

YEAR 9 — REASONING WITH ALGEBRA...

Testing conjectures

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

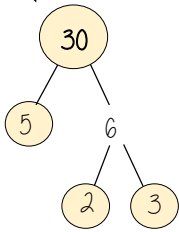
- Use factors, multiples and primes
- Reason True or False
- Reason Always, sometimes never true
- Show that reasoning
- Make conjectures about number
- Expand binomials
- Make conjectures with algebra
- Explore the 100 grid

Keywords

- Multiples:** found by multiplying any number by positive integers
- Factor:** integers that multiply together to get another number.
- Prime:** an integer with only 2 factors.
- HCF:** highest common factor (biggest factor two or more numbers share)
- LCM:** lowest common multiple (the first time the times table of two or more numbers match)
- Verify:** the process of making sure a solution is correct
- Proof:** logical mathematical arguments used to show the truth of a statement
- Binomial:** a polynomial with two terms
- Quadratic:** a polynomial with four terms (often simplified to three terms)

Factors, Multiples and Primes

Multiplication part-whole models



All three prime factor trees represent the same decomposition

HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

Common factors are factors two or more numbers share

LCM – Lowest common multiple

LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54

12: 12, 24, 36, 48, 60

Common multiples are multiples two or more numbers share



True or False?

Conjecture

A pattern that is noticed for many cases

1, 2, 4, ...
The numbers in the sequence are doubling each time.

Counterexamples



This sequence isn't doubling it is adding 2 each time

Only **one** counterexample is needed to disprove a conjecture

Always, Sometimes, Never true.

Always Every value always supports the statement

Sometimes Examples show the statement being true and counter examples to show when it is false.

Never No example supports the statement

Examples to try

- 0 and 1
- Fractions
- Negative numbers

Show that

Numerical verification

Show the stages to a solution with numerical values

Algebraic verification

Show algebraic properties of the solution
You may want to use pictorial images to support this

Proof

Simple proofs using algebra

Compare the left hand side of an equation with the right hand side – are they the same or different?

Conjectures

Even

(2n)

Multiple of 2

Odd

(2n + 1)

One more than any even

Use numerical verification first
Use pictorial verification – the representations of numbers of odd and even

Exploring the 100 square

In terms of 'n' is used to make generalisations about relationships between numbers

Positions of numbers in relation to n form expressions

Eg one space to the right of n
 $n + 1$

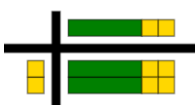
Eg One row below n
 $n + 10$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

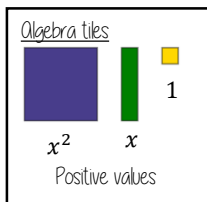
The size of the grid for generalisation changes the relationship statements

Expanding binomials

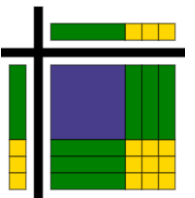
$$2(x + 2) \equiv 2x + 4$$



Algebra tiles can represent a binomial expansion
Has two terms



$$(x + 3)(x + 3) \equiv x^2 + 6x + 9$$



This is a quadratic
It has four terms which simplified to three terms

The order of the binomial has no impact on the outcome.
eg $(x + 3)(3 + x)$

YEAR 9 — REASONING WITH ALGEBRA...

Forming and Solving Equations

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve inequalities with negative numbers
- Solve equations with unknowns on both sides
- Solve inequalities with unknowns on both sides
- Substitute into formulae and equations
- Rearrange formulae

Keywords

Inequality: an inequality compares two values showing if one is greater than, less than or equal to another

Variable: a quantity that may change within the context of the problem

Rearrange: Change the order

Inverse operation: the operation that reverses the action

Substitute: replace a variable with a numerical value

Solve: find a numerical value that satisfies an equation

Solve equations with brackets



$$3(2x + 4) = 30$$

$$6x + 12 = 30$$

$$6x = 18$$

$$x = 3$$

$$3(2x + 4) = 30$$

Expand the brackets

$$6x + 12 = 30$$

$$-12 \quad -12$$

$$6x = 18$$

$$-6 \quad -6$$

$$x = 3$$

Form and solve inequalities



Two more than treble my number is greater than 11

Find the possible range of values

$$3x + 2 > 11$$

Solve

$$x \leftarrow -3 \leftarrow -2 \leftarrow 11$$

$$x > 3$$

Inequalities with negatives

Method 1 Make x positive first

$$2 - 3x > 17$$

$$+ 3x \quad + 3x$$

$$2 > 17 + 3x$$

$$-17 \quad -17$$

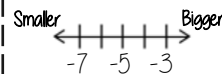
$$-15 > 3x$$

$$\div 3 \quad \div 3$$

$$-5 > x$$

x is true for any value smaller than -5

✓ CHECK IT!
 $2 - 3(-6) = 20$
 TRUE/ CORRECT



Equations with unknown on both sides

$$4x + 5 = 3x + 24$$

$$-3x \quad -3x$$

$$x + 5 = 24$$

$$-5 \quad -5$$

$$x = 19$$

$$x \quad x \quad x \quad x \quad 5$$

$$x \quad x \quad x \quad 24$$

Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$$5(x + 4) < 3(x + 2)$$

$$5x + 20 < 3x + 6$$

$$2x + 20 < 6$$

$$2x < -14$$

$$x < -7$$

Check it!

