Year 7 Maths guide

Introduction

This is a guide of all the maths we cover in year 7 to help with any homework or revision questions that may arise.

These are all the topics that we will be covering over the year

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Algebraic Thinking**						**Place Value and Proportion**					
	Sequences		Understand and use algebraic notation		Equality and equivalence		Place value and ordering integers and decimals			Fraction, decimal and percentage equivalence		
Spring	**Applications of Number**						**Directed Number**			**Fractional Thinking**		
	Solving problems with addition & subtraction		Solving problems with multiplication and division			Fractions & percentages of amounts	Four operations with directed number			Addition and subtraction of fractions		
Summer	**Lines and Angles**						**Reasoning with Number**					
	Constructing, measuring and using geometric notation			Developing geometric reasoning			Developing number sense		Sets and probability		Prime numbers and proof	

Algebraic Thinking

Sequences

Arithmetic Sequences

Term to term rule

Term to term rules

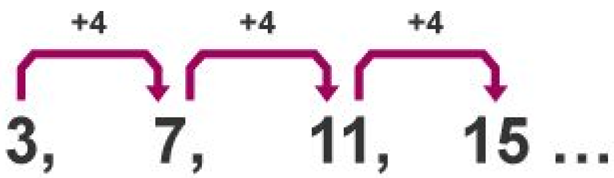
The **term to term rule** of a sequence describes how to get from one term to the next.

Example 1

Write down the term to term rule and then work out the next two **terms** in the following sequence.

3, 7, 11, 15, ...

Firstly, work out the **difference** in the terms.



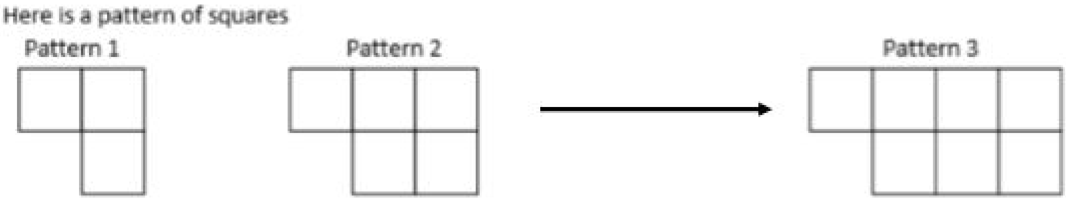
This sequence is going up by four each time, so add 4 on to the last term to find the next term in the sequence.

3, 7, 11, 15, **19**, **23**, ...

Arithmetic with pictures

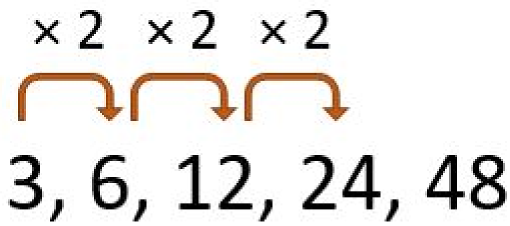
It’s the same principle as the one before we just represent it in a picture form rather than a number

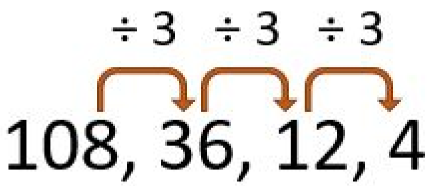
1. Here is a pattern of squares



Geometric Sequences

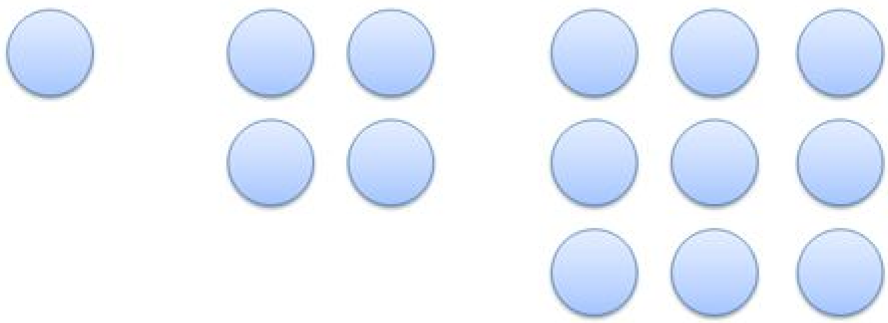
In a $geometric$ sequence, the term to term rule is to multiply or divide by the same value.





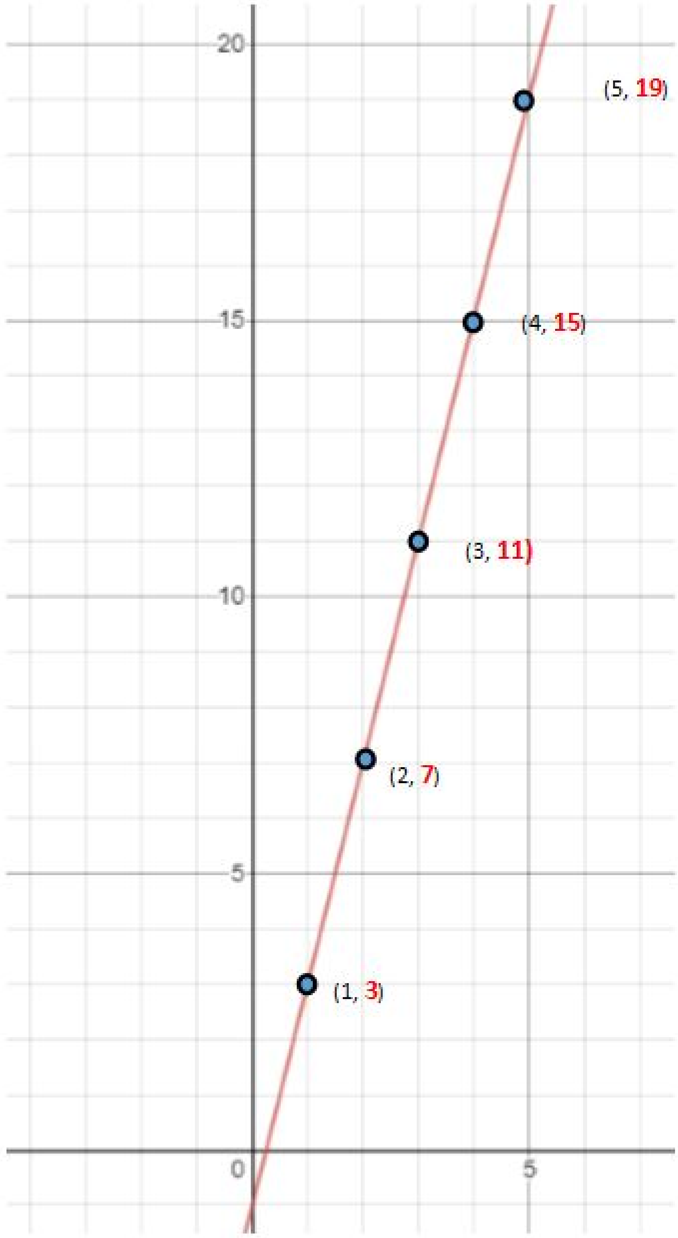
Geometric with pictures

This is the same as the previous section but we represent it with pictures



Sequences represented as graphs

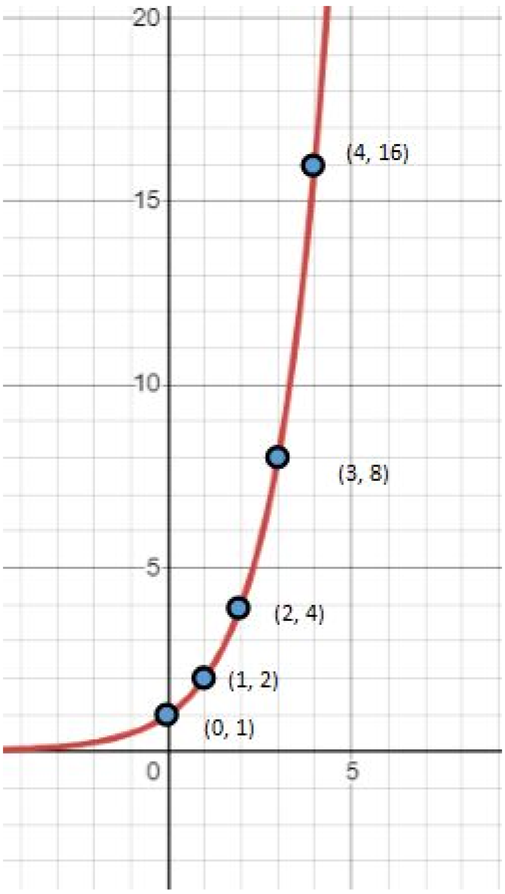
If we take the sequence 3, 7, 11, 15, 19 and plot it on a graph we would get this



Our first term (number in the sequence) is 3 so its plotted at (1,3) our second term is 7 so its plotted at (2,7)

Geometric Sequences

If we have the sequence 1, 2, 4, 8, 16 and we plotted it we would get the following



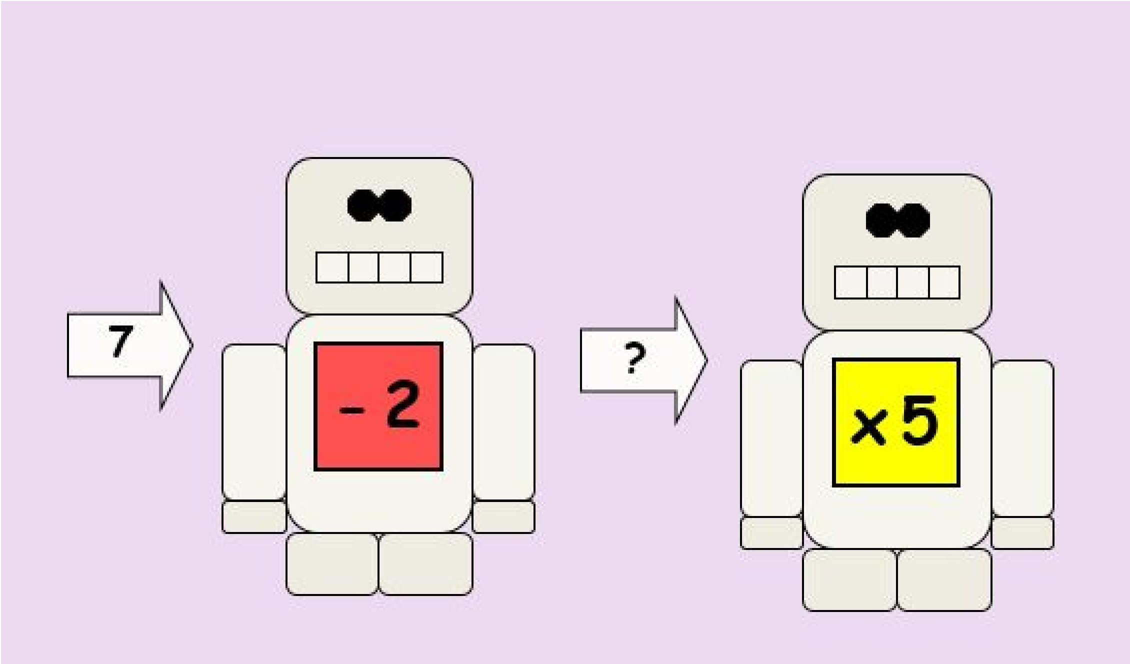
Arithmetic produces a straight line and geometric produces a curve

Understand and use algebraic notation

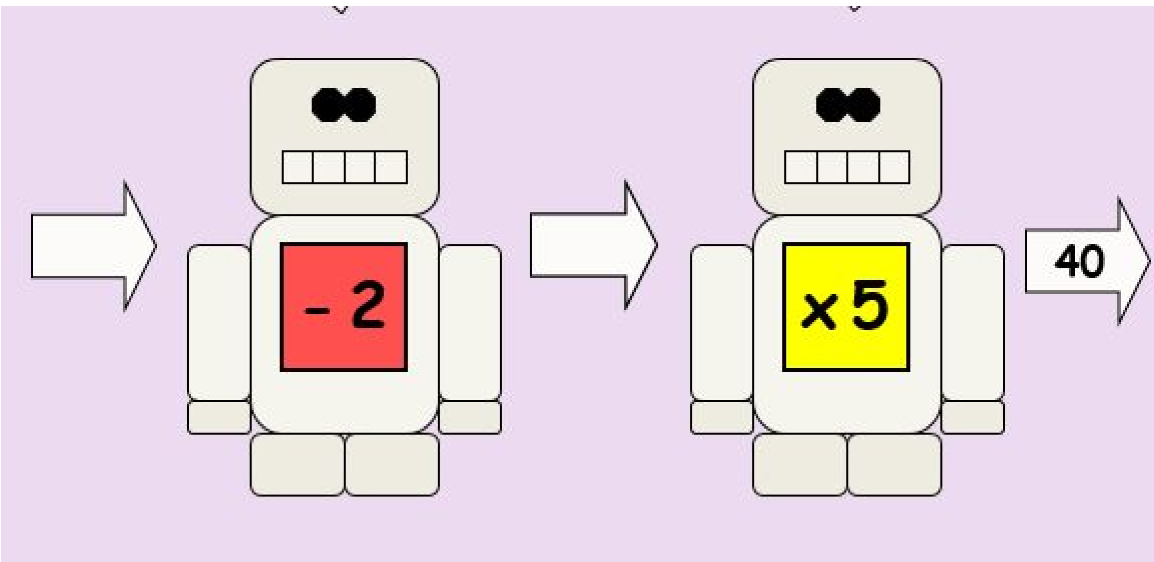
Function machines

Function machines work by putting a number into a “machine” and a process happening and then you get another number out

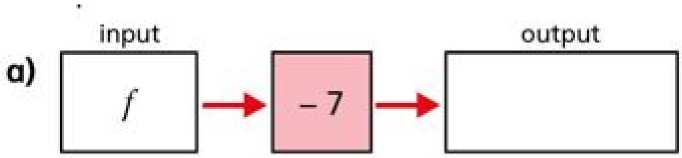
In the diagram below we are putting 7 in. it then goes through a machine that takes away 2. This gives us 5. This new number now goes into the second machine and gets multiplied by 5 to give us 25



We can also be given the final answer and be asked for the input number. For the example below we are told 40 is our final answer. So we put it backwards through the yellow machine which also reverses the operation. This means we need to divide by 5 to give 8 and then the 8 goes into the red machine and we reverse that operation to add 2 giving us 10.



We also use them with algebra



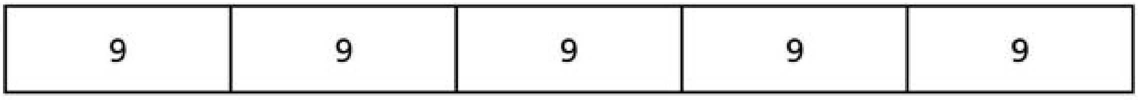
If we put f in and then take 7 away we would get f-7

Bar model

The bar model is a new method used to help students visualise the question.

Looking at the diagram below it is showing that 5x9 is the same as 9+9+9+9+9

d)

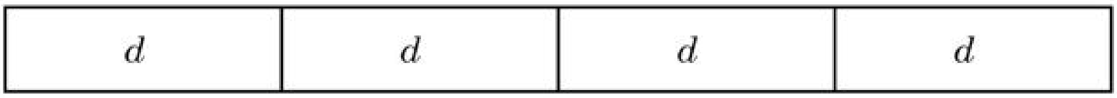


$9 + 9 + 9 + 9 + 9 = \square \times \square = \square \times \square$

The bar model is used a lot in algebra to help students understand the basic concepts.

Just the same as the number example above it can be used in the same way with letters

f)



$d + d + d + d = \square \times \square = \square \times \square$

This is showing that we would write this as 4 x d or 4d

Substitution

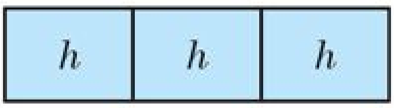
Substitution is where we swap a letter for a number. We can do this with or without the bar model.

Using the bar model we would replace all the h's with 7's and then add them

Substitute $h = 7$ into each of these expressions.

Use the bar models to help you.

a) $3h = \square$



7+7+7 = 21

We can also do it without using the bar model. p=8

c) $4p = \square$ **d)** $\frac{p}{2} = \square$

For the first one we need to multiply p by 4 and as p is 8 we do 8x4 = 32. For the second it means p divided by 2 so we do 8÷2 = 4

Generating a sequence

To find numbers in a sequence we have to substitute numbers into the expression. If we wanted to find the first number in the sequence we would substitute in 1. If we wanted to find the 10th number we would substitute in 10.

Find the first three terms of these sequences by substituting $n = 1$, $n = 2$ and $n = 3$ into them.

n	1	2	3
$2n + 3$			

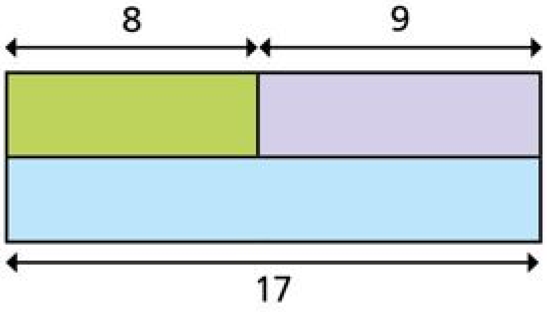
Our first number would be 2 x 1 + 3 = 5
Our second number would be 2 x 2 + 3 = 7
Our third number would be 2 x 3 + 3 = 9

Equality and Equivalence

Fact families

Fact families are a list of equations that are all true for the same scenario.

Write the fact families for these bar models.



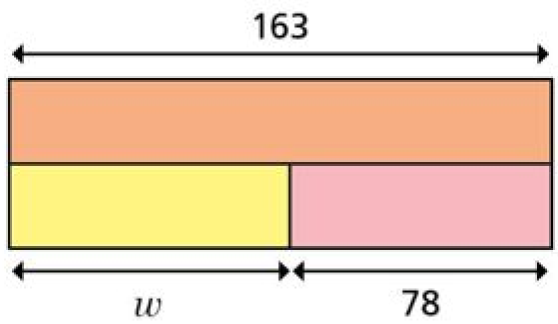
8+9=17
9+8=17

17-8=9
17-9 = 8

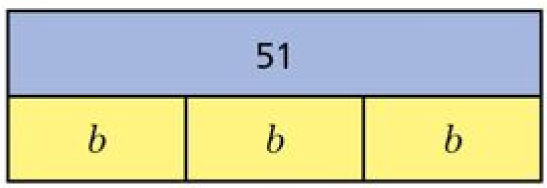
There are all the different ways of writing the same calculation

Solving equations

Solving equations can either be done with the bar model or without. The first example is using the bar model



Visually you can see that w is equal to 163 - 78 so therefore w= 65



Here 3b’s = 51 so to find b we have to divide by 3. b=17

The other method is the balancing method

W+78 = 163
We want w to be alone so we have to -78 from both sides to make that happen

W+78 = 163
-78 -78
w=65

a) $\frac{n}{4} = 12$

This means n is being divided by 4 so the reverse operation is multiplying by 4.

a) $\frac{n}{4} = 12$

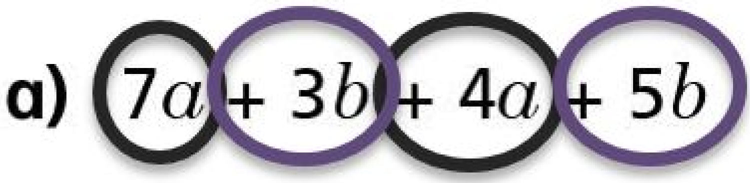
X4 x4

n=48

Simplifying expressions

a) $7a + 3b + 4a + 5b$

First we need to group the like terms (the ones with the same letters)



Once we have done that we add the a’s together and then the b’s

11a+8b

Place value and proportion

Place value and ordering

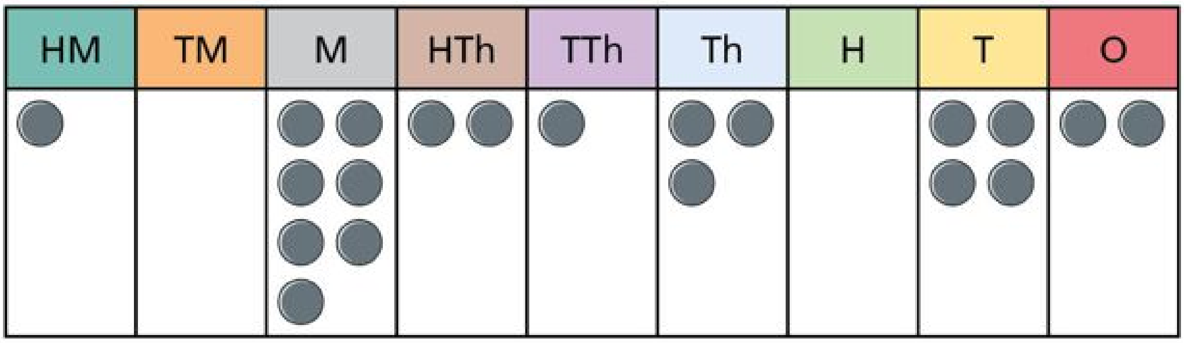
Place Value

We use this table method to read and write number

HM	TM	M	HTh	TTh	Th	H	T	O

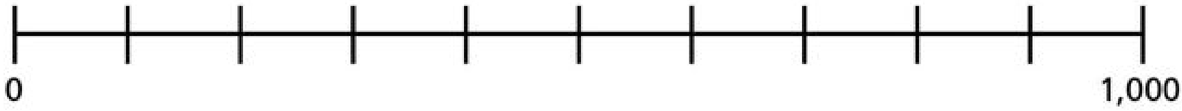
HM = Hundred Million, T M = Thousand million, M= Million, HTh= Hundred thousand, TTh = ten thousand, Th= thousand, H=Hundred, T= Tens and O = Ones

This can be used with numbers written inside or counter representation as shown below

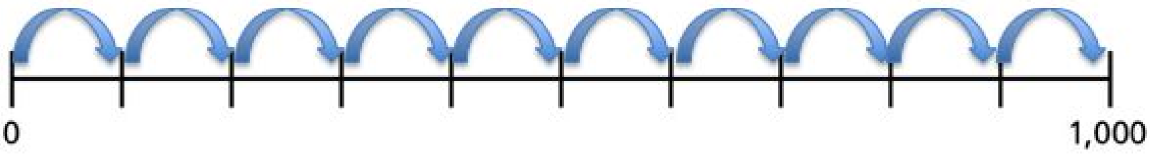


Scales

Students will need to read off a scale and work out what the scale goes up in.



We need to look at the amount of jumps between 0 and 1000



We can see there are 10 jumps from 0-1000 so we need to divide 1000 by 10. Each jump goes up by 100.

Rounding

Rounding to the nearest 1,000

What is $1,484.5$ to the nearest $1,000$?

It is between $1,000$ and $2,000$, but it is closer to $1,000$, so round down.

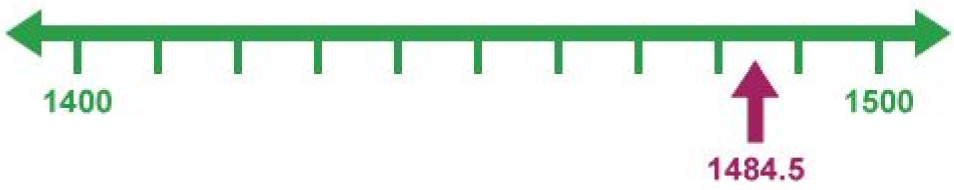
$1,484.5$ rounded to the nearest thousand is $1,000$.

Rounding to the nearest 100

What is $1,484.5$ to the nearest 100?

$1,484.5$ is between $1,400$ and $1,500$, but it is closer to $1,500$, so round up.

$1,484.5$ rounded to the nearest hundred is $1,500$.



Rounding to the nearest 10

$1,484.5$ lies between $1,480$ and $1,490$, but it is closer to $1,480$, so round down.

$1,484.5$ rounded to the nearest ten is $1,480$.

Rounding to the nearest whole number

$1,484.5$ lies between $1,484$ and $1,485$ and it is exactly halfway between them. In this situation round up.

$1,484.5$ rounded to the nearest whole number is $1,485$.

Inequalities

In an equation the ' =' sign means the two sides are identical. But what happens when the two sides are **not** identical?

If this is the case you need to use inequalities to show the relationship between the two sides.

- $<$ means **'less than'**
- $\leq$ means **'less than or equal to'**
- $>$ means **'greater than'**
- $\geq$ means **'greater than or equal to'**

Eg:

978 $<$ 1,111

Range

Range is the biggest number in the list minus the smallest number in the list.

For example if we were given the numbers below. We would first need to find the biggest and smallest number

6, 8, 2, 1, 6, 4, 3, 1

Biggest is 8. Smallest is 1. 8-1=7
So my range is 7

Median

The median is the middle number of a list of numbers when they are put in order of size.

For example if we were given the following numbers the first thing we would have to do is put them in order

4, 9, 1, 2, 12

1,2,4,9,12

We would then need to cross out the biggest and smallest

~~1~~,2,4,9,~~12~~

And keep doing that until there was one (or two if there are an even amount of numbers) in the middle

~~1~~,~~2~~,4,~~9~~,~~12~~

If there was an even amount of numbers and you were left with 2 numbers in the middle

~~1~~, ~~2~~, ~~3~~, 5, 7, ~~10~~, ~~11~~, ~~12~~

You would need to find the number in the middle of those two numbers and in this case it would be 6.

Rounding to significant figures

The method of **rounding to a significant figure** is often used as it can be applied to any kind of number, regardless of how big or small it is. When a newspaper reports a lottery winner has won £3 million, this has been rounded to one significant figure. It rounds to the most important figure in the number.

To round to a significant figure:

1. look at the first non-zero digit if rounding to one significant figure
2. look at the digit after the first non-zero digit if rounding to two significant figures
3. draw a vertical line after the place value digit that is required
4. look at the next digit
5. if it's **5 or more**, increase the previous digit by one
6. if it's **4 or less**, keep the previous digit the same
7. fill any spaces to the **right** of the line with zeros, stopping at the decimal point if there is one

Examples

Round 53,879 to 1 significant figure, then 2 significant figures.

- 5|**3**879 to 1 significant figure is 50,000
- 53|**8**79 to 2 significant figures is 54,000

Round **50 790** to 2 significant figures.

Answer

Find the second significant figure from the number above:

5**0** 790

This is the zero between the 5 and the 7.

As the digit to the right is a 7, then it goes up by one, which changes the 0 to a 1.

Therefore 50 790 rounds to **51 000** to 2 significant figures.

Now try the example questions below.

Powers

The notation 3^2 and 2^3 is known as **index form**. The small digit is called the index number or **power**.

You have already seen that $3^2 = 3 \times 3 = 9$ and that $2^3 = 2 \times 2 \times 2 = 8$.

Similarly, 5^4 (five to the power of 4) = $5 \times 5 \times 5 \times 5 = 625$

and 3^5 (three to the power of 5) = $3 \times 3 \times 3 \times 3 \times 3 = 243$.

The index number tells you how many times the number should be multiplied.

- When the index number is two, the number has been **squared**.
- When the index number is three, the number has been **cubed**.
- When the index number is greater than three you say that it has been multiplied **to the power of**.

For example:

7^2 is 'seven squared'.

3^3 is 'three cubed'.

3^7 is 'three to the power of seven'.

4^5 is 'four to the power of five'.

Standard form

This is a way we write really big or really small numbers to avoid missing out zeros when we are doing calculations

Powers of 10

Standard form uses the fact that the decimal place value system is based on powers of 10:

$10^0 = 1$

$10^1 = 10$

$10^2 = 100$

$10^3 = 1000$

$10^4 = 10000$

$10^5 = 100000$

$10^6 = 1000000$

Large numbers

Example

Write 50,000 in standard form.

50,000 can be written as: $5 \times 10,000$

$10,000 = 10 \times 10 \times 10 \times 10 = 10^4$

So: $50,000 = 5 \times 10^4$

It is useful to look at patterns to try to understand negative indices:

$10^0 = 1$

$10^{-1} = 0.1$

$10^{-2} = 0.01$

$10^{-3} = 0.001$

$10^{-4} = 0.0001$

$10^{-5} = 0.00001$

$10^{-6} = 0.000001$

A negative power does not mean that the number is negative. It means that we have gone from multiplying by 10 to dividing by 10.

Example

Write 0.0005 in standard form.

0.0005 can be written as 5×0.0001.

$0.0001 = 10^{-4}$

So $0.0005 = 5 \times 10^{-4}$

Examples

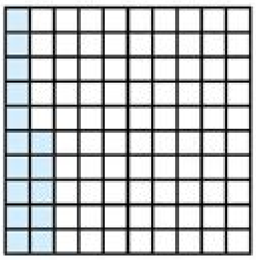
1.34×10^3 is 1,340, since $1.34 \times 10 \times 10 \times 10 = 1,340$.

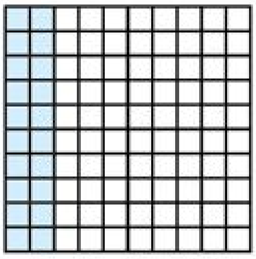
4.78×10^{-3} is 0.00478, as $4.78 \times 0.001 = 0.00478$.

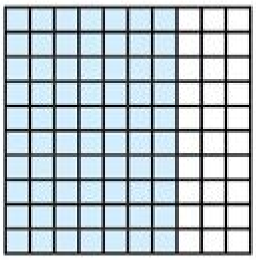
Fraction, decimal and percentage equivalence

Tenths and hundredths as diagrams

We can show 10ths and 100ths using a diagram.







The first picture shows 15 hundreths, the second shows 20 hundredths which we can simplify down to 2 tenths and the last show 70 hundredths which can be simplified down to 7 tenths.

Fractions to decimals

Yet another method you may like is to follow these steps:

- **Step 1**: Find a number you can multiply by **the bottom of the fraction** to make it 10, or 100, or 1000, or any 1 followed by 0s.
- **Step 2**: Multiply both top and bottom by that number.
- **Step 3**. Then write down just the top number, putting the decimal point in the correct spot (one space from the right hand side for every zero in the bottom number)

Example: Convert $\frac{3}{4}$ to a Decimal

Step 1: We can multiply 4 by 25 to become 100

Step 2: Multiply top and bottom by 25:

×25

$$\frac{3}{4} = \frac{75}{100}$$

×25

Step 3: Write down 75 with the decimal point 2 spaces from the right (because 100 has 2 zeros);

Answer = 0.75

Decimals to fractions

Example: Convert 0.75 to a fraction

Step 1: Write down 0.75 divided by 1:

$$\frac{0.75}{1}$$

Step 2: Multiply both top and bottom by **100** (because there are 2 digits after the decimal point so that is 10×10=100):

× 100

$$\frac{0.75}{1} = \frac{75}{100}$$

× 100

(Do you see how it turns the top number into a whole number?)

Step 3: Simplify the fraction (this took me two steps):

÷5 ÷ 5

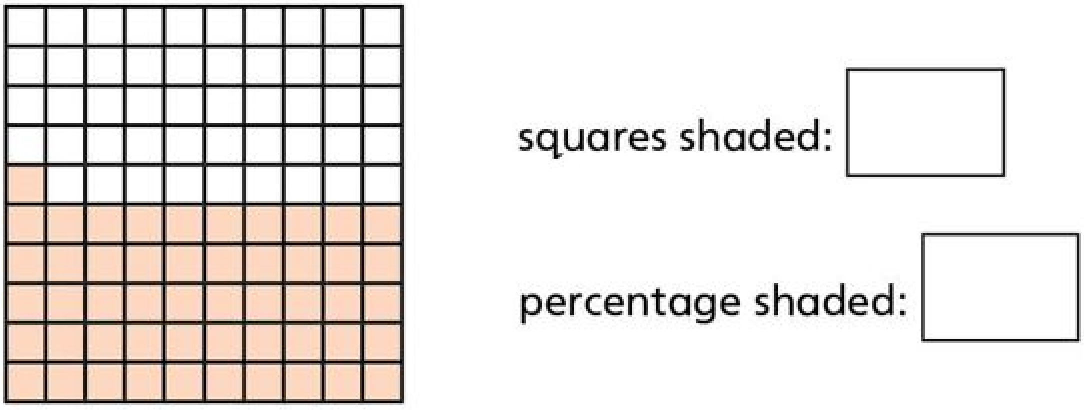
$$\frac{75}{100} = \frac{15}{20} = \frac{3}{4}$$

÷5 ÷ 5

Answer = $\frac{3}{4}$

Percentages

We can use 100 square to help with finding a percentage. Percentage means out of 100 so if we shade blocks in 100 grid it will give us a percentage.



There are 51 boxes shaded out of a possible 100 so the percentage shaded is 51%

Decimals to percentages

Example: Convert 0.125 to percent

Multiply 0.125 by 100:

$$0.125 \times 100 = 12.5$$

Put the percent sign:

$$0.125 \times 100 = \mathbf{12.5\%}$$

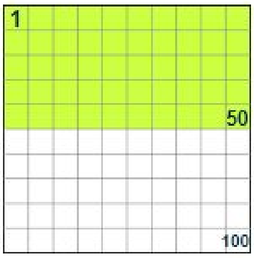
Answer **0.125 = 12.5%**

The easy way to multiply by 100 is to **move the decimal point 2 places to the right**. so:

From Decimal		To Percent	
0.125	0.1.2.5 (2 Places)	12.5%	move the decimal point **2 places to the right** (and add the "%" sign!)

Percentage to decimal

Converting From Percent to Decimal



Percent means "per 100", so **50%** means 50 per 100, or simply $^{50}/_{100}$

When we divide 50 by 100 we get **0.5** (a decimal number).

So, to convert from percent to decimal: divide by 100, and remove the "%" sign.

The Easy Way

The easy way to divide by 100 is to **move the decimal point 2 places to the left**, so:

From Percent		To Decimal
75%	0.7.5. 2 Places	0.75
move the decimal point 2 places to the left, and remove the "%" sign		

Example: Convert 8.5% to decimal

Move the decimal point two places to the left: 8.5 → 0.85 → 0.085

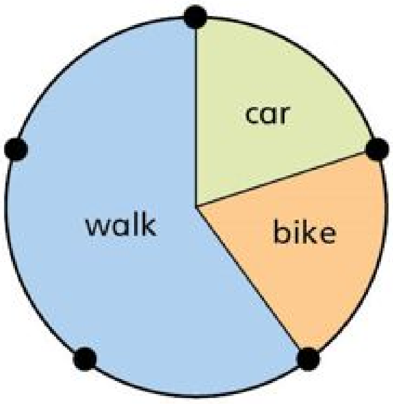
(Note how we inserted an extra "0" as needed)

Answer **8.5% = 0.085**

Pie Charts

We look at pie charts in relation to fractions, decimals and percentages. Looking at the example below we need to first workout how many equal sections the pie chart can be broken down into.

The pie chart shows how students travel to school.



a) What percentage of students travelled to school by car?

b) What proportion of children walk to school?

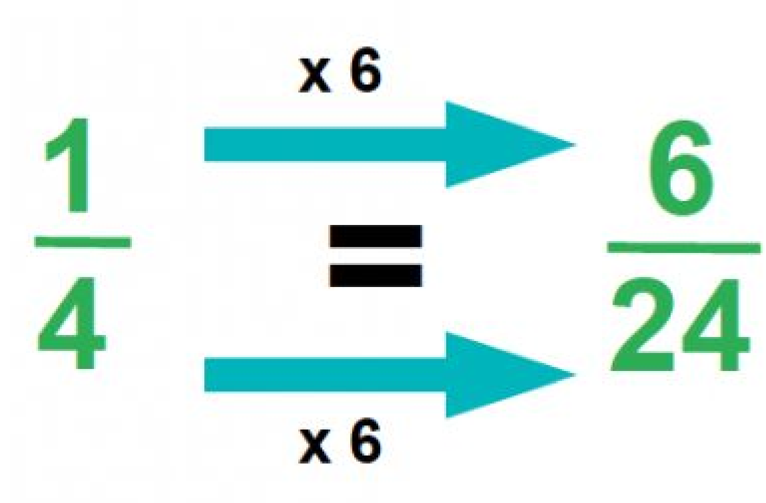
Give your answer as a fraction, decimal and percentage.

fraction = ☐ decimal = ☐ percentage = ☐

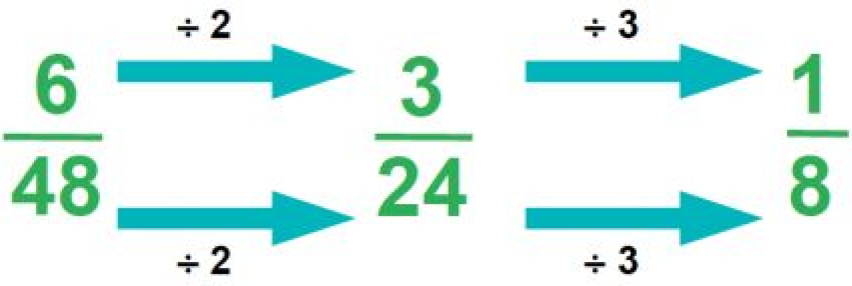
The pie chart can be broken into 5 equal parts (between the points). If we look at the car sector it is worth one of the 5 sections. So we know it's worth $\frac{1}{5}$ and if we make that into a percentage we need it over 100 so we multiply both top and bottom by 20 and get 20/100 which is 20%. Walking to school is worth $\frac{3}{5}$ we now need to get it over 100. It would be 60/100 which is 0.6 and 60%.

Equivalent fractions

Equivalent fractions are all about doing the same calculation to the top and the bottom of the fraction. This can either be multiplying or dividing.



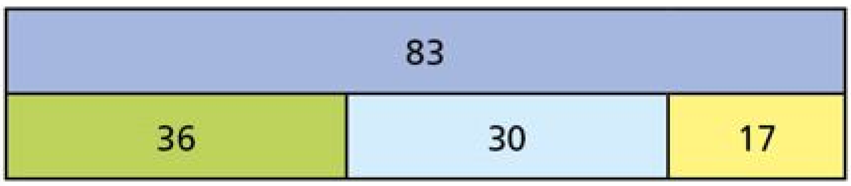
It can be done in one big step or lots of little ones until you find what you are looking for.



Application of number

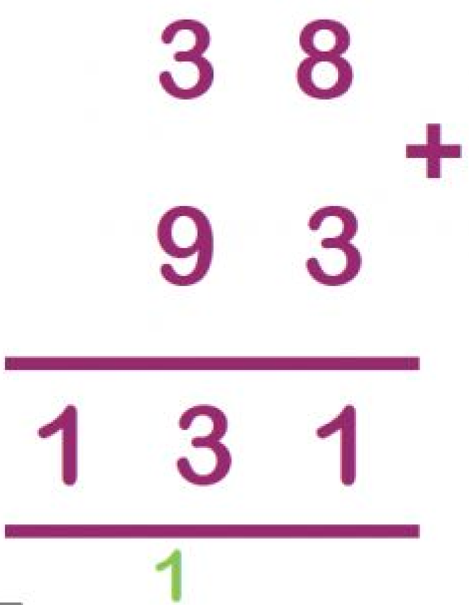
Addition and subtraction

Adding with bar model



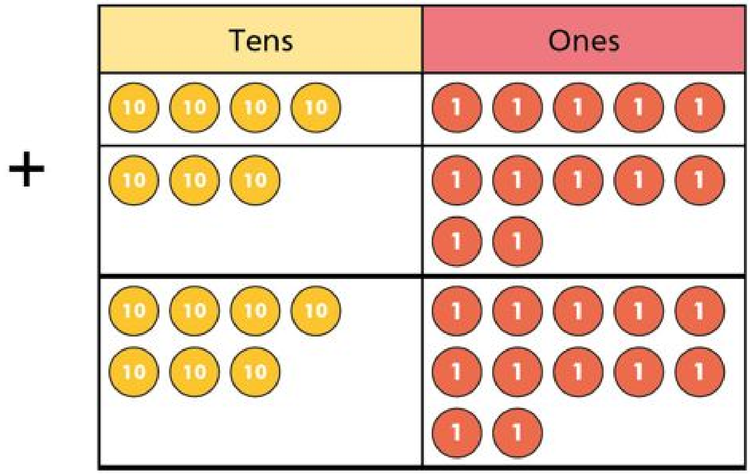
The bar model show that 36+30+17 = 83

Adding with the column method



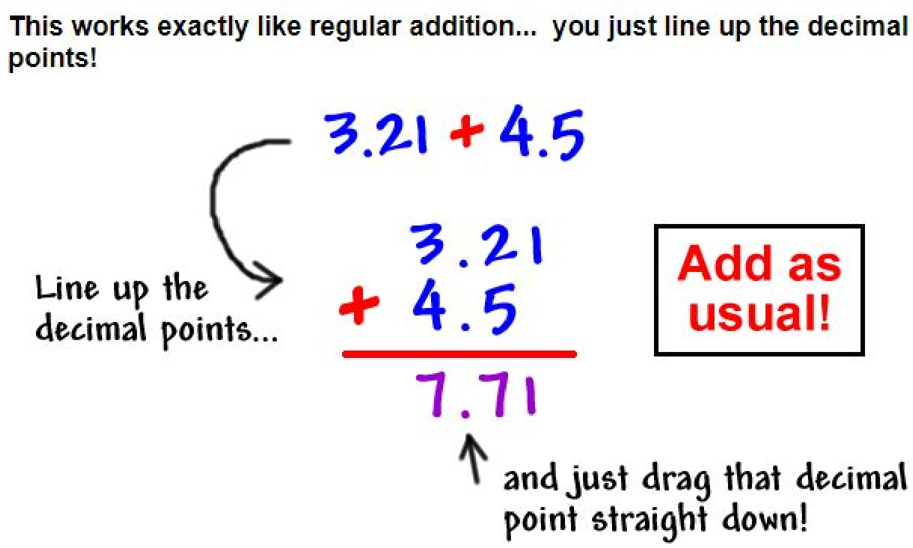
For this method we would add the ones first (numbers in left hand column) which in this case make 11 so we would need to carry the tens over to the next column and add that to the 9 and 3 to make 13. As there are no hundreds to add the 1 to it just goes next to the 3.

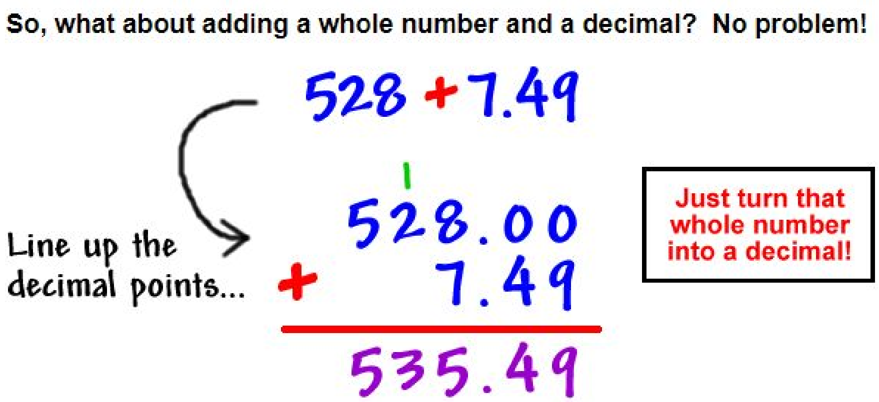
Grid method for addition



This is another way to represent the addition. All the ones can be added together to make 24 and all the 10s can be added to make 140 and then we can add the 140 and 24 together to make 164.

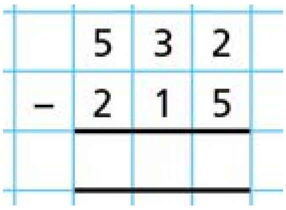
Adding decimals



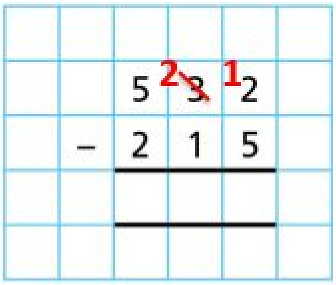


Subtraction using the column method

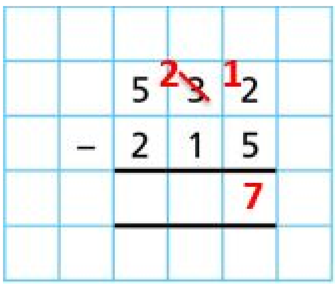
If we were doing the following calculation using the column method we would start with the left hand side numbers



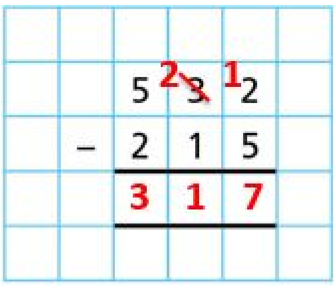
We can't do 2 - 5 so we have to borrow from the 10's column and make the 3 into a 2 and 2 in the left column into a 12 as shown below



Now we can do 12 - 5 to give 7



2-1 to give 1 and 5-2 to give 3



Subtracting Decimals

Example: Subtract 0.03 from 1.1

Line the decimals up:

```
  1.1
– 0.03
```

"Pad" with zeros:

```
  1.10
– 0.03
```

Subtract:

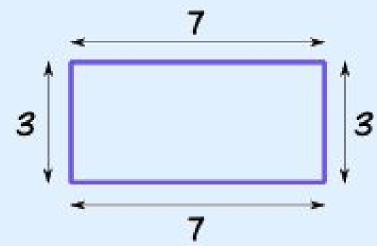
```
  1.10
– 0.03
  ----
  1.07
```

Answer: **1.07**

Perimeter

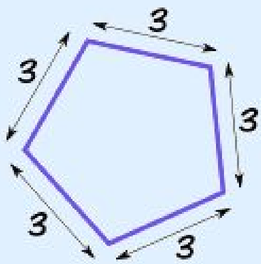
Perimeter is the length that goes around a shape.

Example: the perimeter of this rectangle is 7+3+7+3 = 20



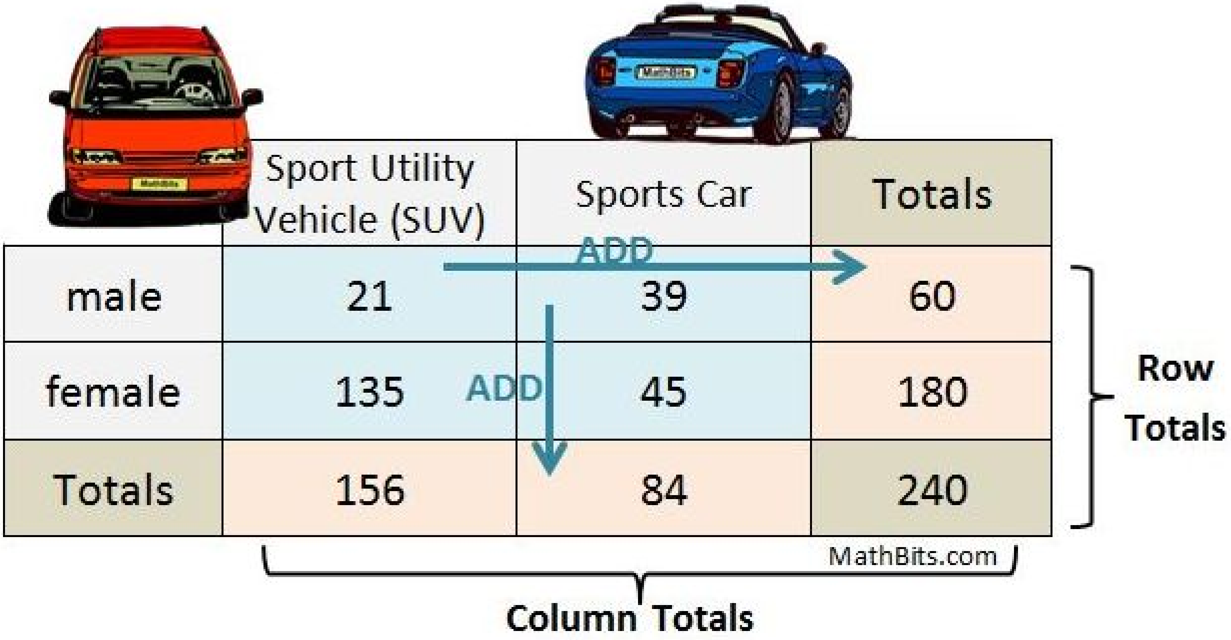
Example: the perimeter of this regular pentagon is:

3+3+3+3+3 = 5×3 = 15



Two way tables

Students will need to add and subtract to find missing numbers in a two way table. At the end of each row and column there is a total box



For example

1 Work out the missing values in this table.

	Men	Women	Total
Prefer cats	22	31	53
Prefer dogs	24	23	47
Total	46	54	100

53-31 = 22

100-53 = 47

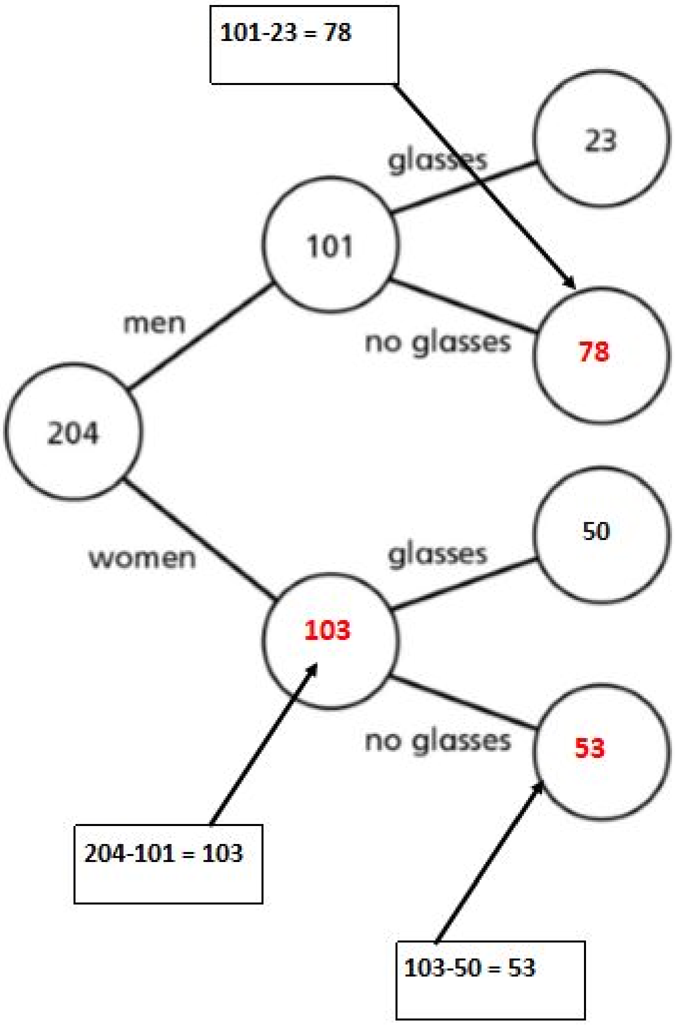
46-22 = 24

100-46 = 54

54-31=23

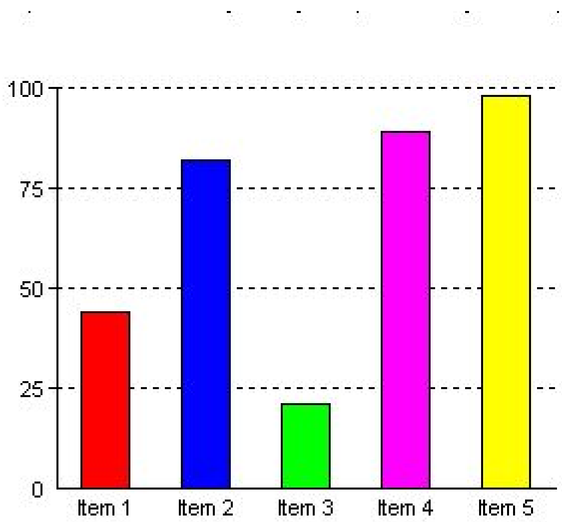
Frequency Trees

Frequency trees are similar to two way tables. The circles coming off a circle must add to give the number in the singular circle. For example



Bar Charts

Bar charts show data. The height of the bars show the frequency (how many) for each different option. The bars must not touch and they need to be an equal width and an equal width apart



Adding and subtracting standard form

Adding and subtracting

When adding and subtracting **standard form** numbers, an easy way is to:

1. convert the numbers from standard form into decimal form or ordinary numbers
2. complete the calculation
3. convert the answer back into standard form

Example

Calculate $(4.5 \times 10^4) + (6.45 \times 10^6)$.

