AQA Level 2 Certificate in Further Mathematics

Worksheets - Teacher Booklet

Level 2

Specification

Level 2 Certificate in Further Mathematics 8360

Worksheets - Teacher Booklet

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Contents

Glossary for Mark Schemes

These examinations are marked in such a way as to award positive achievement wherever possible. Thus, for these papers, marks are awarded under various categories.

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
M Dep	A method mark dependent on a previous method mark being awarded.
B Dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$

1 Coordinate Geometry - Circles

Question 1

Write down the equation of each of these circles.

(a) Centre (0, 3) radius 2 *(2 marks)*

(b) Centre (1, –5) radius 4 *(2 marks)*

(c) Centre (–3, 4) radius $\sqrt{7}$ *(2 marks)*

(d) Centre (8, 15) radius 17

Does this circle pass through the origin?

Show working to support your answer. *(4 marks)*

Mark Scheme

(a)	$x^2 + (y - 3)^2 = 4$	B2	B1 LHS, B1 RHS
(b)	$(x - 1)^2 + (y + 5)^2 = 16$	B2	B1 LHS, B1 RHS
(c)	$(x + 3)^2 + (y - 4)^2 = 7$	B2	B1 LHS, B1 RHS
(d)	$(x - 8)^2 + (y - 15)^2 = 289$	B2	B1 LHS, B1 RHS
	$(–8)^2 + (–15)^2$	M1	oe
	64 + 225 = 289, Yes	A1	

Question 2

Write down the centre and radius of each of these circles.

(a) $x^2 + y^2 = 36$ *(2 marks)*

(b) $(x - 3)^2 + (y - 4)^2 = 100$ *(2 marks)*

(c) $(x + 5)^2 + y^2 = 3$ *(2 marks)*

Mark Scheme

(a)	$(r) = 6$ (centre =) (0, 0)	B2	B1 For each
(b)	$(r) = 10$ (centre =) (3, 4)	B2	B1 For each
(c)	$(r) = \sqrt{3}$ (centre =) (–5, 0)	B2	B1 For each

Question 3 (non-calculator)

AB is the diameter of a circle.

A is (–3, 6) and *B* is (5, 12).

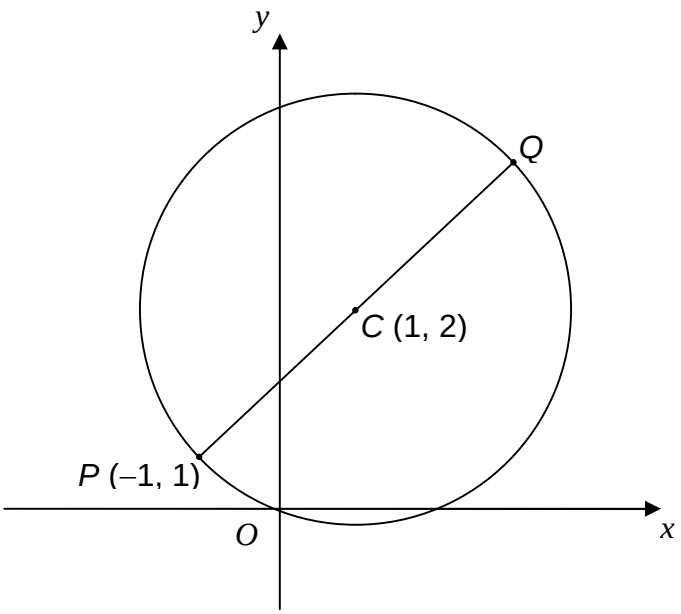
Work out the equation of the circle. *(5 marks)*

Mark Scheme

$\frac{-3+5}{2}$ or $\frac{6+12}{2}$	M1	
(1, 9)	A1	
$\sqrt{(5-1)^2+(12-9)^2}$	M1	oe ft Their centre
5	A1	
$(x-1)^2+(y-9)^2=25$	A1 ft	ft Their centre and radius

Question 4 (non-calculator)

PQ is a diameter of a circle, centre *C*.



Not drawn accurately

(a) Work out the coordinates of *Q*. *(1 mark)*

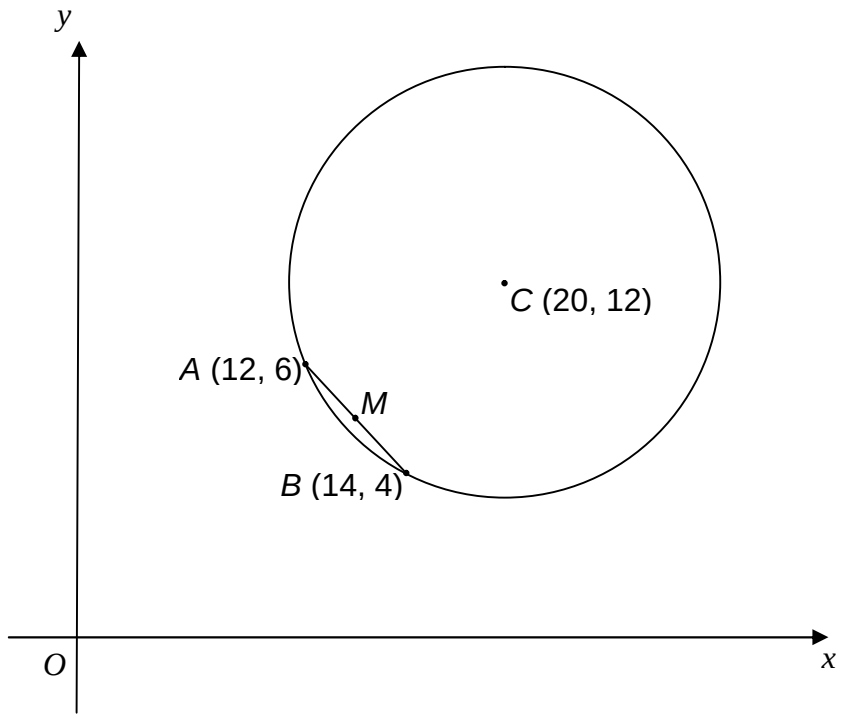
(b) Work out the equation of the circle. *(3 marks)*

Mark Scheme

(a)	(3, 3)	B1	
(b)	$\sqrt{2^2 + 1^2}$	M1	oe
	$\sqrt{5}$	A1	
	$(x - 1)^2 + (y - 2)^2 = 5$	B1 ft	ft Their radius

Question 5 (non-calculator)

A (12, 6) and *B* (14, 4) are two points on a circle, centre *C* (20, 12).



Not drawn accurately

(a) Work out the coordinates of the midpoint *M*, of *AB*. *(2 marks)*

(b) Show that the length $CM = 7\sqrt{2}$ (*3 marks*)

(c) Work out the radius of the circle. *(2 marks)*

Mark Scheme

(a) $\frac{12+14}{2}$ or $\frac{6+4}{2}$ M1

(13, 5) A1

(b) $\sqrt{(20-13)^2+(12-5)^2}$ M1 ft Their M

$\sqrt{98}$ A1 $\sqrt{7^2+7^2}$

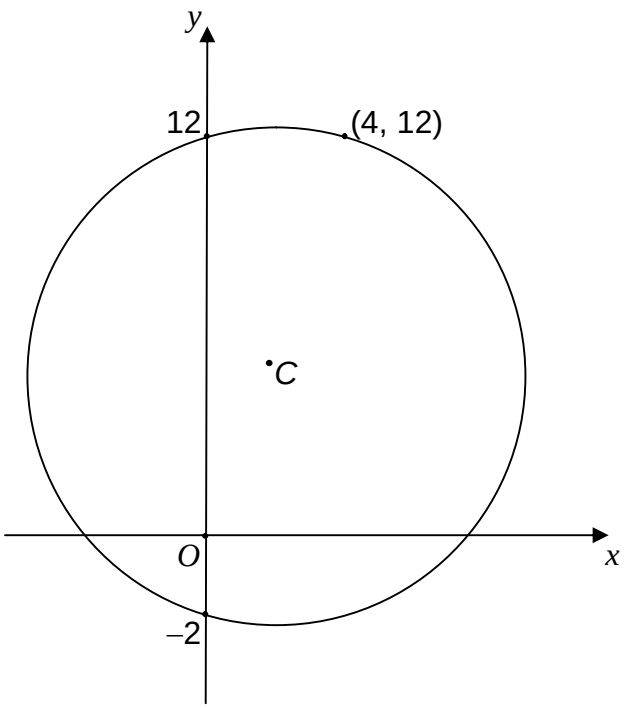
$\sqrt{49\times 2} = 7\sqrt{2}$ A1 $\sqrt{7^2(1+1)} = 7\sqrt{2}$

(c) $\sqrt{(20-12)^2+(12-6)^2}$ M1 oe

10 A1

Question 6

(0, –2), (0, 12) and (4, 12) are three points on a circle, centre *C*.



Not drawn accurately

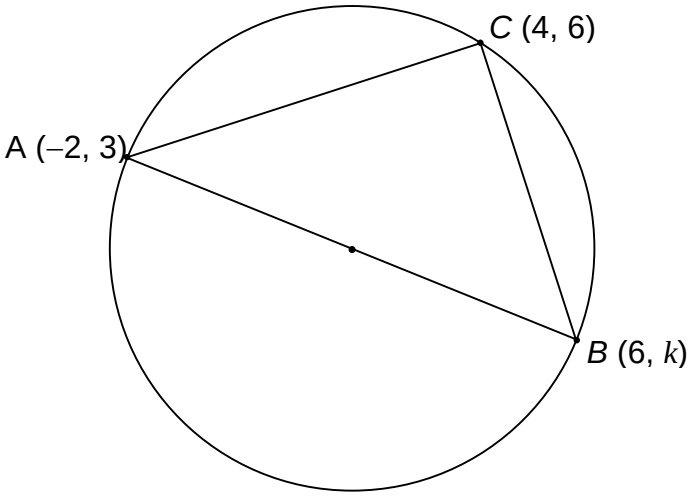
Work out the coordinates of *C*. *(3 marks)*

Mark Scheme

$\frac{-2+12}{2}$	M1
$\frac{0+4}{2}$	M1
C (2, 5)	A1

Question 7

AB is a diameter of the circle *ABC*.



Not drawn accurately

Work out the value of k. *(5 marks)*

Mark Scheme

Gradient $AC = \frac{6-3}{4--2}$	M1	oe
$= \frac{3}{6}$ $\left(= \frac{1}{2}\right)$	A1	oe
Gradient $BC = -2$	B1 ft	
$\frac{6-k}{4-6} = -2$	M1	
$k = 2$	A1	

Question 8

A circle has equation $(x - 5)^2 + (y - 4)^2 = 100$

Show that the point (13, –2) lies on the circle. *(2 marks)*

Mark Scheme

$(13 - 5)^2 + (-2 - 4)^2$	M1	
$64 + 36 = 100$	A1	

Question 9

The point (13, –2) lies on the circle $(x - a)^2 + (y - 4)^2 = 100$

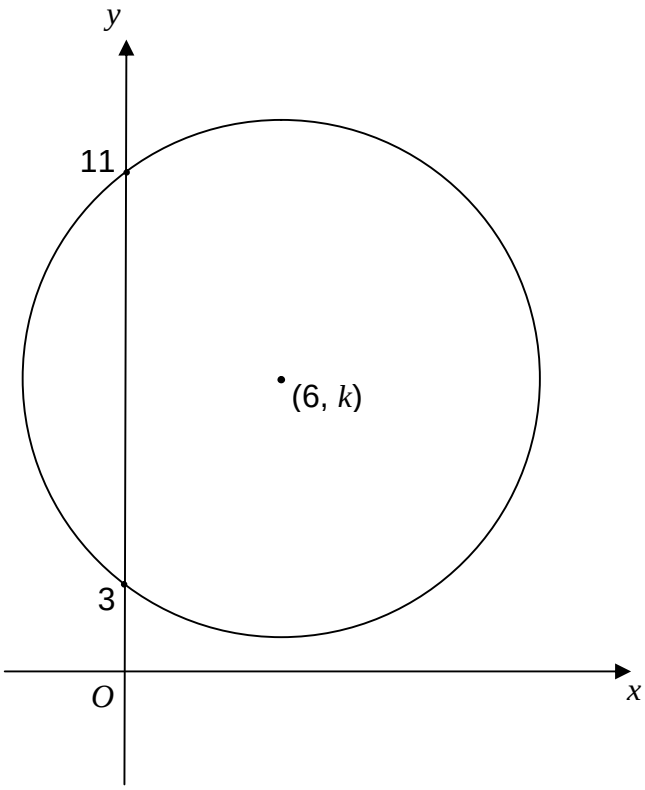
Work out the two possible values of a. *(5 marks)*

Mark Scheme

$(13 - a)^2 + (-2 - 4)^2 = 100$	M1	
$169 - 13a - 13a + a^2 + 36$ (= 100)	M1	Allow 1 error
$a^2 - 26a + 105 = 0$	A1	
$(a - 5)(a - 21) = 0$	M1	
$a = 5$ and $a = 21$	A1	

Question 10

A circle passes through the points (0, 3) and (0, 11) and has centre (6, k)



Not drawn accurately

(a) Work out the value of k.

(b) Hence find the equation of the circle. *(5 marks)*

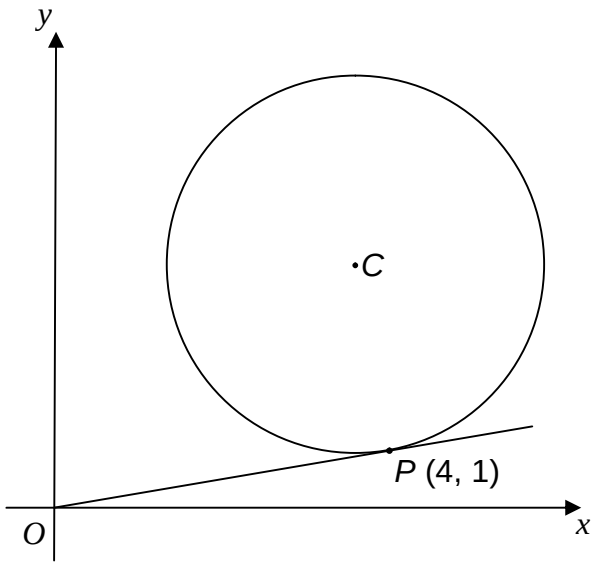
Mark Scheme

(a)	$\frac{3 + 11}{2}$	M1	oe eg, 3 + 4
	$k = 7$	A1	
(b)	$\sqrt{6^2 + (7 - 3)^2}$	M1	oe ft Their k
	$\sqrt{52}$	A1	
	$(x - 6)^2 + (y - 7)^2 = 52$	A1 ft	ft Their k and their radius

Question 11 (non-calculator)

The equation of this circle, centre C, is $(x - 3)^2 + (y - 5)^2 = 17$

P (4, 1) is a point on the circle.



