YEAR 10 – USING NUMBER...

@whisto_maths

Indices & Roots

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify square and cube numbers
- Calculate higher powers and roots
- Understand powers of 10 and standard form
- Know the addition and subtraction rule for indices
- Understand power zero and negative indices
- Calculate with numbers in standard form

Keywords

Standard (index) Form: a system of writing very big or very small numbers
Commutative: an operation is commutative if changing the order does not change the result.
Base: The number that gets multiplied by a power
Power: The exponent – or the number that tells you how many times to use the number in multiplication
Exponent: The power – or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Negative: a value below zero.
Coefficient: The number used to multiply a variable

Square and cube numbers

R

Square numbers

1, 4, 9, 16…

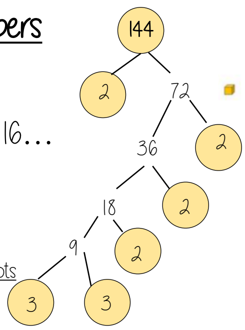
144 = 2 x 2 x 2 x 2 x 3 x 3

2 x 2 x 3 x 2 x 2 x 3

12 x 12

Prime factors can find square roots

$\sqrt{144} = 12$



Cube numbers

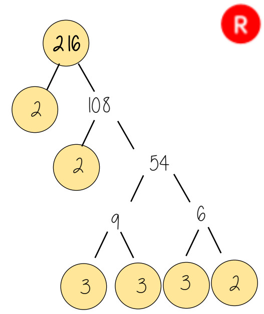
1, 8, 27, 64, 125…

216 = 2 x 2 x 2 x 3 x 3 x 3

2 x 3 x 2 x 3 x 2 x 3

6 x 6 x 6

$\sqrt[3]{216} = 6$



Higher powers and roots

x^n — n – power (number of times multiplied by itself)

x – the base number.

$\sqrt[n]{x}$ — Finding the nth root of any value

Other mental strategies for square roots

$$\sqrt{810000} = \sqrt{81} \times \sqrt{10000}$$
$$= 9 \times 100$$
$$= 900$$

Standard form

R

$$A \times 10^n$$

Any number between 1 and less than 10 (A)

Any integer (n)

0.001

$1 \times \frac{1}{1000}$

1×10^{-3}

10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^1	10^0	10^{-1}	10^{-2}	10^{-3}
10	1	0.1	0.01	0.001

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Example

3.2×10^4
$= 3.2 \times 10 \times 10 \times 10 \times 10$
$= 32000$

Non-example

0.8×10^4

$5.3 \times 10^{0.7}$

Numbers in standard form with negative powers will be less than 1

$3.2 \times 10^{-4} = 3.2 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = 0.00032$

Addition/ Subtraction Laws

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

Zero and negative indices

$$x^0 = 1$$

Any number divided by itself = 1

$$\frac{a^6}{a^6} = a^6 \div a^6$$
$$= a^{6-6} = a^0 = 1$$

Negative indices do not indicate negative solutions

$2^2 = 4$
$2^1 = 2$
$2^0 = 1$
$2^{-1} = \frac{1}{2}$
$2^{-2} = \frac{1}{4}$

Looking at the sequence can help to understand negative powers

Powers of powers

$$(x^a)^b = x^{ab}$$

$$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$$

The same base and power is repeated. Use the addition law for indices

$$(2^3)^4 = 2^{12}$$

$a \times b = 3 \times 4 = 12$

NOTICE the difference

$$(2x^3)^4 = 2x^3 \times 2x^3 \times 2x^3 \times 2x^3$$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$$(2x^3)^4 = 16x^{12}$$

Standard form calculations

R

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard from at the end

$$6 \times 10^5 + 8 \times 10^5$$

Method 1

$= 600000 + 800000$
$= 1400000$
$= 1.4 \times 10^6$

Method 2

$= (6 + 8) \times 10^5$
$= 14 \times 10^5$ — This is not the final answer
$= 1.4 \times 10^1 \times 10^5$
$= 1.4 \times 10^6$

Multiplication and division

Division questions can look like this

$$\frac{1.5 \times 10^5}{0.3 \times 10^3}$$

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$1.5 \div 0.3 \times 10^5 \div 10^3$

$= 5 \times 10^2$

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

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Types of number & sequences

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand factors and multiples
- Express numbers as a product of primes
- Find the HCF and LCM
- Describe and continue sequences
- Explore sequences
- Find the nth term of a linear sequence

Keywords

Factor: numbers we multiply together to make another number
Multiple: the result of multiplying a number by an integer.
HCF: highest common factor. The biggest factor that numbers share.
LCM: lowest common multiple. The first multiple numbers share.
Arithmetic: a sequence where the difference between the terms is constant
Geometric: a sequence where each term is found by multiplying the previous one by a fixed nonzero number
Sequence: items or numbers put in a pre-decided order

Multiples

The "times table" of a given number R

All the numbers in this lists below are multiples of 3.

