for 93p
£0.93 ÷ 3
1 can is £0.31
Or 31p

Cost per item

Shop A is the best value as it is 1p cheaper per can of pop

Shop A
4 cans for £1.20
4 ÷ £1.20
£1 buys 3.333 cans of pop

3 cans for 93p
3 ÷ £0.93
£1 buys 3.23 cans of pop

Cost per pound

Shop A is still shown as being the best value but pay attention to the unit you are calculating, per item or per pound

Best value is the most product for the lowest price per unit

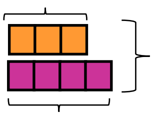
Sharing a whole into a given ratio

R

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy
3 : 4



Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

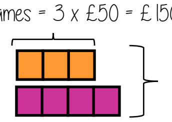
£350 ÷ 7 = £50

□ = one part = £50

Put back into the question

James: Lucy
3 : 4 (x 50)
£150:£200

James = 3 x £50 = £150



Lucy = 4 x £50 = £200

Finding a value given 1:n (or n:1)

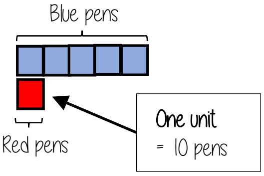
R

Inside a box are blue and red pens in the ratio 5:1
If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red
5 : 1

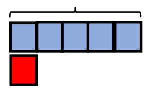
□ = one part = 10 pens



Put back into the question

Blue : Red
5 : 1 (x 10)
50: 10

Blue pens = 5 x 10 = 50 pens



Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens

YEAR 9 – REASONING WITH GEOMETRY…

Enlargement & Similarity

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What do I need to be able to do?

By the end of this unit you should be able to:

- Recognise enlargement and similarity
- Enlarge a shape by a positive SF
- Enlarge a shape from a point
- Enlarge a shape by a fractional SF
- Work out missing sides and angles in a pair of similar shapes.

Keywords

Similar Shapes: shapes of different sizes that have corresponding sides in equal proportion and identical corresponding angles.

Scale Factor: the multiple describing how much a shape has been enlarged

Enlarge: to change the size of a shape (enlargement is not always making a shape bigger)

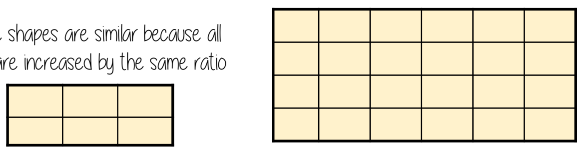
Corresponding: objects (or sides) that appear in the same place in two similar situations.

Image: the picture or visual representation of the shape

Recognise enlargement & similarity

Shapes are similar if all pairs of corresponding sides are in the same ratio

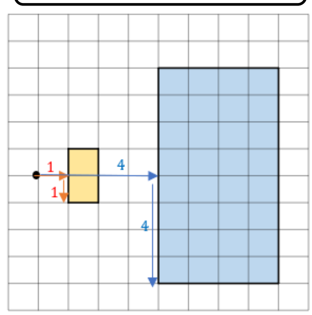
These shapes are similar because all sides are increased by the same ratio



Enlargements are similar shapes with a ratio other than 1

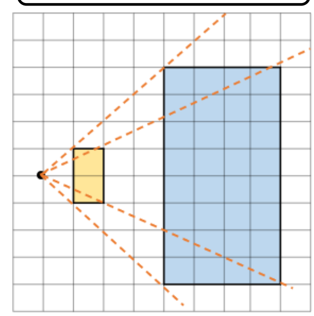
Enlarge a shape from a point

Scaled distances method



Scale the distance between the point of enlargement and each corresponding vertices

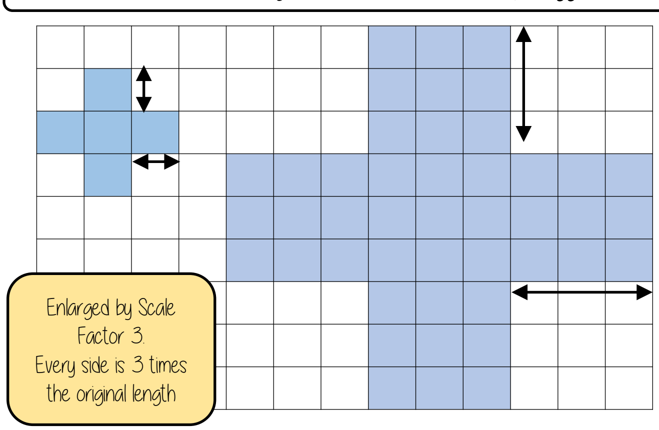
Rays method



Multiply the distance from the centre of corresponding vertices by the scale factor along the ray