Not drawn accurately

(a) Show working to explain why OP is a tangent to the circle. *(5 marks)*

(b) Show that the length OP is equal to the radius of the circle. (*3 marks*)

Mark Scheme

(a)

C is (3, 5)	B1
Gradient $CP = \frac{5 - 1}{3 - 4}$	M1
-4	A1
Gradient $OP = \frac{1}{4}$	B1
$-4 \times \frac{1}{4} = -1$	A1
So perpendicular (ie, tangent)	

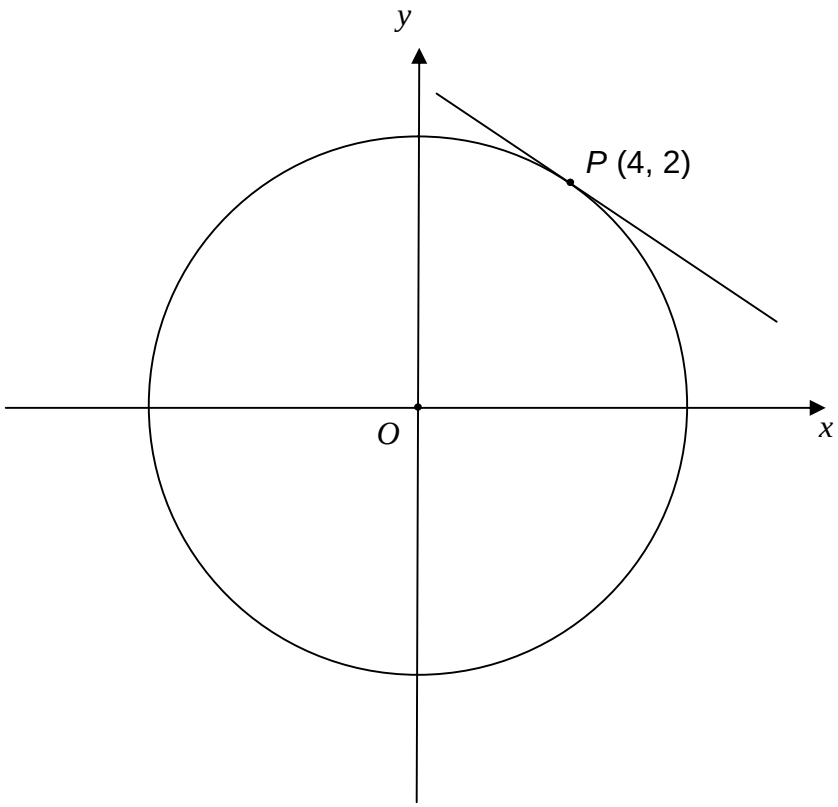
(b)

$r = \sqrt{17}$	B1
$OP = \sqrt{4^2 + 1^2}$	M1
$= \sqrt{17}$	A1

Question 12 (non-calculator)

The equation of this circle is $x^2 + y^2 = 20$

P (4, 2) is a point on the circle.



Not drawn accurately

Work out the equation of the tangent to the circle at P.

Give your answer in the form $y = mx + c$ (*3 marks*)

Mark Scheme

Gradient $OP = \frac{2}{4}$ $\left(= \frac{1}{2}\right)$ B1

Gradient of tangent = –2 B1 ft

$y - 2 = -2(x - 4)$ M1

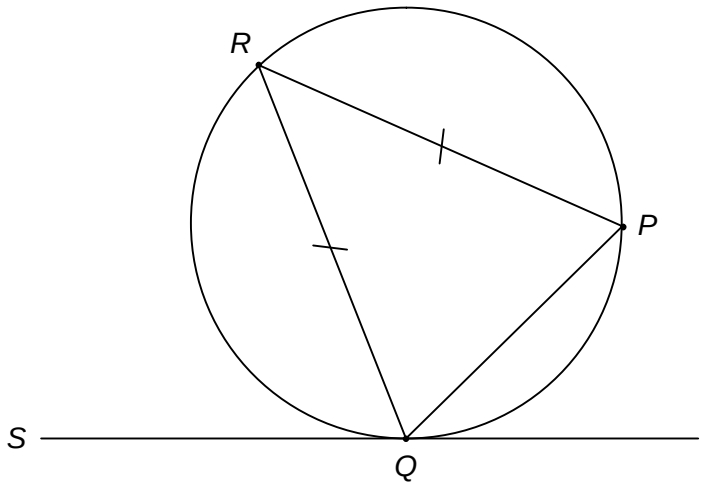
$y = -2x + 10$ A1

2 Geometric Problems and Proof

Question 1

SQ is a tangent to the circle at *Q*.

PR = *QR*



Not drawn accurately

Prove that *RQ* bisects angle *PQS*. *(3 marks)*

Mark Scheme

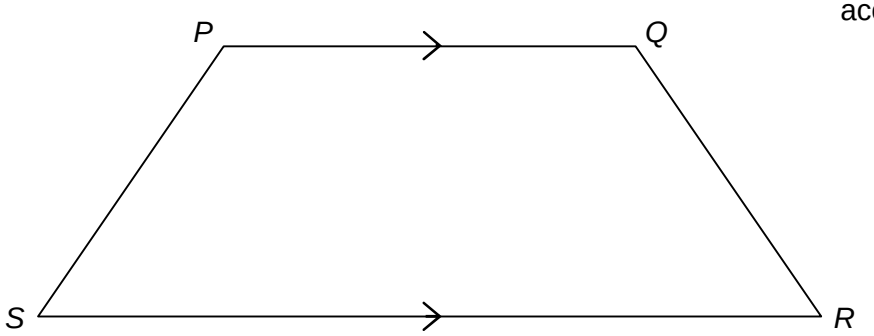
Let angle *SQR* = x	M1	Any order of angles
$\therefore$ angle *RPQ* = x alternate segment		
$\therefore$ angle *RQP* = x isosceles triangle	M1	
$\therefore \angle RQS = \angle RQP$	A1	SC2 'Correct' solution without reasons

Question 2

PQRS is a trapezium.

Angle *PSR* = angle *QRS*

Not drawn accurately



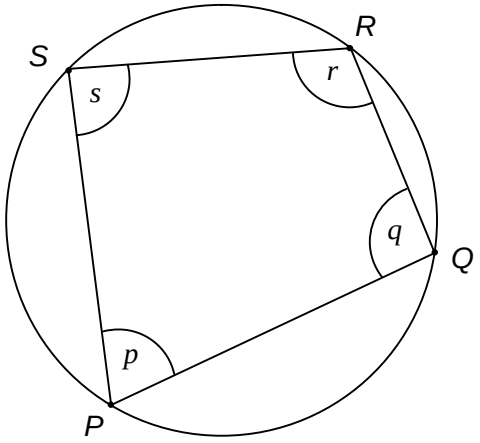
Prove that *PQRS* is a cyclic quadrilateral. *(3 marks)*

Mark Scheme

Let angle *PSR* = x = angle *QRS*	M1	$\angle PQR = 180 - x$
$\therefore \angle SPQ = 180 - x$ Allied angles on parallel lines		
$\therefore \angle SPQ + \angle QRS = 180$	A1	$\angle PSR + \angle PQR = 180$
PQRS is a cyclic quadrilateral (converse of) opposite angles add up to 180°	A1	SC2 'Correct' solution without reasons

Question 3

$p : q : r = 4 : 6 : 5$



Not drawn accurately

Work out s. *(5 marks)*

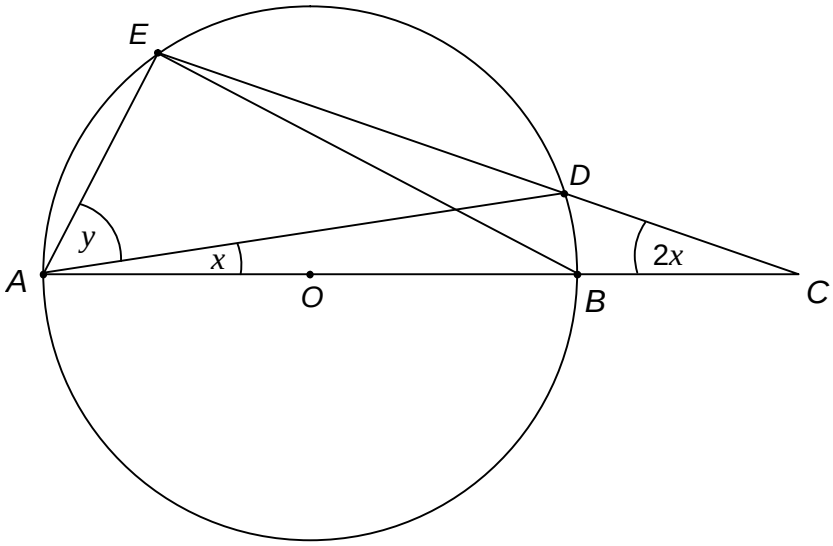
Mark Scheme

$p + r = 180$	M1	
$4x + 5x = 180$	M1	oe
$(9x = 180)$ $x = 20$	A1	
$6x = 120$	M1	ft Their x
$s = 60$	A1 ft	ft Their x

Question 4

O is the centre of the circle.

AOBC and *EDC* are straight lines.



Not drawn accurately

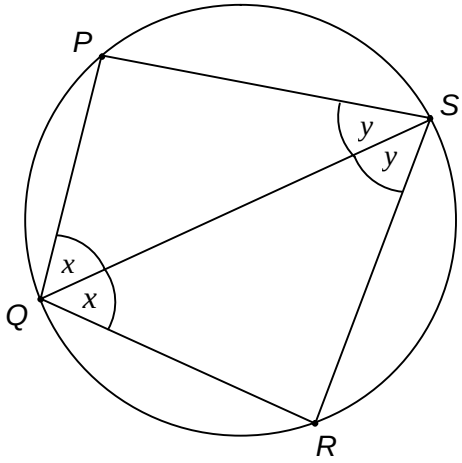
Prove that $4x + y = 90$ *(4 marks)*

Mark Scheme

$\angle BED = x$ angles in same segment	M1	
$\angle AEB = 90°$ angle in semicircle = 90°	A1	
In ΔACE $y + x + 2x + x + 90 = 180$ angle sum of a triangle = 180	A1	
$y + 4x = 180 - 90$ $= 90$	A1	SC2 'Correct' solution without reasons

Question 5

QS bisects both of the angles *PSR* and *PQR*.



Not drawn accurately

Prove that *QS* is a diameter of the circle. *(4 marks)*

Mark Scheme

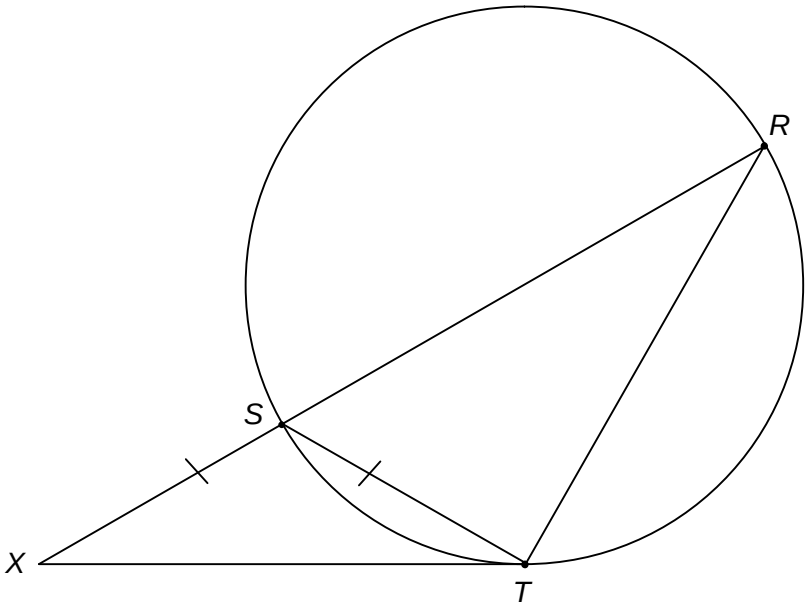
$2x + 2y = 180$ opposite angles of a cyclic quadrilateral = 180	M1	
$x + y = 90$	A1	
$\therefore \angle QPS = 90$ angle sum of triangle = 180	A1	
QS is diameter (converse of) angle in a semicircle = 90)	A1	SC2 'Correct' solution without reasons

Question 6

RSX is a straight line.

XT is a tangent to the circle at *T*.
SX = *ST*

Not drawn accurately



Prove that triangle *RXT* is isosceles. *(3 marks)*

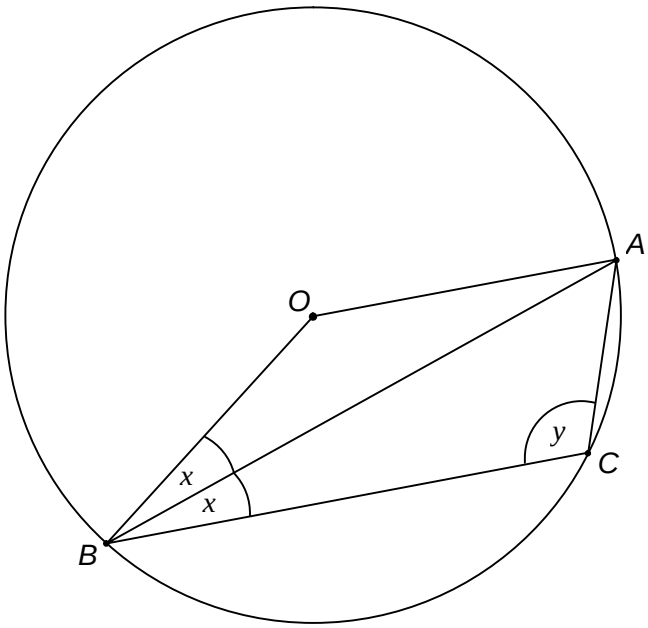
Mark Scheme

Let $\angle SXT = x$	M1	
$\therefore \angle STX = x$ isosceles triangle		
$\therefore \angle SRT = x$ alternate segment	M1	
$\therefore$ triangle *RXT* = is isosceles - 2 base angles equal	A1	SC2 'Correct' solution without reasons

Question 7

O is the centre of the circle.

AB bisects angle *OBC*.