$= 45,000 + 6,450,000$

$= 6,495,000$

$= 6.495 \times 10^6$

Multiplication and division

Multiplying using the grid method

Look at the following number sentence: *38 x 62 =*
Each number would be partitioned into tens and units and placed in the grid. The numbers in the left-hand column would then be multiplied by each of the numbers in the top row and the answers written in the blank squares as follows:

38 x 62 =

X	60	2
30		
8		

X	60	2
30	1800	60
8	480	16

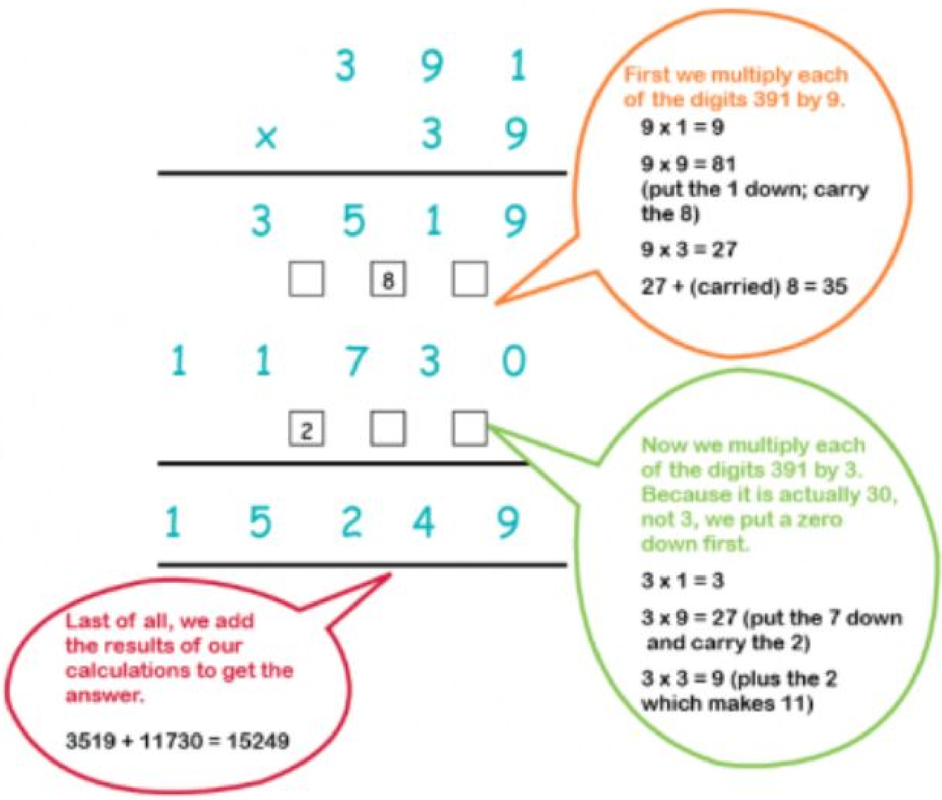
38 x 62 = 1800 + 60 + 480 + 16

38 x 62 = 2356

The four numbers would then be added, using whichever method the child is familiar with.

Multiplying using the column method

As in column addition and column subtraction, the numbers are positioned in a column as follows:



Multiples

Multiples

Multiples are really just extended times tables.

The multiples of 2 are all the numbers in the 2 times table, such as 2, 4, 6, 8, 10 and so on.

Multiples of 2 always end with a 2, 4, 6, 8 or 0. You can tell 2286, for example, is a multiple of 2 because it ends with a 6.

The multiples of 5 are all the numbers in the 5 times table, such as 5, 10, 15, 20, 25 and so on.

Multiples of 5 always end with a 5 or a 0. You can tell 465, for example, is a multiple of 5 because it ends with a 5.

Factors

"Factors" are the numbers we can **multiply together** to get another number:

$$2 \times 3 = 6$$

Factor Factor

2 and 3 are factors of 6

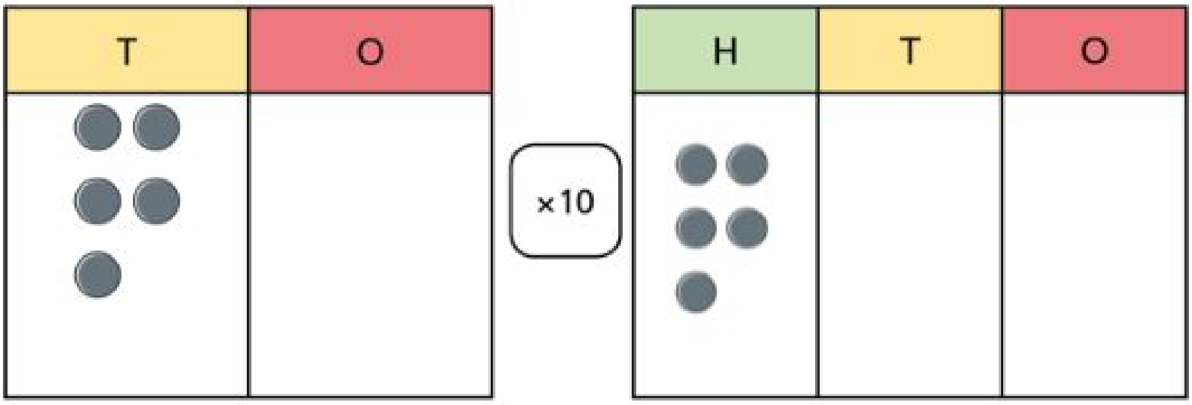
A number can have **many** factors.

Example: 12

- 3 × 4 = 12, so **3** and **4** are factors of 12
- Also 2 × 6 = 12, so **2** and **6** are also factors of 12,
- And 1 × 12 = 12, so **1** and **12** are factors of 12 as well.

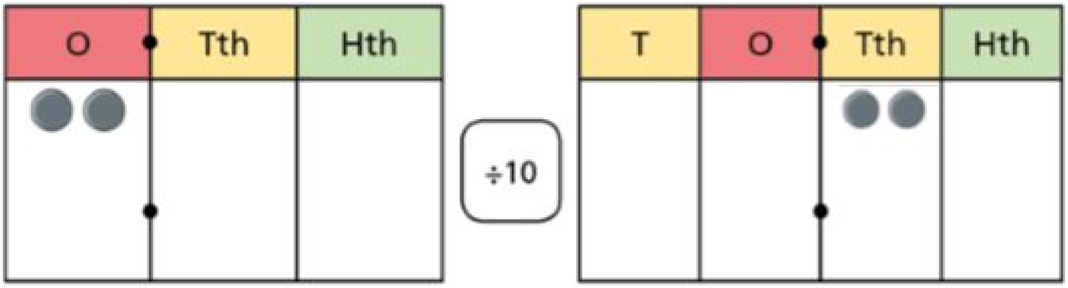
Multiplying and dividing by 10s

When we multiply by 10 we move all the numbers across right one in the place value table. If we look at the example below the 5 counters on the 10s column would go into the hundreds column. If we were multiplying by 100 we would move the numbers over two columns.



=500

With dividing we do the opposite to multiplying we move the counter (or numbers) across to the left. In the example below the two has moved from the ones to the tenths column



=0.2

Multiplying by 0.1 or 0.01

If we were going to multiply by 0.1 we could change that to be 1/10 to make it easier for ourselves and when we times by a fraction we divide by the denominator (bottom number) and multiply by the (numerator) top. As the numerator is 1 in this case we can just divide by 10.

$$87 \times 0.1 = 87 \times \frac{1}{10} = 87 \div \boxed{10} = \boxed{8.7}$$

The same principle can be used for multiplying by 0.01 apart from we change the 0.01 to be 1/100 and divide by 100

$$53 \times 0.01 = 53 \times \frac{1}{100} = 53 \div \boxed{100} = \boxed{0.53}$$

Converting metric units using bar model

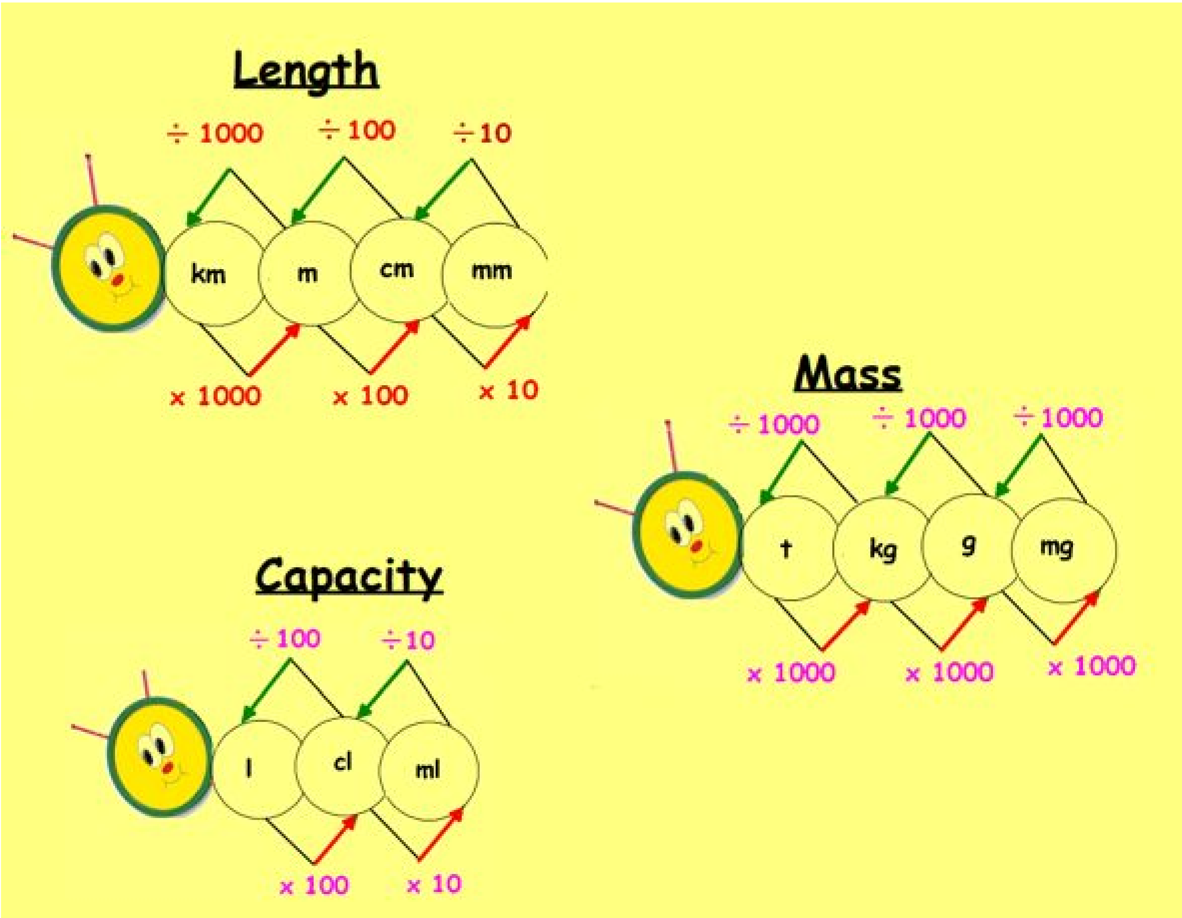
With the bar model students would break it down into parts and find out what each part is worth and then add them together as shown below.

1 m	1 m	1 m	1 m	1 m
100 cm	100 cm			

5 m = 500 cm

Converting metric units by multiplying and dividing

Students can use multiplying and dividing. Using the caterpillars below students can put what they have in the correct part of the body and then do what the legs say to get to the unit they need. For example if we wanted to change 3400g into Kg we would need to divide by 1000 giving us 3.4Kg.



Multiplying decimals

Example: Multiply 0.03 by 1.1

start with:	**0.03 × 1.1**
multiply without decimal points:	**3 × 11 = 33**
0.03 has **2 decimal places**,	
and 1.1 has **1 decimal place**,	
so the answer has **3 decimal places**:	**0.033**

Bus stop division

If we wanted to work out 468 ÷ 4.

Set out the numbers like this;

$$4 \overline{)468}$$

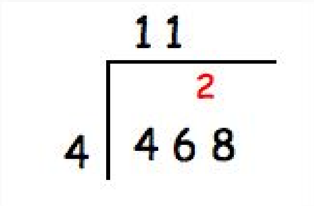
Place your answer directly above the 4.
Work out how many 4's go into 4 (answer 1).

$$\begin{array}{r} 1 \\ 4 \overline{)468} \end{array}$$

Work out how many 4's go into 6 (answer 1 remainder 2).

Place the answer (1) directly above the 6.

Carry the remainder (2) and place it above the next digit 8 to make the number 28.

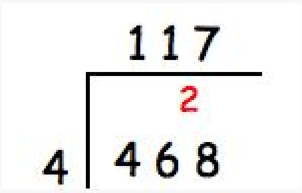


Work out how many 4's go into 28 (answer 7).

Place the answer (7) directly above the 28.

You can then clearly see that 468 ÷ 4 = 117.

If there is anything left over from the final division, this becomes your remainder.



Bus stop division with decimal

$$\begin{array}{r} 1\;3\;7\,.\,7\;1\;4\ldots \\ 7\,\overline{\big)\,9\;{}^{2}6\;{}^{5}4\,.\,{}^{5}0\;{}^{1}0\;{}^{3}0\ldots} \end{array}$$

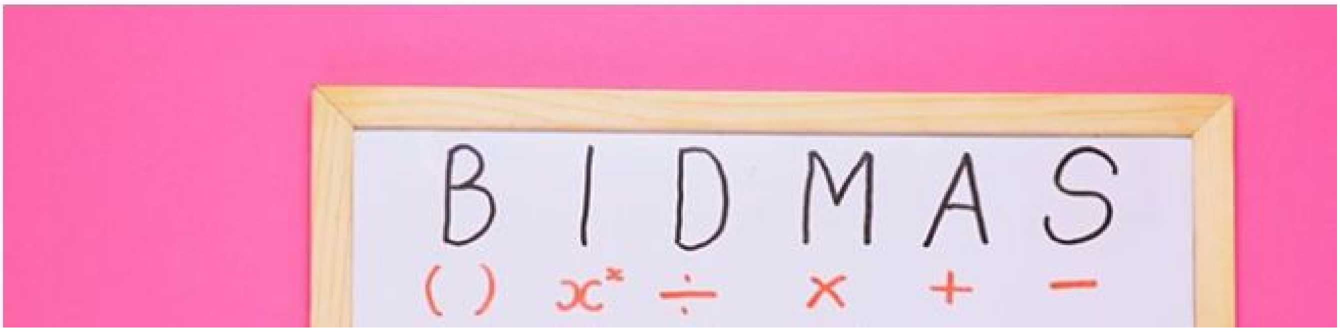
1. 7 goes into 9 once with 2 remaining (remainder 2), so put 1 above the hundreds column and carry the 2 to the tens column.
2. 7 goes into 26 three times, remainder 5, so put 3 above the tens column and carry the 5 to the units column.
3. 7 goes into 54 seven times, remainder 5, so put 7 above the units column and carry the 5 to the tenths column (writing the 964 as 964.000 as 7 does not divide into it exactly).
4. 7 goes into 50 seven times, remainder 1, so put 7 above the tenths column and carry the 1 to the hundredths column.
5. 7 goes into 10 once, remainder 3, etc

So, $964 \div 7 = 137.714...$

$= 137.7$ (correct to 1 decimal place).

