ST AUGUSTINE'S RC PRIMARY SCHOOL

Maths Workshop

Mastery Approach

Addition and Subtraction

DEVELOPING A GROWTH MINDSET

INSTEAD OF.....	TRY THINKING....
I'm not good at this	What am I missing?
I give up	I'll use a different strategy
It's good enough	Is this really my best work?
I can't make this any better	I can always improve
This is too hard	This may take some time
I made a mistake	Mistakes help me to learn
I just can't do this	I am going to train my brain
I'll never be that smart	I will learn how to do this
Plan A didn't work	There's always Plan B
My friend can do it	I will learn from them

AIMS OF TODAY

- **To get an insight into how Maths is taught at St Augustine's, including Maths Mastery.**
- **To explore the key knowledge, skills and understanding children need around number and how Maths is crucial to be successful in today and tomorrow's world.**
- **To begin a journey of understanding around calculation, starting with addition.**
- **To take away some ideas to support your children at home.**

THE MASTERY APPROACH – WHAT IS MASTERY?

The essential idea behind mastery is that **all children** need a ***deep*** understanding of the mathematics they are learning…

- Mastery is all about representing maths so that it makes sense to the children, so the models, images and language that connect the maths are carefully planned.
- Most children access the same content at the same pace.
- All children are given the chance to access the learning with varying support when needed.
- Scaffolded learning is a feature.
- Breadth, depth and challenge are added to the learning.

MASTERY – THERE IS NO 'EASY' OPTION IN LIFE

What does it look like?

efficient

accurate

flexible

appropriate

Maths mastery
=
Fluency

How do we achieve it?

DEPTH

intelligent practice

variation

time

structures

knowledge

reasoning

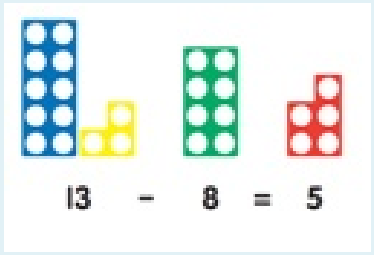
connections

CONCRETE-PICTORIAL-ABSTRACT APPROACH

Concrete, pictorial, abstract (CPA) is a highly effective approach to teaching that develops a deep and sustainable understanding of maths in pupils.

Often referred to as the concrete, representational, abstract framework, CPA was developed by American psychologist Jerome Bruner. It is an essential technique within the Singapore method of teaching maths for mastery.

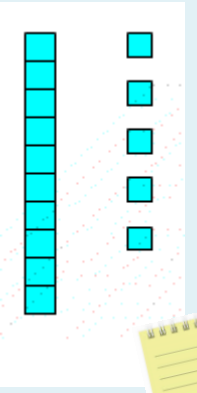
Structuring Learning

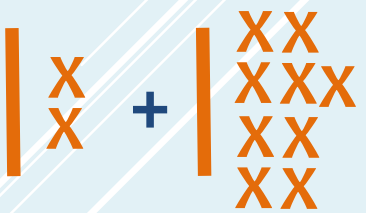


Active / Concrete

Building visual images

Abstract

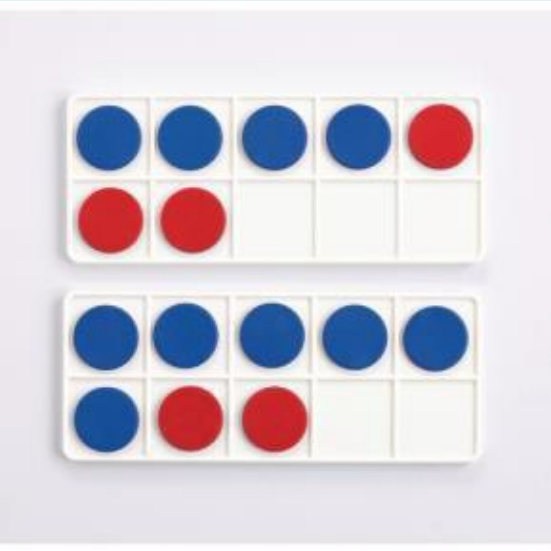


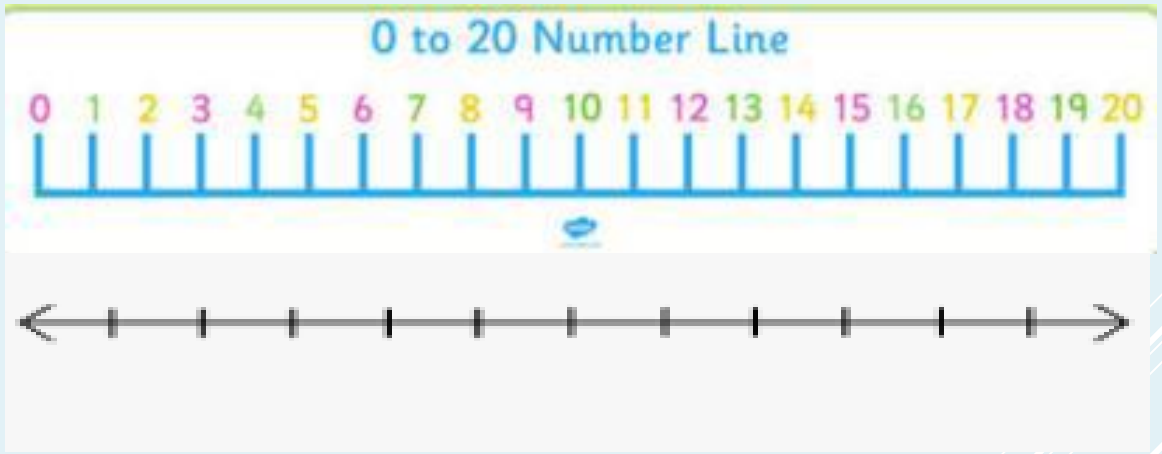


13 - 8

12 + 19

- Just a few of the concrete tools we use to support children that you can use too…

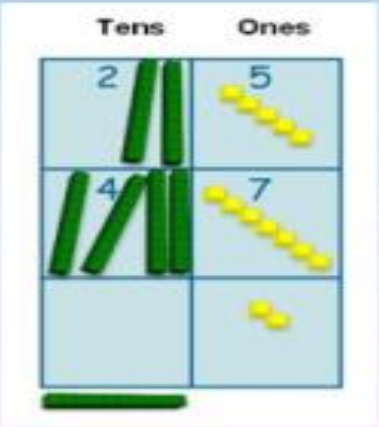


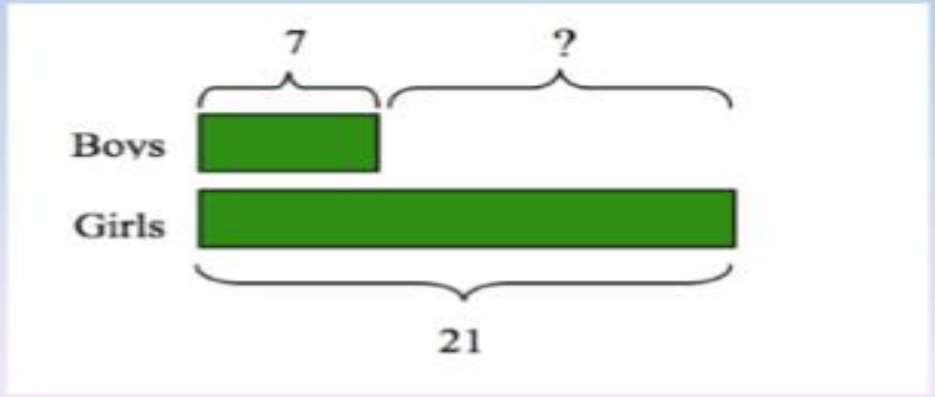


Pictorial Experiences

Pictorial representation

Using representations, such as a diagram or picture of the problem.





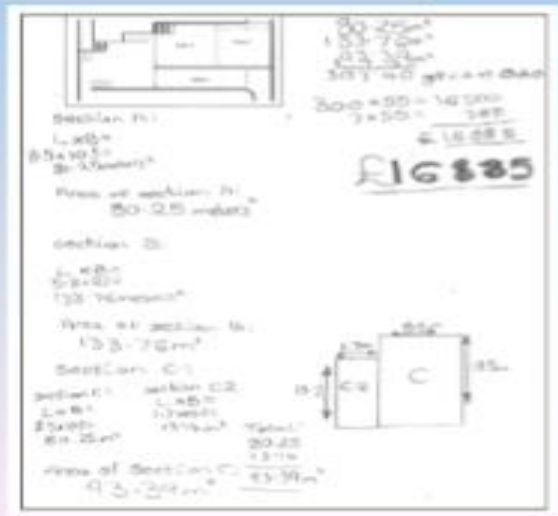
Symbolic/abstract Experiences

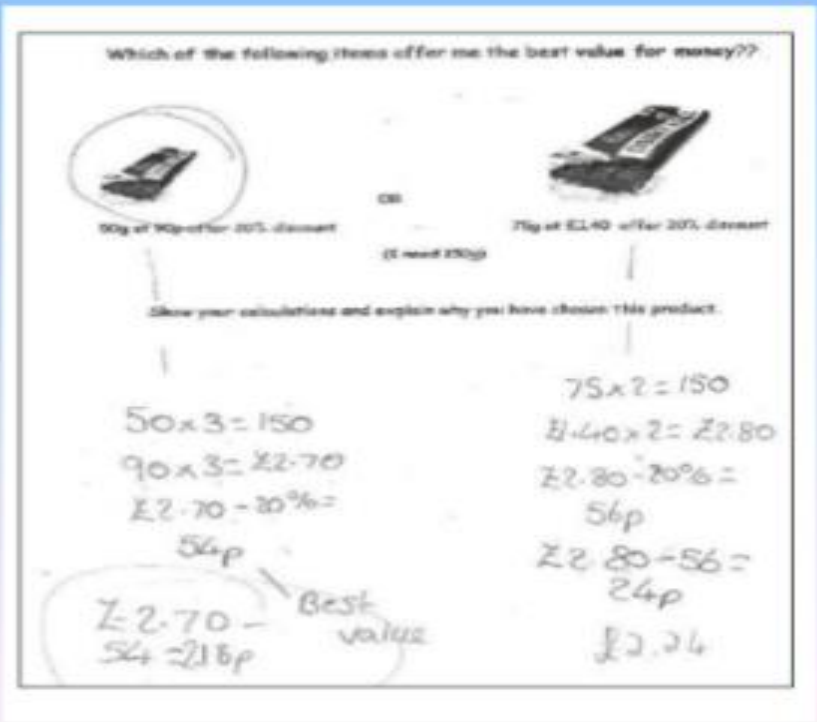
Abstract representation

The symbolic stage - a student is now capable of representing problems by using mathematical notation, for example: 12 ÷ 2 = 6

(3 x 6 = 18, 8 ones and exchange 10 ones for a ten)
(3 x 4 tens plus the 1 exchanged ten = 13 tens)

$$\begin{array}{r} 46 \\ \times\ 3 \\ \hline 138 \\ \hline \end{array}$$





IT'S ALL ABOUT THE WAY WE SEE THINGS…

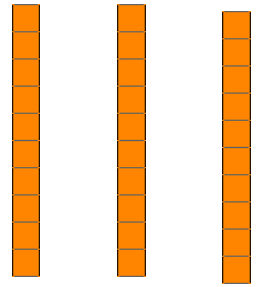
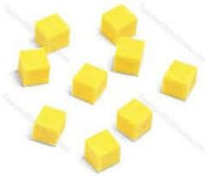
NUMBER SENSE!

Children need to understand our number system, starting with counting numbers, building an understanding of how our numbers work and fit together.

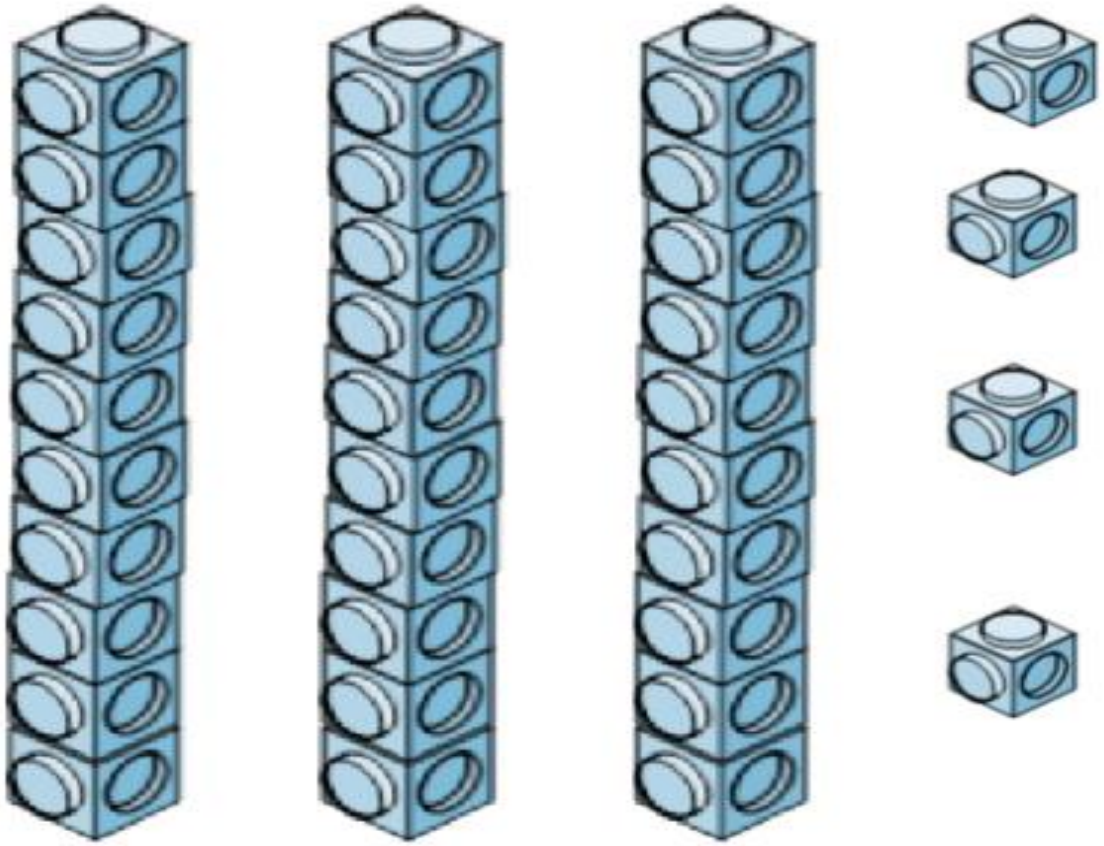
This includes exploring place value and comparing and ordering numbers then applying this understanding in different contexts.

PLACE VALUE

- Place value is at the heart of the number system. All digits have a value and a secure understanding of this will enable children to use and understand different calculation methods.

thousands	hundreds	tens	ones
1	2	3	9
	 		

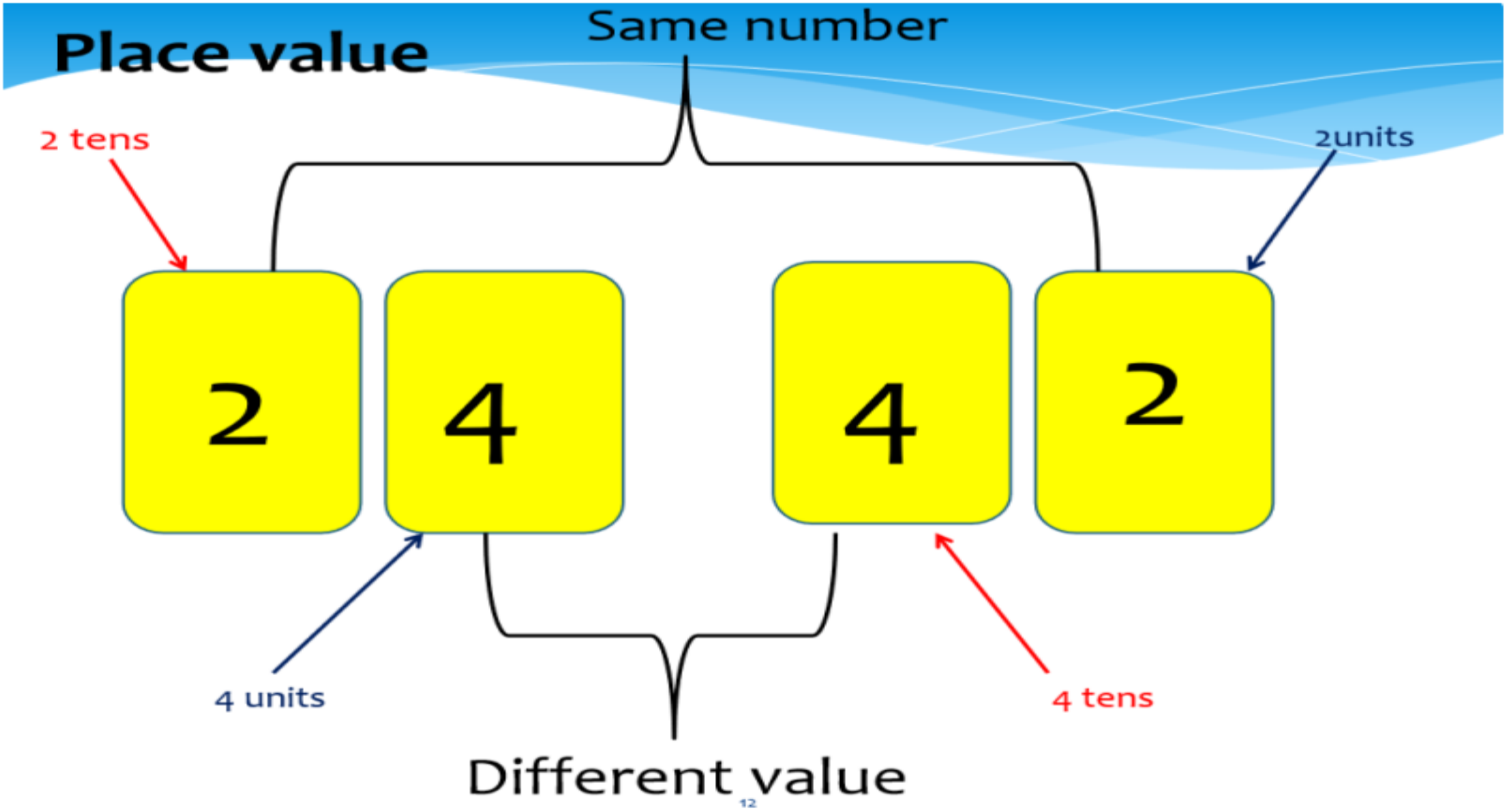
Place value



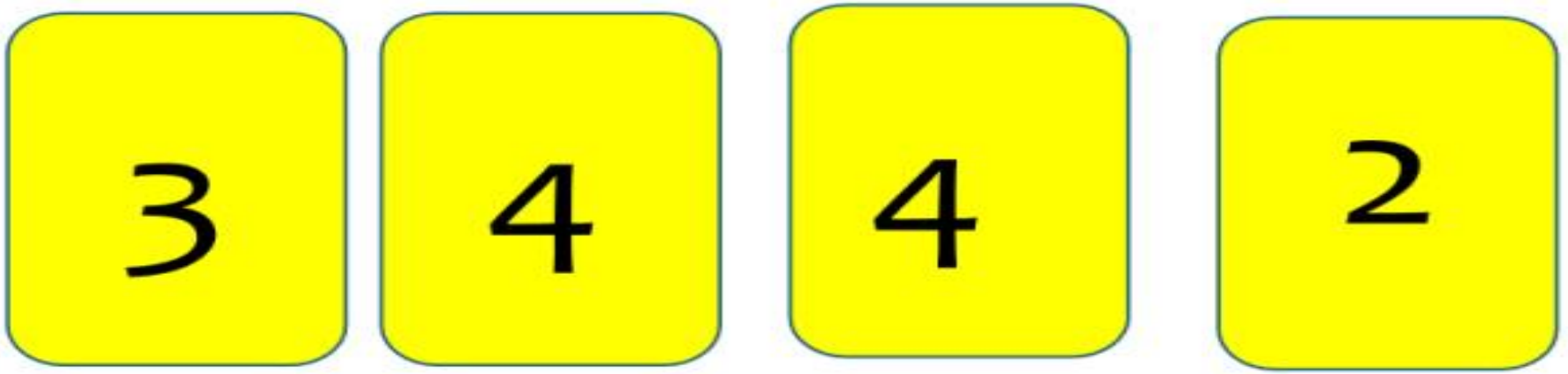
What number do these make?

3 tens and 4 units = 34

Place value


Same number
2 tens
2units
2
4
4
2
4 units
4 tens
Different value

Place value – Your Turn



- What is the biggest number that can be made?
- Smallest?
- How many different numbers can be made?

Addition

Addition vocabulary

+

- altogether
- more than
- count on
- plus
- add
- total
- and

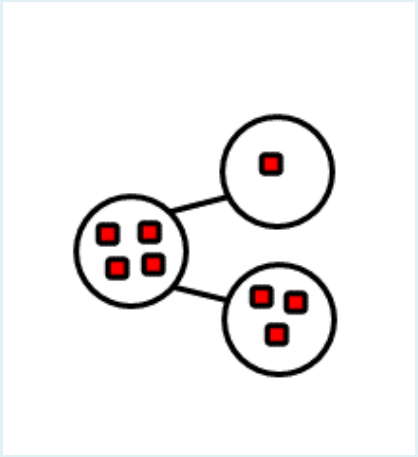
MATHS IN THE EARLY YEARS

- Children are natural problem solvers
- Number and numerals are abstract concepts – 3 can mean very little to a child.
- Children need to build images of numbers – linking numerals to objects
- Maths in the early years is play based. The aim is to build understanding and not written methods.
- Maths games, songs, rhymes and activities help build understanding.

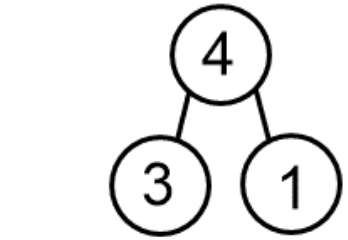
- Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

BY THE END OF RECEPTION
EARLY LEARNING GOAL

Concrete – make a stick of four
Combining two parts to make a whole. At home various other objects can be used for this concept, for example, pasta, pebbles, teddy bears



Pictorial – part whole model



Abstract - $4 = \square + 1$

Recalling facts

* It is important that children recognise number bonds - different pairs of numbers with the same total.

7 + 3

10

6 + 4

6 + 2

8

5 + 3

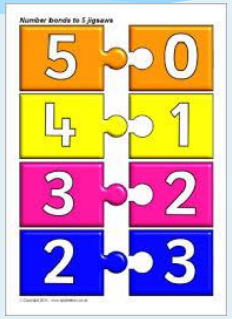
6

3 + 3

3 + 2

5

1 + 4



6 + 3

5 + 4

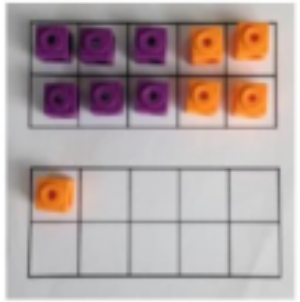
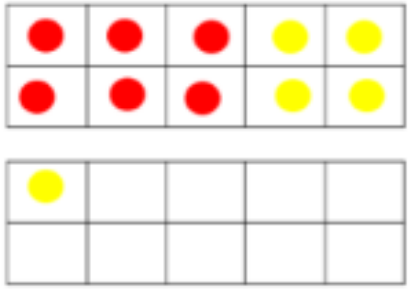
9

7

6 + 1

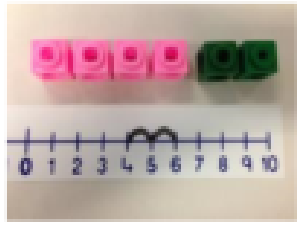
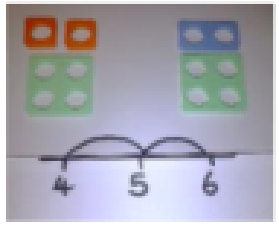
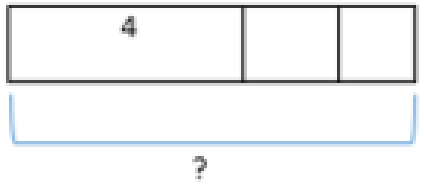
3 + 4

Making 10

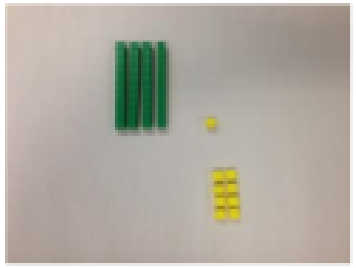
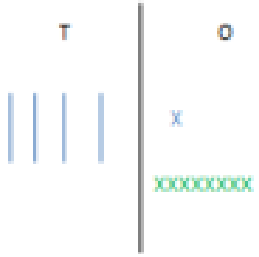
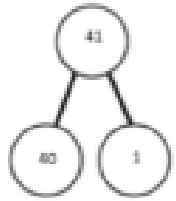
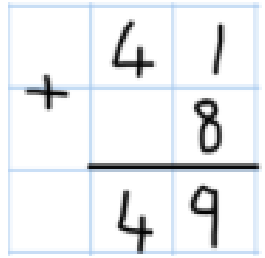
Concrete	Pictorial	Abstract
Regrouping to make 10 by using ten frames and counters/cubes or using numicon: 6 + 5  	Children to draw the ten frame and counters/cubes 	Children to develop an understanding of equality e.g 6 + ☐ = 11 and 6 + 5 = 5 + ☐ 6 + 5 = ☐ + 4

Equals means ‘is the same as’

Counting on…

Concrete	Pictorial	Abstract	
Counting on using number lines by using cubes or numicon  		A bar model which encourages the children to count on 	The abstract number line: What is 2 more than 4? What is the sum of 4 and 4? What's the total of 4 and 2? 4 + 2 

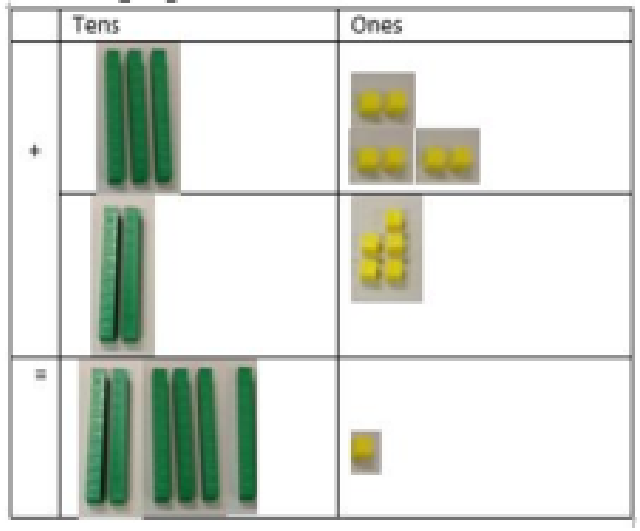
Adding using Dienes within 10

Concrete	Pictorial	Abstract
TO + O using base 10. Continue to develop understanding of partitioning and place value 41 + 8 	Children to represent the concrete using a particular symbol e.g. lines for tens and dot/crosses for ones. 	41 + 8  1 + 8 = 9 40 + 9 = 49 

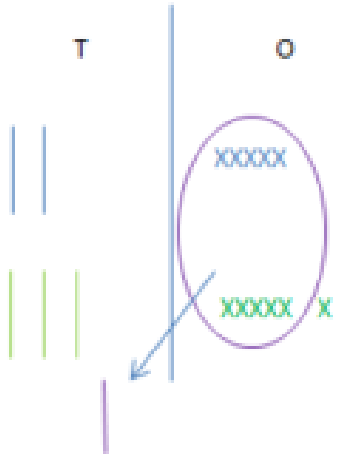
This can be replicated at home using straws to represent sticks of 10 and straws cut into 10 equal parts for ones.

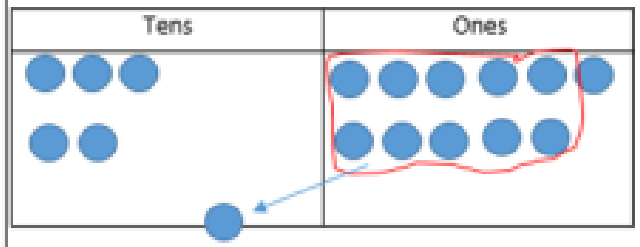
Adding using Dienes acrossing 10

TO + TO using base 10. Continue to develop understanding of partitioning and place value and use this to support addition. Begin with no exchanging. 36 + 25



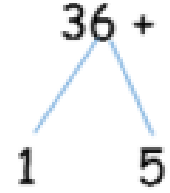
This could be done one of two ways:





Looking for ways to make 10

36 + 25=

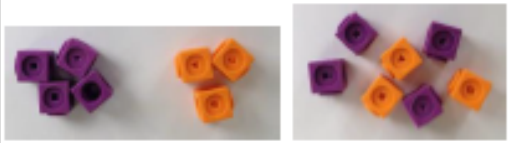
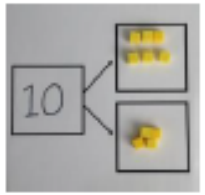
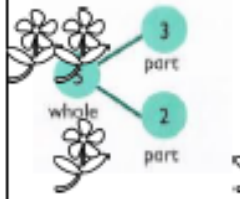
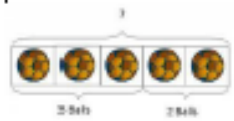
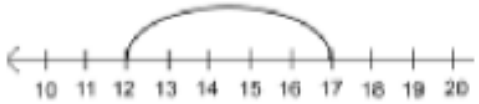
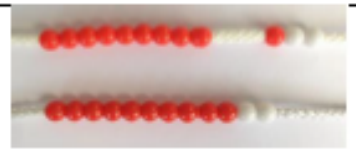


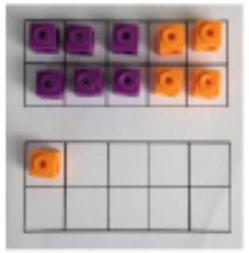
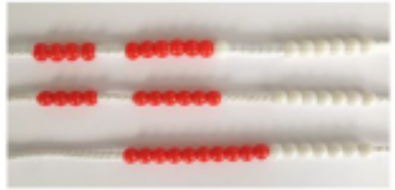
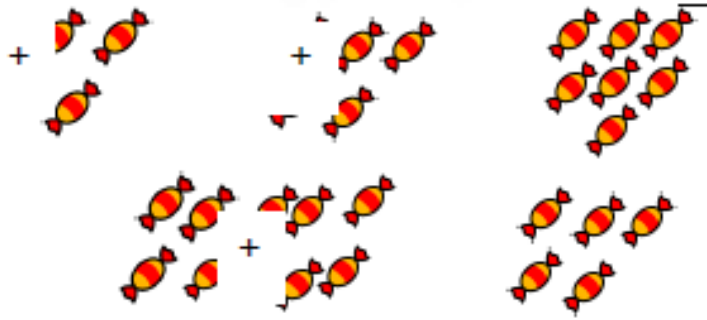
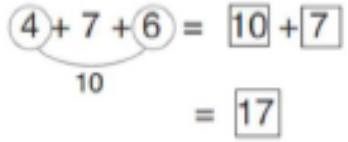
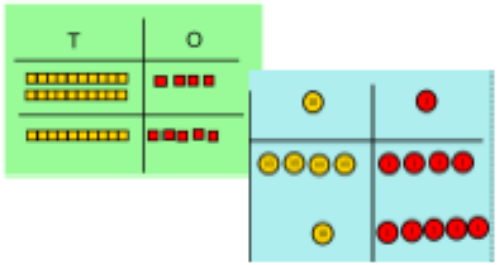
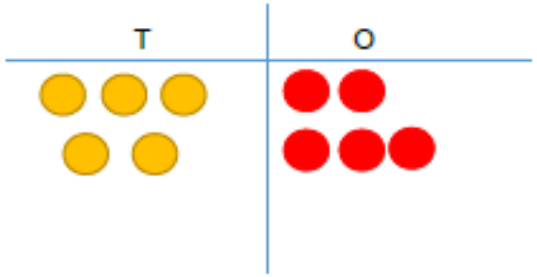
30 + 20 = 50
5 + 5 = 10
50 + 10 + 1 = 61

Formal method:

$$\begin{array}{r} 36 \\ +25 \\ \hline 61 \\ \hline 1 \end{array}$$

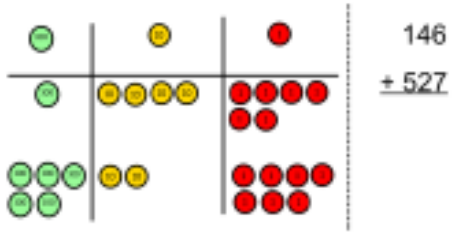
Addition

Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	  Use cubes to add two numbers together as a group or in a bar. 	  Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10.	 6 + 5 = 11	Use pictures or a number line. Regroup or partition the smaller number to make 10.	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?

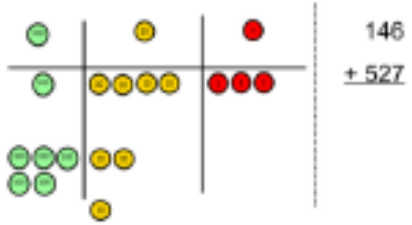
	 Start with the bigger number and use the smaller number to make 10.	 9 + 5 = 14 (1, 4) +1 +4 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	
Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.
Column method- no regrouping	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. 	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. 	Calculations 21 + 42 = 21 + 42

Column method- regrouping

Make both numbers on a place value grid.



Add up the units and exchange 10 ones for one 10.

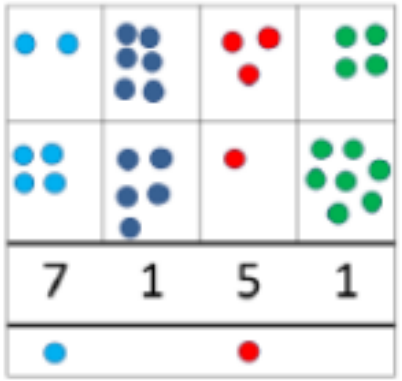


Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.



Start by partitioning the numbers before moving on to clearly show the exchange below the addition.

$$\begin{array}{rrrr} 20 & + & 5 & \\ \underline{40} & + & \underline{8} & \\ 60 & + & 13 & = 73 \end{array}$$

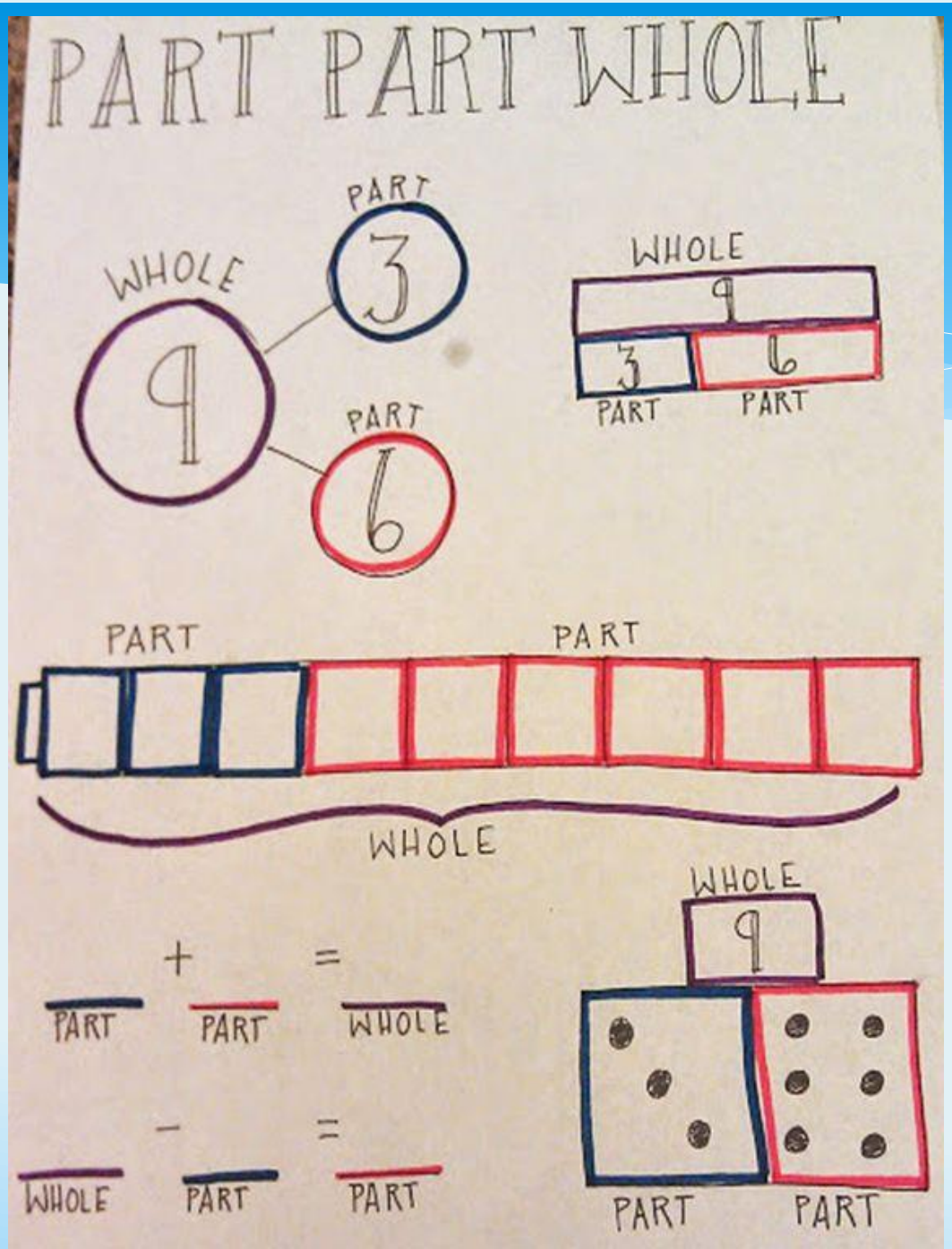
As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

$$\begin{array}{r} 536 \\ +\,85 \\ \hline 621 \\ \scriptstyle 1\,1 \end{array}$$

$$\begin{array}{r} 72.8 \\ +\,54.6 \\ \hline 127.4 \\ \scriptstyle 1\ 1 \end{array}$$

$$\begin{array}{rr} & £23.59 \\ + & £\ \ 7.55 \\ \hline & £31.14 \\ & \scriptstyle 1\ \ 1\ \ \ \ 1 \end{array}$$

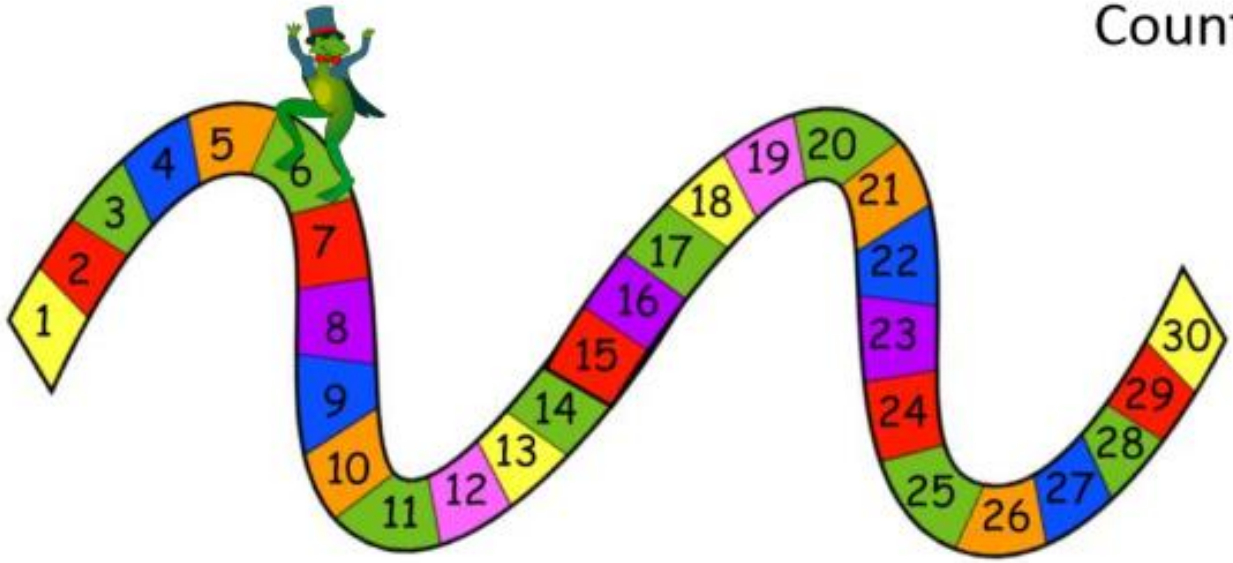
$$\begin{array}{rr} & 23.361 \\ & 9.080 \\ & 59.770 \\ + & 1.300 \\ \hline & 93.511 \\ & \scriptstyle 2\ \ 1\ \ \ \ 2 \end{array}$$


PART PART WHOLE
PART
3
WHOLE
9
PART
6
WHOLE
9
3
6
PART
PART
PART
PART
WHOLE
+
=
PART
PART
WHOLE
−
=
WHOLE
PART
PART
WHOLE
9
PART
PART

Addition

6 + 5 = ?

Number Track

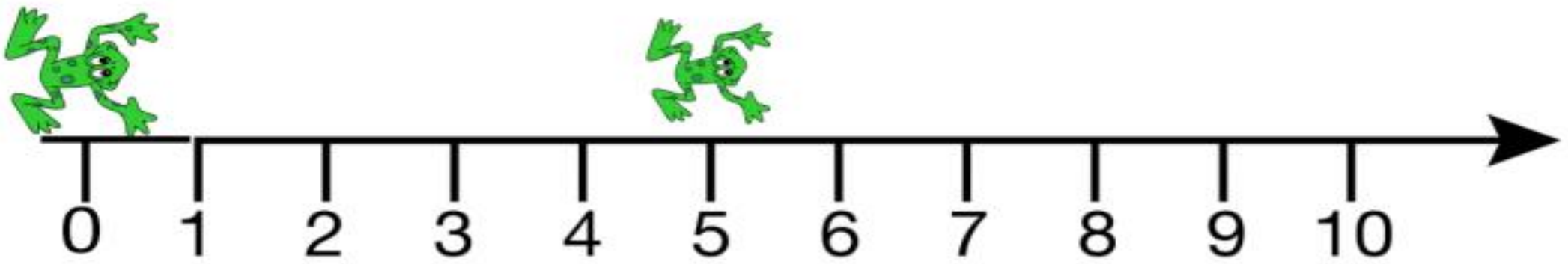


Counting on

Addition

Counting on using a number line

Counting on 4 from zero



Addition

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Using a hundred square to count on and back in 1s and 10s.

We encourage children to do this practically with the hundred square and then using a mental image.

PARTITIONING

Partitioning is a way of working out maths problems that involve large numbers by splitting them into smaller units so they're easier to work with.

Children are taught this method before they learn to add numbers in columns. Partitioning gives children a different way of visualising maths problems, and helps them work out large sums in their head. By breaking numbers down into units that are easy for them to calculate mentally.

432 + 325

400 + 300 = 700

30 + 20 = 50

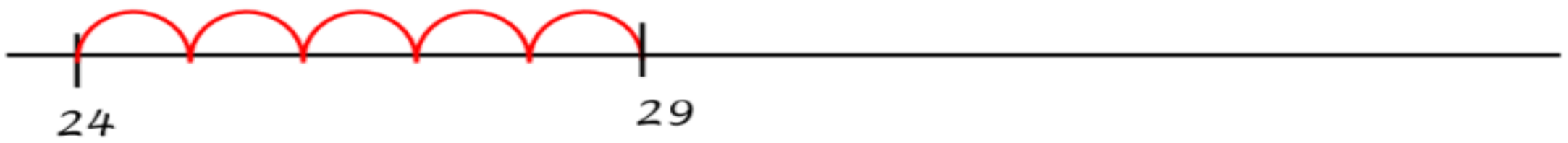
2 + 5 = 7

700 + 50 + 7 = 757

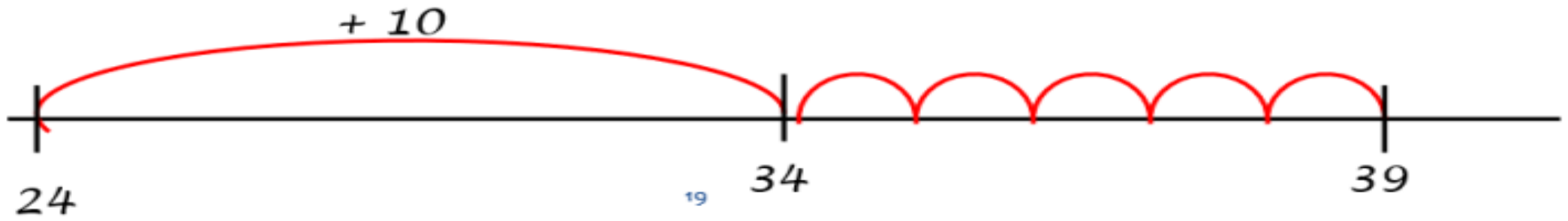
Addition

Once children can partition, they can use this knowledge to add using an empty number line.

Step 1 $24 + 5 = 29$

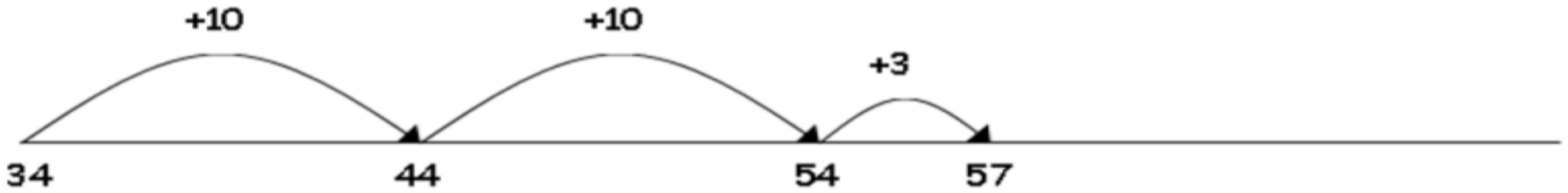


Step 2 $24 + 15 = 39$



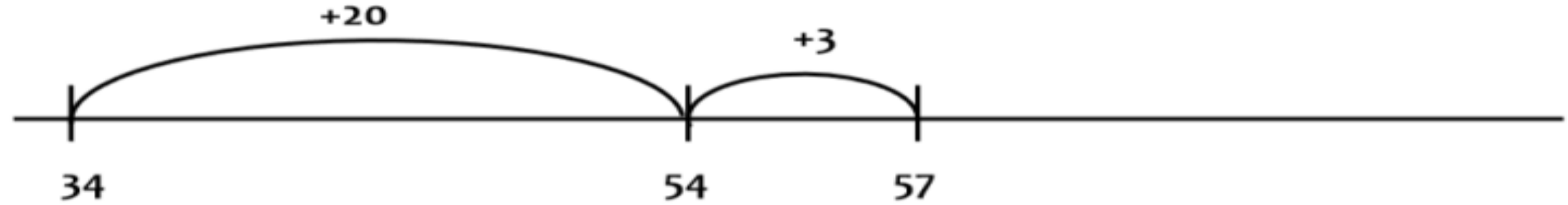
Next step is to become more efficient by adding the units in one jump.

34 + 23 = 57



And then add the tens in one jump

34 + 23 = 57



Your turn!

37 + 56 =

Remember to start from the larger number.

Addition - Expanded

- Adding the least significant digits first.

- Vertical (column) addition is a change from Year 2 where calculations were horizontal.
- Mental strategies still more important – add hundreds or tens first.
- No 'carrying' in this method – children need to understand what they are adding; **ones, tens or hundreds.**

```
  67
+ 24
  11   ( 7 + 4)
  80   (60 + 20)
  91
```

```
  267
+  85
  12  ( 7 +  5)
 140  (60 + 80)
 200
 352
```

Column methods

* Children with a secure understanding of place value will better understand the column method for addition and subtraction.

	HTO
	366
+	458
	824
	1 1

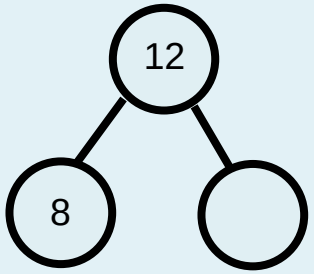
	HTO·t
	132·5
+	156·6
	289·1
	1

* Understanding place value will help children see the relationship between the columns.

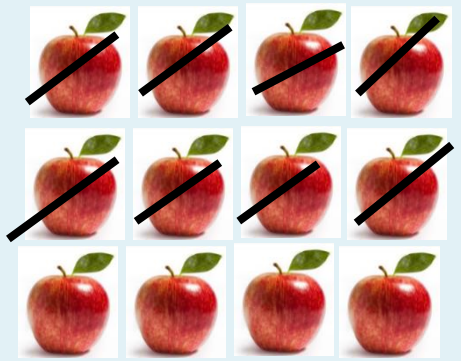
SUBTRACTION

STRUCTURES OF SUBTRACTION

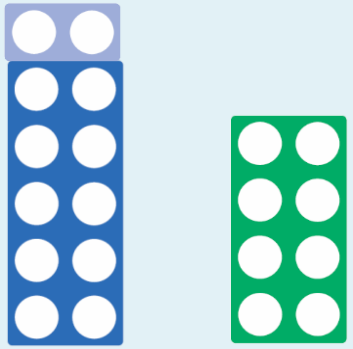
Partition



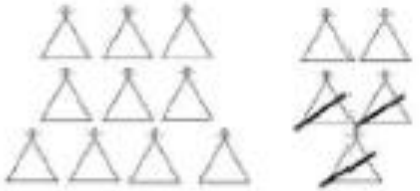
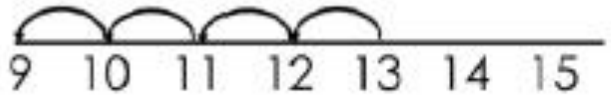
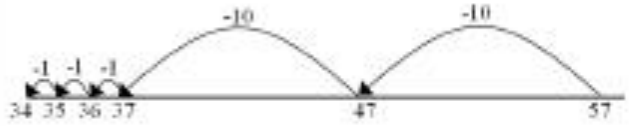
Take away

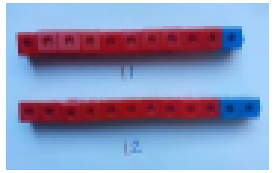
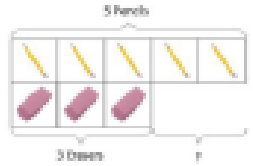
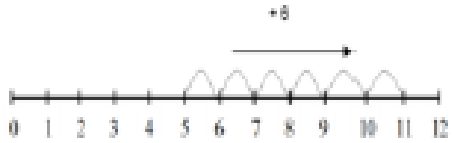
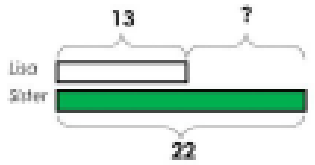
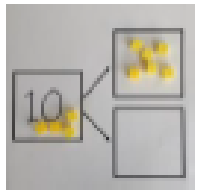
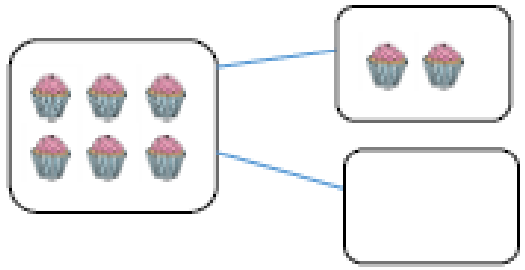
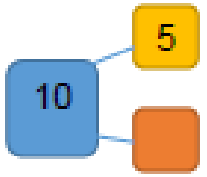
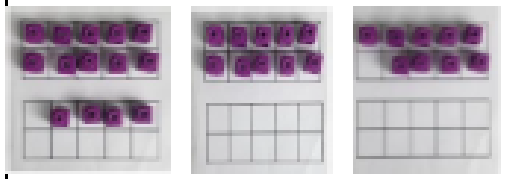
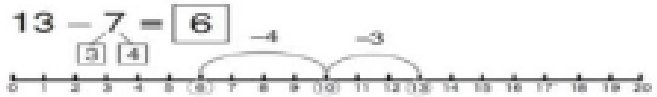


Difference

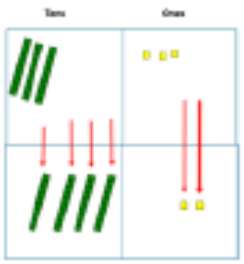


Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4 	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	18 -3= 15 8 – 2 = 6
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4 Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  9 10 11 12 13 14 15 Start at the bigger number and count back the smaller number showing the jumps on the number line.  -10 -10 -1 -1 -1 34 35 36 37 47 57 This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

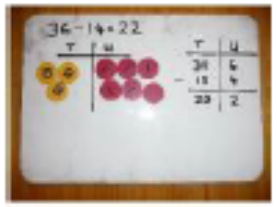
Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference	 Count on to find the difference. Draw bars to find the difference between 2 numbers. **Comparison Bar Models** *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.* 	Hannah has 23 sandwiches. Helen has 15 sandwiches. Find the difference between the number of sandwiches.
Part Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.
Make 10	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?

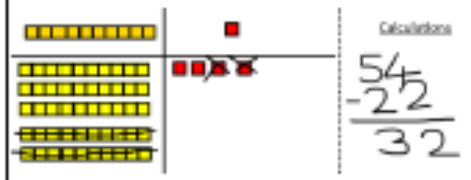
Column method without regrouping



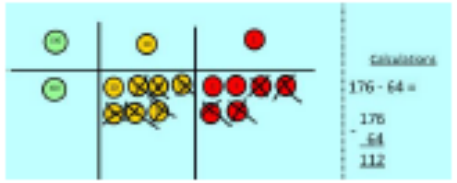
Use Base 10 to make the bigger number then take the smaller number away.

Show how you partition numbers to subtract. Again make the larger number first.





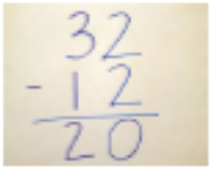
Draw the Base 10 or place value counters alongside the written calculation to help to show working.



47 - 24 = 23

40 + 7
- 20 + 4
20 + 3

This will lead to a clear written column subtraction.

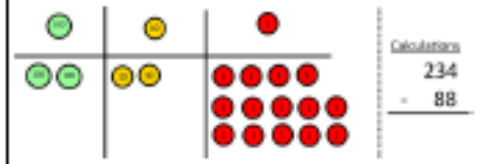


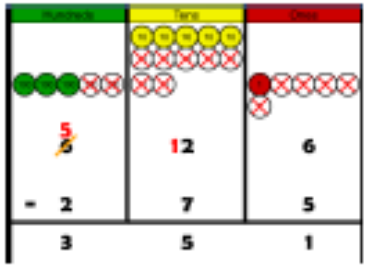
Column method with regrouping

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

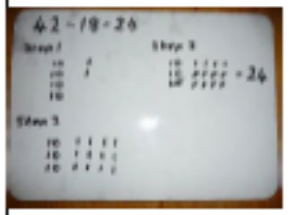
Make the larger number with the place value counters

Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



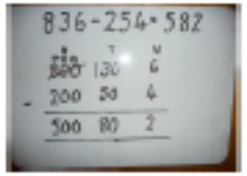


Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

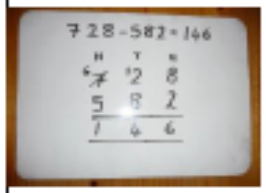


When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

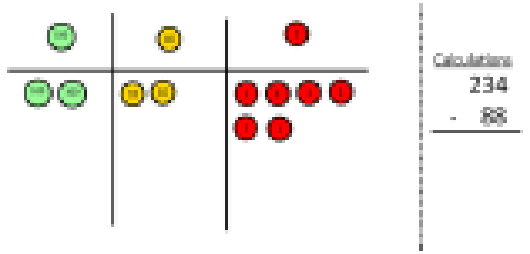


Children can start their formal written method by partitioning the number into clear place value columns.

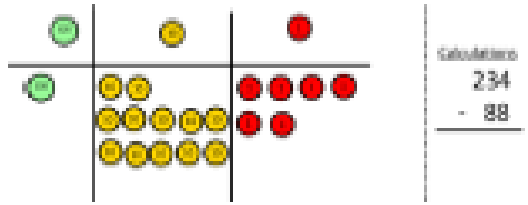


Moving forward the children use a more compact method.

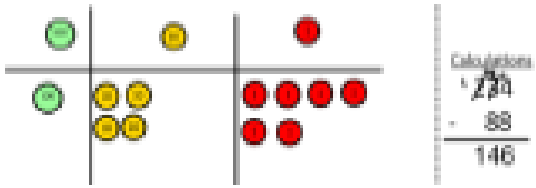
Now I can subtract my ones.



Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



Now I can take away eight tens and complete my subtraction

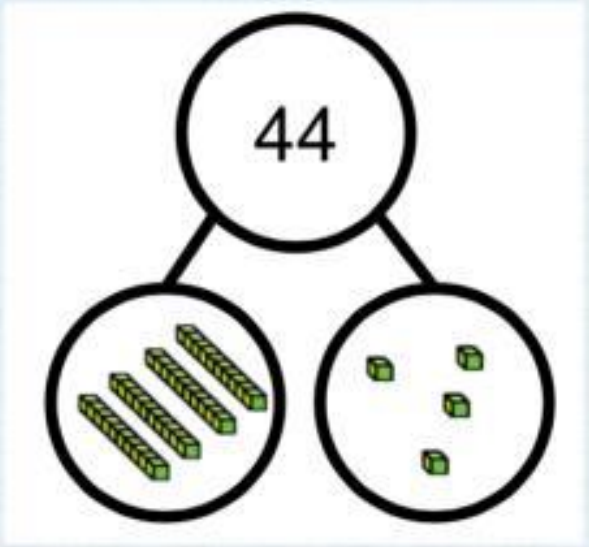


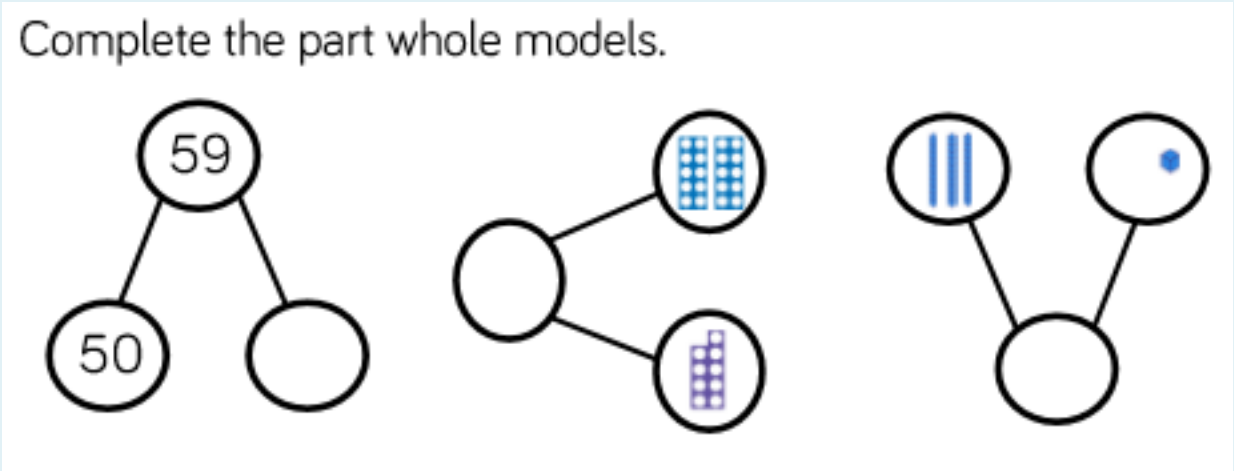
Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

This will lead to an understanding of subtracting any number including decimals.

$$\begin{array}{r} {\scriptstyle 5}\ {\scriptstyle 12}\ \ {\scriptstyle 1} \\ 2\ \not{6}\ \not{3}\ .\ 0 \\ -\quad 2\ 6\ .\ 5 \\ \hline 2\ 3\ 6\ .\ 5 \end{array}$$

A PICTORIAL REPRESENTATION: PART, PART, WHOLE...





It is a mathematical representation of a problem.. A way of modelling ...

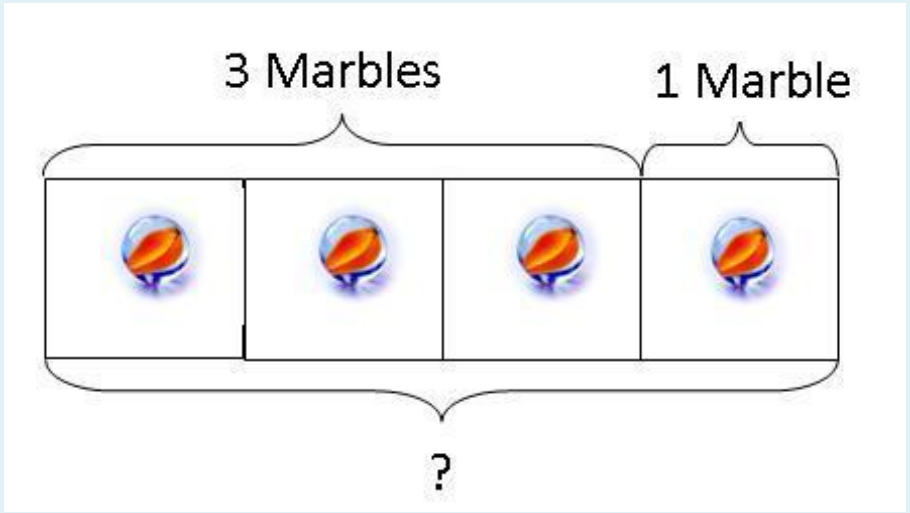
- the knowns and unknowns.
- the parts and the whole.

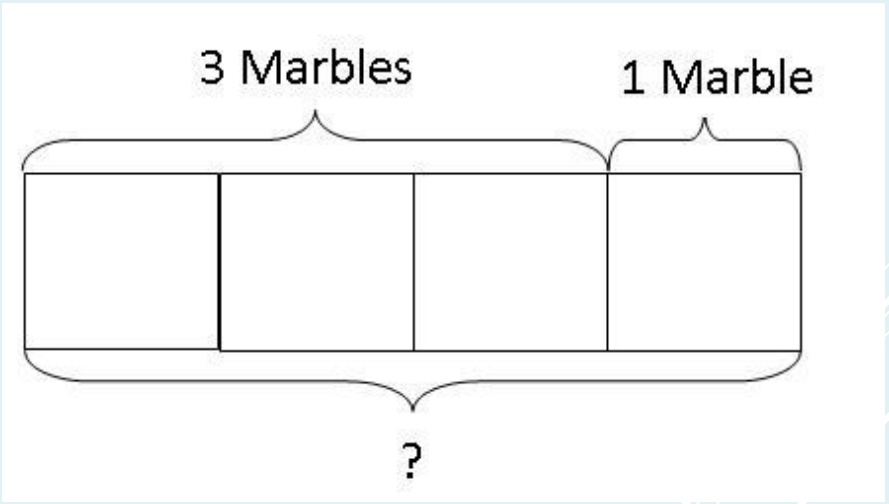
It is a representation that reveals the relationship of the numbers.

A PICTORIAL REPRESENTATION: THE BAR MODEL…

It is a mathematical representation of a problem.. A way of modelling …
the knowns and unknowns.
the parts and the whole.
It is a representation that **reveals the structure** of a word problem.

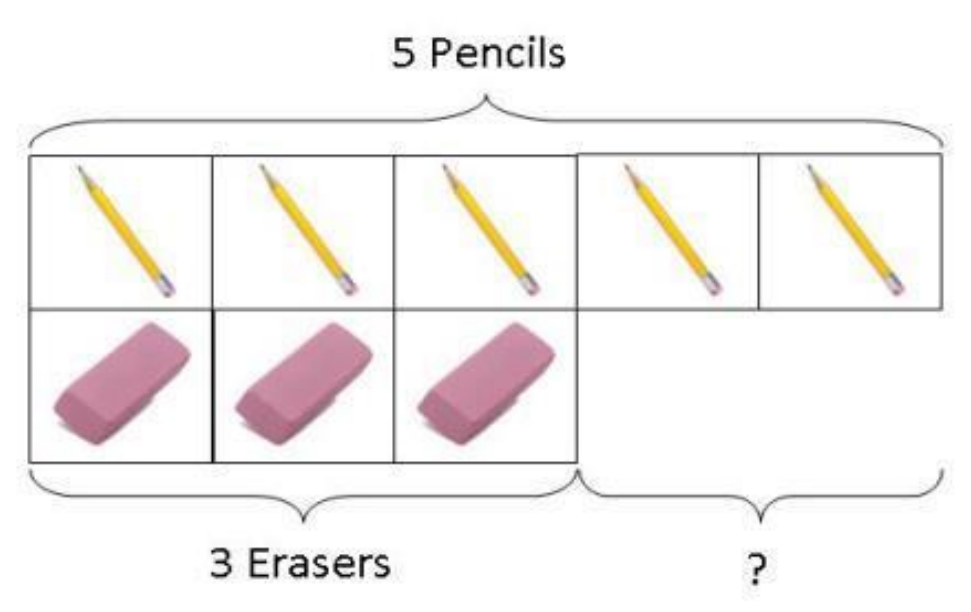
Peter has 3 marbles.
Harry gives Peter 1 more marble.
How many marbles does Peter have now?

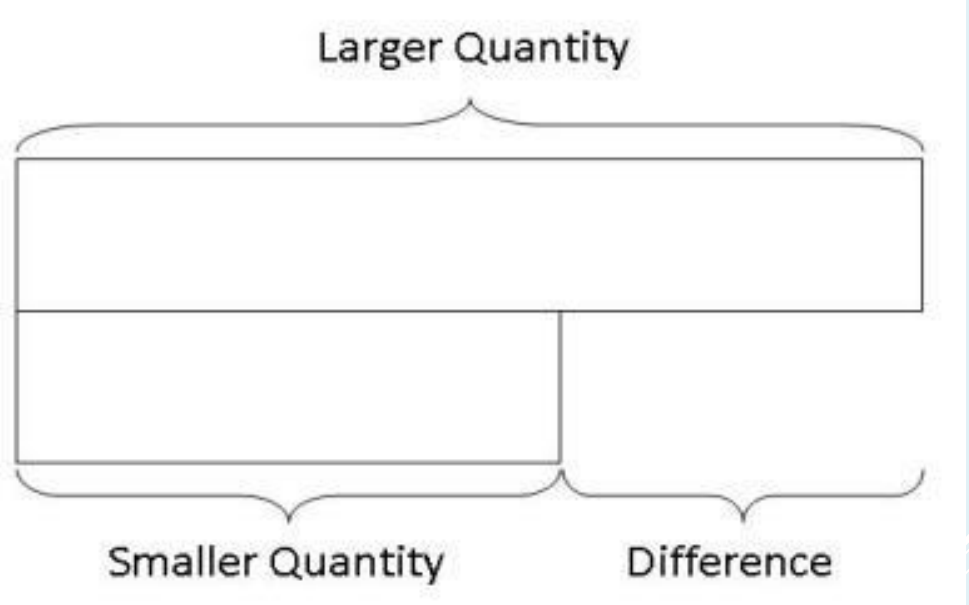




CONCRETE → ABSTRACT

Peter has 5 pencils and 3 rubbers.
How many more pencils than rubbers does he have?



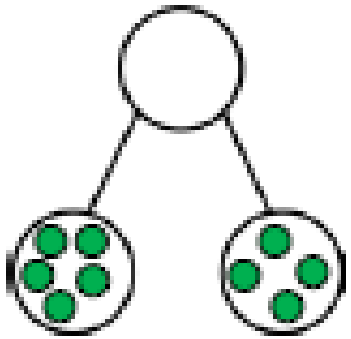


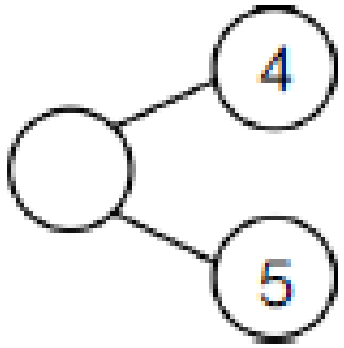
MATHS LESSONS

DO
(FLUENCY)

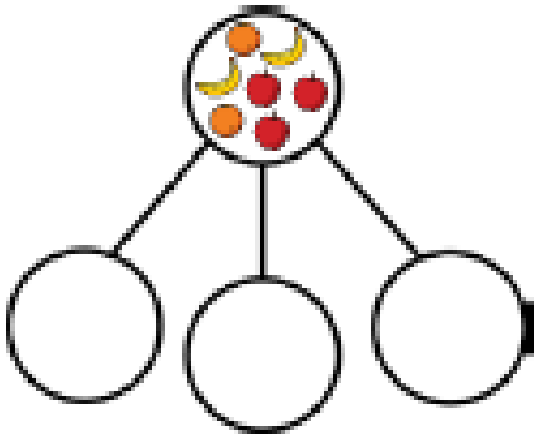
THINK/EXPLAIN & SOLVE
(REASONING AND PROBLEM SOLVING)

Complete the part-whole models by drawing counters and then writing the numerals.





Here are seven pieces of fruit.



Put the fruit into a part-whole model.

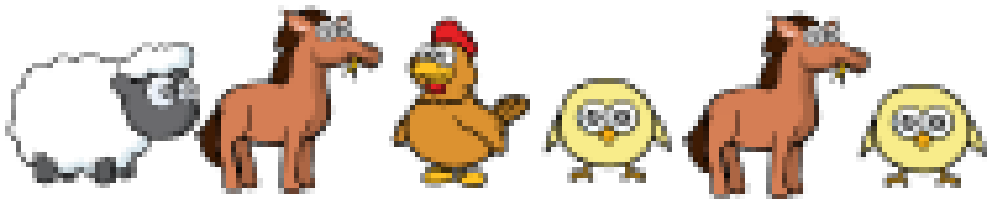
Complete the sentences.

_____ is the whole.

_____ is a part, _____ is a part and _____ is a part.

YEAR 1 DO

There are 6 animals.



How many different ways can you sort the animals?

Complete a part-whole model for each way.

Can you partition the animals into more than 2 groups?

YEAR 1 REASONING

Work in groups of up to 8 children.

Can you split yourselves into different groups?

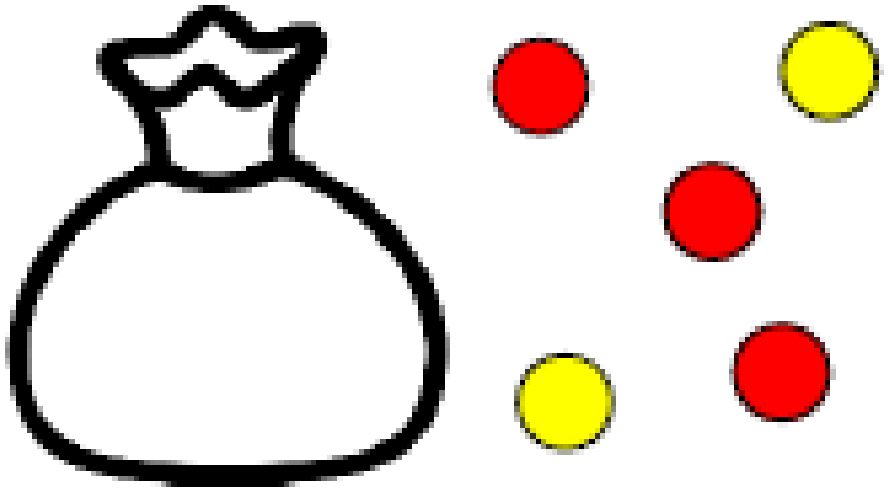
Think of different ways to group yourselves: hair colour, eye colour, gender, shoe size etc.

Complete a part-whole model for each way.

Can you partition into more than 2 groups?

YEAR 1 REASONING

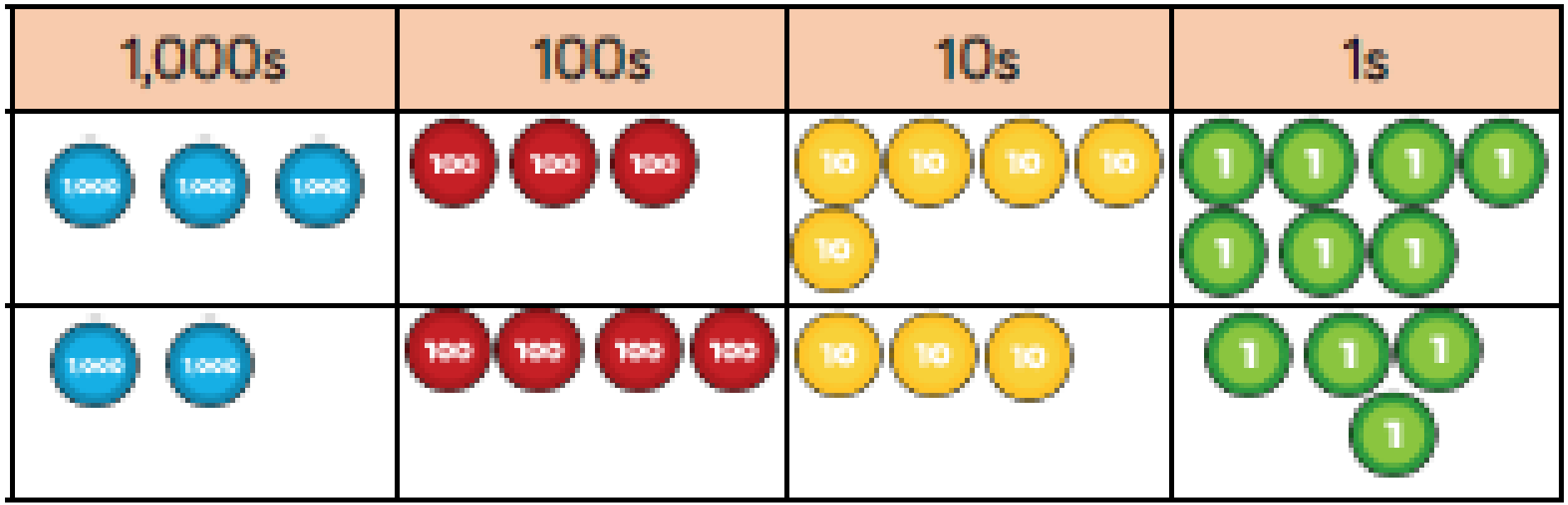
There are no more than 10 counters in total.



How many counters could be in the bag?

Why can't it be six?

Use the place value grid to calculate 3,357 + 2,434



YEAR 4 DO

Complete the calculation.

	Th	H	T	O
	4	5	7	8
−	3	6	4	3

YEAR 4 DO

YEAR 4 REASONING

What is the missing 4-digit number?

	Th	H	T	O
	____	____	____	____
+	6	3	9	5
	8	9	4	9

YEAR 4 REASONING

Three primary schools join together to go on a school visit to The Deep in Hull.
1,235 people go on the trip.

There are 1,179 children and 27 teachers.
The rest are parents.

How many parents are there?

What do you need to do first?

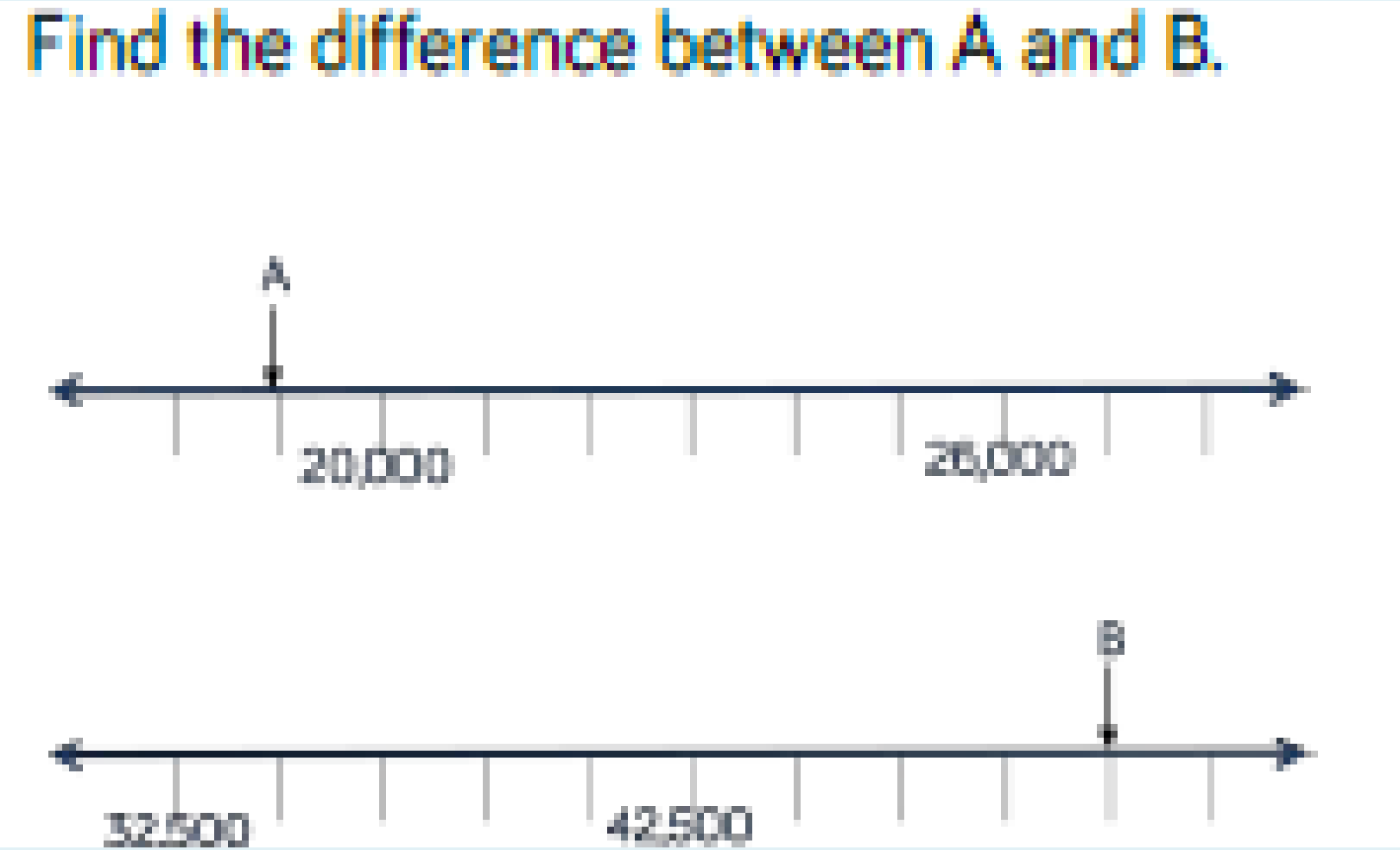
Which operation do you use?

Calculate the missing digits. What do you notice?

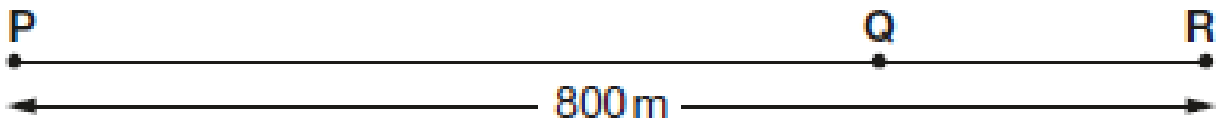
	5	2	2	4	7	?
+	3	?	5	9	0	4
	9	0	?	3	?	2

77

YEAR 6 DO



YEAR 6 REASONING

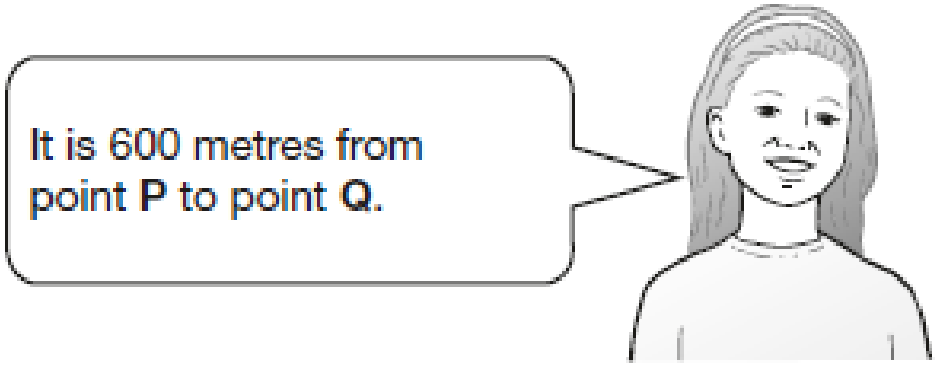


Not to scale

The distance from point **P** to point **R** is 800 metres.

The distance from point **P** to point **Q** is **4 times** the distance from point **Q** to point **R**.

Olivia says,



Explain why Olivia is **not** correct.

HELPING AT HOME

- Cooking or baking – measuring and weighing
- Look at numbers in the environment e.g. telephone keys, number plates, door numbers, book pages, sleeps until Christmas!
- Money - shopping
- Comparing heights
- Birthdays, Months of the year, Days of the week
- Homework