Not drawn accurately

Prove that $y = 90 + x$ *(5 marks)*

Mark Scheme

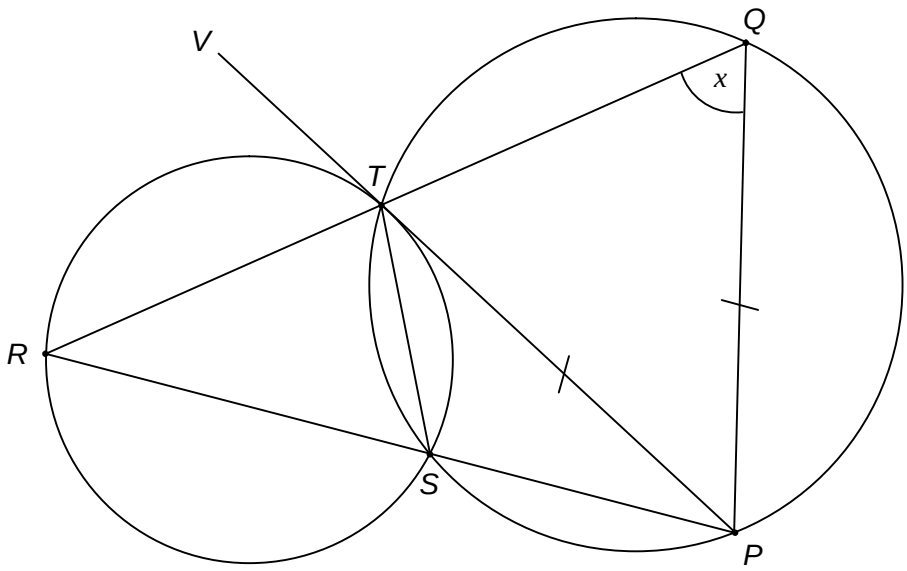
$\angle OAB = x$ isosceles triangle	M1	
$\angle BOA = 180 - 2x$ angle sum of triangle = 180	M1	
Reflex $\hat{BOA} = 360 - (180 - 2x)$	M1	
(Angles at a point = 360) $= 180 + 2x$	A1	
$y = 90 + x$ Angle at centre = 2 × angle at circumference	A1	SC3 'Correct' solution without reasons

Question 8

RTQ, RSP and *PTV* are all straight lines.

PT = *PQ*

Not drawn accurately



Prove that *PTV* is a tangent to circle *RST* at *T* *(5 marks)*

Mark Scheme

$\angle QTP = x$ isosceles triangle	M1	
$\angle VTR = x$ vertically opposite angles equal	M1	
$\angle TQP = x = \angle RST$ exterior angle of cyclic quadrilateral = opposite interior angle	M1	oe
$\therefore \angle VTR = \angle RST$	A2	SC3 'Correct' solution without reasons

PVT is tangent

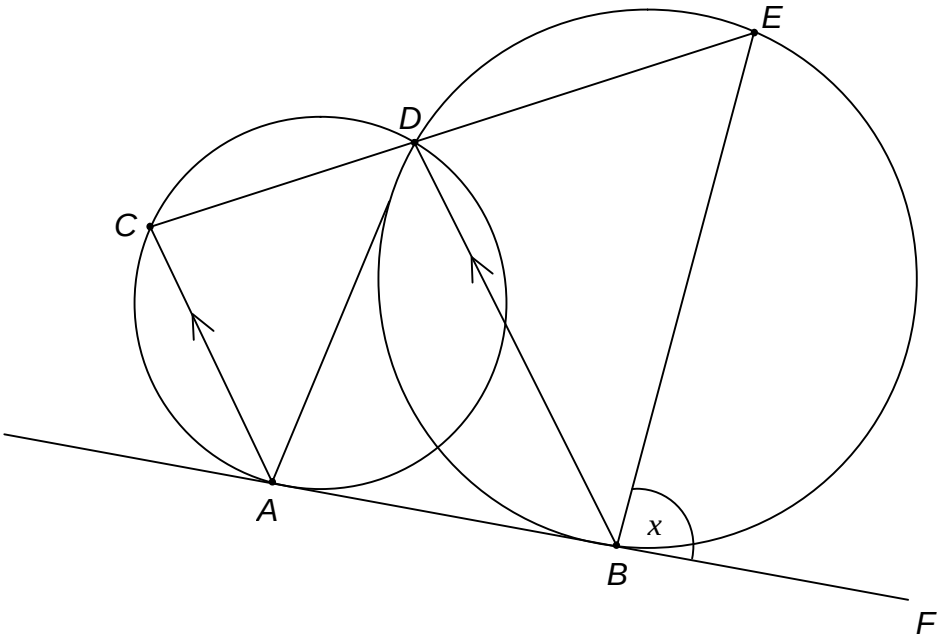
(converse of) alternate segment theorem

Question 9

ABF is a common tangent to the two circles at *A* and *B*.

CDE is a straight line.

AC is parallel to *BD*.



Not drawn accurately

Prove that *AD* is parallel to *BE*. *(5 marks)*

Mark Scheme

$\angle EDB = x$ alternate segment	M1	
$\therefore \angle DCA = x$ corresponding angles equal	M1	
$\therefore \angle DAB = x$ alternate segment	M1	
ie, $\angle DAB = \angle EBF$	A2	SC3 'Correct' solution without reasons
$\therefore$ *AD* is parallel to *BE*		
(converse of) corresponding angles equal		

3 Algebraic Proof

Question 1

Prove that $4(p - 3) - 2(2p - 1)$ is always a negative integer. *(2 marks)*

Mark Scheme

$4p - 12 - 4p + 2$	M1	4 terms with 3 correct
-10	A1	

Question 2

Prove that $8(y + 3) + 3(2 - y)$ is a multiple of 5 when y is a positive integer. *(3 marks)*

Mark Scheme

$8y + 24 + 6 - 3y$ or $5y + 30$	M2	M1 4 terms with 3 correct
$5y + 30$ and $5(y + 6)$	A1	oe eg, $5y + 30$ and states both terms divisible by 5

Question 3

a is a positive integer.

Prove that $4a^2(2a + 1) - (2a)^2$ is a cube number. *(3 marks)*

Mark Scheme

$8a^3 + 4a^2 - 4a^2$ or $8a^3$	M2	M1 3 terms with 2 correct
$8a^3$ and $(2a)^3$	A1	oe eg, $8a^3$ and states that 8 is a cube number

Question 4

a and b are positive integers.
$a < b$

Prove that $\frac{ax+3a}{bx+3b} < 1 \qquad x \neq -3$ *(3 marks)*

Mark Scheme

$a(x + 3)$ or $b(x + 3)$	M1	
$\frac{a(x+3)}{b(x+3)}$ and cancelling seen	A1	
$\frac{a}{b}$ and explains that as numerator is smaller than denominator value will be < 1	A1	oe

Question 5

(a) Express $x^2 + 6x + 11$ in the form $(x + a)^2 + b$ where a and b are integers. *(2 marks)*

(b) Hence, prove that $x^2 + 6x + 11$ is always positive. *(2 marks)*

Mark Scheme

(a)	$a = 3$	B1	
	$b = 2$	B1 ft	ft 11 – their a^2
(b)	$(x + 3)^2 \geq 0$	M1	oe Allow their a
	Adding 2 means always positive	A1	Must have $a = 3$ and $b = 2$

Question 6

Prove that, for all values of x, $x^2 + 2x + 6 > 0$ *(4 marks)*

Mark Scheme

$(x + 1)^2$	B1	
$(x + 1)^2 + 5$	B1 ft	ft Their $(x + 1)^2$
$(x + 1)^2 \geq 0$	M1	oe Allow their 1
Adding 5 means always positive	A1	Must have $(x + 1)^2 + 5$

Question 7

$f(x) = (2x + 3)^2 + 8(x + 2)$ for all values of x.

Prove that there is exactly one value of x for which $f(x) = 0$ *(4 marks)*

Mark Scheme

$4x^2 + 6x + 6x + 9 + 8x + 16$ or $4x^2 + 20x + 25$	M2	M1 Allow one error in expansions
$4x^2 + 20x + 25$ and $(2x + 5)^2$	A1	oe eg, $4x^2 + 20x + 25$ and $(2x + 5)(2x + 5)$
Explains that only solution is $(x =) -2.5$	A1	oe eg, explains that because the brackets are the same there is exactly one solution

Question 8

The nth term of a sequence is $\frac{1}{2}n(n+1)$

(a) Work out an expression for the $(n-1)$th term of the sequence.

Give your answer in its simplest form. *(2 marks)*

(b) Hence, or otherwise, prove that the sum of any consecutive pair of terms of the sequence is a square number. *(3 marks)*

Mark Scheme

(a)	$\frac{1}{2}(n-1)(n-1+1)$	M1	
	$\frac{1}{2}n(n-1)$	A1	oe eg, $\frac{1}{2}n^2 - \frac{1}{2}n$
(b)	$\frac{1}{2}n(n+1) + \frac{1}{2}n(n-1)$	M1	$\frac{1}{2}n(n+1)$ + their (a)
	$\frac{1}{2}n^2 + \frac{1}{2}n + \frac{1}{2}n^2 - \frac{1}{2}n$	M1	Expands brackets ft Their (a)
	n^2	A1	
Alt (b)	$\frac{1}{2}n(n+1) + \frac{1}{2}(n+1)(n+1+1)$	M1	oe
	$\frac{1}{2}n^2 + \frac{1}{2}n + \frac{1}{2}n^2 + n + \frac{1}{2}n + 1$	M1	Expands brackets oe eg, $n^2 + 2n + 1$ ft Their $\frac{1}{2}(n+1)(n+1+1)$
	$(n+1)^2$	A1	

Question 9

Prove that $\frac{x^2-4}{5x-10} \times \frac{10x^2}{x+2}$ is always positive. *(5 marks)*

Mark Scheme

$\frac{(x+2)(x-2)}{5(x-2)}$	M2	M1 For either numerator or denominator factorised correctly
At least one correct cancellation in the product	M1	
$2x^2$	A1	oe eg, $\frac{10x^2}{5}$
Explains that 2 > 0 and $x^2 \geq 0$ so $2x^2$ always positive	A1	oe eg, Explains that 10 > 0 and 5 > 0 and $x^2 \geq 0$ so $\frac{10x^2}{5}$ always positive

Question 10

f(n) = $n^2 - n$

Prove that f($3n$) + f($n + 1$) = $kn(5n - 1)$ where k is an integer. *(3 marks)*

Mark Scheme

$(3n)^2 - 3n + \{(n+1)^2 - (n+1)\}$	M1	oe $9n^2 - 3n$ or $n^2 + n + n + 1 - n - 1$
$9n^2 - 3n + n^2 + n + n + 1 - n - 1$	A1	oe eg, $10n^2 - 2n$
$10n^2 - 2n$ and $2n(5n - 1)$	A1	oe eg $10n^2 - 2n$ and $k = 2$

4 Trigonometry

Question 1 (non-calculator)

Work out the exact value of $\sin 60° + \sin 120° + \sin 270°$.

Give your answer in its simplest form. *(3 marks)*

Mark Scheme

$\sqrt{3}/2 + \sqrt{3}/2 - 1$	M1	Any 2 values correctly stated in surd form
$\sqrt{3}/2 + \sqrt{3}/2 - 1$	M1	All 3 values correctly stated in surd form
$\sqrt{3} - 1$	A1	

Question 2 (non-calculator)

Are these statements true or false?

	True	**False**
$\sin 37° = \sin 127°$	☐	☐
$\cos 54° = \cos 306°$	☐	☐
$\sin 135° = \cos 135°$	☐	☐
$\tan 126° = \tan 306°$	☐	☐

(4 marks

Mark Scheme

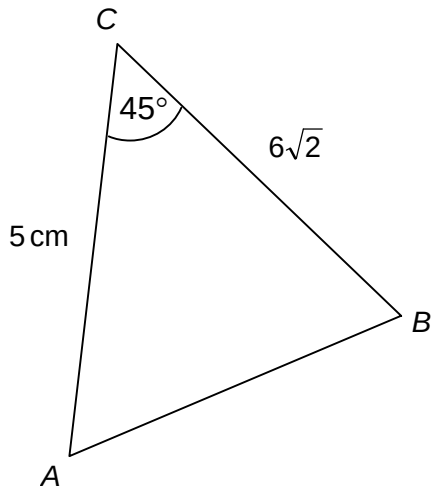
False	A1
True	A1
False	A1
True	A1

Question 3 (non-calculator)

Work out the area of triangle *ABC*.

Write your answer in its simplest form.

Not drawn accurately



(3 marks)

Mark Scheme

Evidence that sin 45° = 1/√2	B1
Area = $\frac{1}{2} \times 5 \times 6\sqrt{2} \times \sin 45°$	M1
15	A1

Question 4 (calculator or non-calculator)

Show that $\tan^2\theta \equiv \dfrac{1}{\cos^2\theta} - 1$ *(3 marks)*

Mark Scheme

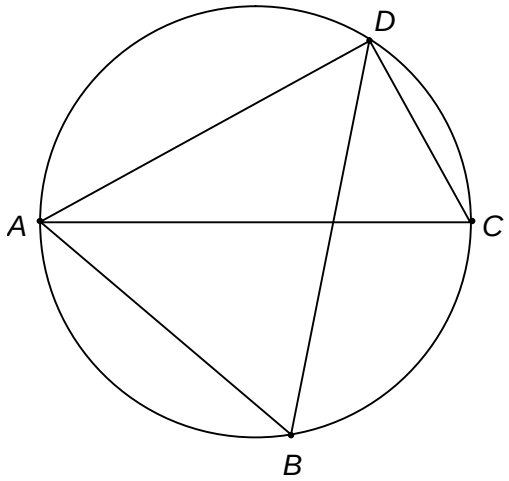
	$\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$ seen	M1	
	$\dfrac{\sin^2\theta}{\cos^2\theta} \equiv \dfrac{1-\cos^2\theta}{\cos^2\theta}$	M1	
	$\tan\theta \equiv \dfrac{1}{\cos^2\theta} - 1$	A1	Accurate method with clear steps is required for all 3 marks
Alt	$\dfrac{1-\cos^2\theta}{\cos^2\theta}$	M1	oe
	$\dfrac{\sin^2\theta}{\cos^2\theta}$	M1	
	$\tan^2\theta$	A1	Accurate method with clear steps is required for all 3 marks

Question 5 (calculator)

AC is a diameter of the circle.

AC = 5 cm, *AD* = 4 cm

Not drawn accurately



Work out angle *ABD*. (*4 marks*)

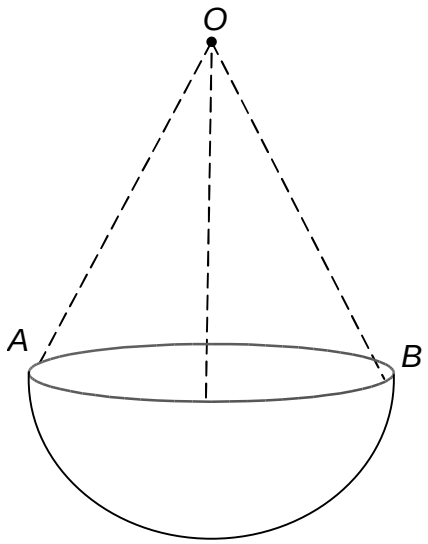
Mark Scheme

Evidence that angle *ADC* is a right angle	M1	
$\sin ACD = \frac{4}{5}$	M1	
ACD = [53.1, 53.13010235]	A1	Allow 53 with method seen
Angle ABD = [53.1, 53.13010235]	B1 ft	ft From 3rd mark their angle *ACD*

Question 6 (calculator)

A hanging basket is made from a hemisphere and three chains.
The radius of the hemisphere is 10 cm.
Each chain is 30 cm long.
The chains are equally spaced around the rim of the hemisphere.

Work out angle *AOB*.



(5 marks)

Mark Scheme

A triangle formed with *A*, *B* and the centre of the hemisphere with 2 sides of 10 cm and an angle of 120°	M1	
$(AB^2 =)\ 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos 120$	M1	$2 \times 10 \times \sin 60$
$(AB =)$ [17.3, 17.321]	A1	oe eg, $\sqrt{300}$
$(\cos AOB =)\ \dfrac{30^2 + 30^2 - \text{their } AB^2}{2 \times 30 \times 30}$	M1	$2 \times \sin^{-1}(0.5 \text{ their } AB \div 30)$
[33.557, 33.6]	A1 ft	ft Their *AB* Accept 34 with correct method seen

Question 7 (calculator)

Solve the following equation for $0 < \theta < 360°$.

$\tan^2 \theta = 2$

(4 marks)

Mark Scheme

$\tan \theta = +\sqrt{2}$ or $\tan \theta = -\sqrt{2}$	M1	
[54.7,54.74] or [125.26,125.3]	A1	
180 + their [54.7,54.74] or 180 + their [125.26,125.3]	M1	
[54.7,54.74] and [125.26,125.3] and 180 + their [54.7,54.74] and 180 + their [125.26,125.3]	A1ft	All 4 solutions [54.7,54.74] and [125.26,125.3] must be correct ft For other two solutions

Question 8 (calculator)

Solve the following equation for $0 < \theta < 360°$.

$3\cos^2 \theta + 2\cos \theta - 1 = 0$

(5 marks)

Mark Scheme

$(3\cos \theta - 1)(\cos \theta + 1)$	M2	M2 Fully correct use of quadratic formula M1 $(a\cos \theta + b)(c\cos \theta + d)$ where $ac = 3$ and $bd = \pm 1$ or quadratic formula with one sign error
$\cos \theta = -1$ so $\theta = 180°$	A1	
$\cos \theta = \frac{1}{3}$ so $\theta =$ [70.5, 70.53]	A1	
$\theta = 289.5^0$	A1 ft	ft 360 – their [70.5, 70.53]

5 Matrices 1

Question 1

Work out

(a) $\begin{pmatrix} 4 & 2 \\ -3 & 5 \end{pmatrix}\begin{pmatrix} 7 \\ 1 \end{pmatrix}$ **(b)** $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} -3 \\ -4 \end{pmatrix}$ **(c)** $2\begin{pmatrix} 5 & -2 \\ 6 & -3 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ **(e)** $6\begin{pmatrix} -4 & 7 \\ -1 & -3 \end{pmatrix}$ **(f)** $\begin{pmatrix} 8 & 4 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} -3 \\ 6 \end{pmatrix}$

(12 marks)

Mark Scheme

Each question 2 marks. M1 for a correct row by column multiplication. A1 for the correct answer.

(a) $\begin{pmatrix} 30 \\ -16 \end{pmatrix}$ **(b)** $\begin{pmatrix} -15 \\ -20 \end{pmatrix}$ **(c)** $\begin{pmatrix} 10 & -4 \\ 12 & -6 \end{pmatrix}$

(d) $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ **(e)** $\begin{pmatrix} -24 & 42 \\ -6 & -18 \end{pmatrix}$ **(f)** $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Question 2

Work out

(a) $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 0 & 3 \\ 1 & -4 \end{pmatrix}$ **(b)** $\begin{pmatrix} -3 & -2 \\ -1 & 5 \end{pmatrix}\begin{pmatrix} -2 & 4 \\ 3 & 4 \end{pmatrix}$ **(c)** $\begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}\begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix}$

(d) $\begin{pmatrix} 10 & -7 \\ 9 & 8 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ -2 & 3 \end{pmatrix}$ **(e)** $\begin{pmatrix} 1 & -2 \\ 3 & -5 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ **(f)** $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 3 & -5 \end{pmatrix}$

(12 marks)

Mark Scheme

Each question 2 marks. M1 for a correct row by column multiplication. A1 for the correct answer.

(a) $\begin{pmatrix} -1 & 10 \\ 3 & -9 \end{pmatrix}$ **(b)** $\begin{pmatrix} 0 & -20 \\ 17 & 16 \end{pmatrix}$ **(c)** $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 34 & 19 \\ 2 & 60 \end{pmatrix}$ **(e)** $\begin{pmatrix} 0 & -5 \\ 1 & -11 \end{pmatrix}$ **(f)** $\begin{pmatrix} 11 & -19 \\ 13 & -22 \end{pmatrix}$

Question 3 (non-calculator)

Work out, giving your answers as simply as possible.

(a) $\begin{pmatrix} \sqrt{2} & 1 \\ -1 & 3\sqrt{2} \end{pmatrix}\begin{pmatrix} \sqrt{2} & 0 \\ -3 & -2\sqrt{2} \end{pmatrix}$

(b) $\begin{pmatrix} -\frac{1}{2} & -1 \\ \frac{3}{2} & 5 \end{pmatrix}\begin{pmatrix} -2 & 4 \\ -\frac{1}{2} & 3 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}^2$

(d) $\begin{pmatrix} 3\sqrt{3} & -4 \\ 2 & 3\sqrt{3} \end{pmatrix}\begin{pmatrix} \sqrt{3} & 1 \\ -4 & 0 \end{pmatrix}$

(e) $\begin{pmatrix} \frac{1}{3} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{4} \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

(f) $\begin{pmatrix} \sqrt{2} & 2 \\ 7 & \sqrt{3} \end{pmatrix}^2$

(17 marks)

Mark Scheme

3 marks per question. 1 mark for multiplication of row by column, 1 mark for 2 simplified elements, 1 for other 2 elements correct. Part **(c)** 2 marks.

(a) $\begin{pmatrix} -1 & -2\sqrt{2} \\ -10\sqrt{2} & -12 \end{pmatrix}$

(b) $\begin{pmatrix} \frac{3}{2} & -5 \\ -\frac{11}{2} & 21 \end{pmatrix}$

(c) $\begin{pmatrix} 23 & 16 \\ 56 & 39 \end{pmatrix}$

(d) $\begin{pmatrix} 25 & 3\sqrt{3} \\ -10\sqrt{3} & 2 \end{pmatrix}$

(e) $\begin{pmatrix} \frac{7}{6} & 3 \\ \frac{19}{12} & 3 \end{pmatrix}$

(f) $\begin{pmatrix} 16 & 2\sqrt{2}+2\sqrt{3} \\ 7\sqrt{2}+7\sqrt{3} & 17 \end{pmatrix}$

Question 4

Work out, giving your answers as simply as possible.

(a) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} p \\ p+1 \end{pmatrix}$

(b) $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} m \\ 2m \end{pmatrix}$

(d) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -a & 0 \\ 0 & a \end{pmatrix}$

(e) $\begin{pmatrix} 4t & 0 \\ 0 & 4t \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

(f) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

(13 marks)

Mark Scheme

Each question 2 marks. M1 for a correct row by column multiplication. A1 for the correct answer.

(f) 3 marks. 2 for 1 pair correctly multiplied, 1 for final answer.

(a) $\begin{pmatrix} -p \\ -p-1 \end{pmatrix}$

(b) $\begin{pmatrix} 3x \\ 3y \end{pmatrix}$

(c) $\begin{pmatrix} 2m \\ m \end{pmatrix}$

(d) $\begin{pmatrix} -2a & 0 \\ 0 & 2a \end{pmatrix}$

(e) $\begin{pmatrix} 12t & 0 \\ 0 & 12t \end{pmatrix}$

(f) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

Question 5

Work out, giving your answers as simply as possible.

(a) $\begin{pmatrix} 2x & -3 \\ -5 & 4x \end{pmatrix}\begin{pmatrix} x & 3x \\ -3 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} a & 3a \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 7 & 8 \\ -10 & 11 \end{pmatrix}$

(c) $\begin{pmatrix} x & 0 \\ 1 & x \end{pmatrix}^2$

(d) $\begin{pmatrix} y & y \\ -3 & x \end{pmatrix}\begin{pmatrix} 2 & 3y \\ 0 & 1 \end{pmatrix}$

(e) $\begin{pmatrix} a+1 & a \\ a+2 & a+1 \end{pmatrix}\begin{pmatrix} a+1 & -a \\ -a-2 & a+1 \end{pmatrix}$

(f) $\begin{pmatrix} 3x & -3 \\ -9 & x+1 \end{pmatrix}^2$

(14 marks)

Mark Scheme

(a) to **(d)** 2 marks each

(e) and **(f)** 3 marks each, 1 for a correct multiplication, 1 for two elements correct, 1 for all correct.

(a) $\begin{pmatrix} 2x^2+9 & 6x^2 \\ -17x & -15x \end{pmatrix}$

(b) $\begin{pmatrix} -23a & 41a \\ -24 & -5 \end{pmatrix}$

(c) $\begin{pmatrix} x^2 & 0 \\ 2x & x^2 \end{pmatrix}$

(d) $\begin{pmatrix} 2y & 3y^2+y \\ -6 & -9y+x \end{pmatrix}$

(e) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(f) $\begin{pmatrix} 9x^2+27 & -12x-3 \\ -36x-9 & x^2+2x+28 \end{pmatrix}$

6 Matrices 2

Question 1

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 7 & 4 \\ 5 & 3 \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix}$$

Work out

(a) **AB** **(b)** **BC** **(c)** 3**A**

(d) **BA** **(e)** –**C** **(f)** $\mathbf{B}\begin{pmatrix} 1 & -4 \\ -5 & 7 \end{pmatrix}$

(12 marks)

Mark Scheme

Each question 2 marks. M1 for a correct row by column multiplication. A1 for the correct answer.

(a) $\begin{pmatrix} 9 & 5 \\ 41 & 24 \end{pmatrix}$ **(b)** $\begin{pmatrix} -10 & 17 \\ -7 & 12 \end{pmatrix}$ **(c)** $\begin{pmatrix} 6 & -3 \\ 9 & 12 \end{pmatrix}$

(d) $\begin{pmatrix} 26 & 9 \\ 19 & 7 \end{pmatrix}$ **(e)** $\begin{pmatrix} 2 & -3 \\ -1 & 1 \end{pmatrix}$ **(f)** $\begin{pmatrix} -13 & 0 \\ -10 & 1 \end{pmatrix}$

Question 2

$$\mathbf{P} = \begin{pmatrix} -2 & 0 \\ 5 & 1 \end{pmatrix} \qquad \mathbf{Q} = \begin{pmatrix} -4 & 1 \\ 3 & -2 \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

Work out

(a) $\mathbf{P}^2$ **(b)** **QP** **(c)** 5**Q**

(d) **PC** **(e)** **IQ** **(f)** 3**I**

(12 marks)

Mark Scheme

Each question 2 marks. M1 for a correct row by column multiplication. A1 for the correct answer.

(a) $\begin{pmatrix} 4 & 0 \\ -5 & 1 \end{pmatrix}$ **(b)** $\begin{pmatrix} 13 & 1 \\ -16 & -2 \end{pmatrix}$ **(c)** $\begin{pmatrix} -20 & 5 \\ 15 & -10 \end{pmatrix}$

(d) $\begin{pmatrix} -6 \\ 13 \end{pmatrix}$ **(e)** $\begin{pmatrix} -4 & 1 \\ 3 & -2 \end{pmatrix}$ **(f)** $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

Question 3

$$\begin{pmatrix} -2 & a \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 3 \\ 7 \end{pmatrix} = \begin{pmatrix} 22 \\ 9 \end{pmatrix}$$

Work out the value of a. *(2 marks)*

Mark Scheme

$-6 + 7a = 22$	M1
$a = 4$	A1

Question 4

Work out the values of a, b and c.

$$\begin{pmatrix} 2 & a \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 2 & b \end{pmatrix} = \begin{pmatrix} 12 & 26 \\ c & 13 \end{pmatrix}$$

(3 marks)

Mark Scheme

$a = 5$	B1
$b = 4$	B1
$c = 5$	B1

Question 5

Work out the image of the point D (–1, 2) after transformation by the matrix $\begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$

(2 marks)

Mark Scheme

(4, 3)	B2	B1 For (4, ?), (?, 3) or $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

Question 6

The point $A(m, n)$ is transformed to the point A' $(-2, 0)$ by the matrix $\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$

Work out the values of m and n.

(4 marks)

Mark Scheme

$2m + 3n = -2$, $m + n =$	M1, A1	M1 For either A1 For both
Attempt to solve	M1	
$m = 2$, $n = -2$	A1	

Question 7

The matrix A represents a reflection in the line $y = x$.

Write down the matrix A.

The unit square is transformed by the matrix A and then by rotation through $-90°$ about O.

Work out the matrix representing the combined transformation.

(4 marks)

Mark Scheme

$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	B1	
Rotation $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	B1	
Combined $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	M1	Multiplication in correct order
$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	A1	

Question 8

Describe fully the transformation given by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

(2 marks)

Mark Scheme

Reflection, in the line $y = -x$ B1, B1

Question 9 (non-calculator)

The unit square *OABC* is transformed by the matrix $\begin{pmatrix} h & 0 \\ 0 & h \end{pmatrix}$ to the square *OA′B′C′*.

The area of *OA′B′C′* is 27.

Work out the exact value of h.

(3 marks)

Mark Scheme

Vertices of image A' $(h, 0)$ B' (h, h) $C'(0, h)$	B1	Any one correct
Area of $OA'B'C' = h^2$	M1	
$h = 3\sqrt{3}$	A1	

Question 10

$\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

The point P (2, 7) is transformed by matrix **BA** to **P′**.

Show that P lies on the line $7x + 2y = 0$

(3 marks)

Mark Scheme

$BA = \begin{pmatrix} -3 & 0 \\ 0 & 3 \end{pmatrix}$ B1

$\begin{pmatrix} -3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 7 \end{pmatrix} = \begin{pmatrix} -6 \\ 21 \end{pmatrix}$ B1

Show this satisfies $7x + 2y = 0$ M1

7 Inequalities

Question 1

$-6 < 3x \leq 6$

x is an integer

Write down all the possible values for x. *(2 marks)*

Mark Scheme

$-2 < x \leq 2$	M1	
$-1 \quad 0 \quad 1 \quad 2$	A2	A1 3 correct with none incorrect or 4 correct with one incorrect

Question 2

Solve $6x > 24 - 2x$ *(2 marks)*

Mark Scheme

$6x + 2x > 24$	M1	oe
$x > 3$	A1	

Question 3

Solve $4(2x - 1) < 2$ *(3 marks)*

Mark Scheme

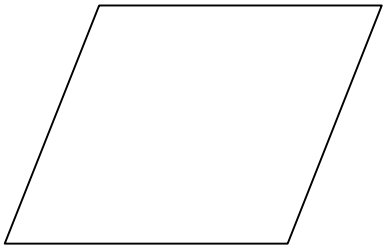
$8x - 4 < 2$	M1	oe $2x - 1 < \frac{2}{4}$ oe
$8x < 2 + 4$	M1	oe $2x < \frac{2}{4} + 1$ oe
$x < \frac{3}{4}$	A1	

Question 4

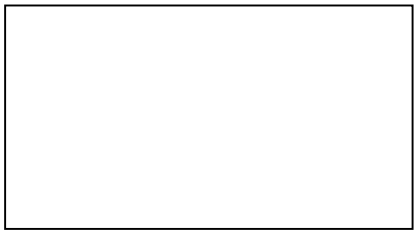
A rhombus and a rectangle are shown.

The perimeter of the rhombus is greater than the perimeter of the rectangle.

Not drawn accurately



$2y + 6$



$y + 4$

$2y + 10$

Show that $y > k$ where k is an integer. *(4 marks)*

Mark Scheme

$4(2y + 6) > 2y + 10 + 2y + 10 + y + 4 + y + 4$	M2	oe eg, $8y + 24 > 6y + 28$ M1 $4(2y + 6)$ or $2y + 10 + 2y + 10 + y + 4 + y + 4$
$8y - 6y > 28 - 24$	M1	oe
$y > 2$ or $k = 2$	A1	

Question 5

$p < -1$ and $q > 1$

Tick the correct box for each statement.

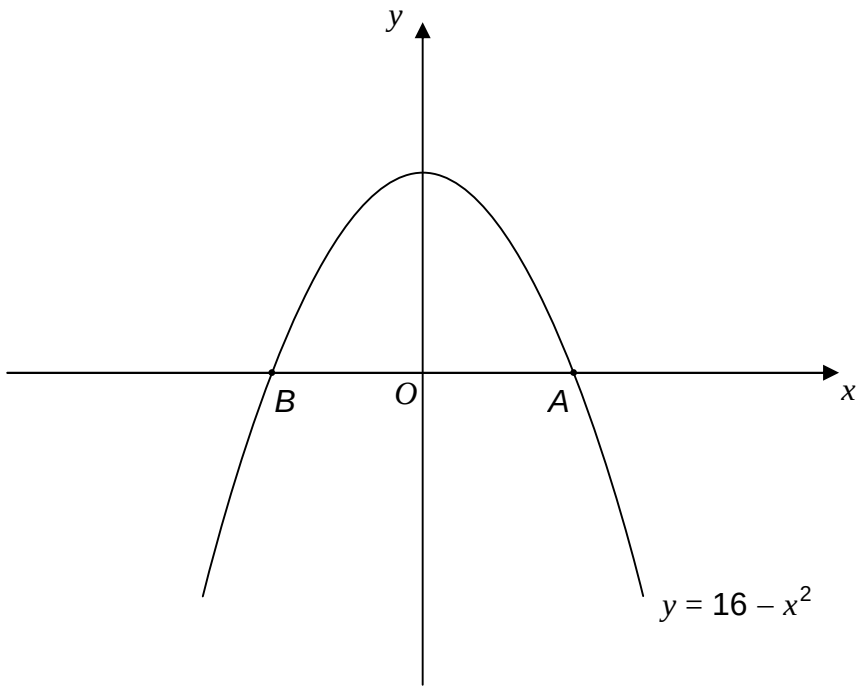
	Always true	Sometimes true	Never true
$5p < 0$	☐	☐	☐
$p^2 < 0$	☐	☐	☐
$p + q > 0$	☐	☐	☐
$-1 < \frac{q}{p} < 0$	☐	☐	☐

(4 marks)

Mark Scheme

Always Never Sometimes Sometimes	B4	B1	For each correct part

Question 6



(a) Write down the coordinates of points *A* and *B*. *(2 marks)*

(b) Hence, or otherwise, solve $16 - x^2 \geq 0$ *(2 marks)*

Mark Scheme

(a)	(4, 0)	B1	
	(–4, 0)	B1	SC1 4 and –4 seen
(b)	$-4 \leq x \leq 4$	B2 ft	ft Their 4 and their –4
			B1 ft $-4 < x < 4$
Alt (b)	$(4 + x)(4 - x)$ and –4 and 4	M1	
	$-4 \leq x \leq 4$	A1	

Question 7

(a) Factorise $x^2 + 3x$ *(1 mark)*

(b) Sketch $y = x^2 + 3x$

Label the x values of the points of intersection with the x-axis. *(2 marks)*

(c) Hence, or otherwise, solve $x^2 + 3x < 0$ *(2 marks)*

Mark Scheme

(a)	$x(x + 3)$	B1	
(b)	U-shaped parabola	M1	
	0 and – 3 labelled on x-axis	A1 ft	ft Their factors in (a)
(c)	$x < -3$ and $x > 0$	B2 ft	ft Their factors in (a)
			B1 ft $x \leq -3$ and $x \geq 0$

Question 8

Solve $(x – 5)(x + 2) \geq 0$ *(3 marks)*

Mark Scheme

5 and –2	B1	
Sketch of graph $y = (x – 5)(x + 2)$	M1	Sign diagram using their 5 and their –2
$x < -2$ and $x > 5$	A1	

Question 9

Solve $x^2 + 4x - 12 < 0$ *(4 marks)*

Mark Scheme

$(x + 6)(x - 2)$	M1	$(x + a)(x + b)$ where $ab = \pm 12$ or $a + b = \pm 4$
-6 and 2	A1	
Sketch of graph $y = (x + 6)(x - 2)$	M1	Sign diagram using their -6 and their 2
$-6 < x < 2$	A1	

Question 10

Solve $2x^2 - x - 3 < 0$ *(4 marks)*

Mark Scheme

$(2x - 3)(x + 1)$	M1	$(2x + a)(x + b)$ where $ab = \pm 3$ or $a + 2b = \pm 1$
$\frac{3}{2}$ and -1	A1	oe
Sketch of graph $y = (2x - 3)(x + 1)$	M1	Sign diagram using their $\frac{3}{2}$ and their -1
$-1 < x < \frac{3}{2}$	A1	

Question 11

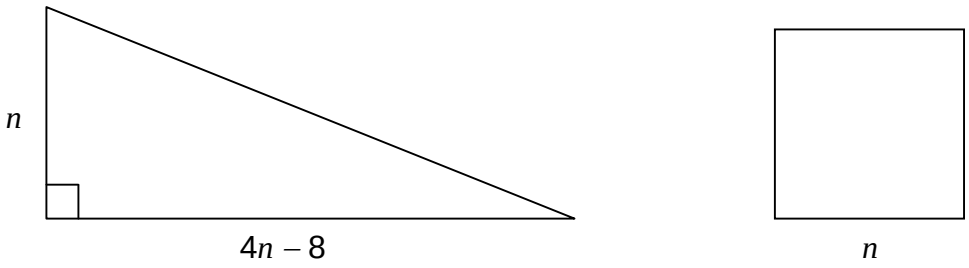
Solve $3x^2 > 14x - 8$ *(4 marks)*

Mark Scheme

$(3x - 2)(x - 4)$	M1	$(3x + a)(x + b)$ where $ab = \pm 8$ or $a + 3b = \pm 14$
$\frac{2}{3}$ and 4	A1	
Sketch of graph $y = (3x - 2)(x - 4)$	M1	Sign diagram using their $\frac{2}{3}$ and their 4
$x < \frac{2}{3}$ and $x > 4$	A1	

Question12

A triangle and a square are shown.



Work out the range of values of n for which

area of triangle < area of square

(5 marks)

Mark Scheme

$n^2 > \frac{1}{2}(4n - 8)n$	M1	oe
$0 > n^2 - 4n$	A1	
$n(n - 4)$	M1	Factorises their quadratic expression
Sketch of graph of $y = n(n - 4)$	M1	Sign diagram using their 0 and their 4
$0 < n < 4$	A1	

8 Functions

Question 1 (non-calculator)

$f(x) = 2x^3 - 250$

Work out x when $f(x) = 0$ *(3 marks)*

Mark Scheme

$2x^3 - 250 = 0$	M1	
$x^3 = \frac{250}{2}$	M1	oe
$x = 5$	A1	

Question 2

$f(x) = x^2 + ax - 8$

$f(-3) = 13$

Work out the value of a. *(3 marks)*

Mark Scheme

$(-3)^2 + a(-3) - 8 = 13$	M1	
$9 - 8 - 13 = 3a$	M1	oe Allow 1 error
$a = -4$	A1	

Question 3

$f(x) = x^2 + 3x - 10$

Show that $f(x + 2) = x(x + 7)$ *(3 marks)*

Mark Scheme

$(x + 2)^2 + 3(x + 2) - 10$	M1	
$x^2 + 2x + 2x + 4 + 3x + 6 - 10$	M1	oe Allow 1 error
$x^2 + 7x$ $= x(x + 7)$	A1	

Question 4

Work out the range for each of these functions.

(a) $f(x) = x^2 + 6$ for all x *(1 mark)*

(b) $f(x) = 3x - 5$ $-2 \leqslant x \leqslant 6$ *(2 marks)*

(c) $f(x) = 3x^4$ $x < -2$ *(1 mark)*

Mark Scheme

(a)	$f(x) \geqslant 6$	B1	
(b)	$-11 \leqslant f(x) \leqslant 13$	B1	B1 For –11 or 13 seen
(c)	$f(x) > 48$	B1	

Question 5

(a) $f(x) = \frac{x+2}{x-3}$

Give a reason why $x > 0$ is **not** a suitable domain for $f(x)$. *(1 mark)*

(b) Give a possible domain for $f(x) = \sqrt{x-5}$ *(1 mark)*

Mark Scheme

(a)	Not defined when $x = 3$ or cannot divide by 0 when $x = 3$	B1	oe
(b)	$x \geqslant a$ where $a \geqslant 5$ or $x > a$ where $a \geqslant 5$	B1	eg $x \geqslant 5$ $x > 6$ Allow list of x values if all are $\geqslant 5$

Question 6

f(x) = 3 – 2x $\quad a < x < b$

The range of f(x) is $\quad -5 < f(x) < 5$

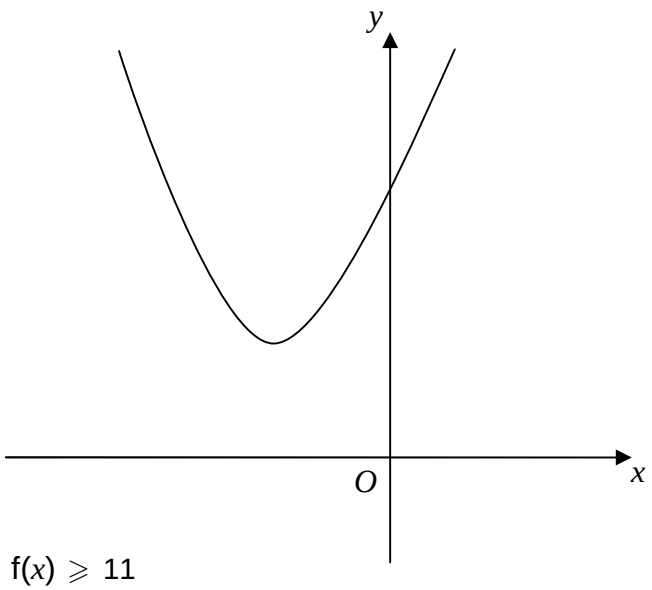
Work out a and b. *(3 marks)*

Mark Scheme

Either	$3 - 2x = -5$ or $3 - 2x = 5$	M1	
$a = -1$		A1	
$b = 4$		A1	SC2 $a = 4$, $b = -1$

Question 7

Here is a sketch of $f(x) = x^2 + 6x + a$ for all x, where a is a constant



The range of f(x) is $f(x) \geqslant 11$

Work out the value of a. *(3 marks)*

Mark Scheme

Attempt to complete the square in the form $(x + 3)^2$	M1	
$(x + 3)^2 - 9 + a$	A1	oe
$a = 20$	A1	

Question 8

(a) Factorise $x^2 - 5x - 14$ *(2 marks)*

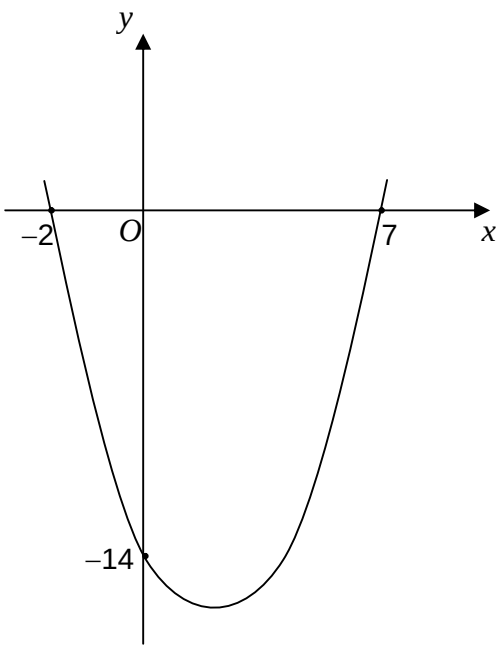
(b) Sketch the function $f(x) = x^2 - 5x - 14$ for all x.

Label the points of intersection with the x and y axes. *(3 marks)*

Mark Scheme

(a) $(x + a)(x + b)$ M1 $ab = -14$ or $a + b = -5$

$(x - 7)(x + 2)$ A1

(b) 

B3 B1 Curve through their (7, 0) and (–2, 0) (from 8(a))

B1 Curve through (0, –14)

B1 Smooth U shape

Question 9

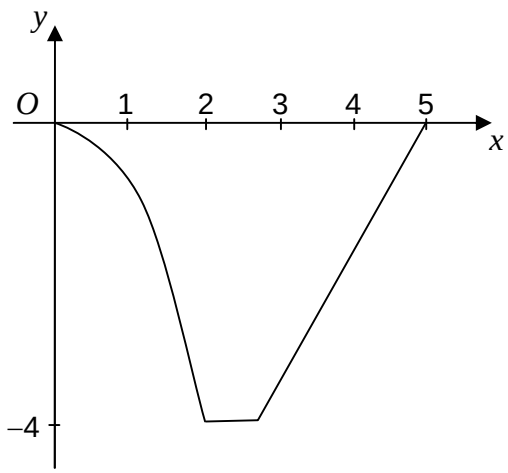
$f(x) = -x^2 \quad 0 \leqslant x < 2$

$-4 \quad 2 \leqslant x < 3$

$2x - 10 \quad 3 \leqslant x \leqslant 5$

Draw the graph of $f(x)$ for values of x from 0 to 5 *(3 marks)*

Mark Scheme

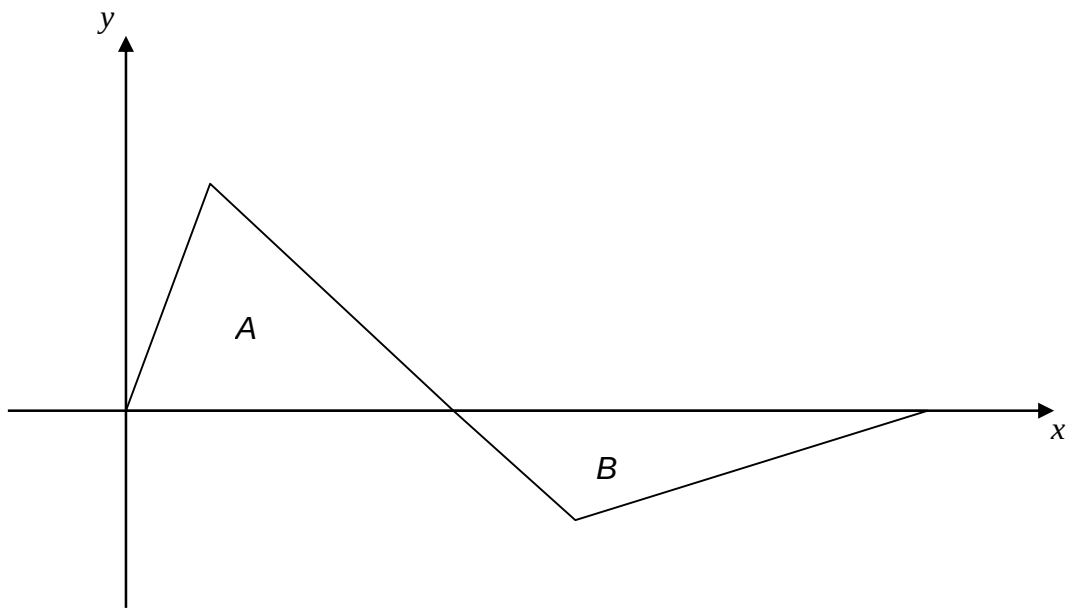


B3 B1 For each part

Question 10

Here is a sketch of the function f(x) for values of x from 0 to 7.

$$f(x) = \begin{cases} 2x & 0 \leqslant x < 1 \\ 3 - x & 1 \leqslant x < 4 \\ \frac{x-7}{3} & 4 \leqslant x \leqslant 7 \end{cases}$$



Show that

area of triangle A : area of triangle B = 3 : 2 *(4 marks)*

Mark Scheme

(3, 0) and (7, 0) marked or used	M1
(1, 2) and (4, –1) marked or used	M1
Either of their triangular areas calculated correctly	M1
$\frac{1}{2} \times 3 \times 2$ and $\frac{1}{2} \times 4 \times 1$ = 3 : 2	A1

9 Coordinate Geometry - Calculus

Question 1

For each of these straight lines, work out

(i) The gradient of the line *(1 mark for each part)*

(ii) The gradient of the line that is perpendicular to the given line *(1 mark for each part)*

(iii) The y-intercept of the line *(1 mark for each part)*

(a) $y = 5x - 4$ **(b)** $3y = 9 - 6x$ **(c)** $3y - 12 = 2x$

(d) $5x - 2y + 15 = 0$ **(e)** $\frac{x}{4} - \frac{y}{3} = 2$

Mark Scheme

(a)

5	B1	
$-\frac{1}{5}$	B1 ft	ft $\frac{-1}{\text{their } 5}$
-4	B1	

(b)

-2	B1	
$\frac{1}{2}$	B1 ft	ft $\frac{-1}{\text{their } -2}$
3	B1	

(c)

$\frac{2}{3}$	B1	
$-\frac{3}{2}$	B1 ft	ft $\frac{-1}{\text{their } \frac{2}{3}}$
4	B1	

(d)

$\frac{5}{2}$	B1	
$-\frac{2}{5}$	B1 ft	ft $\frac{-1}{\text{their } \frac{5}{2}}$
$\frac{15}{2}$	B1	

(e)

$\frac{3}{4}$	B1	
$-\frac{4}{3}$	B1 ft	ft $\frac{-1}{\text{their } \frac{3}{4}}$
-6	B1	

Question 2

For each of these straight line segments, *AB*, work out

(i) The mid-point of *AB* *(2 marks for each part)*

(ii) The gradient of *AB* *(1 mark for each part)*

(iii) The length of *AB*, giving your answer as an integer or a surd *(2 marks for each part)*

(a) $A = (-3, -4)$ $B = (4, 3)$ **(b)** $A = (-4, 1)$ $B = (1, 5)$ **(c)** $A = (5, -2)$ $B = (0, 10)$

(d) $A = (-2, -6)$ $B = (-6, 0)$ **(e)** $A = (1, 9)$ $B = (9, -6)$ **(f)** $A = (7, 1)$ $B = (-5, -3)$

Mark Scheme

(a)	$\left(\frac{1}{2}, -\frac{1}{2}\right)$	B2	B1 For each coordinate	**(b)**	$(-1\frac{1}{2}, 3)$	B2	B1 For each coordinate
	1	B1			$\frac{4}{5}$	B1	
	$\sqrt{(7^2 + 7^2)}$	M1			$\sqrt{(5^2 + 4^2)}$	M1	
	$\sqrt{98}$ or $7\sqrt{2}$	A1			$\sqrt{41}$	A1	
(c)	$(2\frac{1}{2}, 4)$	B2	B1 For each coordinate	**(d)**	$(-4, -3)$	B2	B1 For each coordinate
	$-\frac{12}{5}$	B1			$-\frac{3}{2}$	B1	
	$\sqrt{(5^2 + 12^2)}$	M1			$\sqrt{(4^2 + 6^2)}$	M1	
	13	A1			$\sqrt{52}$ or $2\sqrt{13}$	A1	
(e)	$(5, 1\frac{1}{2})$	B2	B1 For each coordinate	**(f)**	$(1, -1)$	B2	B1 For each coordinate
	$-\frac{15}{8}$	B1			$\frac{1}{3}$	B1	
	$\sqrt{(8^2 + 15^2)}$	M1			$\sqrt{(12^2 + 4^2)}$	M1	
	17	A1			$\sqrt{160}$ or $4\sqrt{10}$	A1	

Question 3

In each of these line segments, B lies between A and C.

Work out the coordinates of C in each case. *(2 marks for each part)*

(a) $A = (-1, 3)$ $B = (1, 1)$ and $AB : BC = 1 : 2$

(b) $A = (-4, -2)$ $B = (2, -5)$ and $AB : BC = 3 : 1$

(c) $A = (11, 0)$ $B = (1, -5)$ and $AB : BC = 5 : 3$

(d) $A = (-6, 2)$ $B = (0, 4)$ and $AB : BC = 2 : 3$

(e) $A = (2, -9)$ $B = (-3, 1)$ and $AB : BC = 5 : 4$

Mark Scheme

(a)	(5, –3)	B2	B1 For each coordinate
(b)	(4, –6)	B2	B1 For each coordinate
(c)	(–5, –8)	B2	B1 For each coordinate
(d)	(9, 7)	B2	B1 For each coordinate
(e)	(–7, 9)	B2	B1 For each coordinate

Question 4

Work out the coordinates of the points of intersection of the curve $y = x^2 + 7$ and the straight line $y = 5x + 1$ *(4 marks)*

Mark Scheme

$x^2 + 7 = 5x + 1$ or $x^2 - 5x + 6 = 0$	M1	
$(x - 2)(x - 3) = 0$	M1	Attempt to factorise the quadratic
(2, 11) or (3, 16)	A1 ft	ft Their factors
(2, 11) and (3, 16)	A1	

Question 5

Line L has equation $y + 3x = 7$

Line N is perpendicular to line L and passes through (3, –1).

Work out the equation of line N.

Give your answer in the form $y = ax + b$ *(4 marks)*

Mark Scheme

Gradient of $L = -3$	B1
Gradient of $N = \frac{1}{3}$	M1
$y - (-1) = \frac{1}{3}(x - 3)$	M1
$y = \frac{1}{3}x - 2$	A1

Question 6

Work out $\frac{dy}{dx}$ for each of the following

(a) $y = 7x + 3$ *(1 mark)*

(b) $y = 8 - 5x + x^2$ *(2 marks)*

(c) $y = 3x^3 + 4x$ *(2 marks)*

(d) $y = x^3 - 7x^2 + 10x - 1$ *(2 marks)*

(e) $y = 4x(x^2 + 2x - 3)$ *(3 marks)*

(f) $y = (3x - 5)(x + 8)$ *(3 marks)*

(g) $y = x(7 - x)(6 - 2x)$ *(3 marks)*

(h) $y = (x + 3)(x - 1)(x - 6)$ *(4 marks)*

Mark Scheme

(a)	$\frac{dy}{dx} = 7$	B1	
(b)	$\frac{dy}{dx} = 2x - 5$	B2	B1 For each term
(c)	$\frac{dy}{dx} = 9x^2 + 4$	B2	B1 For each term
(d)	$\frac{dy}{dx} = 3x^2 - 14x + 10$	B2	B1 For two terms correct
(e)	$y = 4x^3 + 8x^2 - 12x$	B1	
	$\frac{dy}{dx} = 12x^2 + 16x - 12$	B2 ft	B1 For two terms correct ft Their y = ...
(f)	$y = 3x^2 + 19x - 40$	B1	
	$\frac{dy}{dx} = 6x + 19$	B2 ft	B1 For one term correct ft Their y = ...
(g)	$y = 42x - 20x^2 + 2x^3$	B1	
	$\frac{dy}{dx} = 42 - 40x + 6x^2$	B2 ft	B1 For two terms correct ft Their y = ….
(h)	$y = x^3 - 4x^2 - 15x + 18$	B2	B1 For four terms, three of which are correct
	$\frac{dy}{dx} = 3x^2 - 8x - 15x$	B2 ft	B1 For two terms correct ft Their y = ...

Question 7

A curve has equation $y = x^3 + x^2 + 2x - 4$

Work out the equation of the tangent to this curve where $x = -2$

Give your answer in the form $y = ax + b$ *(5 marks)*

Mark Scheme

$\frac{dy}{dx} = 3x^2 + 2x + 2$	M1	
(when $x = -2$) gradient tgt = 10	A1	
(when $x = -2$) $y = -12$	B1	
$y - (-12) = 10(x - (-2))$	M1	oe
$y = 10x + 8$	A1 ft	ft Their m and c

Question 8

A curve has equation $y = x^3 + 2x^2 - 9x + 3$

Work out the equation of the normal to this curve at the point (1, –3)

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. *(5 marks)*

Mark Scheme

$\frac{dy}{dx} = 3x^2 + 4x - 9$	M1	
(when $x = 1$) gradient tgt = –2	A1	
(when $x = 1$) gradient nl = $\frac{1}{2}$	A1 ft	ft Their –2
$y - (-3) = \frac{1}{2}(x - 1)$	M1	oe
$x - 2y - 7 = 0$	A1ft	ft Their m and c

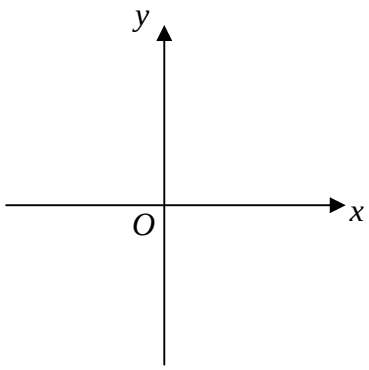
Question 9

A curve has equation $y = x^3 - 6x^2 + 20$

(a) Write down an expression for $\frac{dy}{dx}$ *(1 mark)*

(b) Work out the coordinates of the stationary points and determine whether they are maximum or minimum.

Sketch the curve on the axes clearly labelling the stationary points. *(5 marks)*



Mark Scheme

(a) $\frac{dy}{dx} = 3x^2 - 12x$ M1

(b) $3x^2 - 12x = 0$ or $3x(x - 4) = 0$ M1

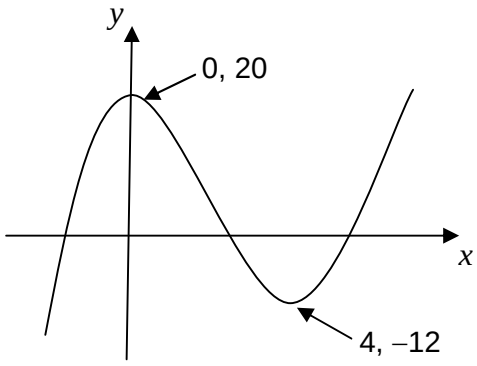
$x = 0$ and $x = 4$ A1

(0, 20) and (4, –12) A1

Testing the sign of $\frac{dy}{dx}$ for values of x either side of 0 and 4 M1

Maximum at (0, 20) Minimum at (4, –12) A1 If previous M1 earned

(c) B2 B1 For correct general shape
B1 ft For labelling the stationary points



Question 10

A curve has equation $y = x^3 - x^2 + k\,x - 2$

(a) Write down an expression for $\frac{dy}{dx}$ *(1 mark)*

(b) The curve has a minimum point at the point where $x = 2$

Work out the value of k. *(2 marks)*

(c) Work out the x coordinate of the maximum point on the curve. *(3 marks)*

Mark Scheme

(a) $\frac{dy}{dx} = 3x^2 - 2x + k$ B1

(b) $3(2)^2 - 2(2) + k = 0$ M1

$k = -8$ A1

(c) $3x^2 - 2x - 8 = 0$ M1

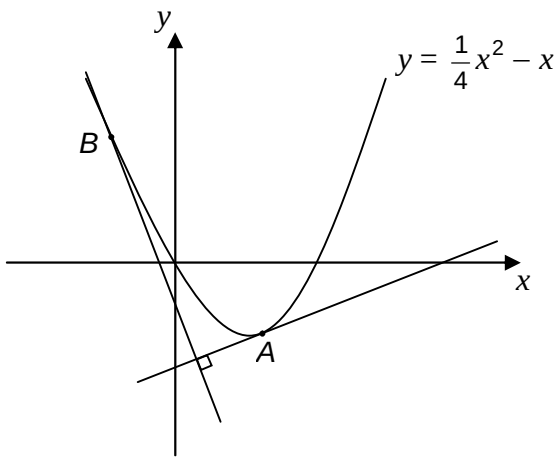
$(3x + 4)(x - 2) = 0$ A1

Maximum at $x = -\frac{4}{3}$ A1

Question 11

(a) Show that the line $y = \frac{1}{2}x - \frac{9}{4}$ is the tangent to the curve $y = \frac{1}{4}x^2 - x$ at the point A $(3, -\frac{3}{4})$. *(4 marks)*

(b) The point *B* on the curve is such that the tangent at *B* is perpendicular to the tangent at *A*, as shown in the diagram.



