YEAR 7 – ALGEBRAIC THINKING...

Equality and Equivalence

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What do I need to be able to do?

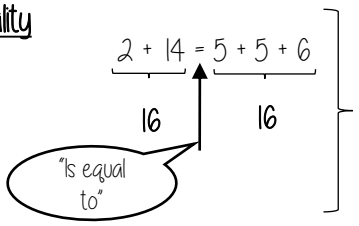
By the end of this unit you should be able to:

- Form and solve linear equations
- Understand like and unlike terms
- Simplify algebraic expressions

Keywords

Equality: two expressions that have the same value
Equation: a mathematical statement that two things are equal
Equals: represented by '=' symbol – means the same
Solution: the set or value that satisfies the equation
Solve: to find the solution
Inverse: the operation that undoes what was done by the previous operation. (The opposite operation)
Term: a single number or variable
Like: variables that are the same are 'like'
Coefficient: a multiplicative factor in front of a variable e.g. 5x (5 is the coefficient, x is the variable)
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

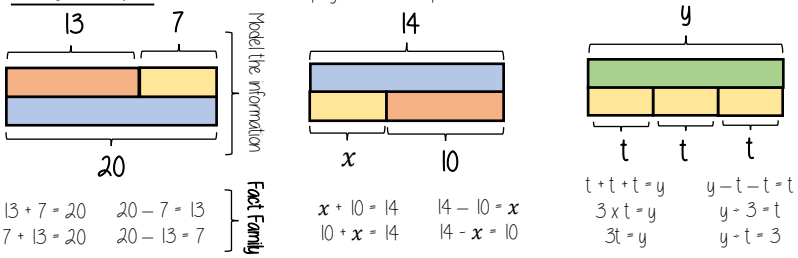
Equality



Saying it out loud sometimes helps you to understand equality

Fact Families

Use a bar model to display the relationships between terms and numbers



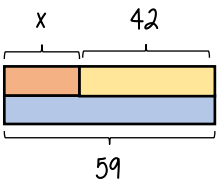
Solve one step equations (+/-)

There is more to this than just spotting the answer

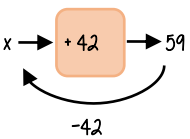
x + 42 = 59

x + 42 = 59
42 + x = 59

59 – x = 42
59 – 42 = x



Don't forget you know how to use function machines

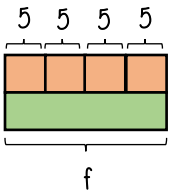


Solve one step equations (x/÷)

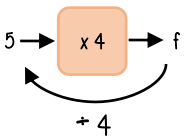
$\frac{f}{4} = 5$

f ÷ 4 = 5
f ÷ 5 = 4

5 x 4 = f
4 x 5 = f

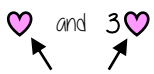


Don't forget you know how to use function machines



Like and unlike terms

Like terms are those whose variables are he same



are **like** terms

are **unlike** terms

Examples and non-examples

Like terms	Un-like terms
y, 7y	y, 7x
$2x^2$, x^2	$2x^2$, $2c^2$
ab, 10ba	ab, 10a
5, -2	5, -2t

Note here ab and ba are commutative operations, so are still like terms

Equivalence

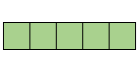
Check equivalence by substitution
e.g. m= 10

5m	2 x 2m	7m – 3m
5 x 10	2 x (2x 10)	(7x 10) – (3x 10)
= 50	= 2 x 20	= 70 – 30
	= 40	= 40

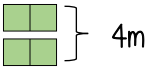
Equivalent expressions

Repeat this with various values for m to check

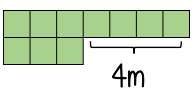
5m



2 x 2m



7m – 3m

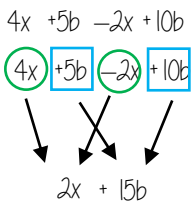


Collecting like terms ≡ symbol

The ≡ symbol means equivalent to.
It is used to identify equivalent expressions

Collecting like terms

Only **like terms** can be combined



Common misconceptions

$2x + 3x^2 + 4x \equiv 6x + 3x^2$

Although they both have the x variable, x2 and x terms are un-like terms so can not be collected

YEAR 7 – ALGEBRAIC THINKING...