Order of operations (BIDMAS)

- The **order of operations** is the order you work out the parts of an equation to give you the correct answer.
- **BIDMAS** is an acronym used to tell you the correct order to complete a equation when there are different operations.
- BIDMAS stands for **B**rackets, **I**ndices, **D**ivision, **M**ultiplication, **A**ddition, **S**ubtraction.



- **Brackets** refers to any part of the equation that is in brackets. These should always be complete first.
- **Indices** simply means to the power of. For example, 3^2 or 5^3.
- **Division** and **Multiplication**: Starting from the left, work these out in the order that they appear in the equation. If multiplication appears first you should complete this before division.
- **Addition** and **Subtraction**: Also start from the left and work these out in the order that they appear in the equation. If subtraction appears before addition, you should complete this first.

It can be helpful to write 'BIDMAS' in the margin of your paper and use it as a checklist.

Example 1

What is $4 + 2 \times 3$?

If you calculate the $4 + 2$ part first you get:

$4 + 2 \times 3 = 6 \times 3 = 18$

If you calculate the 2×3 part first you get:

$4 + 2 \times 3 = 4 + 6 = 10$

These are two very different answers, but only one is correct.

In BIDMAS, multiplication comes before addition, so multiply 2 by 3 first:

$4 + 2 \times 3 = 4 + 6 = 10$, so this is the right answer.

Example 2

What is $9 - 4 + 3$?

This sum has only addition and subtraction. So work them out from left to right:

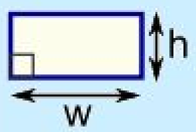
$9 - 4 + 3 = 5 + 3 = 8$

Notice that if you didn't go from left to right you would get a different answer:

$9 - 4 + 3 = 9 - 7 = 2$

That would be incorrect, which is why we work them out from left to right.

Area of rectangles and parallelograms

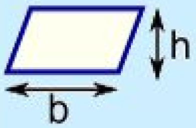


Rectangle

Area = $w \times h$

w = width

h = height



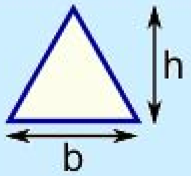
Parallelogram

Area = $b \times h$

b = base

h = vertical height

Area of a triangle



Triangle

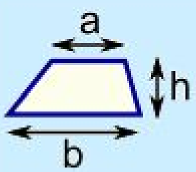
Area = $\frac{1}{2} \times b \times h$

b = base

h = vertical height

Area of a trapezium

There are two methods for this. The first is using the formula as shown below.



Trapezoid (US)

Trapezium (UK)

Area = $\frac{1}{2}(a+b) \times h$

h = vertical height

Or for the second method you can break the trapezium into a rectangle and triangles and work them out separately and add them together.

Mean

Mean is the most common average we use in day to day lives.

Example 1: What is the Mean of these numbers?

6, 11, 7

- Add the numbers: **6 + 11 + 7 = 24**
- Divide by *how many* numbers (there are 3 numbers): **24 / 3 = 8**

The Mean is 8

Fraction and percentage of amounts

Fraction of amount

Find $\frac{2}{5}$ of £35.

First find $\frac{1}{5}$ by dividing £35 by 5 to get £7.

Now to find $\frac{2}{5}$, multiply by 2 to get £7 × 2 = £14.

Example 2 - Increasing or decreasing an amount by a fraction

Increase £240 by $\frac{1}{6}$.

£240 ÷ 6 = £40. So $\frac{1}{6}$ of £240 is £40.

Since we want to increase the amount by $\frac{1}{6}$ we add this on to the original amount of £240. So £240 + £40 = £280.

Example 3 - Multiplying a whole number by a fraction

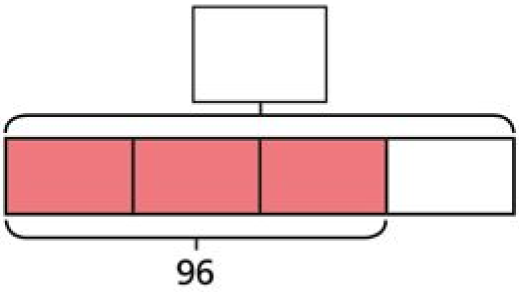
Calculate $\frac{3}{7}$ × 140.

Multiplying a fraction by a whole number is exactly the same as finding a fraction of an amount. In fact, when dealing with fractions the '×' symbol and the word 'of' usually mean the same thing.

So, 140 ÷ 7 = 20 and then 20 × 3 = 60.

Find the whole given a fraction

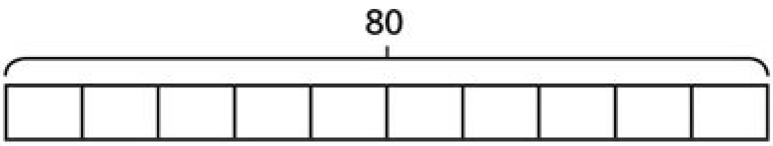
We can do this using the bar model. If we are told that ¾ of something equals 96 we can draw is using the bar model shown below. We can see that 3 parts equal 96 so if we divide that by 3 we get one part and then multiply that by 4 to get what the original number is. 96 divided by 3 is 32 and then 32 multiplied by 4 is 128 b)



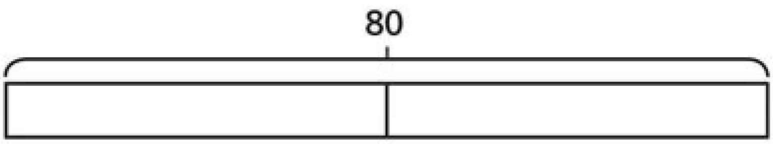
The other method would be to find ¼ by dividing by 3 and then find 4/4 (one whole) by multiplying by 4

Percentage of amounts

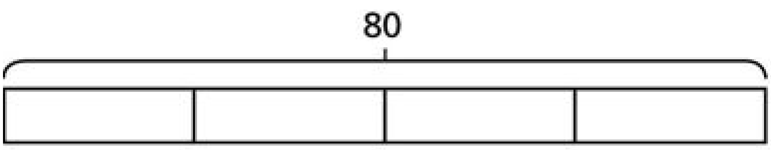
We can use the bar model initially to understand how to find the percentage of amounts. If we wanted to find 10% we could split the 100% into 10 equal boxes and one of those would be worth 10%. For the example below we would divide 80 by 10 to get 8



If we wanted to find 50% we would split the 100% into two boxes



If we wanted to find 25% we could split the box into 4 or half 50%.



Once we have 10%, 50% and 25% we can work out all the rest. If we wanted to find 1% we could divide the 10% value by 10. If we wanted to find 20% we could double the 10% value. If we wanted to find 75% we would add the 25% and 50%.

Percentages using a calculator

To find a percentage of amount using a calculator you first need to change the percentage you want to find out into a decimal. Once you have done that you can multiply your number by the decimal.

If we wanted to find 37% of 60 using a calculator we would change 37% into a decimal (by dividing it by 100) giving us 0.37 and then multiply 60 by it. 0.37 x 60 = 22.2

Directed numbers and fractional thinking

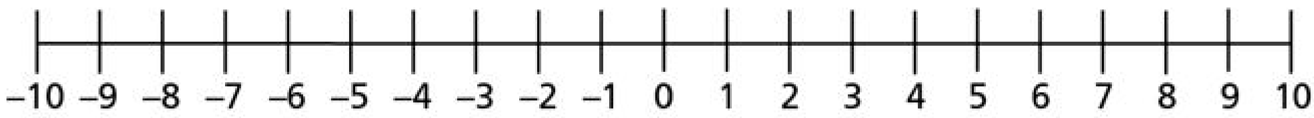
Directed numbers

Representing positive and negative numbers

We represent positive and negative numbers with different colours to represent calculations.

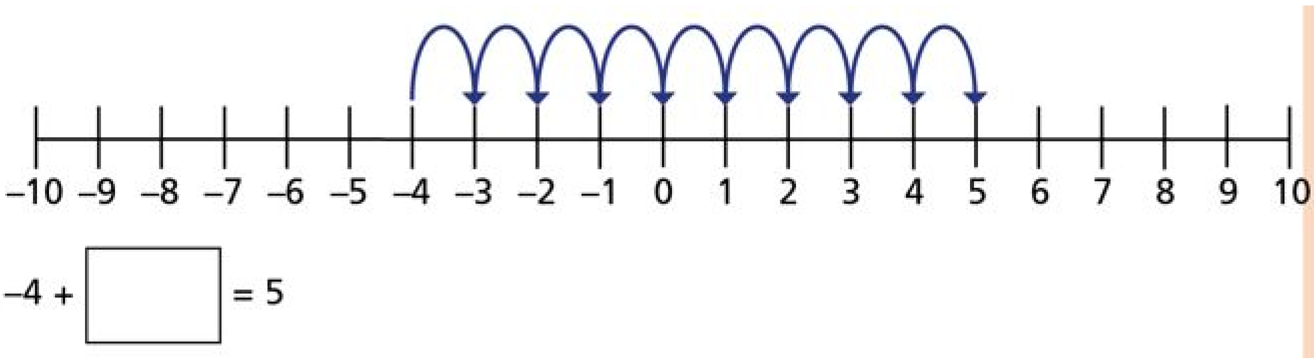
Negative numbers on a numberline

We can represent negative numbers on a numberline making it easier to order them



Adding and subtracting negative numbers

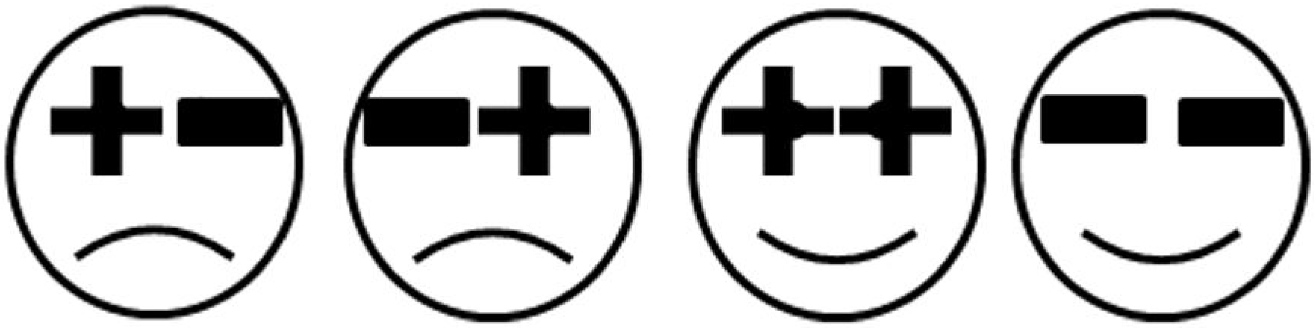
We can use a numberline to help with adding and subtracting negatives as shown below



If we count the amount of jumps we can see what we have to add to negative 4 to get positive 5.

Face method for adding and subtracting negatives

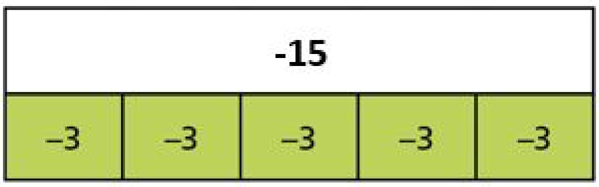
In the picture below it shows a positive and a negative with a sad face meaning it would be negative and a positive and positive or negative and negative with a happy face meaning it would be positive.



5 - + 3 would turn into 5-3
5 - -3 would turn into 5+3

Multiplying using the bar method

We can look at multiplying negative numbers using the bar model to represent it. If we were doing 5 x -3 that would be 5 lots of -3 giving us -15



Multiplying negatives using the faces

If we take the same question as before 5 x -3 we have one positive and one negative which makes a sad face so we know our answer has to be negative. Then we just do 5 x 3 =15 and make it negative =-15

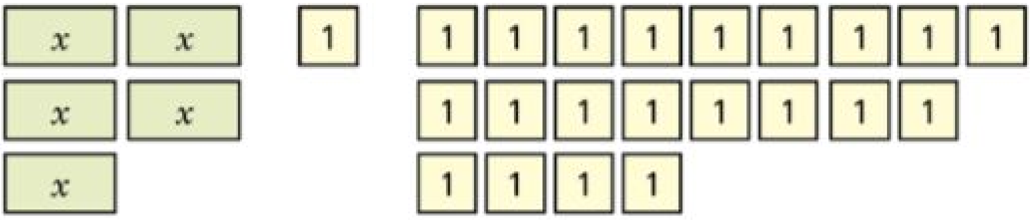
Division with negative numbers

We do division in the same way that we do multiplication. If we had $-30 \div -6$ because they are both negative our answer will be positive. We then do the calculation as if there are no negatives. 30 divided by 6 to give 5 and we have already worked out it was positive so our answer stays as 5. If we had $63 \div -7$ there is one positive and one negative so our answer will be negative then we do 63 divided by 7 to give 9. We already worked out it would be negative so our answer is -9.

Two step equations

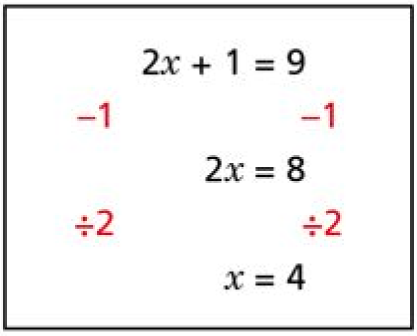
There are three methods we can use to solve two step equations the first is with algebra tiles as shown below

Huan is using algebra tiles to solve the equation **5x + 1 = 21**

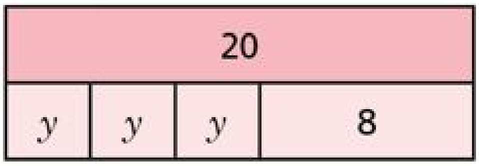


We can cancel a one from each side and then share the remaining ones between the x's

The second method is the balancing method and it is shown below. Whatever you do to one side you must do to the other. The aim is to get x by itself.



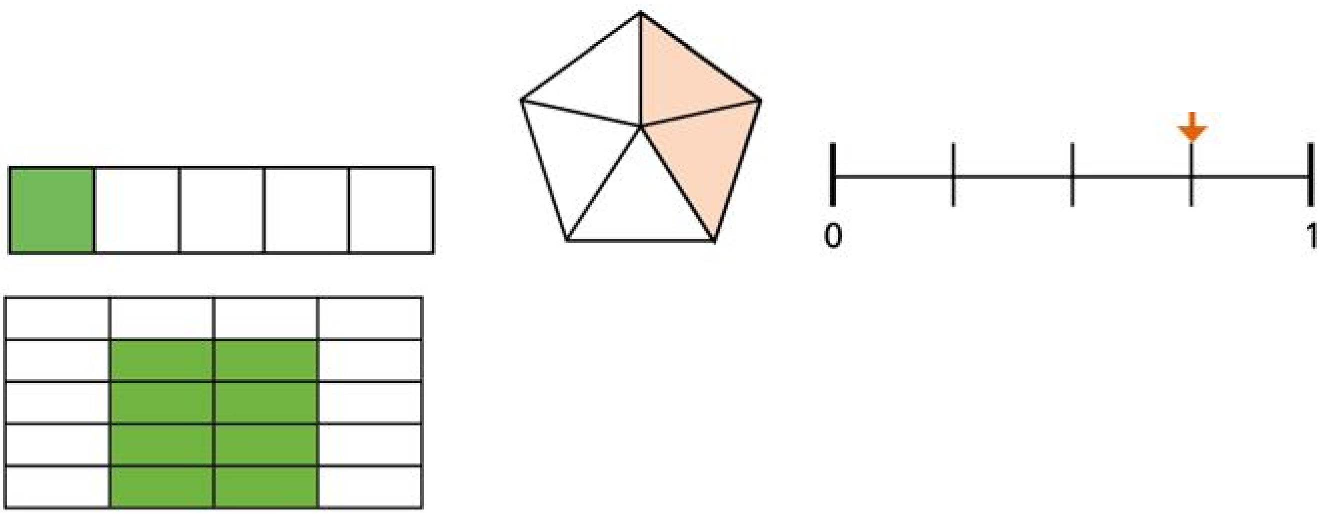
The third method is the bar method as shown below. To get y by itself we need to take 8 away from the 20 and then divide the remaining by 3 as there are 3 y's.



Addition and subtraction of fractions

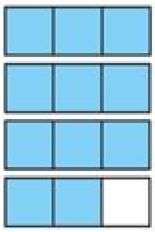
Representing fractions

We can represent fractions on diagrams either on a bar, grid, polygon or number line as shown below.



Converting between mixed and improper fractions

There are two methods for doing this the first is the diagram method as shown below. We can see that there are 3 whole and ⅔. If we split the 3 wholes into 3's we can see that we have 11 sections so as an improper fraction (top heavy) it would be $\frac{11}{3}$



The other method is to do it all numerically

Example: Convert $3\frac{2}{5}$ to an improper fraction.

Multiply the whole number part by the denominator:

$3 \times 5 = 15$

Add that to the numerator:

$15 + 2 = 17$

Then write that result above the denominator:

$$\frac{17}{5}$$

To go from a improper fraction to a mixed number

Example: Convert $\frac{11}{4}$ to a mixed fraction.