Not drawn accurately

Work out the coordinates of *B*. *(4 marks)*

Mark Scheme

(a) $\frac{dy}{dx} = \frac{1}{2}x - 1$ M1

(when $x = 3$) $\frac{dy}{dx} = \frac{3}{2} - 1 = \frac{1}{2}$ A1

$y - (-\frac{3}{4}) = \frac{1}{2}(x - 3)$ M1

$y = \frac{1}{2}x - 1\frac{1}{2} - \frac{3}{4}$ A1 Clearly shown since $y = \frac{1}{2}x - \frac{9}{4}$ answer given

(b) Gradient tangent at $B = -2$ B1

$\frac{1}{2}x - 1 = -2$ M1

$x = -2$ A1 ft ft Their tangent gradient

$B = (-2, 3)$ A1

10 Factor Theorem

Question 1

(a) Show that $x(x + 4)(x – 9) = x^3 – 5x^2 – 36x$ *(1 mark)*

(b) Write down the x values of the three points where the graph of $y = x^3 – 5x^2 – 36x$ crosses the x-axis. *(2 marks)*

Mark Scheme

(a) $x(x^2 – 5x – 36)$ B1

(b) $x = 0$, $x = –4$, $x = 9$ B2 B1 For two solutions

Question 2

$f(x) = x^3 + 2x^2 – 5x – 6$

(a) Work out f(1) and f(–1) *(2 marks)*

(b) Work out f(2) and f(–2) *(2 marks)*

(c) Work out f(3) and f(–3) *(2 marks)*

(d) Write down the three linear factors of $f(x)$. *(1 mark)*

Mark Scheme

(a) f(1) = 1 + 2 – 5 – 6 = –8 B1

f(−1) = –1 + 2 + 5 – 6 = 0 B1

(b) f(2) = 8 + 8 – 10 – 6 = 0 B1

f(–2) = –8 + 8 + 10 – 6 = 4 B1

(c) f(3) = 27 + 18 – 15 – 6 = 24 B1

f(–3) = –27 + 18 + 15 – 6 = 0 B1

(d) $(x + 1)$, $(x – 2)$ and $(x + 3)$ B1

Question 3

(a) Show that $(x + 5)$ is a factor of $x^3 + 7x^2 + 2x - 40$ *(2 marks)*

(b) Work out the other two linear factors of $x^3 + 7x^2 + 2x - 40$ *(3 marks)*

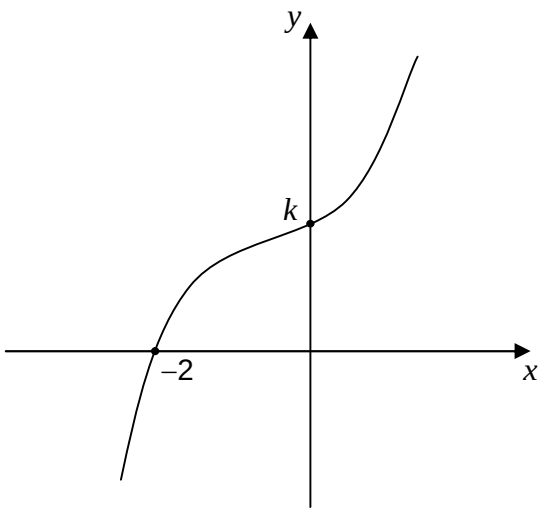
(c) Hence, solve $x^3 + 7x^2 + 2x - 40 = 0$ *(1 mark)*

Mark Scheme

(a)	$(-5)^3 + 7(-5)^2 + 2(-5) - 40$	M1	oe
	$-125 + 175 - 10 - 40 = 0$	A1	Clearly shown to = 0
(b)	$x^3 + 7x^2 + 2x - 40$ $\equiv (x + 5)(x^2 + kx - 8)$	M1	Sight of x^2 and -8 in a quadratic factor
	$(x - 2)$	A1	
	$(x + 4)$	A1	
Alt 1 (b)	Substitutes another value into the expression and tests for '= 0'	M1	
	$(x - 2)$	A1	
	$(x + 4)$	A1	
Alt 2 (b)	Long division of polynomials getting as far as $x^2 + 2x$	M1	
	$(x - 2)$	A1	
	$(x + 4)$	A1	
(c)	$(x =) -5, -4$ and 2	B1	

Question 4

A sketch of $y = x^3 + 5x^2 + 9x + k$ where k is an integer, is shown.



Work out the value of k. *(3 marks)*

Mark Scheme

$(-2)^3 + 5(-2)^2 + 9(-2) + k = 0$	M1
$-8 + 20 - 18 + k = 0$	A1
$k = 6$	A1

Question 5

(a) $(x + 3)$ is a factor of $f(x) = x^3 + x^2 + ax - 72$ where a is an integer.

Work out the value of a. *(3 marks)*

(b) Work out the other linear factors of $f(x)$. *(3 marks)*

Mark Scheme

(a)	$(-3)^3 + (-3)^2 + (-3)a - 72 = 0$	M1	
	$-27 + 9 - 3a - 72 = 0$	A1	
	$a = -30$	A1	
(b)	$x^3 + x^2 - 30x - 72$ $\equiv (x + 3)(x^2 + kx - 24)$	M1	Sight of x^2 and -24 in a quadratic factor
	$(x + 4)$	A1	
	$(x - 6)$	A1	
Alt 1 (b)	Substitutes another value into the expression and tests for '= 0'	M1	
	$(x + 4)$	A1	
	$(x - 6)$	A1	
Alt 2 (b)	Long division of polynomials getting as far as $x^2 - 2x$	M1	
	$(x + 4)$	A1	
	$(x - 6)$	A1	

Question 6

$(x - 3)$ and $(x + 4)$ are factors of $f(x) = x^3 + ax^2 + bx + 24$ where a and b are integers.

(a) Work out the third linear factor of $f(x)$. *(2 marks)*

(b) Work out the values of a and b. *(4 marks)*

Mark Scheme

(a)	$(x - 3)(x + 4)(x + k)$ $\equiv x^3 + ax^2 + bx + 24$	M1	or $-3 \times 4 \times k = 24$
	$(x - 2)$	A1	
(b)	$(x - 3)(x + 4)(x - 2)$	M1	
	$(x - 3)(x^2 + 2x - 8)$	M1	oe
	$x^3 - x^2 - 14x + 24$	A1	
	$a = -1$ and $b = -14$	A1 ft	ft Their expansion
Alt (b)	Substitutes any two of $x = -4$, $x = 2$ or $x = 3$ into $x^3 + ax^2 + bx + 24$ to create simultaneous equations	M1	
	Any two of $-64 + 16a - 4b + 24 = 0$ or $8 + 4a + 2b + 24 = 0$ or $27 + 9a + 3b + 24 = 0$	M1	
	$a = -1$	A1	
	$b = -14$	A1 ft	ft Their first solution

Question 7

(a) $(x - 5)$ is a factor of $f(x) = x^3 + kx^2 + 9x - 20$ where k is an integer.

Work out the value of k. *(3 marks)*

(b) Express $f(x)$ as a product of $(x - 5)$ and a quadratic factor. *(2 marks)*

(c) Show that $(x - 5)$ is the only linear factor of $f(x)$. *(2 marks)*

Mark Scheme

(a)	$(5)^3 + k(5)^2 + 9(5) - 20 = 0$	M1	
	$125 + 25k + 45 - 20 = 0$	A1	
	$k = -6$	A1	
(b)	$x^3 - 6x^2 + 9x - 20$ $\equiv (x - 5)(x^2 + kx + 4)$	M1	Sight of x^2 and 4 in a quadratic factor
	$(x - 5)(x^2 - x + 4)$	A1	
(c)	Tests '$b^2 - 4ac$' for the quadratic	M1	ft Their quadratic or attempts to solve their quadratic = 0
	Shows '$b^2 - 4ac$' = −15 (or < 0) and states no more linear factors	A1	States 'no solutions' to their quadratic = 0

Question 8

Solve $x^3 - 6x^2 - 25x - 18 = 0$ *(5 marks)*

Mark Scheme

	Substitutes a value of x into the expression and tests for '= 0'	M1	
	Works out first linear factor $(x + 1)$, $(x + 2)$ or $(x - 9)$	A1	
	$x^3 - 6x^2 - 25x - 18 \equiv (x + 1)(x^2 + kx - 18)$ or $(x + 2)(x^2 + kx - 9)$ or $(x - 9)(x^2 + kx + 2)$	M1	Attempts to work out the quadratic factor Sight of x^2 and -18 in a quadratic factor or sight of x^2 and -9 in a quadratic factor or sight of x^2 and 2 in a quadratic factor
	2nd and 3rd linear factors	A1	
	-1, -2 and 9	A1	
Alt 1	Substitutes a value of x into the expression and tests for '= 0'	M1	
	Works out first linear factor $(x + 1)$, $(x + 2)$ or $(x - 9)$	A1	
	Substitutes another value into the expression and tests for '= 0'	M1	
	2nd and 3rd linear factors	A1	
	-1, -2 and 9	A1	
Alt 2	Substitutes a value of x into the expression and tests for '= 0'	M1	
	Works out first linear factor $(x + 1)$, $(x + 2)$ or $(x - 9)$	A1	
	Long division of polynomials getting as far as $x^2 - 7x$ or $x^2 - 8x$ or $x^2 + 3x$	M1	Depending on first linear factor
	2nd and 3rd linear factors	A1	
	-1, -2 and 9	A1	

11 Sequences

Question 1

A linear sequence starts

250 246 242 238 .....

Which term is the first to have a negative value? *(4 marks)*

Mark Scheme

For the nth terms of quadratic sequences two methods are shown (see example 2).
Other valid methods may be used.

$-4n$	M1	
$254 - 4n$	A1	
$254 - 4n < 0$	M1	oe
64th	A1	

Question 2

Work out the nth term of this quadratic sequence.

8 9 14 23 36 .....

(4 marks)

Mark Scheme

2 **Method A** M1

8 9 14 23 36

1 5 9 13

4 4 4

Subtract $2n^2$ from sequence A1

6 1 –4 ……

nth term of this sequence is M1

$11 - 5n$

Giving $2n^2 - 5n + 11$ A1

Alt **Method B** M1

Using $an^2 + bn + c$

$a + b + c = 8$

$4a + 2b + c = 9$

$9a + 3b + c = 14$

$3a + b = 1$ M1 oe

$5a + b = 5$

$a = 2$ and $b = -5$ A1

Giving $2n^2 - 5n + 11$ A1

Question 3

(a) Show that the nth term of the quadratic sequence

4 10 18 28 …… is $n^2 + 3n$

(3 marks)

(b) Hence, write down the nth term of these quadratic sequences.

(b) (i) 5 11 19 29 ……

(1 mark)

(b) (ii) 5 12 21 32 ……

(1 mark)

Mark Scheme

(a) Use Method A or B from Q2 3 marks or any other valid method

(b)(i) $n^2 + 3n + 1$ B1

(b)(ii) $n^2 + 4n$ B1

Question 4 (non calculator)

(a) Write down the nth term of the linear sequence

4 7 10 13 ……

(1 mark)

(b) Hence, write down the nth term of the quadratic sequence.

16 49 100 169 ……

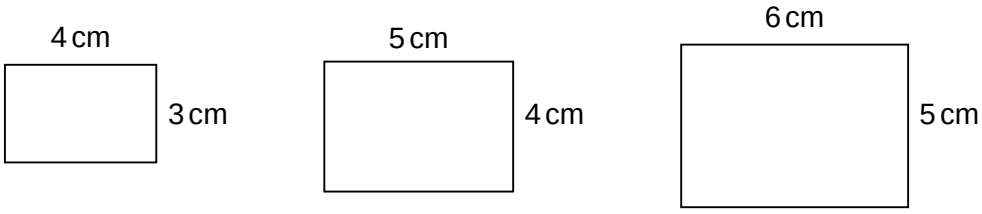
(1 mark)

(c) For the sequence in part 4(b), show that the 30th term is equal to the product of the 2nd and 4th terms *(3 marks)*

Mark Scheme

(a)	$3n + 1$	B1	
(b)	$(3n + 1)^2$	B1	oe
(c)	$49 \times 169 = 7^2 \times 13^2$	B1	oe 8281
	30th is 91^2	M1	
	$= (7 \times 13)^2$ $= 7^2 \times 13^2$	A1	oe 8281

Question 5



This pattern of rectangles continues.

Show that the sequence of numbers formed by the areas of these rectangles has nth term

$$n^2 + 5n + 6$$ *(4 marks)*

Mark Scheme

	nth term of lengths is $n + 3$	M1	
	nth term of widths is $n + 2$	M1	
	Area is $(n + 3)(n + 2)$	M1	
	$n^2 + 3n + 2n + 6$ $= n^2 + 5n + 6$	A1	
Alt	nth term of 12 20 30 by Method A or Method B	4 marks	or any other valid method

Question 6

A linear sequence starts

$a + b$ $a + 3b$ $a + 5b$ $a + 7b$ ……

The 5th and 8th terms have values 35 and 59.

(a) Work out a and b. *(4 marks)*

(b) Work out the nth term of the sequence. *(2 marks)*

Mark Scheme

(a) $a + 9b = 35$ M1

$a + 15b = 59$

$6b = 24$ M1 oe

$b = 4$ A1

$a = -1$ A1 ft

(b) 3 11 19 …… B1 ft

$8n - 5$ B1 ft

Question 7

A sequence has nth term $\dfrac{3n + 1}{n}$

(a) Show that the difference between the nth and $(n + 1)$th terms is $\dfrac{1}{n(n + 1)}$ *(3 marks)*

(b) Which are the first two consecutive terms with a difference less than 0.01? *(2 marks)*

(c) Write down the limiting value of the sequence as $n \to \infty$ *(1 mark)*

Mark Scheme

(a)	$\frac{3n+1}{n} - \frac{3(n+1)+1}{n+1}$	M1	oe eg subtracts in different order
	$\frac{(3n+1)(n+1) - n(3n+4)}{n(n+1)}$	M1	oe
	$\frac{3n^2+n+3n+1-3n^2-4n}{n(n+1)}$	A1	
	$= \frac{1}{n(n+1)}$		
Alt (a)	$\frac{3n+1}{n} = 3 + \frac{1}{n}$	M1	oe eg subtracts in different order
	$(3 + \frac{1}{n}) - (3 + \frac{1}{n+1})$	M1	oe
	$\frac{n+1-n}{n(n+1)}$	A1	
	$= \frac{1}{n(n+1)}$		
(b)	Any substitution and evaluation for $1 \leqslant n \leqslant 10$	M1	oe eg $1 < 0.01n^2 + 0.01n$ and attempt to solve
	eg $\frac{1}{9 \times 10} = \frac{1}{90}$		
	or $\frac{1}{10 \times 11} = \frac{1}{110}$		
	10th and 11th	A1	
(c)	3	B1	

Question 8

A sequence has nth term $\dfrac{5n+2}{2n}$

Show that the limiting value of the sequence, S, as $n \to \infty$ is 2.5 *(2 marks)*

Mark Scheme

$\dfrac{5n+2}{2n} = \dfrac{5n}{2n} + \dfrac{2}{2n}$ M1 oe

$\left(\dfrac{5}{2} + \dfrac{1}{n}\right)$

$\dfrac{1}{n} \to 0$ as $n \to \infty$ $S = \dfrac{5}{2}$ (= 2.5) A1

Question 9

Here is the sequence of odd numbers

1 3 5 7 9 ……

A quadratic sequence is formed by multiplying consecutive odd numbers in successive pairs.

3 15 35 63 ……

Work out the nth term of this sequence. *(3 marks)*

Mark Scheme

Odd number is $2n + 1$ **or** $2n - 1$ M1

$2n - 1$ **and** $2n + 1$ M1

Sequence is $(2n - 1)(2n + 1)$ A1

$(= 4n^2 - 1)$

Alt Using Method A or Method B giving 3 marks or any other valid method

$4n^2 - 1$

eg

1	4	9	16	→	n^2
4	16	36	64	→	$4n^2$
3	15	35	63	→	$4n^2 - 1$

Question 10

The nth term of a sequence is $\dfrac{2n^2 - 1}{3n^2 + 2}$

(a) Show that the difference between the first two terms is $\dfrac{3}{10}$ *(3 marks)*

(b) Write down the limiting value of the sequence as $n \to \infty$ *(1 mark)*

Mark Scheme

(a) $T_1 = \dfrac{1}{5}$ B1

$T_2 = \dfrac{7}{14}$ B1 oe

$\left(= \dfrac{1}{2}\right)$

$\dfrac{5}{10} - \dfrac{2}{10} = \dfrac{3}{10}$ B1 oe

(b) $\dfrac{2}{3}$ B1

12 Algebraic problems – including ratio

Note

- If $x : y = 4 : 7$, then $\frac{x}{y} = \frac{4}{7}$
- If, in a problem, two numbers are in the ratio 4 : 7, use $4x$ and $7x$ as the numbers (usually leading to a linear equation); otherwise, use x and y as the numbers (which will lead to simultaneous equations).
- If $x : y = 4 : 7$, what is $x + 2y : 3x$?

  Think in terms of 'parts', ie 4 parts and 7 parts, so $x + 2y : 3x = 4 + 14 : 12$

  $= 18 : 12$

  $= 3 : 2$

Question 1

Work out the possible values of $\frac{2n - 1}{3n + 2}$ if $n^2 = 16$

Give your answers as fractions in their simplest form. *(4 marks)*

Mark Scheme

$n = 4$	M1
$\frac{1}{2}$	A1
$n = -4$	M1
$\frac{9}{10}$	A1

Question 2

$x : y = 6 : 5$

(a) Express x in terms of y. *(2 marks)*

(b) Show that $x + 3y : 2x - y = 3 : 1$ *(2 marks)*

Mark Scheme

(a) $\frac{x}{y} = \frac{6}{5}$ M1

$x = \frac{6y}{5}$ A1 oe

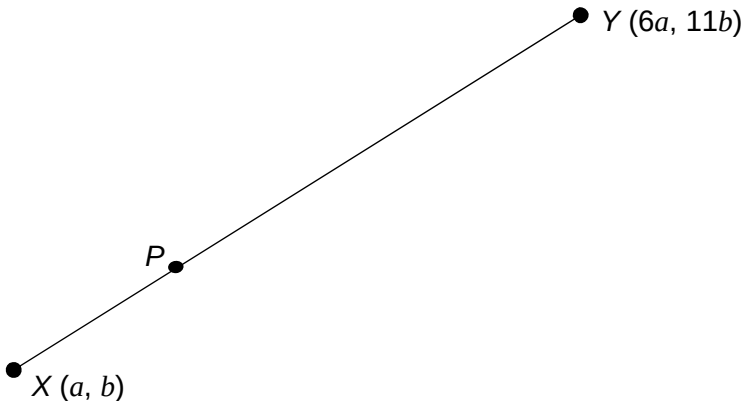
(b) $\frac{6y}{5} + \frac{15y}{5} : \frac{12y}{5} - \frac{5y}{5}$ M1 oe $6 + 3 \times 5 : 2 \times 6 - 5$

$\frac{21(y)}{(5)} : \frac{7(y)}{(5)}$ A1

Question 3

A point P divides XY in the ratio 3 : 7

Not drawn accurately



Work out the coordinates of P, in terms of a and b. *(3 marks)*

Mark Scheme

$\frac{3}{10}$ of $(6a - a)$ **or** $\frac{3}{10}$ of $(11b - b)$	M1	oe
$(2.5a, 4b)$	A2	oe A1 For each coordinate SC2 $(1.5a, 3b)$

Question 4

Here is a linear sequence

$$a + b \qquad a + 3b \qquad a + 5b \qquad a + 7b \qquad \ldots\ldots\ldots$$

Given that

- 2nd term : 4th term = 2 : 5
- 1st term = – 4

Work out a and b. *(5 marks)*

Mark Scheme

$\frac{a + 3b}{a + 7b} = \frac{2}{5}$	M1	
$5a + 15b = 2a + 14b$	M1	Allow one error
$3a + b = 0$	A1	oe
$a + b = -4$ $2a = 4$	A1 ft	
$a = 2$ and $b = -6$	A1 ft	

Question 5

You are given that $ab + a = 5$ and $a : b = 4 : 3$

Work out the possible pairs of values of a and b. *(7 marks)*

Mark Scheme

$\frac{a}{b} = \frac{4}{3}$	M1	oe
$b = \frac{3a}{4}$	A1	$a = \frac{4b}{3}$
$a \times \frac{3a}{4} + a = 5$	M1	$\frac{4b}{3} \times b + \frac{4b}{3} = 5$
$3a^2 + 4a - 20 = 0$	A1	$4b^2 + 4b - 15 = 0$
$(3a + 10)(a - 2) = 0$	M1	$(2b + 5)(2b - 3)$
$a = -\frac{10}{3}$ $a = 2$	A1 ft	$b = -\frac{5}{2}$ $b = \frac{3}{2}$
$b = -\frac{5}{2}$ $b = \frac{3}{2}$	A1 ft	$a = -\frac{10}{3}$ $a = 2$

Question 6

The sum of the ages of two people is 90 years.

Six years ago, their ages were in the ratio 8 : 5

How old are they now?

Do **not** use trial and improvement.

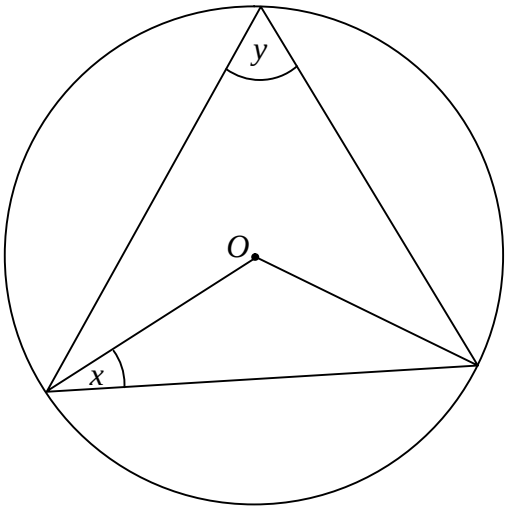
You **must** show your working. *(5 marks)*

Mark Scheme

	Let their ages 6 years ago be $8x$ and $5x$	M1	
	$8x + 5x = 90 - 12$	M1	Allow $90 - 6$ for M1
	$13x = 78$ $(x = 6)$	A1	
	Their 6×8 and their 6×5 (48) (30)	M1	
	54 and 36	A1	
Alt	$x + y = 90$	M1	
	$\frac{x-6}{y-6} = \frac{8}{5}$	M1	
	$18 = 8y - 5x$	A1	
	Eliminates a letter	M1	
	$(x =)$ 54 and $(y =)$ 36	A1	

Question 7

O is the centre of the circle.



Not drawn accurately

Given that $x : y = 4 : 5$

Work out the value of y.

Do **not** use trial and improvement.

You **must** show your working. *(7 marks)*

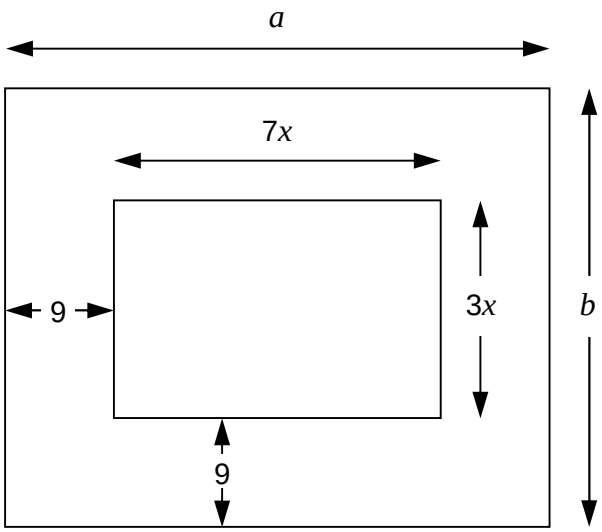
Mark Scheme

x, x and $180 - 2x$ seen or on diagram	M1	
$\frac{x}{y} = \frac{4}{5}$	M1	
$x = \frac{4y}{5}$	A1	oe
$2y = 180 - 2x$ (or $y = 90 - x$)	M1	oe
$y = 90 - \frac{4y}{5}$	M1	oe
$\frac{9y}{5} = 90$	M1	oe
$y = 50$	A1	

Question 8

A rectangular picture is surrounded by a frame of constant width.

All measurements are in centimetres.



Not drawn accurately

Given that $a : b = 3 : 2$

Work out x. *(5 marks)*

Mark Scheme

$a = 7x + 18$ or $b = 3x + 18$	B1	oe
$\frac{\text{their}\,(7x + 18)}{\text{their}\,(3x + 18)} = \frac{3}{2}$	M1	
$14x + 36 = 9x + 54$	M1	Rearranging
$5x = 18$	M1	Solving
$x = 3.6$	A1	

Question 9

If $x : y = 3 : 5$ and $y : z = 10 : 9$

Find, in its simplest form

(a) $x : z$ *(3 marks)*

(b) $10x : 7y$ *(2 marks)*

(c) $x + y : y$ *(2 marks)*

Mark Scheme

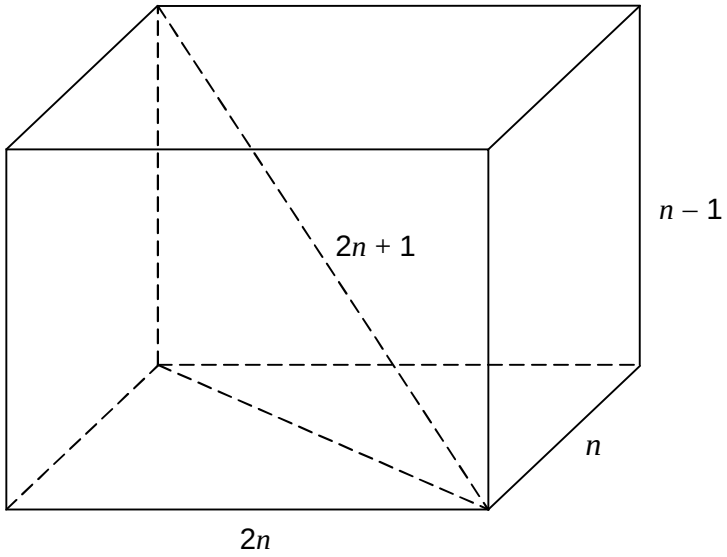
(a)	$x : y = 6 : 10$	M1	oe
	$x : y : z = 6 : 10 : 9$	M1	
	$x : z = 2 : 3$	A1	
(b)	$3 \times 10 : 7 \times 5$	M1	oe
	$6 : 7$	A1	
(c)	$3 + 5 : 5$	M1	$\frac{x+y}{y} = \frac{x}{y} + 1$ or $\frac{3}{5} + 1$
	$8 : 5$	A1	

Question 10

A cuboid has dimensions $2n$, n and $n - 1$ cm.

A diagonal has length $2n + 1$ cm.

Not drawn accurately



Work out n. *(6 marks)*

Mark Scheme

$(2n)^2 + n^2$	M1	oe
$(2n)^2 + n^2 + (n - 1)^2 = (2n + 1)^2$	M1	
$4n^2 + n^2 + n^2 - n - n + 1$ $= 4n^2 + 2n + 2n + 1$	M1	Allow one error
$2n^2 - 6n = 0$	M1	Rearranging ; or $2n^2 = 6n$
$2n(n - 3) = 0$	M1	(allow ÷ by n) $2n = 6$
$n = 3$	A1	

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