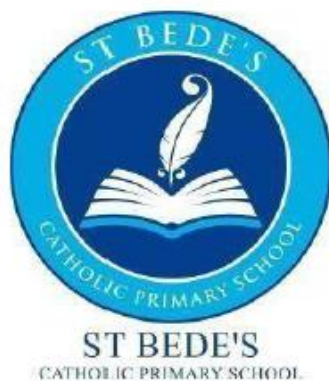
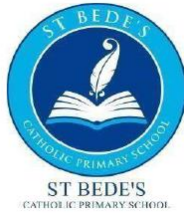


St Bede's Primary School a Catholic Voluntary Academy

Calculation Policy

Reviewed September 2018
Review Date September 2019





Our Mission Statement



“In St Bede’s we welcome everyone, grow and learn together as God’s family, and use our gifts to serve with love.”

Our Vision Statement

We are guided by our Mission Statement and we aim to:

- develop and care for the **whole child** through our teaching and sharing of the Catholic faith.
- show our love of God in the way we care and value each other and ourselves.
- aspire and equip children with those skills necessary to become contributing members of society and responsible adults.
- recognise and encourage all pupils’ individual gifts and talents.
- provide an excellent quality of education striving to achieve the very highest standards for all pupils and, at the same time, develop lively critical minds.
- develop each pupil’s appreciation of education as a lifelong and enjoyable process.
- work in partnership with the Parish and families, local schools and community groups recognising that only by working together can the school make its contribution towards the development of committed Christians and active members of the Church.

Our Values

WELCOME - *Welcome everyone into our school family through our kind words and caring actions.*

LOVE - *Let the love of God shine in everything we do.*

INSPIRE - *Inspire each other to achieve our very best.*

RESPECT - *Respect each unique member of our family.*

PATIENCE - *Show patience and tolerance towards each other.*

COMMUNITY - *Work together to serve our community with love.*

Written Methods of Calculation Policy

This calculation policy has been written in line with the programmes of study taken from the revised National Curriculum for Mathematics (2014). It provides guidance on appropriate calculation methods and progression. The content is set out in yearly blocks under the following headings: addition, subtraction, multiplication and division

Children will use mental methods as their first port of call when appropriate, but for calculations that they cannot do in their heads, they will need to use an efficient written method accurately and with confidence.

Aims of the Policy:

- To ensure consistency and progression in our approach to calculation
- To ensure that children develop an efficient, reliable, formal written method of calculation for all operations
- To ensure that children can use these methods accurately with confidence and understanding

General Progression:

- Establish mental methods based on a good understanding of place value
- Use of informal jottings to aid mental calculations
- Develop use of empty number line to help mental imagery and aid recording
- Use models and images to promote understanding.
- Use partitioning and recombining to aid informal methods
- Use inverses to check calculations
- Introduce expanded methods
- Develop expanded methods into compact standard written form

Once a child has become secure in a particular form they will move on to learn other methods, so that they build a bank of methods. Children will be encouraged to then select an appropriate method to solve problems.

Vocabulary

Misconceptions will occur, and need to be rectified quickly. It is necessary to use clear, unambiguous language to minimise misconceptions, and this should be consistent across the whole school. When using column methods for subtraction the children should be told to 'exchange' a ten for units, or a hundred for tens, not to 'borrow' a ten or a hundred. When teaching multiplication / division by 10, 100 etc., the figures move, not the decimal point. The decimal point always resides between the units and the tenths! When multiplying an integer by 10 we do not ADD a 0. Adding implies addition and the addition of a number and zero means the original number remains unchanged.

How to use this policy:

- Use the policy as the basis of your planning but ensure you use previous or following years' guidance to allow for personalised learning
- Always use Assessment for Learning to identify suitable next steps in calculation for groups of children
- If, at any time, children are making significant errors, return to the previous stage in calculation
- Cross reference with the mental maths policy for guidance on key facts, key vocabulary and mental methods
- Always use suitable resources, models and images to support children's understanding of calculation and place value, as appropriate
- Encourage children to make sensible choices about the methods they use when solving problems

Early Years Foundation Stage (EYFS)

Whilst EYFS works developmentally to support the children to play and explore, actively learn, create and think critically the expectation is that children will meet the Early Learning Goals (ELGs) in Mathematics by the end of EYFS. In acknowledging this we realise, that if children are not ready to meet these requirements, further provision at this level is needed to ensure a smooth transition.

Key Stage One and Two

Using Written Methods

Written methods enable children to demonstrate their approach to calculations for the four operations which they cannot complete mentally and help pupils to improve their methods of working out;

- It is good practice when first introducing a method for the range of numbers to be within what the pupil can calculate mentally so that they can self-assess their success at using a method.
- Once pupils are able to perform a written method successfully they should be encouraged to complete calculations independently choosing the most appropriate way of doing so.


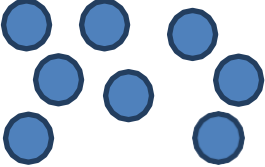
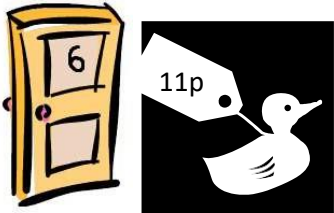

Progression of Written Methods

Taking account of visual, auditory and kinaesthetic (VAK) learning approaches, the written methods for each of the four operations demonstrate progression by building upon skills and knowledge learnt in each year at school;

- *A pupil should not be targeted at achieving an age-expected method if they are not able to successfully use the method for a previous age-group.
- *As a school we are taking an approach which ensures consistency across the school using the same few methods across both key stages.
- *With this in mind, it should be easier for pupils to work on calculations using the method for their appropriate ability.

Early Learning Goal

Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling,

Elements of Number Early Learning Goal (ELG)	Progression	Notes
<p>1. Children count reliably with numbers from 1 to 20.</p>	<p>Uses some number names accurately in play.</p> <p>Recites numbers in order to 10.</p> <p>Counts up to 3 or 4 objects saying one number name for each item.</p>  <p>One, two, three, four... there are four teddies.</p> <p>Counts out up to six objects from a larger group.</p> <p>Counts objects or actions which cannot be moved.</p> <p>Counts an irregular arrangement of up to 10 objects.</p>  <p>Children count reliably with numbers from 1 to 20.</p> <p>Using number names beyond 20.</p>	<p>Children need to see and hear numbers being used in their learning environment.</p> <p>As their knowledge of number increases they will begin to recognise that numbers identify how many objects are in a set.</p>
<p>2. They place them in order.</p>	<p>Shows an interest in numerals in the environment.</p>  <p>Shows an interest in representing numbers.</p> <p>Recognise some numerals of personal significance.</p> 	<p>Note: Numicon supports number recognition, ordering and counting.</p>

	<p>Recognise and order numbers to 5.</p> <p>Recognise and order numbers to 10.</p> <p>Recognise and order numbers to 20.</p>	
<p>3. They say which number is one more or one less than a given number.</p>	<p><u>One more</u> Saying one more using songs and rhymes.</p> <p>Continue to count on in 1's by saying the next number.</p> <div data-bbox="421 577 995 763" data-label="Text"> <p>5, 6, 7, 8...</p> </div> <p>Using equipment (including objects and fingers) ask the children to say one more.</p> <p>Using groups of objects, ask the children one more.</p> <div data-bbox="354 972 552 1106" data-label="Image"> </div> <div data-bbox="651 943 1129 1160" data-label="Text"> <p>If I give you one more, how many will you have?</p> </div> <p><u>One less</u> Saying one less using songs and rhymes.</p> <p>Continue to count back in 1's by saying the next number.</p> <div data-bbox="399 1375 970 1559" data-label="Text"> <p>9, 8, 7, 6...</p> </div> <p>Using groups of objects, ask the child to find one more.</p> <div data-bbox="651 1621 1129 1839" data-label="Text"> <p>If I give you one less, how many will you have?</p> </div> <p>Using equipment (including objects and fingers) ask the children to say one less.</p>	<p>Note: Find 1 more/less up to 5, then 10, then 20 and beyond.</p> <p>Initially the children may need a prompt to know that "one less" is "take one away."</p>

4. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.

Addition

Addition songs and rhymes.

e.g 1, 2, 3, 4, 5 Once I caught a fish alive



At a party, I eat 2 cakes and my friends eat 3. How many do we eat altogether?



Count out 3 apples. Count out 2 apples. How many apples all together?

Once confident children will be encouraged to draw simple pictures to help them combine sets without objects.

Children will then be introduced to the formal method of recording addition.

$$2+3=5$$

Count along a number line to add numbers together.



Subtraction

Addition vocabulary is introduced:
add
addition
plus
and
count on
more
sum
total
altogether
increase

Early addition is about combining 2 sets of objects physically.

Children are encouraged to "one touch one count."

Children are encouraged to use a large number line counting on in 1's, often using a finger to mark each jump. Initially this method would be used alongside previous methods until children are confident in using a number line.

Subtraction songs and rhymes.
e.g. 5 little speckled frogs

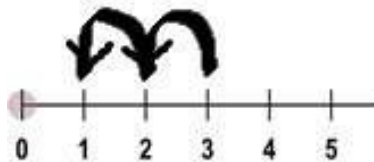
Begin to relate subtraction to "taking away".
3 presents **take away** 2 presents **leaves** 1 present.



Children will then be introduced to the formal method of recording subtraction.

$$3-2=1$$

Using a number line to subtract. Start on 3 count back 2 jumps.



Subtraction vocabulary is introduced:
subtract
take away
minus
count back
less
fewer
difference
between
left/leaves

Early subtraction is about physically removing objects.

5. They solve problems, including doubling, halving and sharing.

Doubling (Multiplication)

Counting songs and rhymes.

Counting on in 2s, 5s and 10s



2

4

6 etc...



5



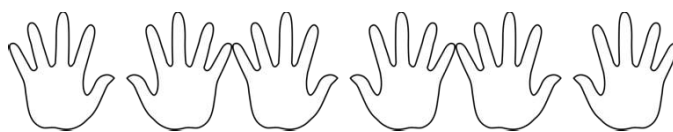
10



15



20



10


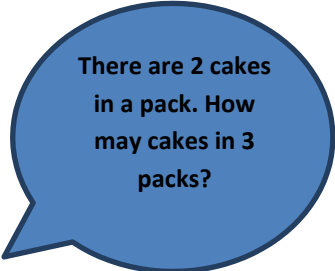


20

30

Multiplication vocabulary is introduced:
lots of
groups of
times
multiply
multiplication
multiple
once, twice,
three times
double

At this early stage children are counting objects or pictures. They are also being introduced to the language of multiplication.

It is important children have the opportunity to count

	<p>Counting in 2's along a number line or track.</p> <p>Adding in groups.</p>   <p><u>Halving and Sharing (Division)</u></p> <p>Number songs and rhymes.</p> <p>Sharing games and activities, including: Sharing fruit</p>  <p>Teddy Bear's Picnic</p>  <p>Grouping activities, including: Asking the children to get into groups (pairs etc...)</p>	<p>repeatedly in groups of the same size.</p> <p>Children physically make sets or groups and then add them by counting up from 1 until all the objects have been used.</p> <p>Division vocabulary is introduced: lots of groups of share group halve half divide division divided by remainder</p> <p>Visual images for division are important.</p> <p>Sorting objects and people into groups is the main skill to be taught.</p>
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A short list of criteria for readiness for written methods of addition and subtraction would include:

- Can children count forwards and backwards in sequence?
- Do they understand the correct vocabulary for addition and subtraction?
- Do children know addition and subtraction facts to 20?
- Do they understand place value and can they partition numbers into hundreds, tens and units?

- Do they use and apply the commutative and associative laws of addition?
- Can they add at least three 1-digit numbers mentally?
- Can they add and subtract any pair of 2-digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

Corresponding criteria to indicate readiness to learn written methods for multiplication and division are:

- Do the children understand multiplication as repeated addition and division as repeated subtraction?
- Do they understand the correct vocabulary for multiplication and division?
- Do the children know the 2, 3, 4, 5 and 10 times tables and corresponding division facts?
- Do they know the result of multiplying by 0 or 1?
- Do they understand place value?
- Do they understand 0 as a place holder?
- Can they multiply 2 and 3 digits mentally by 10 and 100?
- Can they use their knowledge of all the multiplication tables to approximate?
- Can they find products using multiples of 10?
- Do they use the commutative and associative laws of multiplication?
- Can they halve and double 2 digit numbers mentally?
- Can they use multiplication facts to derive mentally, other multiplication facts they don't know?
- Can they explain their mental strategies orally and record them using informal jottings?

Addition



Progression in Calculations for Addition

Mental Calculations

(ongoing)

These are a **selection** of mental calculation strategies:

Mental recall of number bonds

$$6 + 4 = 10$$

$$25 + 75 = 100$$

$$\square + \square = 10$$

$$19 + \square = 20$$

$$\square + \square = 15$$

Using near doubles

$$6 + 7 = \text{double } 6 + 1 = 13$$

Addition using partitioning and recombining

$$34 + 45 = (30 + 40) + (4 + 5) = 79$$

Counting on or back in repeated steps of 1, 10, 100, 1000

$$86 + 57 = 143 \text{ (by counting on in tens and then in ones)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds)}$$

Add the nearest multiple of 10, 100 and 1000 and adjust

$$24 + 19 = 24 + 20 - 1 = 43$$

$$458 + 71 = 458 + 70 + 1 = 529$$

Use the relationship between addition and subtraction

$$36 + 19 = 55$$

$$19 + 36 = 55$$

$$55 - 19 = 36$$

$$55 - 36 = 19$$

Counting on or back in fractional steps appropriate to stage

$\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1, 1\frac{1}{5}, 1\frac{2}{5}$ etc. Also practise with percentages and decimals related to fractions.

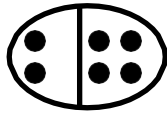
Mental calculation strategies will continue to be used.

They are not replaced by written methods.

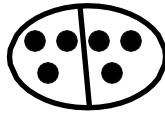
Year 1

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.

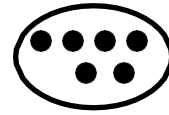
Story of 6 (All story numbers up to 10)



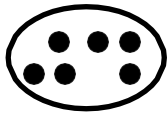
2 and 4



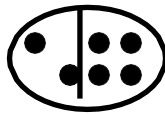
3 and 3



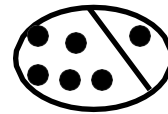
4 and 2



0 and 6



1 and 5

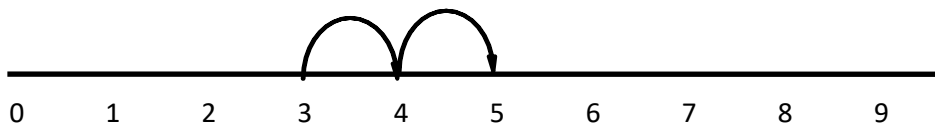


5 and 1

They use numbered lines and practical resources to support calculation and teachers **demonstrate** the use of the number line.

$$3 + 2 = 5$$

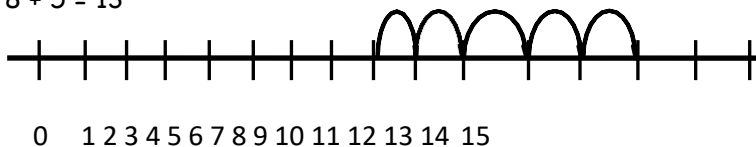
+1 +1



Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones. Increase the numbers - less than 100.

$$8 + 5 = 13$$

+1 +1 +1 +1 +1



0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.



Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line

Key skills for addition at Stage 1:

- Read and write numbers to 100 in numerals, incl. 1–20 in words
- Recall bonds to 10 and 20, and addition facts within 20
- Count to and across 100
- Count in multiples of 1, 2, 5 and 10
- Solve simple 1-step problems involving addition, using objects, number lines and pictorial representations.

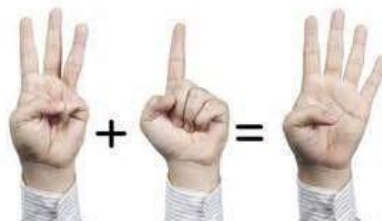
Video clips: Using a range of equipment and strategies to reinforce addition statements / bonds to 10



$5 = 4 + 1$

$10 = 7 + 3$

$3 + 2$

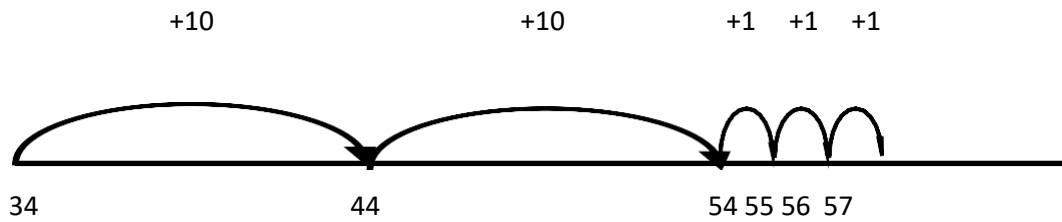


Year 2

Children will then use a number square and practical apparatus, before beginning to use 'empty number lines' themselves starting with the larger number and counting on.

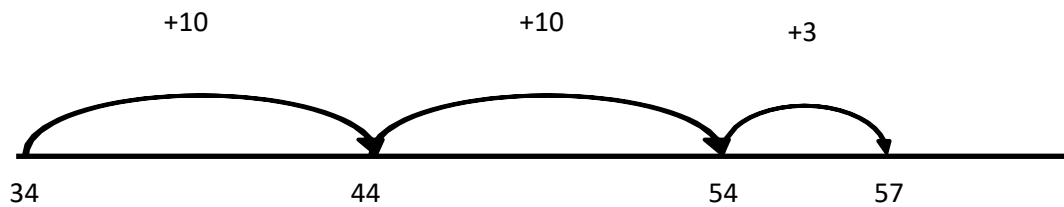
- First counting on in tens and ones.

$$34 + 23 = 57$$



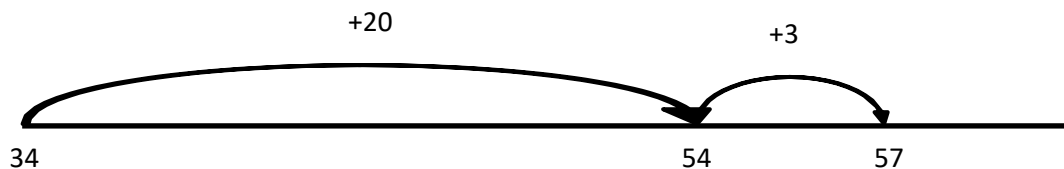
- Then helping children to become more efficient by adding the units in one jump (by using the known fact $4 + 3 = 7$)

$$34 + 23 = 57$$



- Followed by adding the tens in one jump and the units in one jump.

$$34 + 23 = 57$$



Children would move on to partitioning using arrow cards (horizontal).

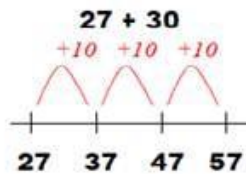
$$32 + 22 = 54$$

$$30 + 20 = 50$$

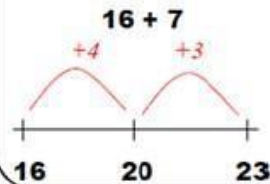
$$2 + 2 = 4$$

$$50 + 4 = 54$$

Add 2-digit numbers and tens:

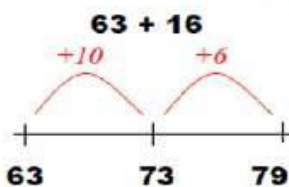


Add 2-digit numbers and units:



Use empty number lines, concrete equipment, hundred squares etc. to build confidence and fluency in mental addition skills.

Add pairs of 2-digit numbers, moving to the partitioned column method when secure adding tens and units:



23 + 34:

20	+ 3		
+ 30	+ 4		
<hr/>			
50	+ 7		
<hr/>			
		=	<u>57</u>

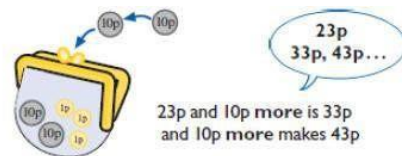
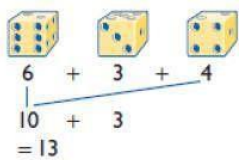
STEP 1: Only provide examples that do NOT cross the tens boundary until they are secure with the method itself.

STEP 2: Once children can add a multiple of ten to a 2-digit number mentally (e.g. $80+11$), they are ready for adding pairs of 2-digit numbers that DO cross the tens boundary (e.g. $58 + 43$).

58 + 43:

50	+ 8		
40	+ 3		
<hr/>			
90	+ 11		
<hr/>			
		=	<u>101</u>

STEP 3: Children who are confident and accurate with this stage should move onto the expanded addition methods with 2 and 3-digit numbers (see Y3).



Use of Hundred square for adding in tens and ones

Extensive use of Dienes apparatus (Base 10 apparatus) for understanding of place value

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

36...46,
56,66

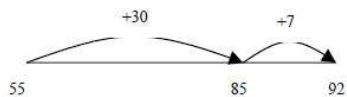
- 6 + 10 = 16
- 16 + 10 = 26
- 26 + 10 = 36
- 36 + 10 = 46
- 36 + 20 = 56



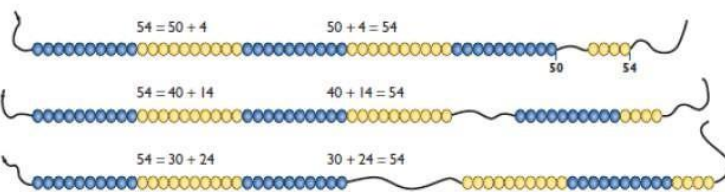
Swap shop - exchanging 10 units for a Ten bar.

55 + 37

Estimate: 60 + 40 = 100



55 + 37 = 92



25 + 47 =

Tens	Units

Tens	Units

Tens	Units

= 72

Children use the Tens and Units chart to use Dienes apparatus vertically to prepare them for column addition.

Recorded as:

$$\begin{array}{r} 20 + 5 \\ 40 + 7 \\ \hline 60 + 12 = 72 \end{array}$$

To support understanding, pupils may physically make and carry out the calculation with Dienes Base 10 apparatus or place value counters, then compare their practical version to the written form, to help them to build an understanding of it.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary

Key skills for addition at Stage 2:

- Add a 2-digit number and ones (e.g. $27 + 6$)
- Add a 2-digit number and tens (e.g. $23 + 40$)
- Add pairs of 2-digit numbers (e.g. $35 + 47$)
- Add three single-digit numbers (e.g. $5 + 9 + 7$)
- Show that adding can be done in any order (the commutative law).
- Recall bonds to 20 and bonds of tens to 100 ($30 + 70$ etc.)
- Count in steps of 2, 3 and 5 and count in tens from any number.
- Understand the place value of 2-digit numbers (tens and ones)
- Compare and order numbers to 100 using $<$ $>$ and $=$ signs.
- Read and write numbers to at least 100 in numerals and words.
- Solve problems with addition, using concrete objects, pictorial representations, involving numbers, quantities and measures, and applying mental and written methods.

Year 3

Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

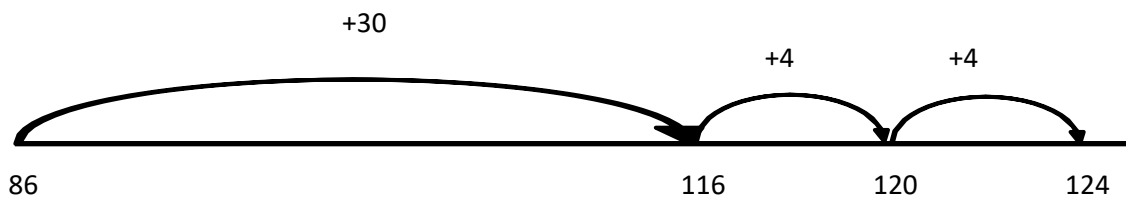
Adding the least significant digits first (this method would begin in the Summer Term of Y2).

$$\begin{array}{r} 67 \\ + 24 \\ \hline 11 (7 + 4) \\ \underline{80} (60 + 20) \\ \underline{91} \end{array}$$

Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.

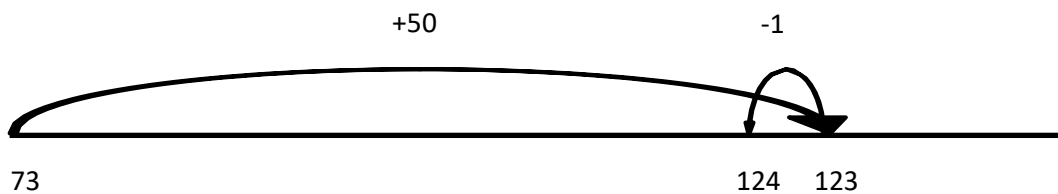
- Count on from the largest number irrespective of the order of the calculation.

$$38 + 86 = 124$$



- Compensation (higher level 3)

$$49 + 73 = 122$$



Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

Adding the least significant digits first.

$$\begin{array}{r}
 + 24 \\
 11 (7 + 4) \\
 \underline{80} (60 + 20) \\
 \underline{91}
 \end{array}$$

$$\begin{array}{r}
 + 85 \\
 12 (7 + 5) \\
 140 (60 + 80) \\
 \underline{200} \\
 \underline{352}
 \end{array}$$

From this, children will begin to carry below the line (higher level 3).

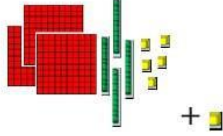
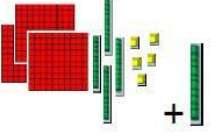
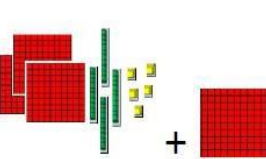
$$\begin{array}{r}
 625 \\
 + 48 \\
 \hline
 673
 \end{array}$$

$$\begin{array}{r}
 783 \\
 + 142 \\
 \hline
 825
 \end{array}$$

$$\begin{array}{r}
 367 \\
 + 185 \\
 \hline
 452
 \end{array}$$

Using similar methods, children will:

- Add several numbers with different numbers of digits (up to three digits)
- Begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds
- Know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, eg £3.59 + 78p.

<p>HTU + 1 $345 + 1 = 346$</p> 	<p>HTU + 10 $345 + 10 = 355$</p> 	<p>HTU + 100 $345 + 100 = 445$</p> 
<p>Continue with expanded method with three digit numbers: HTU + HTU using partitioning and Dienes apparatus $347 + 122 =$ $300 + 40 + 7$ $+ 100 + 20 + 2$ $400 + 60 + 9 = 469$</p>	<p>Then, cross the tens and the hundreds boundary: $264 + 159 =$ $100 + 50 + 9$ $+ 200 + 60 + 4$ $300 + 110 + 13 = 423$</p>	

Introduce the **expanded column addition** method:

$$\begin{array}{r}
 236 \\
 + 73 \\
 \hline
 \\
 100 \\
 200 \\
 \hline
 309
 \end{array}$$

Add the **units** first, in preparation for the compact method.

In order to carry out this method of addition:

- Children need to recognise the value of the hundreds, tens and units without recording the partitioning.
- Pupils need to be able to add in columns.



Move to the compact **column addition** method, with 'carrying':

$$\begin{array}{r}
 236 \\
 + 73 \\
 \hline
 309 \\
 1
 \end{array}$$

Add **units** first.

'Carry' numbers **underneath** the bottom line.

Children who are very secure and confident with 3-digit expanded column addition should be moved onto the **compact column addition** method, being introduced to 'carrying' for the first time. Compare the expanded method to the compact column method to develop an understanding of the process and the reduced number of steps involved.

Remind pupils the actual value is '**thirty add seventy**', but we say '**three add seven**' because it is already in the tens column.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, **hundreds boundary, increase, vertical, carry, expanded, compact**

Key skills for addition at Stage 3:

- Read and write numbers to 1000 in numerals and words.
- Add 2-digit numbers mentally, incl. those exceeding 100.
- **Add a three-digit number and ones mentally (175 + 8)**
- **Add a three-digit number and tens mentally (249 + 50)**
- **Add a three-digit number and hundreds mentally (381 + 400)**
- Estimate answers to calculations, using inverse to check answers.
- Solve problems, including missing number problems, using
- Number facts, place value, and more complex addition.
- Recognise place value of each digit in 3-digit numbers (hundreds, tens, ones.)
- Continue to practise a wide range of mental addition strategies, ie. number bonds, adding the nearest multiple of 10, 100, 1000 and adjusting, using near doubles, partitioning and recombining.

Year 4

Children should extend the carrying method to numbers with at least four digits.

$$\begin{array}{r} 587 \\ + 417.5 \\ \hline 1062 \end{array}$$

$$\begin{array}{r} 3587 \\ + 1617.5 \\ \hline 4262 \end{array}$$

$$\begin{array}{r} 587 \\ + 475 \\ \hline 1062 \end{array}$$

$$\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \end{array}$$

11

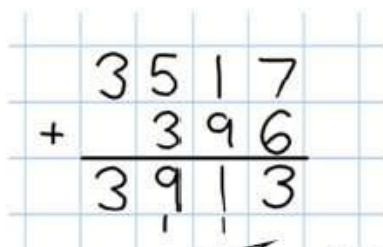
111

Using similar methods, children will:

- Add several numbers with different numbers of digits
- Begin to add two or more decimal fractions with up to three digits and the same number of decimal places
- Know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts eg 3.2m - 280 cm.

Move from expanded addition to the compact column method, **adding units first**, and 'carrying' numbers **underneath** the calculation. Also include money and measures contexts.

e.g. $3517 + 396 = 3913$



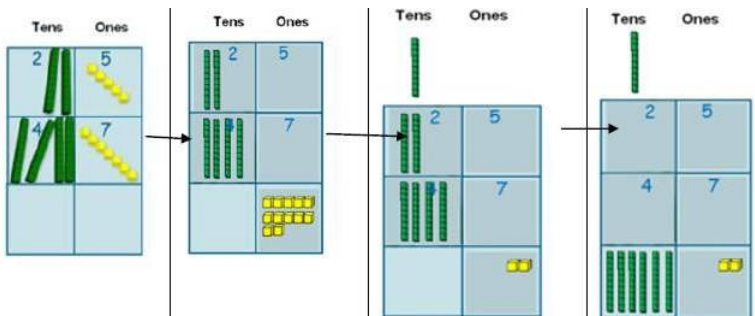
Introduce the **compact column addition** method by asking children to add the two given numbers together using the method that they are familiar with (expanded column addition—see Y3). Teacher models the compact method with carrying, asking children to discuss similarities and differences and establish how it is carried out.

Add **units** first.

'Carry' numbers **underneath** the bottom line.

Reinforce correct place value by reminding them the actual value is 5 hundreds add 3 hundreds, **not 5 add 3**, for example.

Use and apply this method to money and measurement values.



Children use Dienes in a vertical format to prepare them for column addition.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, „carry“, expanded, compact, **thousands, hundreds, digits, inverse**

Key skills for addition at Stage 4:

- Select most appropriate method: mental, jottings or written and explain why.
- Recognise the place value of each digit in a four-digit number.
- Round any number to the nearest 10, 100 or 1000.
- Estimate and use inverse operations to check answers.
- Solve 2-step problems in context, deciding which operations and methods to use and why.
- Find 1000 more or less than a given number.

- Continue to practise a wide range of mental addition strategies, ie. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining.
- Add numbers with up to 4 digits using the formal written method of column addition
- Solve 2-step problems in contexts, deciding which operations and methods to use and why.
- Estimate and use inverse operations to check answers to a calculation.

Year 5

Children should extend the carrying method to number with any number of digits.

$$\begin{array}{r} 7648 \\ + 1486 \\ \hline 9134 \end{array}$$

$$\begin{array}{r} 6584 \\ + 5848 \\ \hline 12432 \end{array}$$

$$\begin{array}{r} 7648 \\ + 1486 \\ \hline 9134 \\ 111 \end{array}$$

$$\begin{array}{r} 6584 \\ + 5848 \\ \hline 12432 \\ 1111 \end{array}$$

Using similar methods, children will

- Add several numbers with different numbers of digits
- Begin to add two or more decimal fractions with up to four digits and either one or two decimal places
- Know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts eg $401.2 + 26.85 + 0.71$

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

1. They are not ready.
2. They are not confident.

Children should be encouraged to approximate their answers before calculating.