3, 6, 9, 12, 15…

This list continues and doesn't end

$3x, 6x, 9x$ …

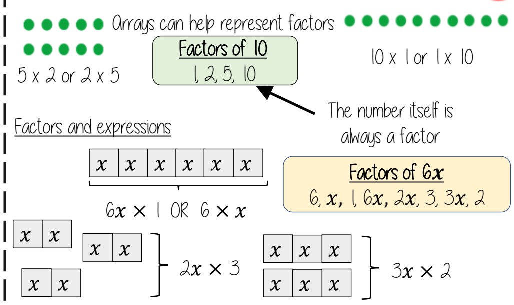
x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Non example of a multiple

4.5 is not a multiple of 3 because it is 3 x 1.5 ← Not an integer

Factors

R



Arrays can help represent factors

5 x 2 or 2 x 5

Factors of 10
1, 2, 5, 10

10 x 1 or 1 x 10

The number itself is always a factor

Factors and expressions

$6x \times 1$ OR $6 \times x$

$2x \times 3$

$3x \times 2$

Factors of $6x$
$6, x, 1, 6x, 2x, 3, 3x, 2$

Prime numbers

- Integer
- Only has 2 factors
- and itself

2 – The first prime number. The only even prime number

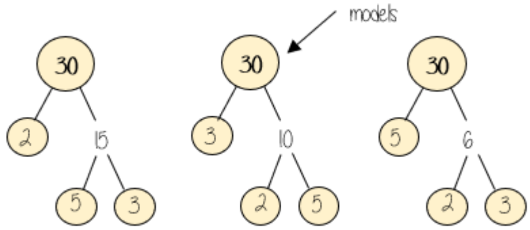
Learn or how-to quick recall…

2, 3, 5, 7, 11, 13, 17, 19, 23, 29…

R

Product of prime factors

R



All three prime factor trees represent the same decomposition

30 = 2 x 3 x 5 ← Multiplication of prime factors

Using prime factors for predictions

eg **60**	30 x 2	2 x 3 x 5 **x 2**
150	30 x 5	2 x 3 x 5 **x 5**

Finding the HCF and LCM

R

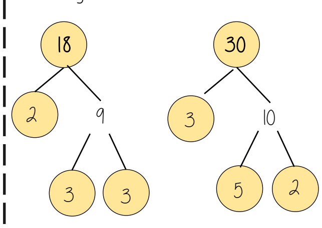
HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

6 is the biggest factor they share — HCF = 6



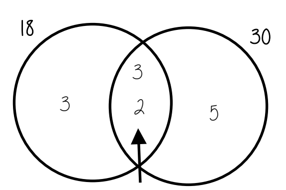
LCM – Lowest common multiple

LCM of 18 and 30

18: 18, 36, 54, 72, 90

30: 30, 60, 90

The first time their multiples match — LCM = 90



LCM = 90

HCF = 6

Arithmetic/ Geometric sequences

Arithmetic Sequences change by a common difference. This is found by addition or subtraction between terms

Geometric Sequences change by a common ratio. This is found my multiplication/ division between terms.

Term to term rule – how you get from one term (number in the sequence) to the next term.

Position to term rule – take the rule and substitute in a position to find a term. Eg. Multiply the position number by 3 and then add 2

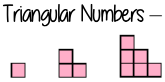
Other sequences

Fibonacci Sequence

1, 1, 2, 3, 5, 8 …

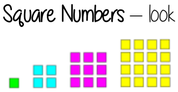
Each term is the sum of the previous two terms

Triangular Numbers – look at the formation



1, 3, 6, 10, 15 …

Square Numbers – look at the formation



1, 4, 9, 16…

Sequences are the repetition of a patten

Finding the nth term

R

This is the 4 times table → 4, 8, 12, 16, 20…

4n

7, 11, 15, 19, 22

This has the same constant difference – but is 3 more than the original sequence

4n + 3

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence