YEAR 8 - PROPORTIONAL REASONING...

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Multiplying and Dividing Fractions

What do I need to be able to do?

By the end of this unit you should be able to:

- Carry out any multiplication or division using fractions and integers.
- Solutions can be modelled, described and reasoned

Keywords

Numerator : the number above the line on a fraction. The top number. Represents how many parts are taken
Denominator: the number below the line on a fraction. The number represent the total number of parts..
Whole: a positive number including zero without any decimal or fractional parts.
Commutative: an operation is commutative if changing the order does not change the result.
Unit Fraction: a fraction where the numerator is one and denominator a positive integer.
Non-unit Fraction: a fraction where the numerator is larger than one.
Dividend : the amount you want to divide up.
Divisor: the number that divides another number.
Quotient: the answer after we divide one number by another. e.g. dividend÷ divisor = quotient
Reciprocal: a pair of numbers that multiply together to give 1.

Representing a fraction

Numerator
Denominator

$\frac{3}{5}$

Number of parts represented
Numerator

Number of parts to make up the whole
Denominator

ALL PARTS of a fraction are of equal size

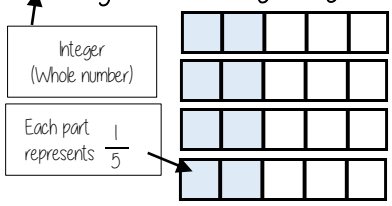
Repeated addition = multiplication by an integer

$4 \times \frac{2}{5} \rightarrow \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$

Integer (Whole number)

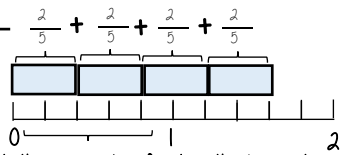
Each part represents $\frac{1}{5}$

How many parts are shaded?
What each part represents



$= \frac{8}{5}$

$= 1\frac{3}{5}$



Each whole is split into the same number of parts as the denominator

Revisit
When adding fractions with the same denominator = add the numerators

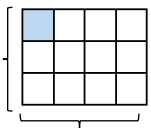
Multiplying unit fractions

$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

Parts shaded

Total number of parts in the diagram

Modelled:



Multiplying non-unit fractions

Shade in 3 parts

Repeat it on this many rows

$\frac{3}{4} \times \frac{2}{3}$

This many columns

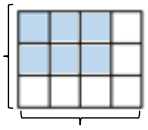
This many rows

$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$

Parts shaded

Total number of parts in the diagram

Modelled:



Quick Multiplying and Cancelling down

$\frac{\cancel{3}^{1}}{5} \times \frac{4}{\cancel{9}_{3}}$

The 3 and the 9 have a common factor and can be simplified

Quick Solving
Multiply the numerators
Multiply the denominators

$\frac{1 \times 4}{5 \times 3} = \frac{4}{15}$

The reciprocal *When you multiply a number by its reciprocal the answer is always 1*

$3 \times \frac{1}{3} = 1$

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

The reciprocal of 3 is $\frac{1}{3}$ and vice versa

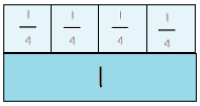
Reciprocals for division

e.g $5 \div \frac{1}{4} = 20$

$5 \times 4 = 20$

Multiplying by a reciprocal gives the same outcome

Dividing an integer by an unit fraction



$1 \div \frac{1}{4} = 4$

How many quarters are in 1?

"There are 4 quarters in 1 whole. Therefore, there are 20 quarters in 5 wholes"

$5 \div \frac{1}{4} = 20$

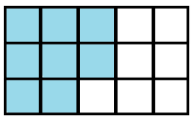
Dividing any fractions *Remember to use reciprocals*

$\frac{2}{5} \div \frac{3}{4}$

$\frac{2}{5} \times \frac{4}{3}$

Multiplying by a reciprocal gives the same outcome

Represented



$= \frac{8}{15}$

YEAR 8 - PROPORTIONAL REASONING...

Multiplicative Change

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What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems and explain direct proportion
- Use conversion graphs to make statements, comparisons and form conclusions.
- Understand and use scale factors for length

Keywords

Proportion: a statement that links two ratios
Variable: a part that the value can be changed
Axes: horizontal and vertical lines that a graph is plotted around
Approximation: an estimate for a value
Scale Factor: the multiple that increases/ decreases a shape in size
Currency: the system of money used in a particular country
Conversion: the process of changing one variable to another
Scale: the comparison of something drawn to its actual size.

Direct Proportion

As one variable changes the other changes at the same rate

This is a multiplicative change

4 cans of pop = £2.40

4 cans of pop = £2.40 → (x 0.5) → 2 cans of pop = £1.20

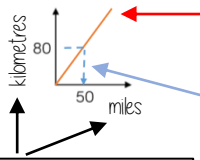
4 cans of pop = £2.40 → (x 3) → 12 cans of pop = £7.20

This multiplier is the same in the same way that this would be for ratio

Sometimes this is easiest if you work out how much one unit is worth first
e.g. 1 can of pop = £0.60

Conversion Graphs

Compare two variables



This is always a straight line because as one variable increases so does the other at the same rate

To make conversions between units you need to find the point to compare – then find the associated point by using your graph.
Using a ruler helps for accuracy
Showing your conversion lines help as a "check" for solutions