Divide:

$11 \div 4 = 2$ with a remainder of 3

Write down the 2 and then write down the remainder (3) above the denominator (4).

Answer:

$$2\frac{3}{4}$$

Adding fractions with the same denominator

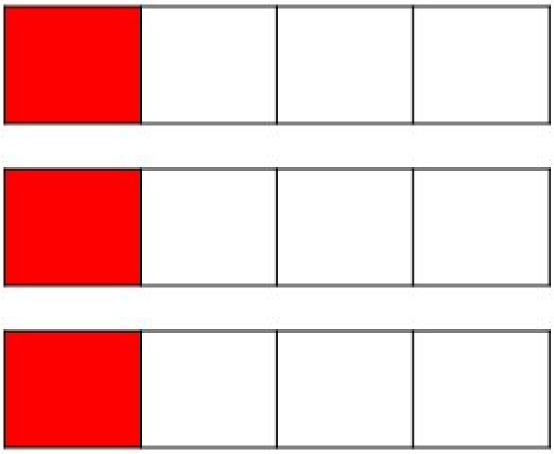
When you add numbers with the same denominator (bottom number) you simply need to add the numerators (top numbers) together and keep the bottom number the same

$$\frac{1}{3} + \frac{1}{3} = \boxed{\frac{2}{3}}$$

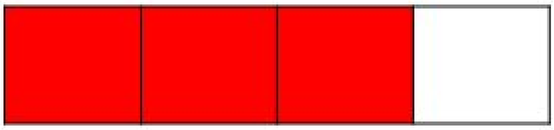
$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \boxed{\frac{3}{4}}$$

$$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \boxed{\frac{3}{5}}$$

This can also be shown with the bar model



This would equal

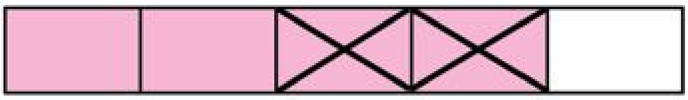


Subtracting fractions with the same denominator

When you subtract numbers with the same denominator (bottom number) you simply need to subtract the numerators (top numbers) together and keep the bottom number the same

$$\frac{4}{5} - \frac{2}{5} = \frac{\boxed{2}}{5}$$

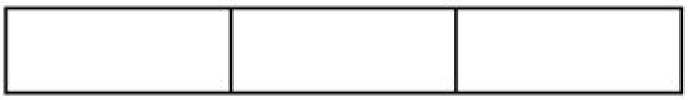
This can also be seen using the bar model



Adding and subtracting fractions from whole numbers

We can do this using the bar model

$$1 - \frac{1}{3} = \square$$



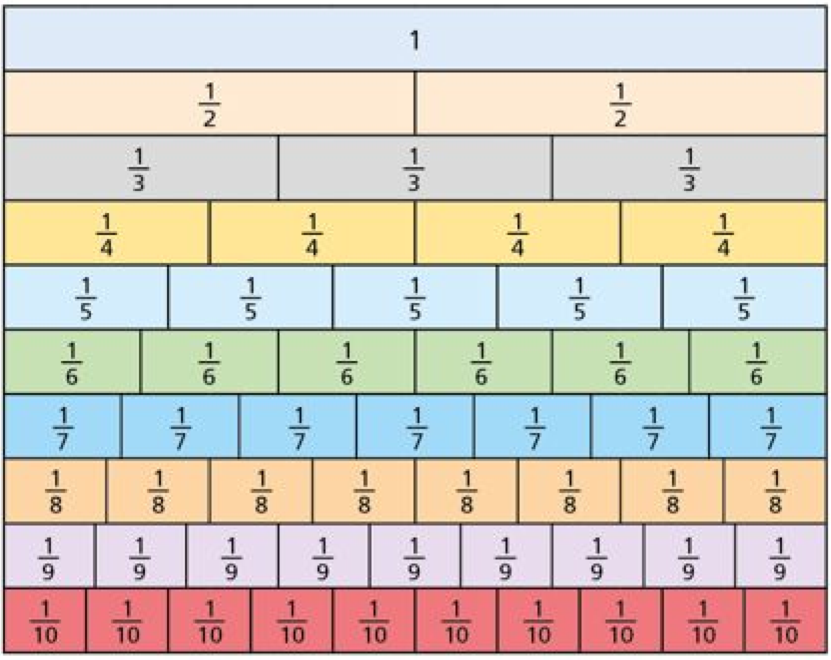
If we split the whole into 3 parts and take one of them away we would be left with 2 parts out of the 3 and that would be $\frac{2}{3}$

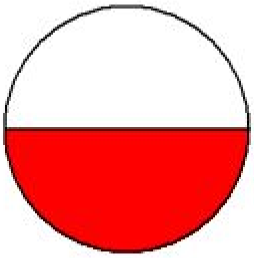
Another method is to change the denominator of our whole number to be the same as the fraction so 3/3 would give one. We could therefore do

$$\frac{3}{3} - \frac{1}{3}$$

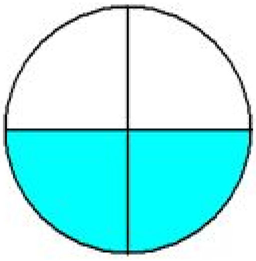
Equivalent fractions

We can use a fraction wall to find equivalent fractions. Equivalent fractions are two fractions that are worth the same amount.

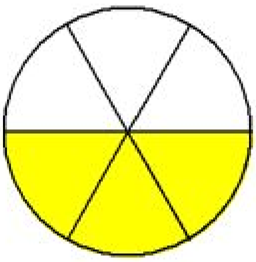




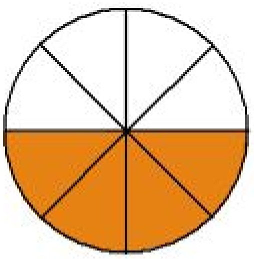
$\frac{1}{2}$



$\frac{2}{4}$

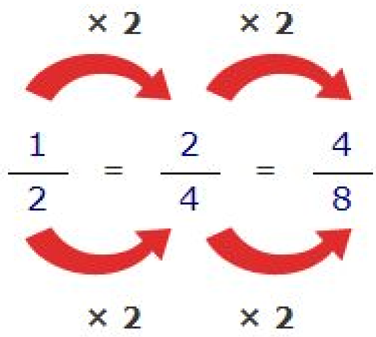


$\frac{3}{6}$



$\frac{4}{8}$

The other method is timesing or dividing the numerator (top number) and denominator (bottom number) by the same amount as shown below



Adding and subtracting fractions with different denominators (bottom numbers)

Equivalent fractions

However, sometimes the denominators are different.

You use equivalent fractions to make them the same.

A **common multiple** of 2 and 3 is **6**.

So, for each fraction we need an equivalent fraction with a denominator of 6.

Now you can add these together.

$$\frac{1}{2} + \frac{1}{3} = ?$$

$$\frac{1}{2}\,{}^{\times 3}_{\times 3} = \frac{3}{6} \qquad \frac{1}{3}\,{}^{\times 2}_{\times 2} = \frac{2}{6}$$

$$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

Adding mixed numbers

Example: What is $2\frac{3}{4} + 3\frac{1}{2}$?

Convert to Improper Fractions:

$$2\frac{3}{4} = \frac{11}{4}$$

$$3\frac{1}{2} = \frac{7}{2}$$

Common denominator of 4:

$$\frac{11}{4} \text{ stays as } \frac{11}{4}$$

$$\frac{7}{2} \text{ becomes } \frac{14}{4}$$

(by multiplying top and bottom by 2)

Now Add:

$$\frac{11}{4} + \frac{14}{4} = \frac{25}{4}$$

Convert back to Mixed Fractions:

$$\frac{25}{4} = 6\frac{1}{4}$$

Example: What is $15\frac{3}{4} - 8\frac{5}{6}$?

Convert to Improper Fractions:

$$15\frac{3}{4} = \frac{63}{4}$$

$$8\frac{5}{6} = \frac{53}{6}$$

Common denominator of 12:

$$\frac{63}{4} \text{ becomes } \frac{189}{12}$$

$$\frac{53}{6} \text{ becomes } \frac{106}{12}$$

Now Subtract:

$$\frac{189}{12} - \frac{106}{12} = \frac{83}{12}$$

Convert back to Mixed Fractions:

$$\frac{83}{12} = 6\frac{11}{12}$$

Lines and angles

Constructing, Measuring & Using Geometric Notation

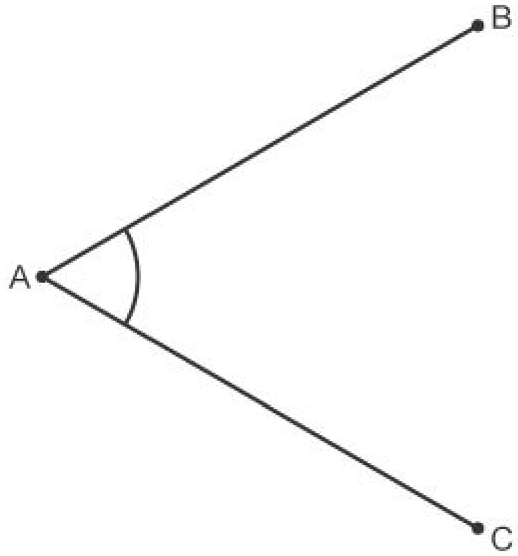
Understand and use letter and labelling conventions including those for geometric figures

Labelling angles and sides

Letters can be used to label angles.

AB and AC are **line segments**, and they meet at point A. AB joins the points A and B.

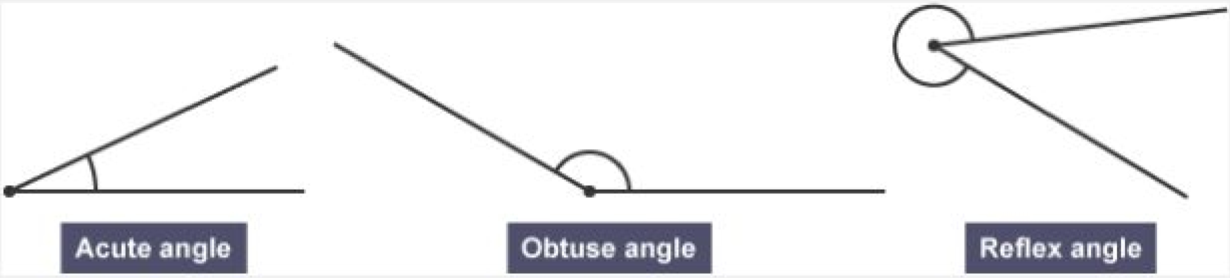
The angle between AB and AC is labelled BAC.



The angle can be written as BAC or BÂC or ∠BAC.

Angle types

There are three different types of angle.



An **acute angle** is an angle less than 90°.

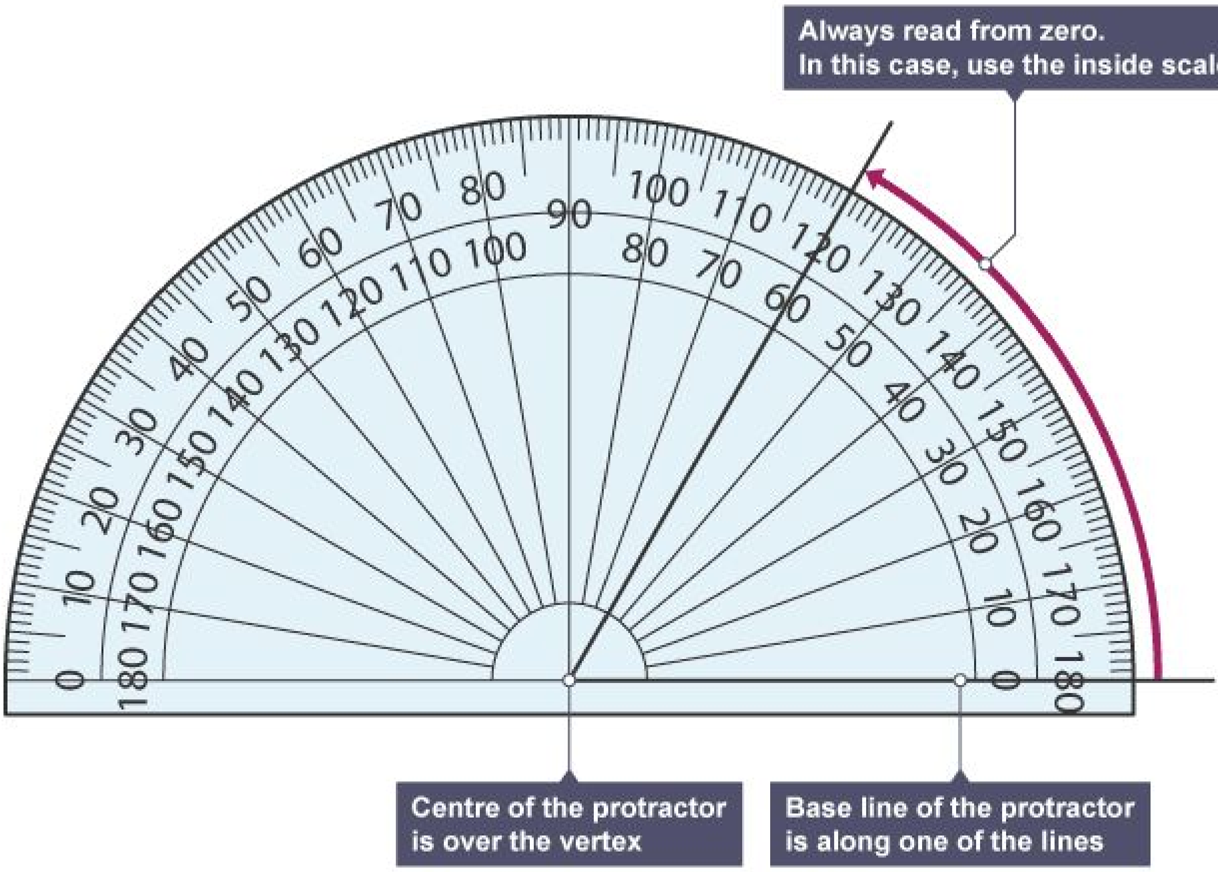
An **obtuse angle** is an angle between 90° and 180°.

A **reflex angle** is an angle between 180° and 360°.

Measuring Angles

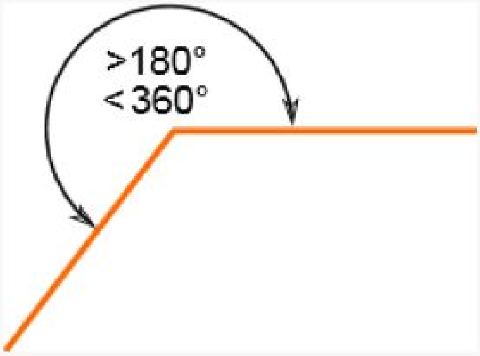
Measuring angles

When measuring angles, make sure that the centre of the protractor is over the **vertex** (corner) of the angle and that the base line of the protractor is along one of the lines of the angle.



How do you measure a Reflex Angle?

First, make sure that it is the reflex angle you're being asked to measure! Lots of children mistakenly, but very carefully measure the wrong angle. Usually, the angle you're being asked to measure will have a little circle drawn around the point to show you which side needs measuring.



You can measure the reflex angle in one of two ways; either measure the inner angle and subtract it from 360° (to give you the measure of the reflex angle) or measure the reflex angle itself.

A reflex angle is probably the trickiest angle to measure because it will be larger than your standard 180° protractor so you'll need to do a bit of accurate drawing.

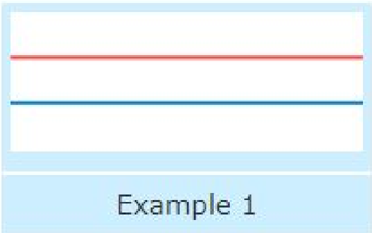
Parallel and Perpendicular lines

Parallel

Lines are parallel if they are always the same distance apart (called "equidistant"), and will never meet. (They also point in the same direction). Just remember:

Always the same distance apart and never touching.

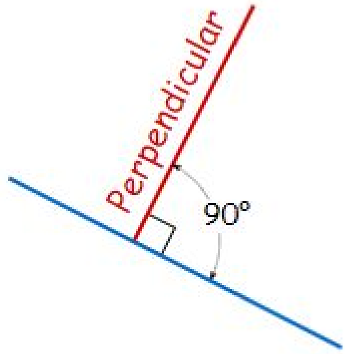
The red line and blue line are parallel in both these examples:



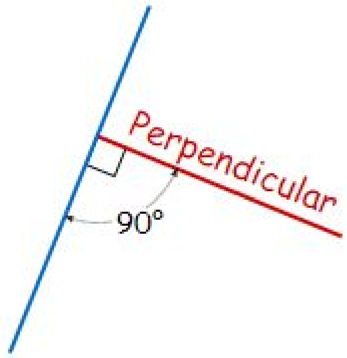
Perpendicular

It just means **at right angles (90°) to**.