Algebraic notation

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What do I need to be able to do?

By the end of this unit you should be able to:

- Be able to use inverse operations and "operation families".
- Be able to substitute into single and two step function machines.
- Find functions from expressions.
- Form sequences from expressions
- Represent functions graphically.

Keywords

Function: a relationship that instructs how to get from an input to an output.
Input: the number/ symbol put into a function.
Output: the number/ expression that comes out of a function.
Operation: a mathematical process
Inverse: the operation that undoes what was done by the previous operation. (The opposite operation)
Commutative: the order of the operations do not matter.
Substitute: replace one variable with a number or new variable.
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
Evaluate: work out
Linear: the difference between terms increases or decreases by the same value each time
Sequence: items or numbers put in a pre-decided order

Single function machines

INPUT → [ ] → OUTPUT

The number that goes IN — The number that comes out

This box gives the calculation instruction

To find the input from the output
Use the **INVERSE** operation

Using letters to represent numbers

5 + 5 + 5	y + y + y + y	20 ÷ h
3 x 5	y x 4	$\frac{20}{h}$
5 x 3	4 x y	
	4y	

Addition and multiplication can be done in any order
Commutative calculations

4 lots of 'y'

20 shared into 'h' number of groups

Single function machines (algebra)

INPUT → [x 10] → OUTPUT

a → 10a
3c → 30c

÷ 10

To find the input from the output
Use the **INVERSE** operation

Find functions from expressions

INPUT 7x → [?] → OUTPUT 14x

Find the relationship between the input and the output

Sometimes there can be a number of possible functions e.g. **+7x** or **x 2** could both be solutions to the above function machine

Substitution into expressions

4y ← 4 lots of 'y'

If y = 7 this means the expression is asking for 4 'lots of' 7

4 x 7 **OR** 7 + 7 + 7 + 7 **OR** 7 x 4 = 28

e.g. $y - 2$
$= 7 - 2 = 5$

Two step function machines

INPUT → [ ] → [ ] → OUTPUT

Calculate the value at the end of each operation

For the input use the **INVERSE** operations

Two step function machines (algebra)

b → [x 5] → 5b → [+ 4] → 5b + 4
[+ 5] [- 4]

IMPORTANT
Calculate the value at the end of each operation

c → [+ 4] → c+4 → [x 5] → 5(c + 4) = 5c + 20
[- 4] [÷ 5]

NOTE:
The whole first output is multiplied by 5

Find functions from expressions

f → [+ 5] → [÷ 3] → $\frac{f + 5}{3}$

f add 5 then divide by 3

f → [÷ 3] → [+ 5] → $\frac{f}{3} + 5$

f divided by then add 5

NOTE: the difference in the two expressions

Sometimes it helps to try to explain the expression in word – and consider what has happened to the input

Substitution into an expression

$2(x + 3)$

Add 3 to the input then times 2

Put the expression into a function machine

INPUT → [+ 3] → [x 2] → OUTPUT

If x = 10
10 + 3 = 13.... 13 x 2 = 26

Forming a sequence

$2(x + 3)$

INPUT	1	2	3
OUTPUT	8	10	12

← The substitution is the 'input' value
The OUTPUT becomes the sequence

Representing functions graphically

Take the function and generate a sequence $2(x + 3)$

INPUT → [+ 3] → [x 2] → OUTPUT

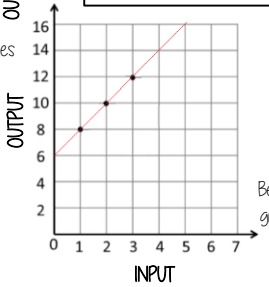
To represent graphically the input becomes x co-ordinates and the output becomes y co-ordinates

$y=2(x + 3)$

INPUT (x)	1	2	3
OUTPUT (y)	8	10	12

This becomes a co-ordinate pair (2, 10) to plot on a graph

Not all graphs will be linear only those with an integer value for x.
Powers and fractions generate differently shaped graphs.