Labelling of both axes is vital

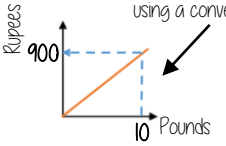
Conversion between currencies

£1 = 90 Rupees ← Currency is directly proportional

For every £1 I have 90 Rupees

£1 = 90 Rupees → (x 10) → £10 = 900 Rupees

Currency can be converted using a conversion graph



Convert 630 Rupees into Pounds

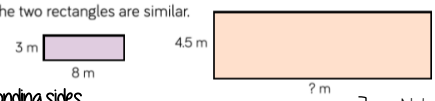
£1 = 90 Rupees → (x 7) → £7 = 630 Rupees

630 ÷ 90 = 7

Ratio between similar shapes

Angles in similar shapes do not change
e.g. if a triangle gets bigger the angles can not go above 180°

The two rectangles are similar.



Corresponding sides

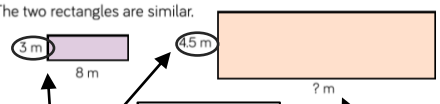
3m : 4.5m → (÷ 3) → 1 m : 1.5 m

8m : 12m → (x 8) → 1 m : 1.5 m

Note
Simplify to the same ratio

Understand Scale Factor

The two rectangles are similar.



3 x 1.5 = 4.5

This is a multiplicative change

Missing length
8 x 1.5 = 12m

Use **corresponding** sides to calculate a scale factor

Scale factor can also be calculated by:

Bigger corresponding side
Smaller corresponding side

Small corresponding side → (x SF) → Big corresponding side

Big corresponding side → (÷ SF) → Small corresponding side

Draw and interpret scale diagrams

A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

The car image is 10cm

Image	Real life
1cm	30cm
10cm	300cm

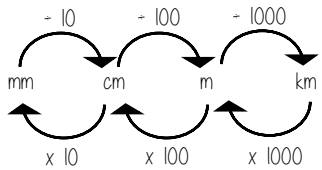
(x 10 / x 10)

The car in real life is 210cm

Image	Real life
1cm	30cm
7cm	210 cm

(x 7 / x 7)

Interpret maps with scale factors



1 cm : 250 m

Ratios need to be in the same units

1 cm : 250m

1 cm : 25000cm

250 x 100 = 25000

For every 1cm on my map is 25000cm in real life.

YEAR 8 - PROPORTIONAL REASONING...

Ratio and Scale

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What do I need to be able to do?

By the end of this unit you should be able to:

- Simplify any given ratio
- Share an amount in a given ratio
- Solve ratio problems given a part

Solutions should be modelled, explained and solved.

Keywords

Ratio: a statement of how two numbers compare
Equal Parts:: all parts in the same proportion, or a whole shared equally
Proportion: a statement that links two ratios
Order: to place a number in a determined sequence
Part: a section of a whole
Equivalent: of equal value
Factors: integers that multiply together to get the original value
Scale: the comparison of something drawn to its actual size.

Representing a ratio

"For every 5 boys there are 3 girls"

5:3

This is the "whole" – boys and girls together

This represents the 5 boys | This represents the 3 girls

Double Number Line

This represents the 5 boys

This is the "whole" – boys and girls together

This represents the 3 girls

Order is Important

"For every dog there are 2 cats"

Dogs: Cats

1:2

The ratio has to be written in the same order as the information is given.

e.g. 2:1 would represent 2 dogs for every 1 cat ✗

Simplifying a ratio

Cancel down the ratio to its lowest form

"For every 6 days of rain there are 4 days of sun"

6:4

÷ by 2

3:2

rain | sun

Find the biggest common factor that goes into all parts of the ratio

For **6** and **4** the biggest factor (number that multiples into them is **2**)

"For every 3 days of rain there are 2 days of sun" – when this happens twice the ratio becomes 6:4

Ratio 1:n (or n:1)

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

The question states that **this part has to be** 1 unit. Therefore **Divide by 4**

4 : 20

1 : 5

This side has to be divided by 4 too – to keep in **proportion**

H the n part does not have to be an integer for this type of question

Units are important:

When using a ratio – all parts should be in the same units

Useful Conversions

mm ⇄ cm ⇄ m ⇄ km (×10, ×100, ×1000 / ÷10, ÷100, ÷1000)

g ⇄ kg (×1000 / ÷1000)

ml ⇄ L (×1000 / ÷1000)

Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4. Work out how much each person earns

Model the Question

James: Lucy

3 : 4

James | Lucy | £350

Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

£350 ÷ 7 = £50

□ = one part = £50

Put back into the question

James: Lucy

3 : 4 (×50)

£150 : £200

James = 3 x £50 = £150

Lucy = 4 x £50 = £200

Finding a value given 1:n (or n:1)

Inside a box are blue and red pens in the ratio 5:1. If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red

5 : 1

□ = one part = 10 pens

Blue pens | Red pens

One unit = 10 pens

Put back into the question

Blue : Red

5 : 1 (×10)

50 : 10

Blue pens = 5 x 10 = 50 pens

Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens

Ratio as a fraction

Trees: Flowers

3 : 7

Trees | Flowers

Ratio

There are **3** parts for trees

Fraction of trees

Number of parts of in group / Total number of parts = 3/10

Fraction

Tree parts **3** + Flower parts **7** = **10**

Pi

Circumference | Diameter

The ratio of a circles circumference to its diameter