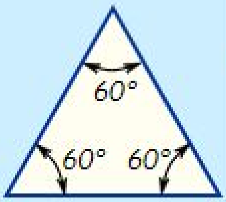
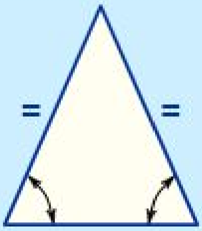
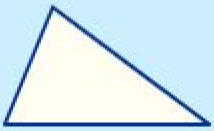
The red line is perpendicular to the blue line:

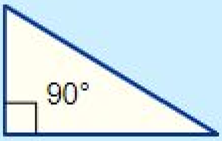
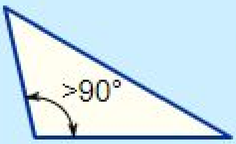


Here also:



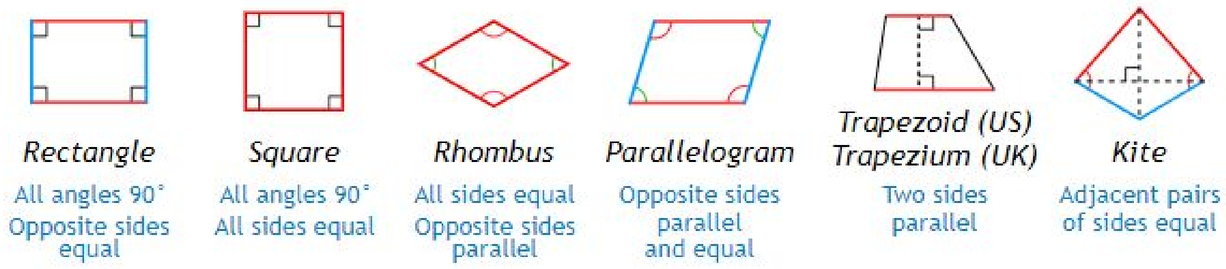
Triangle types

	**Equilateral Triangle** **Three** equal sides **Three** equal angles, always 60°
	**Isosceles Triangle** **Two** equal sides **Two** equal angles
	**Scalene Triangle** **No** equal sides **No** equal angles

	**Acute Triangle** All angles are less than 90°
	**Right Triangle** Has a right angle (90°)
	**Obtuse Triangle** Has an angle more than 90°

Types of quadrilaterals

There are special types of quadrilateral:



Constructing triangles

Here are 3 great animations that we can watch that go through how to draw all the different types of triangles. https://www.bbc.co.uk/bitesize/guides/zx2sb82/revision/4

Drawing pie charts

A **pie chart** is a circular chart. It shows the proportion of each group at a glance. Remember that there are 360° in a circle so each group in the pie chart will be a proportion of 360°.

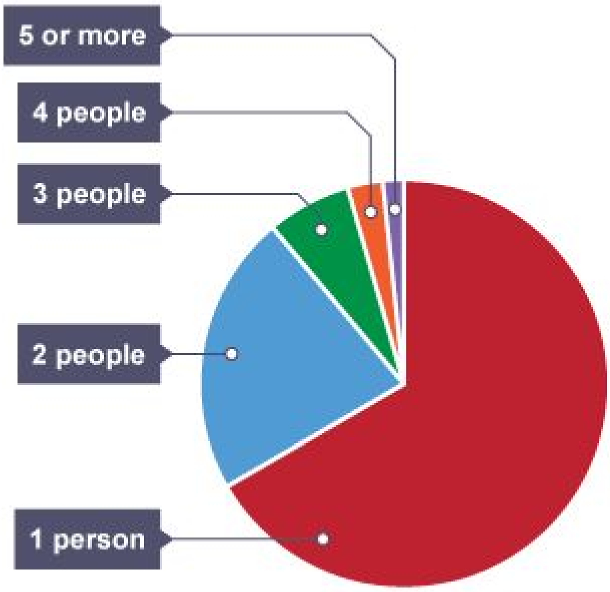
A survey of the number of people in 180 vehicles is taken.

The results are in the table below.

Number of people in a vehicle	Frequency
1	120
2	40
3	13
4	5
5 or more	2
Total	**180**

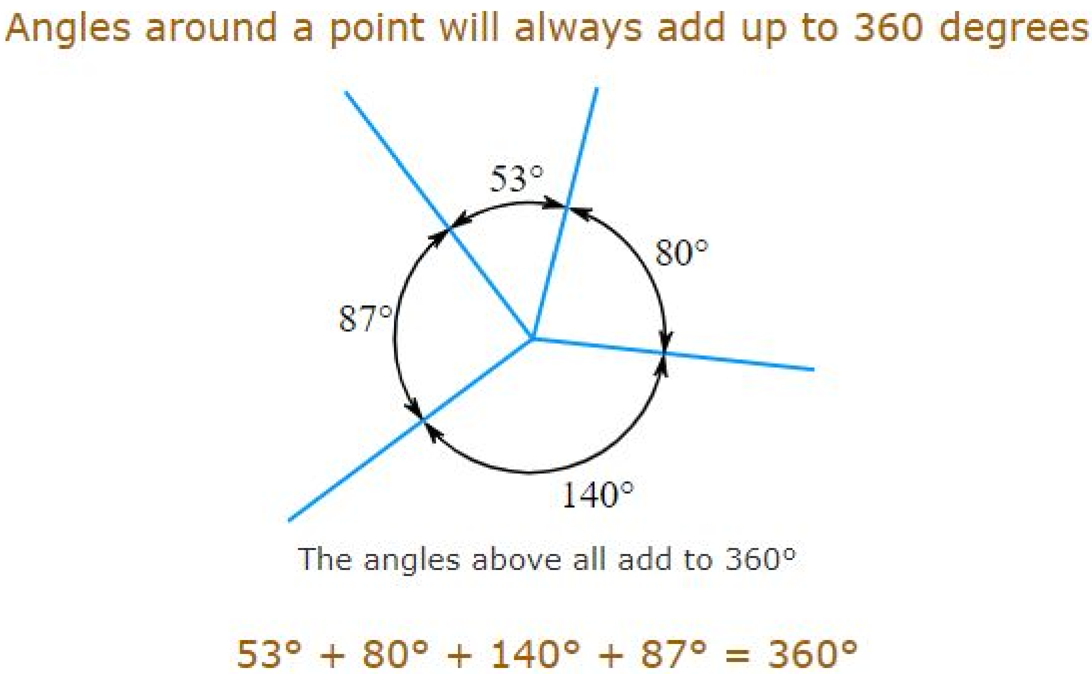
To draw a pie chart for this data, find the proportion of 360.

Number of people in vehicle	Frequency	Calculation	Angle
1 person	120	$\frac{120}{180} \times 360^\circ$	240°
2 people	40	$\frac{40}{180} \times 360^\circ$	80°
3 people	13	$\frac{13}{180} \times 360^\circ$	26°
4 people	5	$\frac{5}{180} \times 360^\circ$	10°
5 or more	2	$\frac{2}{180} \times 360^\circ$	4°
Total	**180**		



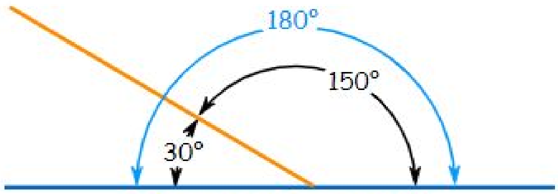
Developing Geometric Reasoning

Angles around a point



Angles on a straight line

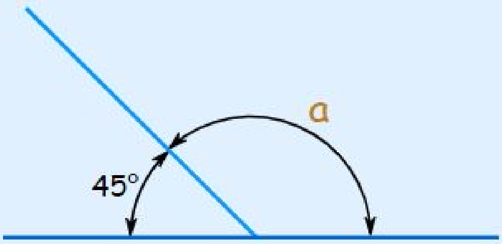
Angles on one side of a straight line always add to **180 degrees**



$30° + 150° = \mathbf{180°}$

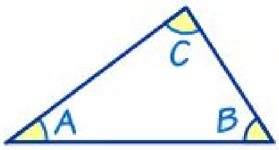
When a line is split into 2 and we know one angle, we can always find the other one.

Example: We know one angle is 45°, what is the other angle "**a**" ?



Angle **a** is $180° - 45° = \mathbf{135°}$

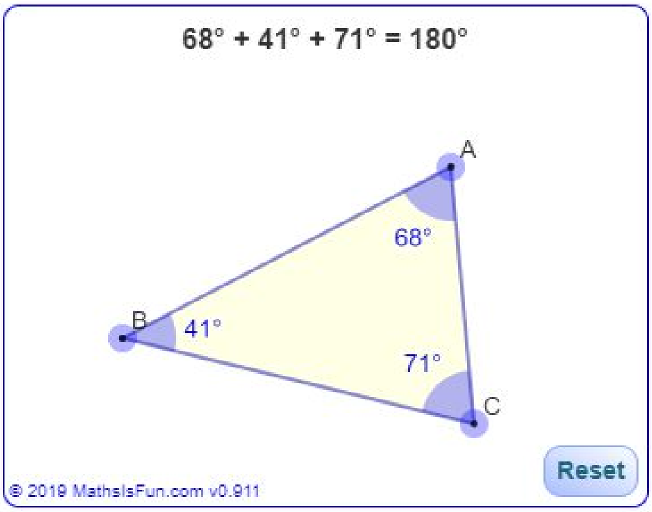
Angles in a triangle



In a triangle, the three interior angles always add to 180°:

A + B + C = 180°

Try it yourself (drag the points):



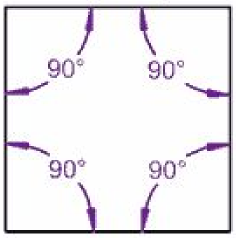
We can use that fact to find a missing angle in a triangle:

Angles in a quadrilateral

Quadrilaterals (Squares, etc)

(A Quadrilateral has 4 straight sides)

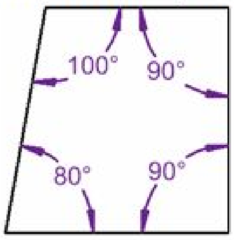
Let's try a square:



90° + 90° + 90° + 90° = 360°

A Square adds up to 360°

Now tilt a line by 10°:



80° + 100° + 90° + 90° = 360°

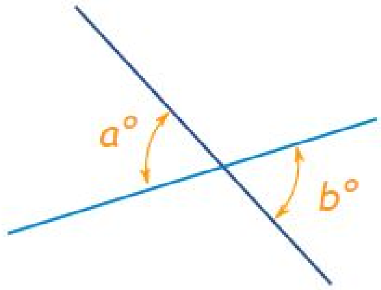
It still adds up to 360°

The Interior Angles of a Quadrilateral add up to 360°

Vertically opposite angles

Vertically Opposite Angles

more ...



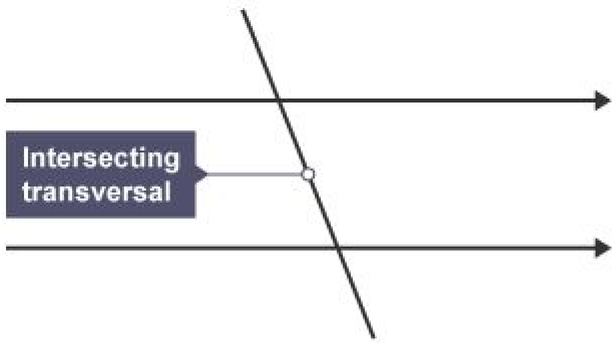
The angles opposite each other when two lines cross. They are always equal.

In this example a° and b° are vertically opposite angles.

"Vertical" refers to the vertex (where they cross), NOT up/down.

Angles in parallel lines

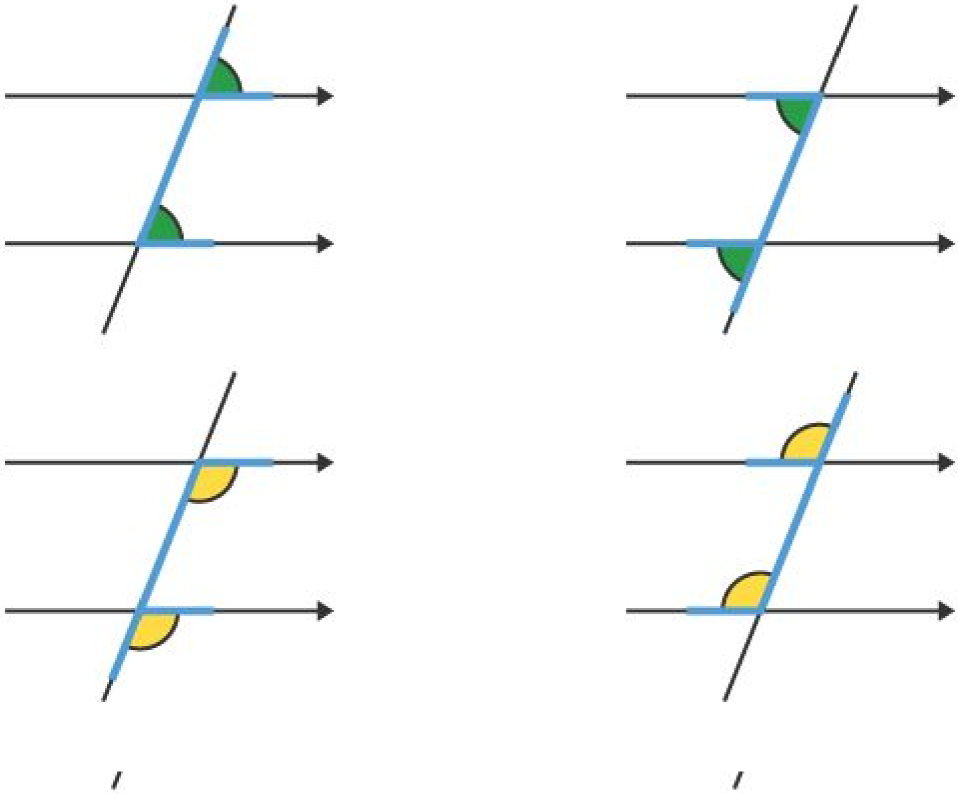
Parallel lines are lines which are always the same distance apart and never meet. Arrowheads show lines are parallel.



When a pair of parallel lines is cut with another line known as an **intersecting transversal**, it creates pairs of angles with special properties.

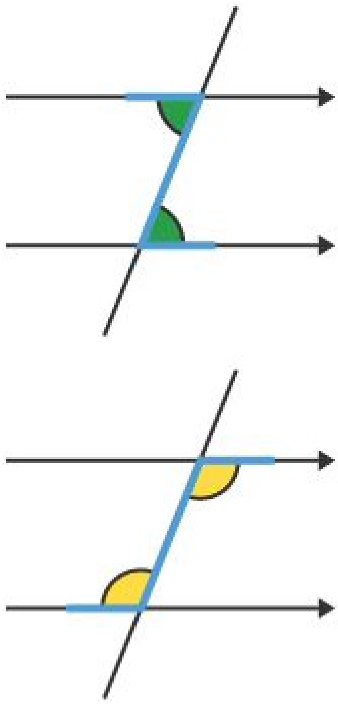
Corresponding angles

Corresponding angles are equal. The lines make an **F shape**.
Notice that the F shape can be upside down or back to front.



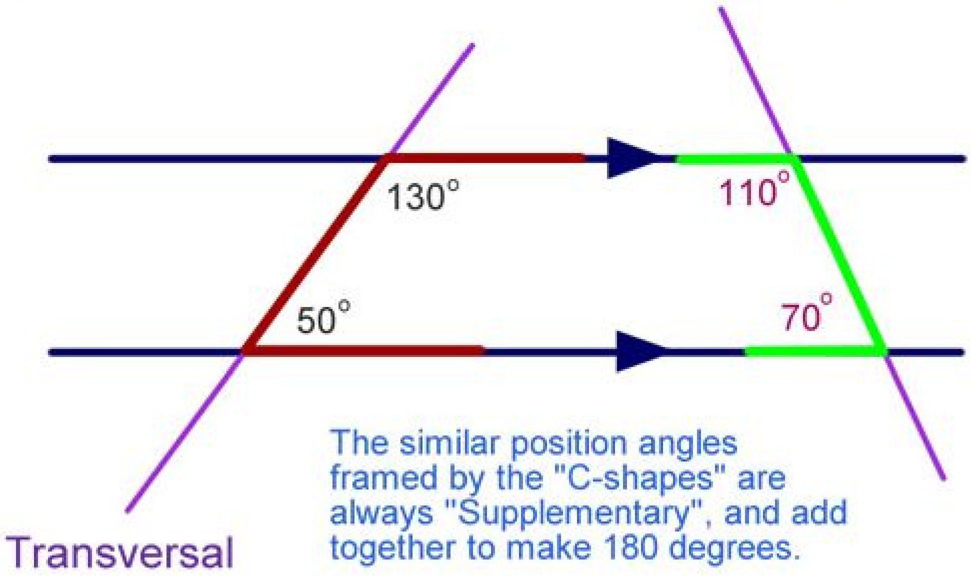
Alternate angles

Alternate angles are equal. The lines make a **Z shape** which can also be back to front.



Co-Interior Angles

Co-Interior Angles exist in an "C" shape and do NOT equal equal each other. However they always **ADD up to equal 180**.