NOTE:
Because this is a linear graph you can predict other values

YEAR 7 – ALGEBRAIC THINKING...

Sequences

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What do I need to be able to do?

By the end of this unit you should be able to:

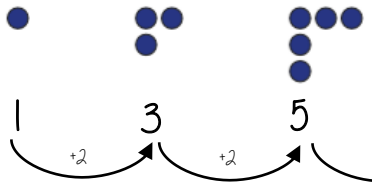
- Describe and continue both linear and non-linear sequences
- Explain term to term rules for linear sequence
- Find missing terms in a linear sequence

Keywords

Sequence: items or numbers put in a pre-decided order
Term: a single number or variable
Position: the place something is located
Rule: instructions that relate two variables
Linear: the difference between terms increases or decreases by the same value each time
Non-linear: the difference between terms increases or decreases in different amounts
Difference: the gap between two terms
Arithmetic: a sequence where the difference between the terms is constant
Geometric: a sequence where each term is found by multiplying the previous one by a fixed non zero number

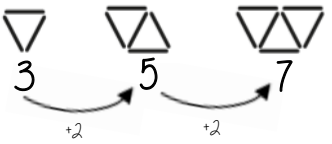
Describe and continue a sequence diagrammatically

Count the number of circles or lines in each image



What will the next number be? Can you draw this?

Predict and check terms



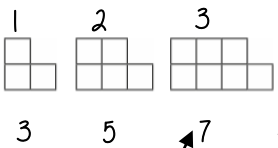
CHECK – draw the next terms

Predictions:
Look at your pattern and consider how it will increase.

e.g. How many lines in pattern 6?
Prediction - 13
If it is increasing by 2 each time - in 3 more patterns there will be 6 more lines

Sequence in a table and graphically

Position: the place in the sequence



"The **term** in **position** 3 has 7 squares"

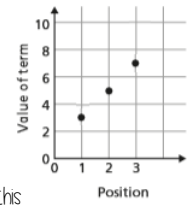
Term: the number or variable
(the number of squares in each image)

In a table

Position	1	2	3
Term	3	5	7

+2 +2

Graphically



Because the terms increase by the same addition each time this is **linear** – as seen in the graph

Linear and Non Linear Sequences

Linear Sequences – increase by addition or subtraction and the same amount each time
Non-linear Sequences – do not increase by a constant amount – quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division.

Fibonacci Sequence – look out for this type of sequence

0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms.

Continue Linear Sequences

7, 11, 15, 19...

How do I know this is a linear sequence?
It increases by adding 4 to each term.
How many terms do I need to make this conclusion?
At least 4 terms – two terms only shows one difference not if this difference is constant (a common difference).
How do I continue the sequence?
You continue to repeat the same difference through the next positions in the sequence.

Continue non-linear Sequences

1, 2, 4, 8, 16 ...

How do I know this is a non-linear sequence?
It increases by multiplying the previous term by 2 – this is a geometric sequence because the constant is multiply by 2
How many terms do I need to make this conclusion?
At least 4 terms – two terms only shows one difference not if this difference is constant (a common difference).
How do I continue the sequence?
You continue to repeat the same difference through the next positions in the sequence

Explain term-to-term rule

How you get from term to term

Try to explain this in full sentences not just with mathematical notation
Use key maths language – doubles, halves, multiply by two, add four to the previous term etc.

To explain a whole sequence you need to include a term to begin at...

The next term is found by tripling the previous term. The sequence begins at 4.

4, 12, 36, 108...
x3 x3 x3
First term