YEAR 8 - ALGEBRAIC TECHNIQUES...

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Brackets, Equations & Inequalities

What do I need to be able to do?

By the end of this unit you should be able to:

- Form Expressions
- Expand and factorise single brackets
- Form and solve equations
- Solve equations with brackets
- Represent inequalities
- Form and solve inequalities

Keywords

Simplify: grouping and combining similar terms

Substitute: replace a variable with a numerical value

Equivalent: something of equal value

Coefficient: a number used to multiply a variable

Product: multiply terms

Highest Common Factor (HCF): the biggest factor (or number that multiplies to give a term)

Inequality: an inequality compares who values showing if one is greater than, less than or equal to another

Form expressions

For unknown variables, a letter is normally used in its place

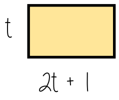
More than – ADD

Less than/ difference – SUBTRACT

eg 4 more than t $\longrightarrow$ t + 4

8 less than k $\longrightarrow$ k - 8

Only similar terms can be grouped together



eg Find the perimeter of this shape

(Perimeter = length around outside of shape)

t + 2t + 1 + t + 2t + 1 $\longrightarrow$ 6t + 2

Directed numbers

$+ \ + \rightarrow +$

$- \ - \rightarrow +$

$+ \ - \rightarrow -$

$- \ + \rightarrow -$

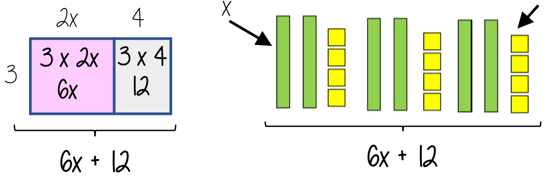
eg a = -5 and b = 2

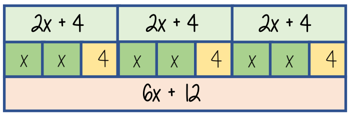
$a^2 = a \times a = -5 \times -5 = 25$

b + a = 2 + - 5 = -3

Multiply single brackets

3 (2x + 4)

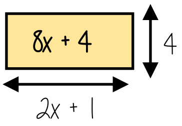




Different representations of 3(2x+4) = 6x + 12

Factorise into a single bracket

8x + 4



Try and make this the **highest common factor**

The two values **multiply** together (also the area) of the rectangle

8x + 4 ≡ 4 (2x + 1)

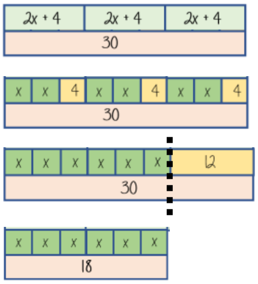
Note:

8x + 4 ≡ 2(4x + 2)

This is factorised but the HCF has not been used

Solve equations with brackets

3 (2x + 4) = 30



3 (2x + 4) = 30

Expand the brackets

6x + 12 = 30

-12 -12

Substitute to check your answer. This could be negative or a fraction or decimal

6x = 18

÷ 6 ÷ 6

x = 3

Simple Inequalities

< less than

≤ Less than or equal to

> More than

≥ More than or equal to

x < 10

Say this out loud

"x is a value less than 10"

10 > x

Say this out loud

"10 is more than the value"

Note:

x<10 and 10>x represent the same values

x + 2 ≤ 20

"my value + 2 is less than or equal to 20"

x ≤ 18

The biggest the value can be is 18

Form and solve inequalities

Two more than treble my number is greater than 11

Find the possible range of values

Form

x $\longrightarrow$ x3 $\longrightarrow$ +2 $\longrightarrow$ 11

3x + 2 > 11

Solve

x $\longleftarrow$ ÷3 $\longleftarrow$ -2 $\longleftarrow$ 11

x > 3

Check

This would suggest any value bigger than 3 satisfies the statement

3 x 3 + 2 = 11 ✓   10 x 3 + 2 = 32 ✓

Algebraic constructs

Expression

A sentence with a minimum of two numbers and one maths operation

Equation

A statement that two things are equal

Term

A single number or variable

Identity

An equation where both sides have variables that cause the same answer includes ≡

Formula

A rule written with all mathematical symbols

eg area of a rectangle A = b x h

YEAR 8 - ALGEBRAIC TECHNIQUES...

Sequences

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What do I need to be able to do?

By the end of this unit you should be able to:

- Generate a sequence from term to term or position to term rules
- Recognise arithmetic sequences and find the nth term
- Recognise geometric sequences and other sequences that arise

Keywords

Sequence: items or numbers put in a pre-decided order
Term: a single number or variable
Position: the place something is located
Linear: the difference between terms increases or decreases (+ or -) by a constant value each time
Non-linear: the difference between terms increases or decreases in different amounts, or by x or ÷
Difference: the gap between two terms
Arithmetic: a sequence where the difference between the terms is constant
Geometric: a sequence where each term is found by multiplying the previous one by a fixed non zero number

Linear and Non Linear Sequences

Linear Sequences – increase by addition or subtraction and the same amount each time

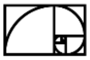
Non-linear Sequences – do not increase by a constant amount – quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division.

Fibonacci Sequence – look out for this type of sequence

0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms.



Sequences from algebraic rules

This is substitution!

$3n + 7$

This will be linear - note the single power of n. The values increase at a constant rate

$3n^2 + 7$

This is not linear as there is a power for n

$2n - 5$ →

Substitute the number of the term you are looking for in place of 'n'

e.g

1st term = 2 (1) – 5 = -3
2nd term = 2 (2) – 5 = -1
100th term = 2 (100) – 5 = 195

Checking for a term in a sequence

Form an equation

Is 201 in the sequence 3n – 4?

$$3n - 4 = 201$$

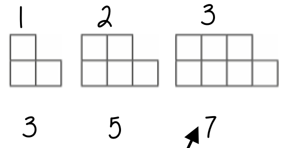
Algebraic rule

Term to check

Solving this will find the position of the term in the sequence.
ONLY an integer solution can be in the sequence.

Sequence in a table and graphically

Position: the place in the sequence



Term: the number or variable
(the number of squares in each image)

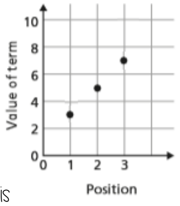
"The **term** in **position** 3 has 7 squares"

In a table

Position	1	2	3
Term	3	5	7

+2 +2

Graphically



Because the terms increase by the same addition each time this is **linear** – as seen in the graph

Complex algebraic rules

Misconceptions and comparisons

$2n^2$

2 tijmes whatever n squared is

eg

1st term = $2 \times 1^2 = 2$
2st term = $2 \times 2^2 = 8$
100th term = $2 \times 100^2 = 2000$

$(2n)^2$

2 times n then square the answer

eg

1st term = $(2 \times 1)^2 = 4$
2st term = $(2 \times 2)^2 = 16$
100th term = $(2 \times 100)^2 = 40000$

$n(n + 5)$ ←

e.g

1st term = 1 (1 + 5) = 6
2st term = 2 (2 + 5) = 14
100th term = 100 (100 + 5) = 10500

You don't need to expand the expression

Finding the algebraic rule

This is the 4 times table → 4, 8, 12, 16, 20...

4n

↓ ↓ ↓

7, 11, 15, 19, 22 ←

This has the same constant difference – but is 3 more than the original sequence

4n + 3

$4n + 3$

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

YEAR 8 - ALGEBRAIC TECHNIQUES...

Indices

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What do I need to be able to do?

By the end of this unit you should be able to:

- Add/ Subtract expressions with indices
- Multiply expressions with indices
- Divide expressions with indices
- Know the addition law for indices
- Know the subtraction law for indices

Keywords

Base: The number that gets multiplied by a power

Power: The exponent – or the number that tells you how many times to use the number in multiplication

Exponent: The power – or the number that tells you how many times to use the number in multiplication

Indices: The power or the exponent.

Coefficient: The number used to multiply a variable

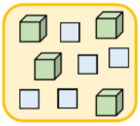
Simplify: To reduce a power to its lowest term

Product: Multiply

Addition/ Subtraction with indices

Coefficient Power

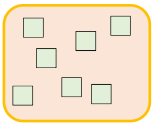
$$\underbrace{\underbrace{5x^2}_{\text{Term}} + \underbrace{4x^4}_{\text{Term}}}_{\text{Expression}} \rightarrow$$



Each square represents x^2 and each cube represents x^4

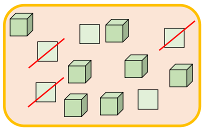
Only similar terms can be simplified
If they have different powers, they are unlike terms

$5x^2 + 2x^2 \rightarrow$



$\rightarrow 7x^2$

$5x^2 + 6x^4 - 3x^2 + x^4 \rightarrow$



$\rightarrow 2x^2 + 7x^4$

Divide expressions with indices

$$\frac{24}{36} \rightarrow \frac{\cancel{2} \times \cancel{2} \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{3} \times 2 \times \cancel{3}} \rightarrow \frac{2}{3}$$

$$\frac{5a^3b^2}{15ab^6} \rightarrow \underbrace{\frac{\cancel{5} \times \cancel{a} \times a \times a \times \cancel{b} \times \cancel{b}}{3 \times \cancel{5} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times b \times b \times b \times b}} \rightarrow \frac{a^2}{3b^4}$$

Cross cancelling factors shows cancels the expression

$$\frac{23a^7y^2}{5db^6}$$

This expression cannot be divided (cancelled down) because there are no common factors or similar terms

Multiply expressions with indices

$4b \times 3a$

$\equiv 4 \times b \times 3 \times a$

$\equiv 4 \times 3 \times b \times a$

$\equiv 12ab$

$5t \times 9t$

$\equiv 5 \times t \times 9 \times t$

$\equiv 5 \times 9 \times t \times t$

$\equiv 45t^2$

$2b^4 \times 3b^2$

$\equiv 2 \times b \times b \times b \times b \times 3 \times b \times b$

$\equiv 2 \times 3 \times b \times b \times b \times b \times b \times b$

$\equiv 6b^6$

There are often misconceptions with this calculation but break down the powers

Addition/ Subtraction laws for indices

$3^5 \times 3^2 \rightarrow 3^7$

$= \underbrace{(3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3)}$

The base number is all the same so the terms can be simplified

Addition law for indices

$$a^m \times a^n = a^{m+n}$$

$3^5 \div 3^2 \rightarrow 3^3$

$$\frac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} \rightarrow \frac{3^3}{3^0} \rightarrow \frac{3^3}{1}$$

Subtraction law for indices

$$a^m \div a^n = a^{m-n}$$