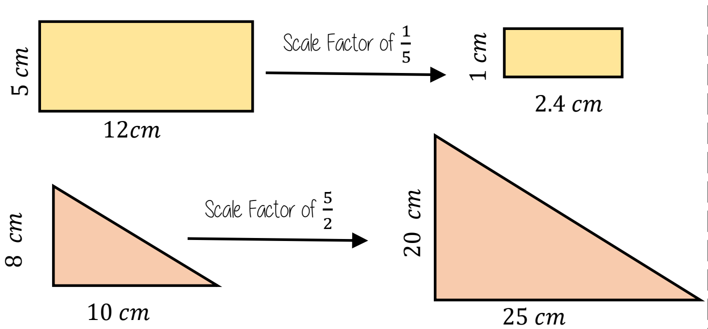
Enlarge by a positive scale factor

With a scale factor larger than 1 it makes the shape **bigger**



Positive fractional scale factor

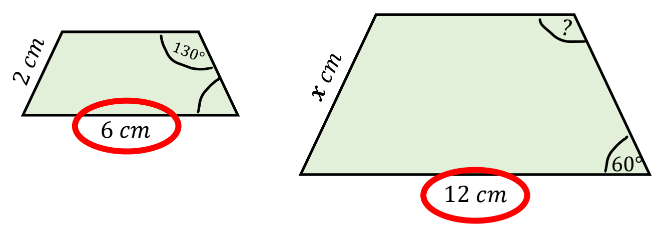
With a scale factor between 0 and 1 it makes the shape **smaller**



Calculations in similar shapes

Don't forget that properties of shapes don't change with enlargements or in similar shapes

The two trapezium are similar find the missing side and angle



Corresponding sides identify the scale factor: $\frac{12}{6} = 2$ Scale Factor = **2**

Calculate the missing side: Length (corresponding side) x scale factor

$2cm \times 2$

$x = 4cm$

Enlargement does not change angle size

Calculate the missing angle: Corresponding angles remain the same

$130°$

YEAR 9 – REASONING WITH GEOMETRY...

Pythagoras' theorem

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What do I need to be able to do?

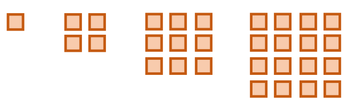
By the end of this unit you should be able to:

- Use square and cube roots
- Identify the hypotenuse
- Calculate the hypotenuse
- Find a missing side in a Right angled triangle
- Use Pythagoras' theorem on axes
- Explore proofs of Pythagoras' theorem

Keywords

Square number: the output of a number multiplied by itself
Square root: a value that can be multiplied by itself to give a square number
Hypotenuse: the largest side on a right angled triangle. Always opposite the right angle.
Opposite: the side opposite the angle of interest
Adjacent: the side next to the angle of interest

Squares and square roots R



This can also be written as 6^2

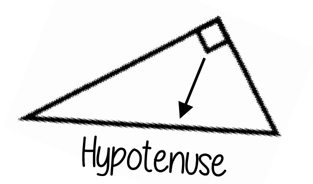
$\sqrt{\ }$ is the square root symbol

eg $\sqrt{64} = 8$
Because $8 \times 8 = 64$

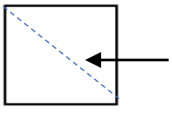
1×1	2×2	3×3	4×4	5×5	6×6	7×7	8×8	9×9	10×10
1	4	9	16	25	36	49	64	81	100

Square numbers

Identify the hypotenuse

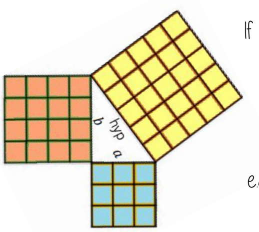


The hypotenuse is always the longest side on a triangle because it is opposite the biggest angle.



Polygons can still have a hypotenuse if it is split up into triangles and opposite a right angle

Determine if a triangle is right-angled



a = 3 b = 4 c = 5

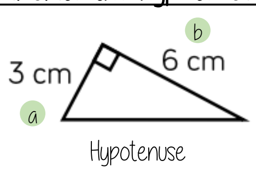
If a triangle is right-angled, the sum of the squares of the shorter sides will equal the square of the hypotenuse.

$a^2 + b^2 = \text{hypotenuse}^2$

eg $a^2 + b^2 = hypotenuse^2$
$3^2 + 4^2 = 5^2$
$9 + 16 = 25$

Substituting the numbers into the theorem shows that this is a right-angled triangle

Calculate the hypotenuse



Either of the short sides can be labelled a or b

$a^2 + b^2 = \text{hypotenuse}^2$

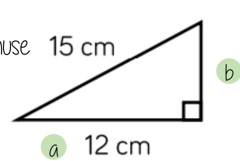
1. Substitute in the values for a and b

$3^2 + 6^2 = \text{hypotenuse}^2$
$9 + 36 = \text{hypotenuse}^2$
$45 = \text{hypotenuse}^2$

2. To find the hypotenuse square root the sum of the squares of the shorter sides.

$\sqrt{45} = \text{hypotenuse}$
$\mathbf{6.71\,cm} = \text{hypotenuse}$

Calculate missing sides



Either of the short sides can be labelled a or b

$a^2 + b^2 = \text{hypotenuse}^2$

$12^2 + b^2 = 15^2$

1. Substitute in the values you are given

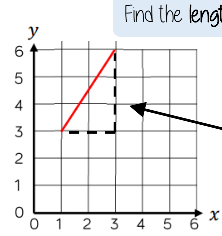
$144 + b^2 = 225$
$-144 \qquad -144$

Rearrange the equation by subtracting the shorter square from the hypotenuse squared

Square root to find the length of the side
$b^2 = 111$
$b = \sqrt{111} = 10.54\ cm$

Pythagoras' theorem on a coordinate axis

Find the **length** of the line segment



The segment can be made into a right-angled triangle by adding the sides on the diagram.

The line segment is the **hypotenuse**

$a^2 + b^2 = \text{hypotenuse}^2$

The lengths of a and b are the sides of the triangle.

Be careful to check the scale on the axes