The similar position angles framed by the "C-shapes" are always "Supplementary", and add together to make 180 degrees.

Developing number sense

Divisibility rules

These rules let you test if one number is divisible by another, without having to do too much calculation!

Example: is 723 divisible by 3?

We could try dividing 723 by 3

Or use the "3" rule: 7+2+3=12, and 12 ÷ 3 = 4 exactly **Yes**

Note: 0 (zero) is a "yes" result to any of these tests.

1 Any integer (not a fraction) is divisible by 1

2 The last digit is even (0,2,4,6,8)

→ 12**8** **Yes**

→ 12**9** **No**

3

The sum of the digits is divisible by 3

- 381 (3+8+1=12, and 12÷3 = 4) **Yes**
- 217 (2+1+7=10, and 10÷3 = $3\ ^{1}/_{3}$) **No**

This rule can be repeated when needed:

- 99996 (9+9+9+9+6 = 42, then 4+2=6) **Yes**

4

The last 2 digits are divisible by 4

- 13**12** is (12÷4=3) **Yes**
- 70**19** is not (19÷4=$4\ ^{3}/_{4}$) **No**

A quick check (useful for small numbers) is to halve the number twice and the result is still a whole number.

- 12/2 = 6, 6/2 = 3, 3 is a whole number. **Yes**
- 30/2 = 15, 15/2 = 7.5 which is not a whole number. **No**

5

The last digit is 0 or 5

- 17**5** **Yes**
- 80**9** **No**

6

Is even and is divisible by 3 (it passes both the 2 rule and 3 rule above)

- 114 (it is even, and 1+1+4=6 and 6÷3 = 2) **Yes**
- 308 (it is even, but 3+0+8=11 and 11÷3 = 3 $^{2}/_{3}$) **No**

7

Double the last digit and subtract it from a number made by the other digits. The result must be divisible by 7. (We can apply this rule to that answer again)

- 672 (Double 2 is 4, 67-4=63, and 63÷7=9) **Yes**
- 105 (Double 5 is 10, 10-10=0, and 0 is divisible by 7) **Yes**
- 905 (Double 5 is 10, 90-10=80, and 80÷7=11 $^{3}/_{7}$) **No**

8

The last three digits are divisible by 8

- 109**816** (816÷8=102) **Yes**
- 216**302** (302÷8=37 $^{3}/_{4}$) **No**

A quick check is to halve three times and the result is still a whole number:

- 816/2 = 408, 408/2 = 204, 204/2 = 102 **Yes**
- 302/2 = 151, 151/2 = 75.5 **No**

9 The sum of the digits is divisible by 9

(Note: This rule can be repeated when needed)

- 1629 (1+6+2+9=18, and again, 1+8=9) **Yes**
- 2013 (2+0+1+3=6) **No**

10 The number ends in 0

- 22**0** **Yes**
- 22**1** **No**

11 Add and subtract digits in an alternating pattern (add digit, subtract next digit, add next digit, etc). Then check if that answer is divisible by 11.

- **1**3**6**4 (+1−3+6−4 = **0**) **Yes**
- **913** (+9−1+3 = **11**) **Yes**
- **3**7**2**9 (+3−7+2−9 = **−11**) **Yes**
- **987** (+9−8+7 = **8**) **No**

12 The number is divisible by both 3 ***and*** 4 (it passes both the 3 rule and 4 rule above)

- 648
  (*By 3?* 6+4+8=18 and 18÷3=6 Yes)
  (*By 4?* 48÷4=12 Yes)
  Both pass, so **Yes**
- 524
  (*By 3?* 5+2+4=11, 11÷3= 3 $^{2}/_{3}$ No)
  (Don't need to check by 4) **No**

Mental maths tips

1. **The "9-trick".**
   To add 9 to any number, first add 10, and then subtract 1. In my Math Mammoth books, I give children this storyline where nine really badly wants to be 10… so, it asks this other number for "one". The other number then becomes one less. For example, we change the addition 9 + 7 to 10 + 6, which is much easier to solve.

But this "trick" expands. Can you think of an easy way to add 76 + 99? Change it to 75 + 100. How about 385 + 999?
How would you add 39 + 28 in your head? Let 39 become 40… which reduces 28 to 27. The addition is now 40 + 27. Yet another way is by thinking of compensation: 39 is one less than 40, and 28 is two less than 30. So, their sum is three less than 70.

2. **Doubles + 1.**
   Encourage children to memorize the doubles from 1 + 1 through 9 + 9. After that, a whole lot of other addition facts are at their fingertips: the ones we can term "doubles plus one more". For example, 5 + 6 is just one more than 5 + 5, or 9 + 8 is just one more than 8 + 8.
3. **Use addition facts when adding bigger numbers.**
   Once you know that 7 + 8 = 15, then you will also be able to do all these additions in your head:
   - 70 + 80 is 15 tens, or 150
   - 700 + 800 is 15 hundreds, or 1500
   - 27 + 8 is 20 and 15, which is 35. Or, think this way: since 7 + 8 is five more than ten, then 27 + 8 is five more than the next ten.
4. **Subtract by adding.**

   This is a very important principle, based on the connection between addition and subtraction. Children really don't need to memorize subtraction facts as such, if they can use this principle. For example, to find 8 − 6, think, "Six plus what number makes 8?" In other words, think of the missing number addition 6 + ___ = 8. The answer to that is also the answer to 8 − 6.
   This principle comes in especially handy with subtractions such as 13 − 7, 17 − 8, 16 − 9, and other basic subtraction facts where the minuend is between 10 and 20. But you can also use it in multitudes of other situations. For example, 63 − 52 is easier to solve by thinking of addition: 52 + 11 makes 63, so the answer to 63 − 52 is 11.
5. **Five times a number.**

   Turning our attention to multiplication now, here's a nifty trick you might not know about. To find 5 times any number, first multiply that number times ten, then take half of that. For example, 5 × 48 can be found by multiplying 10 × 48 = 480, and taking half of the result, which gives us 240. Of course, you can also use this strategy for such multiplication facts as 5 × 7 or 5 × 9.
6. **Four and eight times a number.**

   If you can double numbers, you already have this down pat! To find four times a number, double that number twice. For example, what is 4 × 59? First find double 59, which is 118. Then double that, and you get 236.
   Similarly, eight times a number just means doubling three times. As an example, to find 8 × 35 means doubling 35 to get 70, doubling 70 to get 140, and (once more) doubling 140 to get 280. However, personally I would transform 8 × 35 into 4 × 70 (you double one factor and halve the other), which is easy to solve to be 280.
7. **Multiply in parts.**
   This strategy is very simple, and in fact it is the foundation for the standard multiplication algorithm. You can find 3 × 74 mentally by multiplying 3 × 70 and 3 × 4, and adding the results. We get 210 + 12 = 222. Another example: 6 × 218 is 6 × 200 and 6 × 10 and 6 × 8, which is 1200 + 60 + 48 = 1308.

Estimation

Sometimes it is a good idea to estimate a calculation rather than work it out exactly, if you don't need to know the exact value. In this situation, round the numbers in the question before performing the calculation. Usually, numbers are rounded to one significant figure. The 'approximately equal to' sign, ≈, is used to show that values have been rounded.

Examples

Estimate the value of 23×67.

Rounding to 1 significant figure gives: $20 \times 70 = 1,400$

Therefore: $23 \times 67 \approx 1,400$

Estimate: $\frac{423 - 98}{16.4}$

Rounding to 1 significant figure gives:
$\frac{400 - 100}{20} = \frac{300}{20} = \frac{30}{2} = 15$

Therefore: $\frac{423 - 98}{16.4} \approx 15$

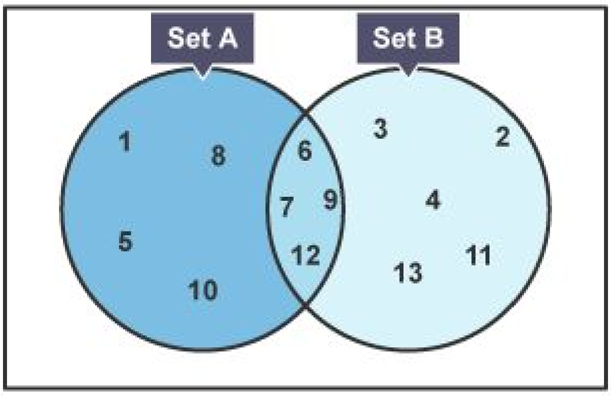
Sets and probability

Venn diagrams

Venn diagrams

Venn diagrams are very useful constructs made of two or more circles that sometimes overlap. Venn diagrams frequently appear in different areas of mathematics but are most common when dealing with sets and probability.

Look at this Venn diagram:



It shows Set A = {1, 5, 6, 7, 8, 9, 10, 12} and Set B = {2, 3, 4, 6, 7, 9, 11, 12, 13}

If we look at the overlapping section of the Venn diagram, this represents $A \cap B = \{6, 7, 9, 12\}$ (The intersection of A and B). This contains the numbers that are in both Set A and Set B.

Taking the two circles in their entirety gives us $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$ (The union of A and B).

Intersection

"Intersection" is when you must be in BOTH sets.

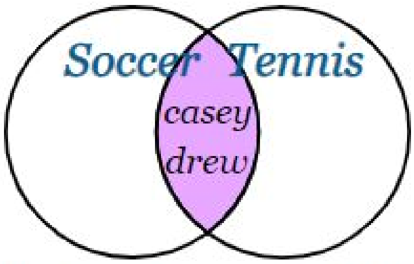
In our case that means **they play both Soccer AND Tennis** ... which is casey and drew.

The special symbol for Intersection is an upside down "U" like this: ∩

And this is how we write it:

Soccer ∩ Tennis = {casey, drew}

In a Venn Diagram:



Venn Diagram: Intersection of 2 Sets

Union

You can now list your friends that play **Soccer OR Tennis**.

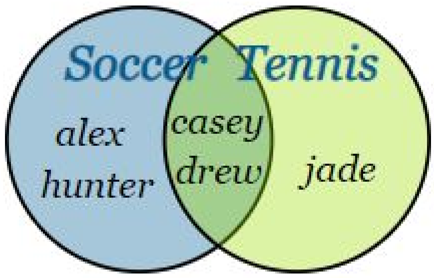
This is called a "Union" of sets and has the special symbol **∪**:

Soccer **∪** Tennis = {alex, casey, drew, hunter, jade}

Not everyone is in that set ... only your friends that play Soccer or Tennis (or both).

In other words we combine the elements of the two sets.

We can show that in a "Venn Diagram":



Venn Diagram: Union of 2 Sets

A Venn Diagram is clever because it shows lots of information:

- Do you see that alex, casey, drew and hunter are in the "Soccer" set?
- And that casey, drew and jade are in the "Tennis" set?
- And here is the clever thing: **casey and drew are in BOTH sets!**

All that in one small diagram.

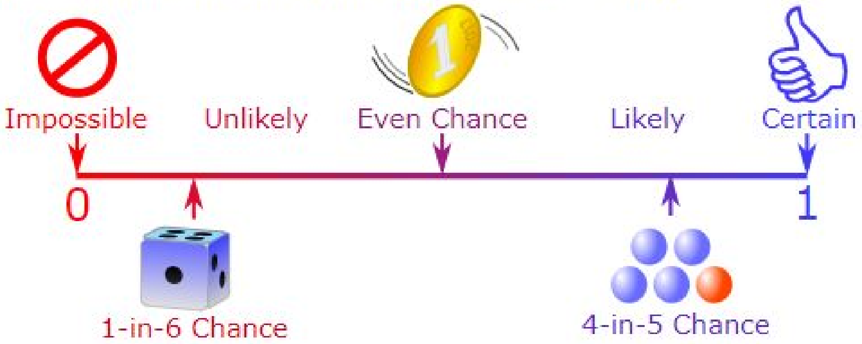
Probability words

The chance that something will happen. How likely it is that some event will occur.

Sometimes we can measure a probability with a number like "10% chance", or we can use words such as impossible, unlikely, possible, even chance, likely and certain.

Example: "It is unlikely to rain tomorrow".

As a number, probability is from 0 (impossible) to 1 (certain).



Prime numbers and proof

Prime numbers

The prime numbers up to 100 are shaded in blue

1	**2**	**3**	4	**5**	6	**7**	8	9	10
11	12	**13**	14	15	16	**17**	18	**19**	20
21	22	**23**	24	25	26	27	28	**29**	30
31	32	33	34	35	36	**37**	38	39	40
41	42	**43**	44	45	46	**47**	48	49	50
51	52	**53**	54	55	56	57	58	**59**	60
61	62	63	64	65	66	**67**	68	69	70
71	72	**73**	74	75	76	77	78	**79**	80
81	82	**83**	84	85	86	87	88	**89**	90
91	92	93	94	95	96	**97**	98	99	100

Prime number decomposition

And a "Factor Tree" can help: find **any factors** of the number, then the factors of those numbers, etc, until we can't factor any more.

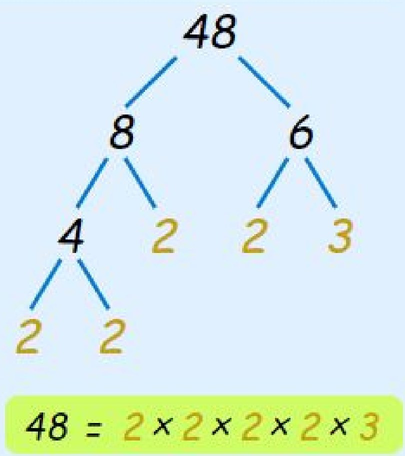
Example: 48

48 = **8** × **6**, so we write down "8" and "6" below 48

Now we continue and factor 8 into **4** × **2**

Then 4 into **2** × **2**

And lastly 6 into **3** × **2**



We can't factor any more, so we have found the prime factors.

Which reveals that **48** = **2** × **2** × **2** × **2** × **3**

(or **48** = $\mathbf{2^4} \times \mathbf{3}$ using exponents)

HCF and LCM (Highest common factor and lowest common multiple)

First you need to do a prime factor tree of both numbers. Once you have done that you need to fit these numbers in to a venn diagram

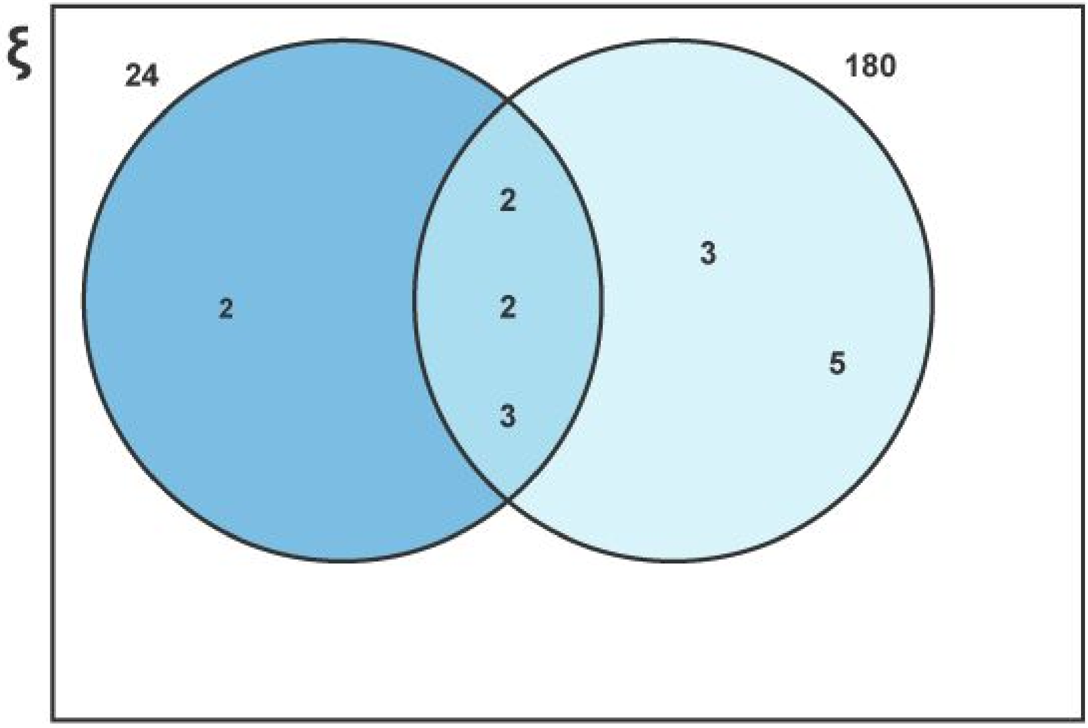
Find the HCF and LCM of 12 and 180.

Break the numbers into the product of prime factors using prime factor trees, as before.

The product of prime factors for 24 are: $2 \times 2 \times 2 \times 3$

The product of prime factors for 180 are: $2 \times 2 \times 3 \times 3 \times 5$

Put each prime factor in the correct place in the Venn diagram. Any common factors should be placed in the intersection of the two circles.



The highest common factor is found by **multiplying together the numbers in the intersection** of the two circles.

HCF = $2 \times 2 \times 3 = 12$

The LCM is found by **multiplying together the numbers from all three sections** of the circles.

LCM = $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$