Children should be encouraged to check their answers after calculation using an appropriate strategy.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

prices.

$$\begin{array}{r} \text{€ } 23.59 \\ + \text{€ } 7.55 \\ \hline \text{€ } 31.14 \end{array}$$

The decimal point should be aligned in the same way as the other place value columns, and must remain in the same column in the answer row.

$$\begin{array}{r} 23481 \\ + 1362 \\ \hline 24843 \end{array}$$

Numbers should exceed 4 digits.

$$\begin{array}{r} 19.01 \\ 3.65 \\ + 0.70 \\ \hline 23.36 \end{array}$$

Pupils should be able to add **more than two values**, carefully aligning place value columns.

Say '6 tenths add 7 tenths' to reinforce place value.

Empty decimal places can be filled with zero to show the place value in each column.

Children should:

- Understand the place value of **tenths and hundredths** and use this to align numbers with different numbers of decimal places.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, „carry“, expanded, compact, vertical, thousands, hundreds, digits, inverse & decimal places, decimal point, tenths, hundredths, thousandths

Key skills for addition at Stage 5:

- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies ie. add the nearest multiple of 10, 100, 1000 and adjust; use near doubles, inverse, partitioning and re-combining; using number bonds.
- Use rounding to check answers and accuracy.
- Solve multi-step problems in contexts, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Round any number up to 1,000,000 to the nearest 10, 100, 1000, 10,000 and 100,000.
- Add numbers with more than 4 digits using formal written method of column addition.

Year 6

$$\begin{array}{r} 23.361 \\ 9.080 \\ 59.770 \\ + 1.300 \\ \hline 93.511 \\ \begin{array}{l} 2 \quad 1 \quad 2 \end{array} \end{array}$$

Adding several numbers with different numbers of decimal places (including money and measures):

- Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.

Empty decimal places should be filled with zero to show the place value in each column.

$$\begin{array}{r} 81,059 \\ 3,668 \\ 15,301 \\ + 20,551 \\ \hline 120,579 \\ \begin{array}{l} 1 \quad 1 \quad 1 \quad 1 \end{array} \end{array}$$

Adding several numbers with more than 4 digits.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, „carry“, expanded, compact, vertical, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths.

Key skills for addition at Y6:

- Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies.
- Solve multi-step problems in context, deciding which operations and methods to use and why.
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy.
- Pupils understand how to add mentally with larger numbers and calculations of increasing complexity.

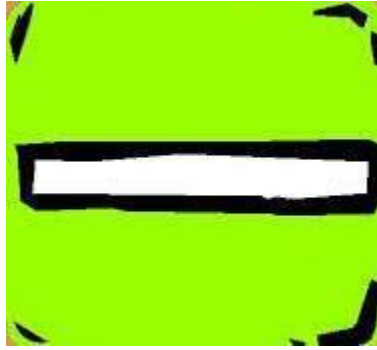
Adding several numbers with different numbers of decimal places (including money and measures):

- Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.

Adding several numbers with more than 4 digits.

Empty decimal places should be filled with zero to show the place value in each column

Subtraction



Progression in Calculations for Subtraction

Mental Calculations

(ongoing)

These are a **selection** of mental calculation strategies:

Mental recall of addition and subtraction facts

$$10 - 6 = 4$$

$$17 - \square = 11$$

$$20 - 17 = 3$$

$$10 - \square = 2$$

Find a small difference by counting up

$$82 - 79 = 3$$

Counting on or back in repeated steps of 1, 10, 100, 1000

$$86 - 52 = 34 \text{ (by counting back in tens and then in ones)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds)}$$

Subtract the nearest multiple of 10, 100 and 1000 and adjust

$$24 - 19 = 24 - 20 + 1 = 5$$

$$458 - 71 = 458 - 70 - 1 = 387$$

Use the relationship between addition and subtraction

$$36 + 19 = 55$$

$$19 + 36 = 55$$

$$55 - 19 = 36$$

$$55 - 36 = 19$$

Mental calculation strategies will continue to be used.

They are not replaced by written methods.

Year 1

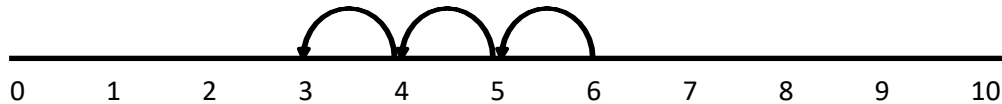
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc. Children understand 'subtraction' as 'taking away' and finding out how many are left.



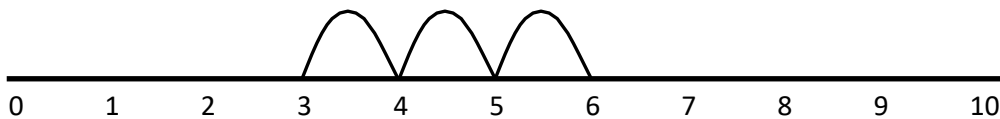
They use number lines and practical resources (eg cubes and Numicon) to support calculation. Teachers **demonstrate** the use of the number line and children use them.

$$6 - 3 = 3$$

-1 -1 -1



The number line should also be used to show that $6 - 3$ means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.



Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones.

$$13 - 5 = 8$$

-1 -1 -1 -1 -1



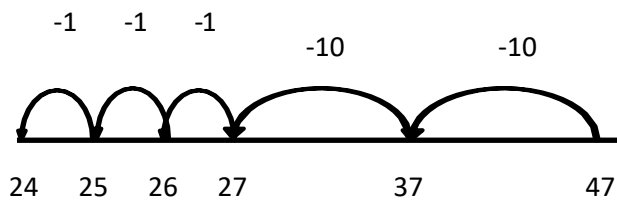
Year 2

Children will use a number square, Dienes, numbered number lines, Numicon, then as they progress, empty number lines to support calculations. These should be differentiated as necessary.

Counting back

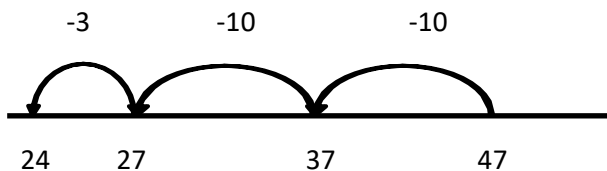
- First counting back in tens and ones.

$$47 - 23 = 24$$



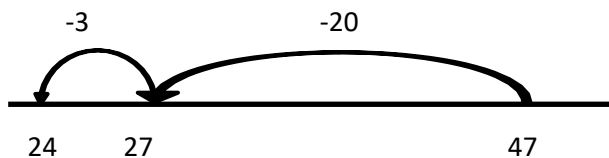
- Then helping children to become more efficient by subtracting the units in one jump (by using the known fact $7 - 3 = 4$).

$$47 - 23 = 24$$



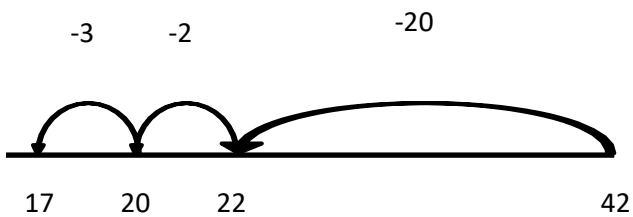
- Subtracting the tens in one jump and the units in one jump.

$$47 - 23 = 24$$



- Bridging through ten can help children become more efficient (higher level 2/3).

$$42 - 25 = 17$$



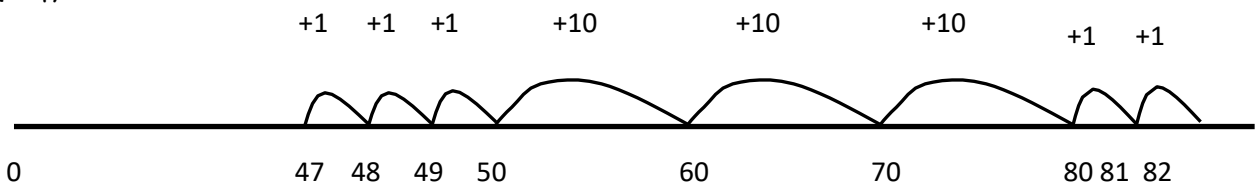
Counting on

If the numbers involved in the calculation are close together or near to multiples of 10, 100, etc, it can be more efficient to count on.

Count up from 47 to 82 in jumps of 10 and jumps of 1.

The number line should still show 0 so children can cross out the section from 0 to the smallest number. They then associate this method with 'taking away.'

$$82 - 47$$



Help children to become more efficient with counting on by:

- Subtracting the units in one jump
- Subtracting the tens in one jump and the units in one jump
- Bridging through ten.

Partitioning and decomposition

This process should be demonstrated using arrow cards to show the partitioning and base 10 materials to show the decomposition of the number.

NOTE when solving the calculation $89 - 57$, children should know the **57 DOES NOT EXIST AS A SEPARATE AMOUNT** it is what you are subtracting from the other number. Therefore, when using base 10 materials, children would need to count out only the 89.

Horizontally first,

$$89 - 57$$

$$80 - 50 = 30$$

$$9 - 7 = 2$$

$$30 + 2 = 32$$

Then vertically,

$$\begin{array}{r} 89 \\ - 57 \\ \hline 2 \\ \hline 30 \\ \hline 32 \end{array}$$

Initially, the children will be taught using examples that do not need them to exchange.

From this the children will begin to exchange (higher level 2/lower3).

$$\begin{array}{r} 71 \\ - 46 \\ \hline \end{array} = \quad =$$

Step 1

$$\begin{array}{r} 70 + 1 \\ - 40 + 6 \\ \hline \end{array}$$

Step 2

$$\begin{array}{r} 60 + 11 \\ - 40 + 6 \\ \hline 20 + 5 \\ = 25 \end{array}$$

The calculation should be read as 'Take 6 from 1.'

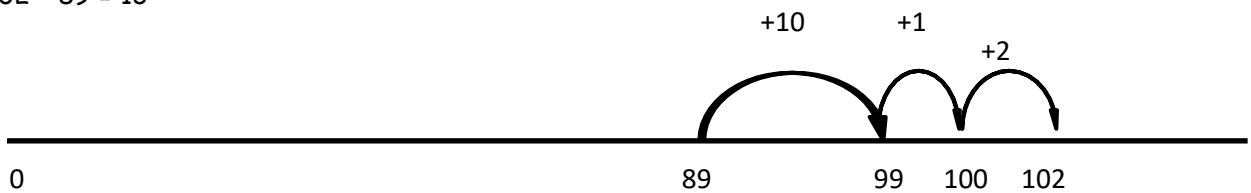
This would be recorded by the children as

$$\begin{array}{r} 60 \\ 70 + 1 \\ - 40 + 6 \\ \hline 20 + 5 \\ = 25 \end{array}$$

Children should know that units line up under units, tens under tens, and so on.

Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

$$102 - 89 = 13$$



Year 3

Partitioning and decomposition

$$\begin{array}{r} 754 \\ - 86 \\ \hline \end{array}$$

$$\begin{array}{l} \text{Step 1} \\ 700 + 50 + 4 \\ - \quad \quad \quad 80 + 6 \\ \hline \end{array}$$

$$\begin{array}{l} \text{Step 2} \\ 700 + 40 + 14 \\ - \quad \quad \quad 80 + 6 \\ \hline \end{array} \quad (\text{adjust from T to U})$$

$$\begin{array}{l} \text{Step 3} \\ 600 + 140 + 14 \\ - \quad \quad \quad 80 + 6 \\ \hline 600 + 60 + 8 \\ = 668 \end{array} \quad (\text{adjust from H to T})$$

This would be recorded by the children as

$$\begin{array}{r} \begin{array}{l} 600 \\ \underline{700} \end{array} + \begin{array}{l} 140 \\ \underline{50} \end{array} + 14 \\ - \quad \quad \quad \underline{80} + 6 \\ \hline 600 + 60 + 8 = 668 \end{array}$$

Decomposition

$$\begin{array}{r} \begin{array}{l} 6 \ 14 \ 1 \\ \cancel{7} \ 5 \ 4 \\ - \quad 8 \ 6 \\ \hline 6 \ 6 \ 8 \end{array} \end{array}$$

Children should

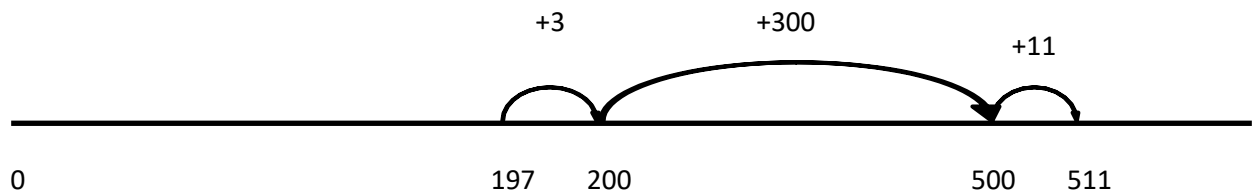
- Be able to subtract numbers with different numbers of digits
- Using this method, children should also begin to find the difference between two three-digit sums of money, with or without 'adjustment' from the pence to the pounds
- Know that decimal points should line up under each other.

When the calculation involves money, children can set the amounts to whole numbers ie 895 - 438 and convert to pounds after the calculation.

NOTE: If your children have reached the concise stage they will then continue this method throughout school. They will not go back to using the expanded methods.

Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

$$511 - 197 = 314$$



Years 4 and 5

Decomposition

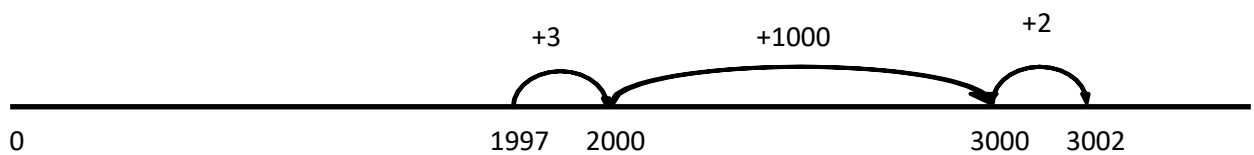
$$\begin{array}{r} \text{5 13 1} \\ \cancel{6} \cancel{4} 6 7 \\ - \quad 2 6 8 4 \\ \hline 3 7 8 3 \end{array}$$

Children should

- Be able to subtract numbers with different numbers of digits
- Be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places
- Know that decimal points should line up under each other.

Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

$$3002 - 1997 = 1005$$



By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

1. They are not ready.
2. They are not confident.

Children should be encouraged to approximate their answers before calculating.

Children should be encouraged to check their answers after calculation using an appropriate strategy.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

Multiplication



Progression in Calculations for Multiplication

Mental Calculations

(ongoing)

These are a **selection** of mental calculation strategies:

Doubling and halving

Applying the knowledge of doubles and halves to know facts.

Eg 8×4 is double 4×4

Using multiplication facts

Tables should be taught every day from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the day.

Using and applying division facts

Children should be able to utilise their times table knowledge to derive other facts.

Eg If I know $3 \times 7 = 21$, what else do I know?

$30 \times 7 = 210$, $300 \times 7 = 2100$, $3000 \times 7 = 21000$, $0.3 \times 7 = 2.1$ etc.

Use closely related facts already known

$$13 \times 11 = (13 \times 10) + (13 \times 1)$$

$$= 130 + 13$$

$$= 143$$

Multiplying by 10 or 100

Knowing that the effect of multiplying by 10 is a shift in the digits **one** place to the left.

Knowing that the effect of multiplying by 100 is a shift in the digits **two** places to the left.

Partitioning

$$23 \times 4 = (20 \times 4) + (3 \times 4)$$

$$= 80 + 12$$

$$= 92$$

Use of factors

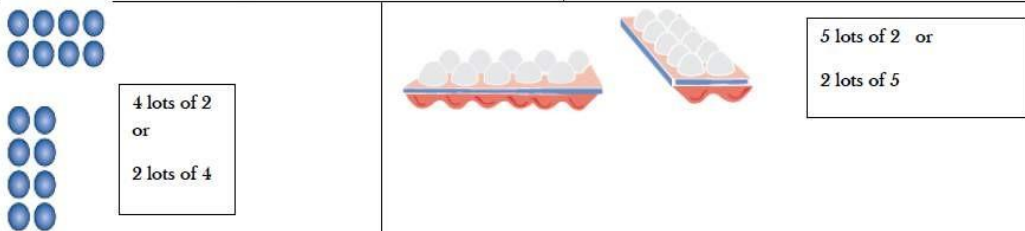
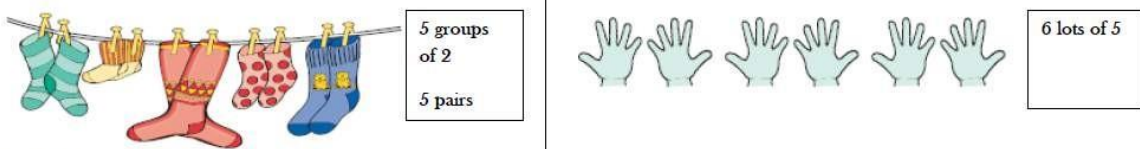
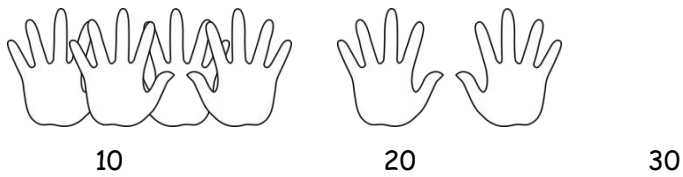
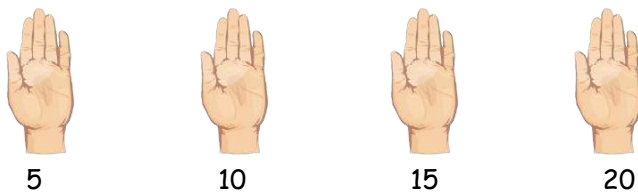
$$8 \times 12 = 8 \times 4 \times 3$$

Mental calculation strategies will continue to be used.

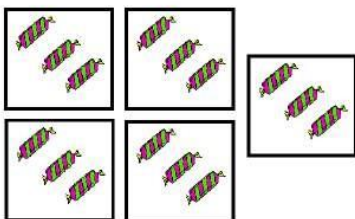
They are not replaced by written methods.

Year 1

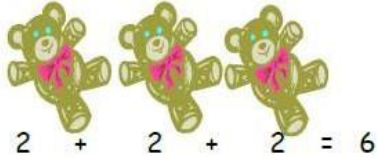
Children will experience equal groups of objects and will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups.



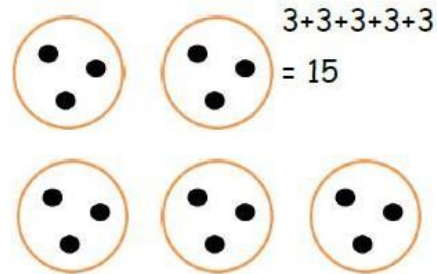
There are 3 sweets in one bag.
How many sweets are in 5 bags
altogether?



How many legs will 3 teddies have?



There are 3 sweets in one bag.
How many sweets are in 5 bags
altogether?



- Give children experience of counting equal group of objects in 2s, 5s and 10s.
- Present practical problem solving activities involving counting equal sets or groups, as above.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count

Key skills for multiplication at Stage 1:

- Count in multiples of 2, 5 and 10.
- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Make connections between arrays, number patterns, and counting in twos, fives and tens.
- Begin to understand doubling using concrete objects and pictorial representations.

Year 2

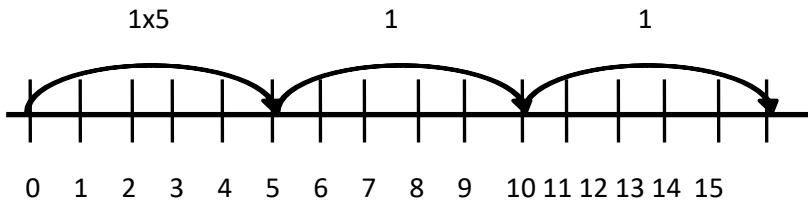
Children will develop their understanding of multiplication and use jottings to support calculation:

➤ Repeated addition

3 times 5 is $5 + 5 + 5 = 15$ or 3 lots of 5 or 5×3

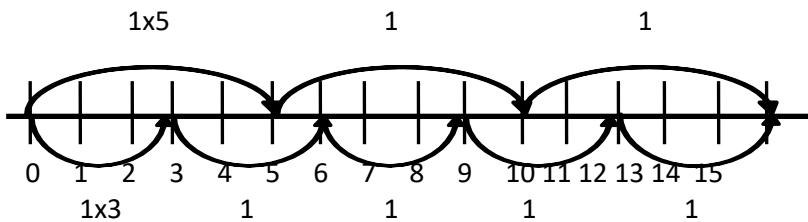
Repeated addition can be shown easily on a number line:

$$3 \times 5 = 5 + 5 + 5$$



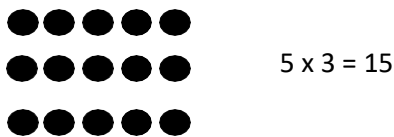
➤ **Commutativity**

Children should know that 3×5 has the same answer as 5×3 . This can also be shown on the number line, Numicon or other practical, tactile equipment to help develop understand the process which gives the same answer, they are different calculations.



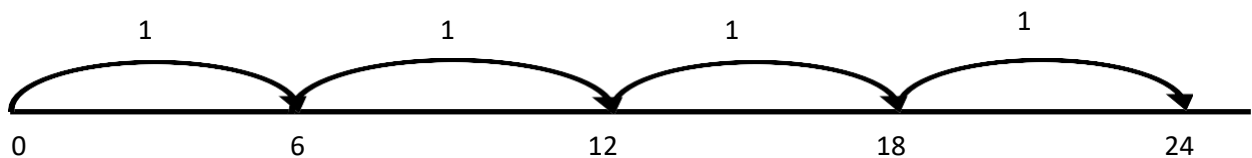
➤ **Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method using appropriate equipment.



Children $3 \times 5 = 15$ empty number lines to support their understanding.

$4 \times 6 =$





$$10p + 10p + 10p + 10p + 10p = 50p$$

$$10p \times 5 = 50p$$

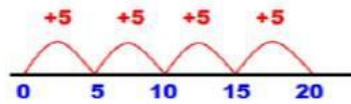


Use repeated addition on a number line:

- Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using \times and $=$ signs.

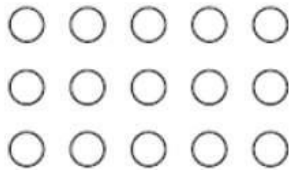
$$4 \times 5 = \dots$$

4 lots of 5



$$4 \times 5 = 20$$

Use arrays:



$$3 \times 5 = 15$$

$$5 \times 3 = 15$$

$$5 \times 3 = 3 + 3 + 3 + 3 = 15$$

$$3 \times 5 = 5 + 5 + 5 = 15$$

Use arrays to help teach children to understand the commutative law of multiplication, and give examples such as $3 \times \underline{\quad} = 6$.

Use practical apparatus:

$$5 \times 3 = 5 + 5 + 5$$



Use mental recall:

- Children should begin to **recall multiplication facts for 2, 5 and 10** times tables through practice in counting and understanding of the operation.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times...

Key skills for multiplication at Stage 2:

- Count in steps of 2, 3 and 5 from zero, and in 10s from any number.
- Recall and use multiplication facts from the **2, 5 and 10** multiplication tables, including recognising odds and evens.
- Write and calculate number statements **using the \times and $=$ signs**.
- Show that multiplication can be done in any order (commutative).
- Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods, and multiplication facts.
- Pupils use a variety of language to discuss and describe multiplication.

Year 3

By end of year 3 (NC 2014)

Note: Grid method is a transition stage towards formal column method.

Grid method

TU x U

(Short multiplication - multiplication by a single digit)

$$23 \times 8$$

Children will approximate first

23×8 is approximately $25 \times 8 = 200$

X	20	3
8	160	24

$$\begin{array}{r} 160 \\ + 24 \\ \hline 184 \end{array}$$

Children represent calculations as arrays, repeated addition and use number lines to calculate.

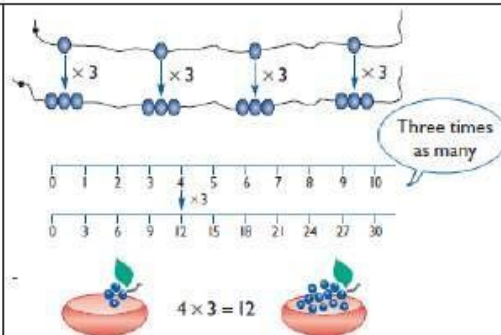
Through doubling, they connect the 2, 4 and 8 multiplication tables.

Pupils develop efficient mental methods, for example, using commutativity and associativity (for example, $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$)

Children use existing knowledge to find related facts:

If $3 \times 2 = 6$ then $3 \times 20 = 60$

Children continue to relate multiplication to scaling contexts.



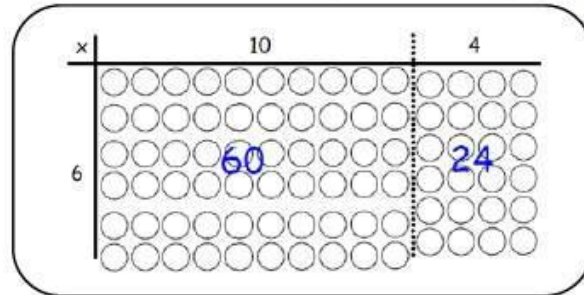
Introduce the **grid method** for multiplying 2-digit by single-digits:

Eg. $23 \times 8 = 184$

X	20	3
8	160	24

$160 + 24 = 184$

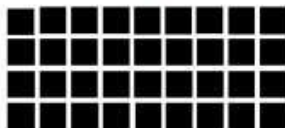
Link the layout of the grid to an array initially:



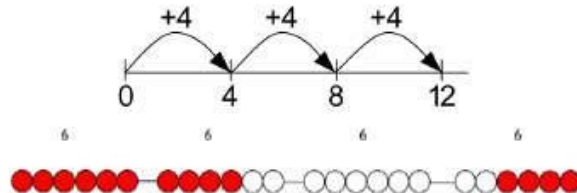
Introduce the grid method with children physically making an array to represent the calculation (e.g. make 8 lots of 23 with 10s and 1s place value counters), then translate this to grid method format (see video clip).

To do this, children must be able to:

- Partition numbers into tens and units
- Multiply multiples of ten by a single digit (e.g. 20×4) using their knowledge of multiplication facts and place value
- Recall and work out multiplication facts in the **2, 3, 4, 5, 8 and 10** times tables.
- Work out multiplication facts not known by repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.) Strategies to support this are repeated addition using a number line, bead bars and arrays:



$9 \times 4 = 36$



Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, _times as big as, once, twice, three times..., **partition, grid method, multiple, product, tens, units, value**

Key skills for multiplication:

- Recall and use multiplication facts for the **2, 3, 4, 5, 8 and 10** multiplication tables, and multiply multiples of 10.
- Write and calculate number statements using the multiplication tables they know, including **2-digit x single digit**, drawing upon mental methods, and progressing to reliable written methods.
- Solve multiplication problems, including missing number problems.
- Develop mental strategies using commutativity (e.g. $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$)
- Solve simple problems in contexts, deciding which operations and methods to use.

- Develop efficient mental methods to solve a range of problems e.g using commutativity ($4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$) and for missing number problems $? \times 5 = 20$, $3 \times ? = 18$, $? \times ? = 32$

Year 4

Grid method (Non statutory)

HTU \times U

(Short multiplication - multiplication by a single digit)

$$346 \times 9$$

Children will approximate first

$$346 \times 9 \text{ is approximately } 350 \times 10 = 3500$$

X	300	40	6
9	2700	360	54

$$\begin{array}{r}
 2700 \\
 360 \\
 \hline
 + 1154 \\
 \hline
 3114
 \end{array}$$

Moving on to:

$$346 \times 22$$

Children will approximate first

$$346 \times 22 \text{ is approximately } 350 \times 20 = 7000$$

X	300	40	6
20	6000	800	120
2	600	80	12

$$\begin{array}{r}
 6000 \\
 800 \\
 120 \\
 600 \\
 80 \\
 \hline
 + 1112 \\
 \hline
 7612
 \end{array}$$

Moving onto the standard written method - multiplying the least significant digits first.

$$\begin{array}{r}
 346 \\
 \times 9 \\
 \hline
 54 \quad (6 \times 9) \\
 360 \quad (40 \times 9) \\
 \underline{2700} \quad (300 \times 9) \\
 \hline
 3114 \\
 11
 \end{array}$$

Using similar methods, they will be able to multiply decimals with one decimal place by a single digit number, approximately first. **They should know that the decimal points line up under each other and never move!**

$$4.9 \times 3$$

Children will approximate first

$$4.9 \times 3 \text{ is approximately } 5 \times 3 = 15$$

$$\begin{array}{r}
 \times \quad 4 \quad 0.9 \\
 3 \quad \boxed{12} \quad \boxed{2.7}
 \end{array}$$

$$\begin{array}{r}
 12.0 \\
 + \underline{2.7} \\
 \hline
 14.7
 \end{array}$$

ThHTU × U

(Short multiplication - multiplication by a single digit)

$$4346 \times 8$$

Children will approximate first

$$4346 \times 8 \text{ is approximately } 4346 \times 10 = 43460$$

$$\begin{array}{r}
 \times \quad 4000 \quad 300 \quad 40 \quad 6 \\
 8 \quad \boxed{32000} \quad \boxed{2400} \quad \boxed{320} \quad \boxed{48}
 \end{array}$$

$$\begin{array}{r}
 32000 \\
 2400 \\
 320 \\
 + \underline{48} \\
 \hline
 34768
 \end{array}$$

Moving onto the standard written method - multiplying the least significant digits first

$$\begin{array}{r}
 4346 \\
 \times \quad 8 \\
 \hline
 48 \quad (6 \times 8) \\
 320 \quad (40 \times 8) \\
 2400 \quad (300 \times 8) \\
 \underline{32000} \quad (4000 \times 8) \\
 \underline{34768}
 \end{array}$$

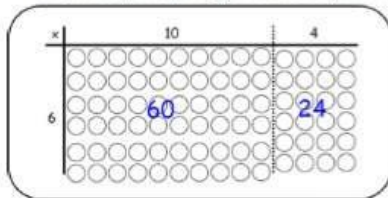
Children use mental strategies to find products and missing numbers.

Children then move on to using grid method for multiplying two and three digit numbers by one number

14 x 6

Estimation: $10 \times 6 = 60$

Link the layout of the grid to an array initially:



x	10	4
6	60	24

$$60 + 24 = 84$$

234 x 6

Estimation: $200 \times 6 = 1200$

$1200 + 180 + 24 = 1404$ using column addition if necessary

x	200	30	4
6	1200	180	24

$$\begin{array}{r}
 1200 \\
 180 \\
 24 \\
 \hline
 1404
 \end{array}$$

Developing the grid method:

Eg. $136 \times 5 = 680$

X	100	30	6
5	500	150	30

$$\begin{array}{r}
 500 \\
 150 \\
 + 30 \\
 \hline
 680
 \end{array}$$

Encourage column addition to add accurately.

Move onto **short multiplication** (see Y5) if and when children are confident and accurate multiplying 2 and 3-digit numbers by a single digit this way, and are already confident in 'carrying' for written addition.

Children should be able to:

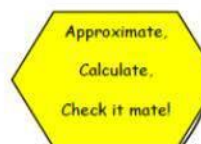
- Approximate before they calculate, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer. e.g:

" 346×9 is approximately $350 \times 10 = 3500$ "

Record an approximation to check the final answer against.

- Multiply multiples of ten and one hundred by a single-digit, using their multiplication table knowledge.

Recall all times tables up to 12×12



This then leads to a compact written method for multiplication e.g.

$$\begin{array}{r}
 246 \\
 \times \quad 7 \\
 \hline
 42 \quad (6 \times 7) \\
 280 \quad (40 \times 7) \\
 1400 \quad (200 \times 7) \\
 \hline
 1722
 \end{array}$$

...notations v
modelled for the c

$$\begin{array}{r}
 246 \\
 \times \quad 7 \\
 \hline
 1722 \\
 34
 \end{array}$$

By end of year 4 (NC 2014)

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times as big as, once, twice, three times... partition, grid method, total, multiple, product, sets of, **inverse**

Key skills for multiplication at Stage 4:

- Count in multiples of 6, 7, 9, 25 and 1000
- Recall multiplication facts for **all multiplication tables up to 12 x 12**.
- Recognise place value of digits in up to 4-digit numbers
- Use place value, known facts and derived facts to multiply mentally, e.g. multiply by 1, 10, 100, by 0, or to multiply 3 numbers.
- Use commutativity and other strategies mentally $3 \times 6 = 6 \times 3$, $2 \times 6 \times 5 = 10 \times 6$, $39 \times 7 = 30 \times 7 + 9 \times 7$.
- Solve problems with increasingly complex multiplication in a range of contexts.
- Count in multiples of 6, 7, 9, 25 and 1000
- Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and units)

Year 5

TU x U

(Long multiplication - multiplication by more than a single digit)

72 x 38

Children will approximate first

23×8 is approximately $70 \times 40 = 2800$

X	70	2
30	2100	60
8	560	16

$$\begin{array}{r} 2100 \\ 560 \\ 60 \\ + \underline{16} \\ \hline 2736 \end{array}$$

Moving onto the standard written method - multiplying the least significant digits first

$$\begin{array}{r} 72 \\ X \underline{38} \\ 16 \quad (2 \times 8) \\ 60 \quad (2 \times 30) \\ 560 \quad (70 \times 8) \\ \underline{2100} \quad (70 \times 30) \\ \hline 2736 \end{array}$$

HTU x TU

Standard written method - multiplying the least significant digits first

$$\begin{array}{r} 372 \\ X \underline{24} \\ 8 \quad (2 \times 4) \\ 40 \quad (2 \times 20) \\ 280 \quad (70 \times 4) \\ 1200 \quad (300 \times 4) \\ 1400 \quad (70 \times 20) \\ \underline{6000} \quad (300 \times 20) \\ \hline 8928 \end{array}$$

Using similar methods, they will be able to multiply decimals with up to two decimal places by a single digit number and then two digit numbers, approximating first. They should know that the decimal points line up under each other.

$$4.92 \times 3$$

Children will approximate first

$$4.92 \times 3 \text{ is approximately } 5 \times 3 = 15$$

$$\begin{array}{r} \times \\ 3 \end{array} \begin{array}{|c|c|c|} \hline 4 & 0.9 & 0.02 \\ \hline 12 & 2.7 & 0.06 \\ \hline \end{array}$$

$$\begin{array}{r} 12.00 \\ 2.70 \\ + 0.06 \\ \hline 14.76 \end{array}$$

Key vocabulary groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, _times as big as, once, twice, three times..., partition, grid method, total, multiple, product, inverse, **square, factor, integer, decimal, short/long multiplication, _carry'**

Key skills for multiplication at Y5:

- Identify multiples and factors, using knowledge of **multiplication tables to 12x12**.
- Solve problems where larger numbers are decomposed into their factors
- Multiply and divide integers and decimals by 10, 100 and 1000
- Recognise and use square and cube numbers and their notation
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately.

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

1. They are not ready.
2. They are not confident.

Children should be encouraged to approximate their answers before calculating.

Children should be encouraged to check their answers after calculation using an appropriate strategy.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

$6481 \times 9 = 58329$

$$\begin{array}{r} 6481 \\ \times 9 \\ \hline 58329 \\ \hline 47 \end{array}$$

$56 \times 27 = 1512$

$$\begin{array}{r} 56 \\ \times 27 \\ \hline 392 \\ 1120 \\ \hline 1512 \end{array}$$

Multiply numbers **up to 4 digits by a one- or two-digit number** using a formal written method, including long multiplication for two-digit numbers

Year 6

$$4346 \times 28$$

Children will approximate first

$$4346 \times 28 \text{ is approximately } 4346 \times 30 = 130380$$

X	4000	300	40	6
20	80000	6000	800	120
8	32000	2400	320	48

$$\begin{array}{r} 80000 \\ 32000 \\ 6000 \\ 2400 \\ 800 \\ 320 \\ 120 \\ + 11 \ 48 \\ \hline 121688 \end{array}$$

Long multiplication (Including decimals in context)

$$134 \times 32 = 4288$$

$$\begin{array}{r} 134 \\ \times 32 \\ \hline 268 \\ 4020 \\ \hline 4288 \end{array}$$

$$124 \times 26 = 3224$$

$$\begin{array}{r} 124 \\ \times 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \\ 11 \end{array}$$

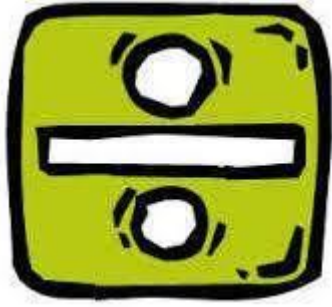
Multiply **multi-digit numbers up to 4 digits by a two-digit whole number** using the formal written method of long multiplication

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times... partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short / long multiplication, „carry“, **tenths, hundredths, decimal**

Key skills for multiplication at Stage 6:

- Recall multiplication facts for all times tables up to 12×12 (as Y4 and Y5).
- Multiply multi-digit numbers, up to 4-digit \times 2-digit using long multiplication.
- Perform mental calculations with mixed operations and large numbers.
- Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods.
- Estimate answers using round and approximation and determine

Division



Progression in Calculations for Division

Mental Calculations

(ongoing)

These are a **selection** of mental calculation strategies:

Doubling and halving

Knowing that halving is dividing by 2.

Deriving and recalling division facts

Tables should be taught every day from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the day.

Using and applying division facts

Children should be able to utilise their times table knowledge to derive other facts.

Eg If I know $3 \times 7 = 21$, what else do I know?

$30 \times 7 = 210$, $300 \times 7 = 2100$, $3000 \times 7 = 21000$, $0.3 \times 7 = 2.1$ etc.

Dividing by 10 or 100

Knowing that the effect of dividing by 10 is a shift in the digits **one** place to the right.

Knowing that the effect of dividing by 100 is a shift in the digits **two** places to the right.

Use of factors

$$378 \div 21$$

$$378 \div 3 = 126$$

$$378 \div 21 = 18$$

$$126 \div 7 = 18$$

Use related facts

Given that $1.4 \times 1.1 = 1.54$

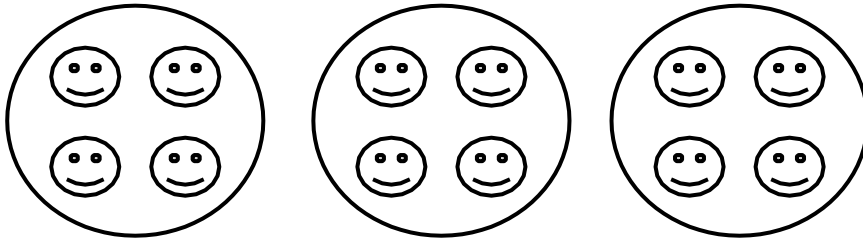
What is $1.54 \div 1.4$ or $1.54 \div 1.1$?

Mental calculation strategies will continue to be used.

They are not replaced by written methods.

Year 1

Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.

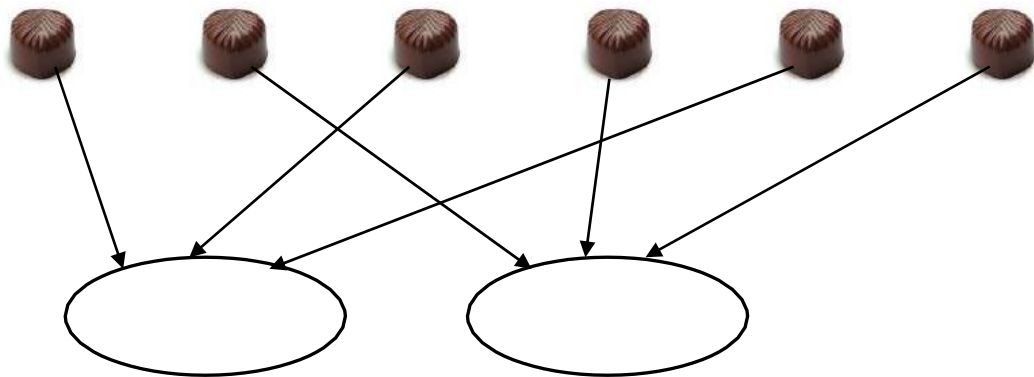


Year 2

Children will develop their understanding of division and use jottings to support calculation.

➤ **Sharing equally**

6 chocolates shared between 2 people, how many do they each get?



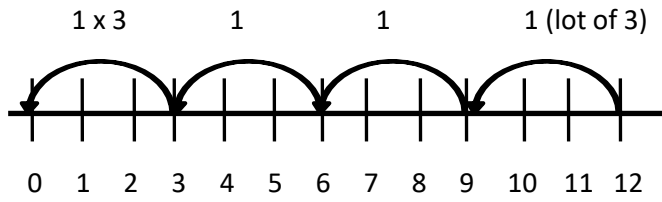
➤ **Grouping or repeated subtraction**

There are 6 chocolates, how many people can have 2 chocolates each?



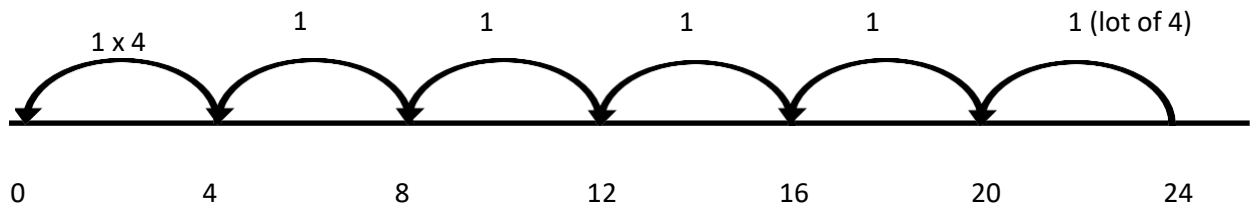
➤ Repeated subtraction using a numbered line

$$12 \div 3 = 4$$



➤ Repeated subtraction using an empty number line

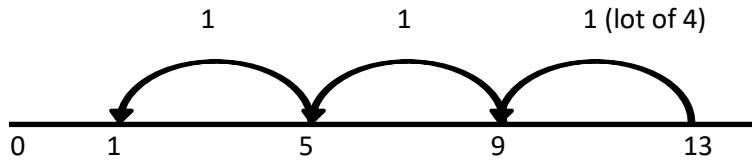
$$24 \div 4 = 6$$



Year 3

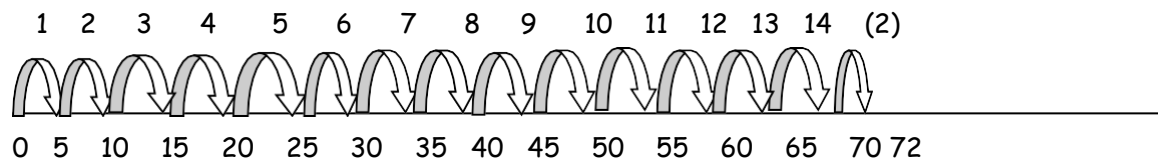
Children should also move onto calculations involving remainders.

$13 \div 4 = 3 \text{ and } \frac{1}{4}$ The so called remainder is still divided by 4 leaving a quarter. The confidence in fractions should be considered when asking children to do divisions.

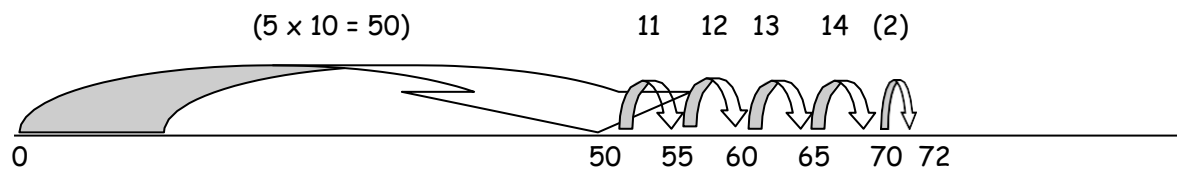


Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s - numbers with which the children are more familiar.

$$72 \div 5 = 14 \frac{2}{5}$$



Moving onto:



Then onto the vertical method:

Short division TU ÷ U

$$72 \div 3$$

$$\begin{array}{r} \underline{24} \\ 3 \overline{)72} \end{array}$$

Answer: 24

Leading to subtraction of other multiples

$$96 \div 6$$

$$\begin{array}{r} \underline{16} \\ 6 \overline{)96} \end{array}$$

Answer: 16

Any remainders should be shown as fractions as they will still be divided, they are not just left alone. They are part of the answer. See apple problem below.

Children need to be able to decide what to do after division and round up or down accordingly. Doing this using fractions deciding if the numerator is closer to the denominator or zero. They should make sensible decisions about rounding up or down after division. For example $62 \div 8$ is $7 \frac{6}{8}$, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context.

Eg I have 62p. Sweets are 8p each. How many can I buy?

Answer: 7 (the remaining 6p is not enough to buy another sweet leaving you with 6 out of seven, children should start to realise they need one more penny or $\frac{1}{7}$ to make a whole).

Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed?

Answer: 8 (the remaining 6 apples still need to be placed into a box, this box will be $\frac{6}{8}$ full).

Year 4

Children will continue to use written methods to solve short division $TU \div U$.

Children can start to subtract larger multiples of the divisor, eg $30 \times$.

Short division $HTU \div U$

$$196 \div 6$$

$$\begin{array}{r} 032\frac{4}{6} \\ 6 \overline{)196} \end{array}$$

Any remainders should be shown as fractions. If a child knows the decimal conversion they could start to use it. This is also a way to lead to a fraction in its lowest term (equivalent).

Informal Method of Long division (Non Statutory)

$$184 \div 7 =$$

184 chairs are needed for a concert. They are arranged in rows of seven. How many rows of chairs are needed?

$$\begin{array}{r} 26 \text{ r } 2 \\ \hline 7 \overline{)184} \\ -140 \text{ (20 lots of 7)} \\ \hline 44 \\ -42 \text{ (6 lots of 7)} \\ \hline 2 \end{array}$$

This method is known as chunking. In this example, you are taking away chunks of seven. First subtract 140 (20 lots of 7) and you are left with 44. Then subtract 42 (six lots of seven) to leave 2. Altogether that is 26 sevens with a remainder of 2. So 26 rows are needed with either a small row of two or two rows with 8 chairs.

Year 5

Children will continue to use written methods to solve short division $TU \div U$ and $HTU \div U$.

Extend to decimals with up to two decimal places. Children should know that decimal points line up under each other.

Non statutory method

$$87.5 \div 7$$

$$\begin{array}{r} \underline{12.5} \\ 7 \overline{)87.5} \\ \underline{-70.0} \quad 10x \\ 17.5 \\ \underline{-14.0} \quad 2x \\ 3.5 \\ \underline{-3.5} \quad 0.5x \\ 0 \end{array}$$

Answer: 12.5

or

$$87.5 \div 7$$

$$\begin{array}{r} \underline{12.5} \\ 7 \overline{)87.5} \end{array}$$

Answer: 12.5

Formal Method

Year 5: Short division - dividing by a one digit number (Decimal division in context to be included)

$$98 \div 7 = 14$$

$$\begin{array}{r} 14 \\ 7 \overline{)98} \\ \underline{7} \\ 28 \\ \underline{28} \\ 0 \end{array}$$

$$432 \div 5 = 86 \text{ r}2$$

$$\begin{array}{r} 86 \text{ r}2 \\ 5 \overline{)432} \\ \underline{40} \\ 32 \\ \underline{30} \\ 2 \end{array}$$

Divide numbers **up to 4 digits by a one-digit number** using a formal written method of short division and interpret remainders appropriately for the context

By the end of year 6, children will have a range of calculation methods, mental and written.
 Selection will depend upon the numbers involved.
 Children should not be made to go onto the next stage if:

1. They are not ready.
2. They are not confident.

Long division HTU ÷ TU (Non Statutory)

$$972 \div 36$$

$$\begin{array}{r}
 \underline{27} \\
 36 \overline{) 972} \\
 \underline{- 360} \quad (36 \times 10) \\
 5\overset{1}{\cancel{6}}12 \\
 \underline{- 360} \quad (36 \times 10) \\
 2\overset{1}{\cancel{5}}2 \\
 \underline{- 180} \quad (36 \times 5) \\
 72 \\
 \underline{- 72} \quad (36 \times 2) \\
 0
 \end{array}$$

Answer: $10 + 10 + 5 + 2 = 27$

Any remainders should be shown as fractions eg if the children were dividing 32 by 10 the answer should be shown as $3 \frac{2}{10}$ which could then be written as $3 \frac{1}{5}$ in its lowest terms.

Formal Method

Year 6: Short and Long division - dividing by a two digit number (Decimal division in context to be included)

$$432 \div 15 \text{ r}12$$

$$\begin{array}{r}
 28 \text{ r}12 \\
 15 \overline{) 432} \\
 \underline{300} \\
 132 \\
 \underline{120} \\
 12
 \end{array}$$

$$496 \div 11 = 45 \text{ r}1$$

$$\begin{array}{r}
 45 \text{ r}1 \\
 11 \overline{) 496} \\
 \underline{440} \\
 56 \\
 \underline{550} \\
 1
 \end{array}$$

Answer: $45 \frac{1}{11}$

Divide numbers **up to 4 digits by a two-digit whole number** using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

Divide numbers **up to 4 digits by a two-digit whole number** using the formal written method of short division, and interpret remainders according to the context

Children should be encouraged to approximate their answers before calculating.

Children should be encouraged to check their answers after calculation using an appropriate strategy.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

Glossary

2- digit - a number with 2 digits like 23, 45, 12 or 60

3- digit - a number with 3 digits like 123, 542, 903 or 561

Addition facts - knowing that $1+1 = 2$ and $1+3 = 4$ and $2+5 = 7$. Normally we only talk about number facts with totals of 20 and under.

Array -An array is an arrangement of a set of numbers or objects in rows and columns -it is mostly used to show how you can group objects for repeated addition or subtraction.

Bridge to ten - a strategy when using number lines. Adding a number that takes you to the next 'tens' number.

Bus Stop Method - traditional method for division with a single digit divisor

Concrete apparatus- objects to help children count - these are most often cubes (multilink) but can be anything they can hold and move.

Dienes (purple hundreds, tens and units blocks), Numicon, Cuisenaire rods are also referred to as concrete apparatus.

Column chunking - method of division involving taking chunks or groups or the divisor away from the larger number

Decimal number - a number with a decimal point
Divisor - the smaller number in a division calculation. The number in each group for chunking.

Double - multiply a number by 2

Exchanging - Moving a 'ten' or a 'hundred' from its column into the next column and splitting it up into ten 'ones' (or 'units') or ten 'tens' and putting it into a different column

Expanded Multiplication - a method for multiplication where each stage is written down and then added up at the end in a column

Find the difference - A method for subtraction involving counting up from the smaller to the larger number

Grid method - a method for multiplying two numbers together involving partitioning

Half - a number, shape or quantity divided into 2 equal parts

Halve - divide a number by 2

Integer - a number with no decimal point

Inverse - the opposite operation. Addition is the inverse of subtraction, multiplication is the inverse of division

Long Multiplication - column multiplication where only the significant figures are noted

Number bonds to ten - 2 numbers that add together to make ten, like 2 and 8, or 6 and 4.

Number bonds to 100 - 2 numbers that add together to make 100 like 20 and 80, or 45 and 65 or 12 and 88

Number line - a line either with numbers or without (a blank number line). Children use this tool to help them count on for addition of subtraction and also in multiplication and division.

Number line Chunking - method of division involving taking chunks or groups or the divisor away from the larger number

Number sentence- writing out a calculation with just the numbers in a line E.G. $2+4=6$ or $35 \div 7 = 5$ or $12 \times 3 = 36$ or $32 - 5 = 27$

Partition - split up a larger number into the hundreds, tens and units. E.G. $342 = 300$ and 40 and 2

Place Value - knowing that in the number 342 - the '3' means '3 hundreds', the '4' means '4 tens' and the '2' means '2'.

Quarter - a number, shape or quantity divided into 4 equal parts

Recombine - for addition, once you have partitioned numbers into hundreds, tens and units then you have to add then hundreds together, then add the tens to that total, and then add the units to that total

Remainder - a whole number left over after a division calculation

Repeated addition - repeatedly adding groups of the same size for multiplication

Significant digit - the digit in a number with the largest value. E.G in 34 - the most significant digit is the 3, as it has a value of '30' and the '4' only has a value of '4'

Single digit - a number with only one digit. These are always less than 10.

Taking away - a method for subtraction involving counting backwards from the larger to the smaller number

Tens number - a number in the ten times tables - 10,20,30,40 50, etc.

Unit - another term for single digit numbers. The right hand column in column methods is the 'units' column