YEAR 10 – SIMILARITY...

Trigonometry

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

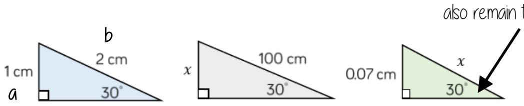
- Work fluently with hypotenuse, opposite and adjacent sides
- Use the tan, sine and cosine ratio to find missing side lengths
- Use the tan, sine and cosine ratio to find missing angles
- Calculate sides using Pythagoras' Theorem

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)
Scale Factor: the multiplier of enlargement
Constant: a value that remains the same
Cosine ratio: the ratio of the length of the adjacent side to that of the hypotenuse. The sine of the complement.
Sine ratio: the ratio of the length of the opposite side to that of the hypotenuse.
Tangent ratio: the ratio of the length of the opposite side to that of the adjacent side.
Inverse: function that has the opposite effect.
Hypotenuse: longest side of a right-angled triangle. It is the side opposite the right-angle.

Ratio in right-angled triangles

When the angle is the same the ratio of sides a and b will also remain the same



a : b
1 : 2

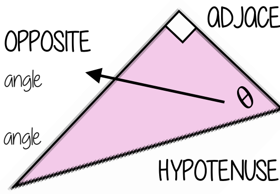
$\frac{a}{b} = \frac{1}{2}$

a : b
x : 100
50 : 100

a : b
0.07 : x
0.07 : 0.14

Hypotenuse, adjacent and opposite

ONLY right-angled triangles are labelled in this way



OPPOSITE
Always opposite an acute angle
Useful to label second
Position depend upon the angle in use for the question

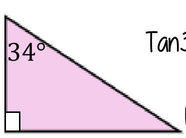
ADJACENT
Next to the angle in question
Often labelled last

HYPOTENUSE
Always the longest side
Always opposite the right angle
Useful to label this first

Tangent ratio: side lengths

$\text{Tan}\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Substitute the values into the tangent formula



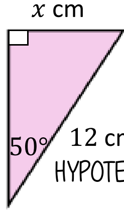
$\text{Tan}34 = \frac{10}{x}$

Equations might need rearranging to solve

$x \times \text{Tan}34 = 10$

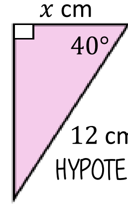
$x = \frac{10}{\text{Tan}34} = 14.8\text{cm}$

Sin and Cos ratio: side lengths



$\text{Sin}\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$

NOTE
The Sin(x) ratio is the same as the Cos(90-x) ratio



$\text{Cos}\theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$

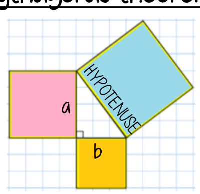
Substitute the values into the ratio formula

Equations might need rearranging to solve

Pythagoras theorem

R

$\text{Hypotenuse}^2 = a^2 + b^2$

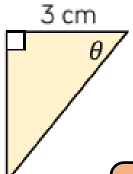


This is commutative – the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras
- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Ony length made from a right angles

Sin, Cos, Tan: Angles

Inverse trigonometric functions



Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$\text{Tan}\theta = \frac{3}{4}$

$\theta = \text{Tan}^{-1}\frac{3}{4}$

$\theta = 36.9°$

$\theta = \text{Tan}^{-1}\frac{\text{opposite side}}{\text{adjacent side}}$

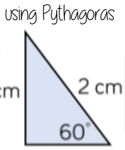
$\theta = \text{Sin}^{-1}\frac{\text{opposite side}}{\text{hypotenuse side}}$

$\theta = \text{Cos}^{-1}\frac{\text{adjacent side}}{\text{hypotenuse side}}$

Key angles

Because trig ratios remain the same for similar shapes you can generalise from the following statements

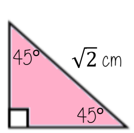
This side could be calculated using Pythagoras



$\text{Tan}30 = \frac{1}{\sqrt{3}}$
$\text{Tan}60 = \sqrt{3}$

$\text{Cos}30 = \frac{\sqrt{3}}{2}$
$\text{Cos}60 = \frac{1}{2}$

$\text{Sin}30 = \frac{1}{2}$
$\text{Sin}60 = \frac{\sqrt{3}}{2}$



$\text{Tan}45 = 1$

$\text{Cos}45 = \frac{1}{\sqrt{2}}$

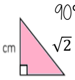
$\text{Sin}45 = \frac{1}{\sqrt{2}}$

Key angles 0° and 90°

$\text{Tan}0 = 0$

~~Tan90~~

This value cannot be defined – it is impossible as you cannot have two 90° angles in a triangle



$\text{Sin}0 = 0$

$\text{Sin}90 = 1$

$\text{Cos}0 = 1$

$\text{Cos}90 = 0$

YEAR 10 – SIMILARITY…

Congruence, similarity & enlargement

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What do I need to be able to do?