$$5(-8 + 4) < 3(-8 + 2)$$

$$5(-4) < 3(-6)$$

$$-20 < -18$$

✓ -20 IS smaller than -18

Method 2 Keep the negative x

$$2 - 3x > 17$$

$$-2 \quad -2$$

$$-3x > 15$$

$$\div -3 \quad \div -3$$

$$x > -5$$

x is true for any value bigger than -5

This cannot be true...

$$x < -5$$

When you multiply or divide x by a negative you need to reverse the inequality

Formulae and Equations

Substitute in values

Formulae — all expressed in symbols

Equations — include numbers and can be solved

Rearranging Formulae (one step)

$$x = y + z$$

$$x = y + z$$

Rearrange to make y the subject.

$$y = x - z$$

$$y \rightarrow +z \rightarrow x$$

$$y \leftarrow -z \leftarrow x$$

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution.

Language of rearranging...

Make XXX the subject

Change the subject

Rearrange

Rearranging Formulae (two step)

In an equation (find x)

$$4x - 3 = 9$$

$$+3 \quad +3$$

$$4x = 12$$

$$\div 4 \quad \div 4$$

$$x = 3$$

In a formula (make x the subject)

$$xy - s = a$$

$$+s \quad +s$$

$$xy = a + s$$

$$\div y \quad \div y$$

$$x = \frac{a + s}{y}$$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

e.g Find the gradient of the line $2y - 4x = 9$

Make y the subject first $y = \frac{4x + 9}{2}$

$$\text{Gradient} = \frac{4}{2} = 2$$

YEAR 9 — REASONING WITH ALGEBRA... Straight Line Graphs

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Compare gradients
- Compare intercepts
- Understand and use $y = mx + c$
- Find the equation of a line from a graph
- Interpret gradient and intercepts of real-life graphs

Keywords

Gradient: the steepness of a line

Intercept: where two lines cross. The y-intercept: where the line meets the y-axis

Parallel: two lines that never meet with the same gradient

Co-ordinate: a set of values that show an exact position on a graph

Linear: linear graphs (straight line) — linear common difference by addition/ subtraction

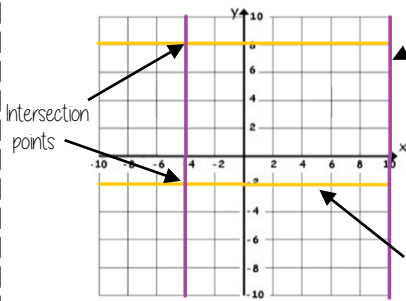
Asymptote: a straight line that a graph will never meet

Reciprocal: a pair of numbers that multiply together to give 1

Perpendicular: two lines that meet at a right angle

Lines parallel to the axes

R



All the points on this line have a x coordinate of 10

Lines parallel to the y axis take the form $x = a$ and are vertical

Lines parallel to the x axis take the form $y = a$ and are horizontal

All the points on this line have a y coordinate of -2 eg (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

'a' can be ANY positive or negative value including 0

Plotting $y = mx + c$ graphs

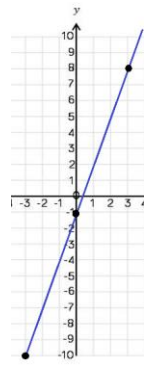
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$y = 3x - 1$ → 3 x the x coordinate then - 1

x	-3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

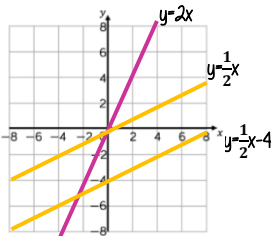
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

Compare Gradients

$y = mx + c$

The coefficient of x (the number in front of x) tells us the gradient of the line



The greater the gradient — the steeper the line

Positive gradients

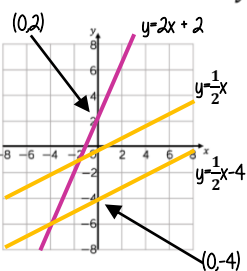
Parallel lines have the same gradient

Negative gradients

Compare Intercepts

$y = mx + c$

The value of c is the point at which the line crosses the y-axis Y intercept



The coordinate of a y intercept will always be (0,c)

Lines with the same y-intercept cross in the same place

$y = mx + c$

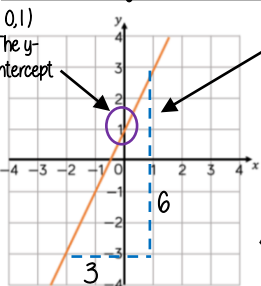
The coefficient of x (the number in front of x) tells us the gradient of the line

$y = mx + c$
y and x are coordinates

The value of c is the point at which the line crosses the y-axis Y intercept

The equation of a line can be rearranged. Eg
 $y = c + mx$
 $c = y - mx$
Identify which coefficient you are identifying or comparing

Find the equation from a graph



The Gradient $\frac{6}{3} = 2$

$y = 2x + 1$

The direction of the line indicates a positive gradient

Positive gradients

Negative gradients

Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

The y-intercept shows the minimum charge.
The gradient represents the price per mile

Direct Proportion graphs

To represent direct proportion the graph must start at the origin

A box of pens costs £2.30

Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

When you have 0 pens this has 0 cost.
The gradient shows the price per pen