By the end of this unit you should be able to:

- Enlarge by a positive scale factor
- Enlarge by a fractional scale factor
- Identify similar shapes
- Work out missing sides and angles in similar shapes
- Use parallel lines to find missing angles
- Understand similarity and congruence

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)
Scale Factor: the multiplier of enlargement
Centre of enlargement: the point the shape is enlarged from
Similar: when one shape can become another with a reflection, rotation, enlargement or translation.
Congruent: the same size and shape
Corresponding: items that appear in the same place in two similar situations
Parallel: straight lines that never meet (equal gradients)

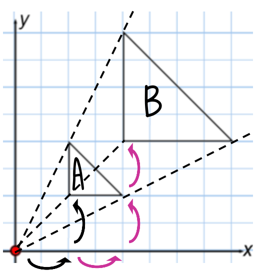
Positive scale factors (R)

Enlargement from a point

Enlarge shape A by SF 2 from (0,0)

The shape is enlarged by 2

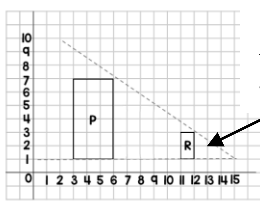
The distance from the point enlarges by 2



Fractional scale factors (R)

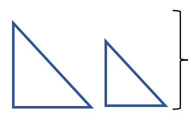
Fractions less than 1 make a shape SMALLER

R is an enlargement of P by a scale factor $\frac{1}{3}$ from centre of enlargement (15,1)



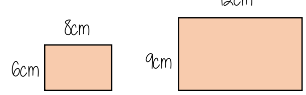
SF: $\frac{1}{3}$ - R is three times smaller than P

Identify similar shapes



Angles in similar shapes do not change.
e.g if a triangle gets bigger the angles can not go above 180°

Similar shapes

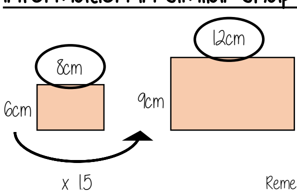


Scale Factor: Both sides on the bigger shape are 1.5 times bigger

Compare sides: 6 : 9 → 2 : 3; 8 : 12 → 2 : 3

Both sets of sides are in the same ratio

Information in similar shapes



Compare the equivalent side on both shapes

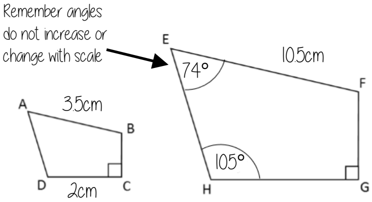
Scale Factor is the multiplicative relationship between the two lengths

Shape ABCD and EFGH are similar

Notation helps us find the corresponding sides

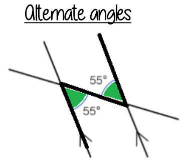
AB and EF are corresponding

Remember angles do not increase or change with scale



Angles in parallel lines (R)

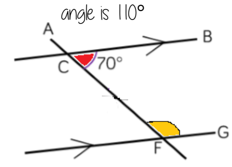
Alternate angles



Because alternate angles are equal the highlighted angles are the same size

Co-interior angles

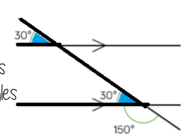
Because co-interior angles have a sum of 180° the highlighted angle is 110°



As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

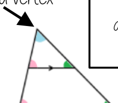
Corresponding angles

Because corresponding angles are equal the highlighted angles are the same size



Similar triangles

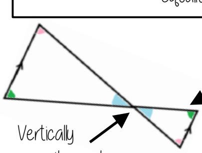
Shares a vertex



Because corresponding angles are equal the highlighted angles are the same size

Parallel lines – all angles will be the same in both triangle

As all angles are the same this is similar – it only one pair of sides are needed to show equality

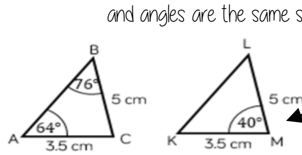


All the angles in both triangles are the same and so similar

Vertically opposite angles

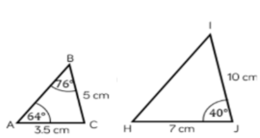
Congruence and Similarity

Congruent shapes are identical – all corresponding sides and angles are the same size



$A\hat{C}B = K\hat{M}L$

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**



Because all angles are the same, but all sides are enlarged by 2 ABC and HIJ are **similar**

Conditions for congruent triangles

Triangles are congruent if they satisfy any of the following conditions

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same