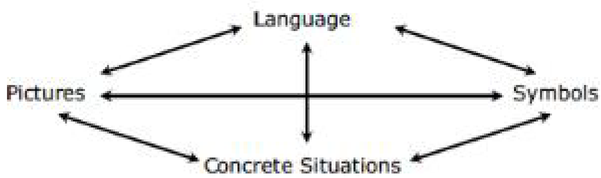
# East Boldon Junior School
# Calculation Policy 2020

**Introduction**
This policy outlines the principles, aims and pedagogy for calculation in addition, subtraction, multiplication and division at East Boldon Junior School. It also demonstrates how children's learning will progress when using mental strategies, working with fractions, and our expectations for the acquisition of times tables facts. This policy has been written to ensure consistency and progression throughout the school.

**<u>Intent</u>**
At the heart of our approach is children's understanding of the calculating process. We believe that children develop conceptual understanding in mathematics when they create links between pictures, language, symbols and resources (concrete situations). We feel that competency within, and fluency between, these elements will enable pupils to understand, use and apply mathematics to everyday life.




*A diagrammatical representation of the connective model of learning mathematics. (Haylock & Cockburn, 2008)*

The National Curriculum for Mathematics highlights the importance of ensuring that children are competent and confident in using an appropriate calculation method when problem solving.

We believe that at all ages, children should have the opportunity to manipulate and experience a variety of models, images, and resources to support their mental calculation and representations, before attempting formal written methods.

Resources available include:
- arrow cards
- bead strings (0-10, 0-20, 0-100)
- numicon,
- counters
- Cuisenaire rods
- Dice
- Dienes (base ten equipment)
- digit cards
- multilink cubes
- number fans
- number lines
- place value cards
- hundred squares
- multiplication squares

**Representations**
Key to successful implementation of a school calculation policy is consistent use of representations (practical resources, models and images that support conceptual understanding of the mathematics) and this policy promotes a range of relevant representations, across the primary years. Mathematical understanding is developed through use of representations that are first of all concrete (e.g. Numicon, Dienes apparatus), and then pictorial (e.g. Array, place value counters) to then facilitate abstract working (e.g. Columnar addition, long multiplication). This policy guides teachers through an appropriate progression of representations, and if at any point a pupil is struggling they should revert to familiar pictorial and/or concrete materials/representations as appropriate.

**Aims**
It is through this approach that we believe children will be able to:
· understand important concepts and make connections within mathematics
· show high levels of fluency in performing written and mental calculations
· be taught consistent calculation strategies
· be ready for the next stage of learning
· have a smooth transition between phases
· be able to add, subtract, multiply and divide efficiently
· be competent in fluency, reasoning and problem solving.

**Pedagogy**
Our class teachers will employ the following pedagogy to support calculation in mathematics:
- Developing pupils' understanding of number and place value should be explored daily.
- Models, images and resources should be used throughout all year groups.
- Pupils should be encouraged to develop independence, and to select resources to support their learning.
- Practical activities should be a regular feature of maths lessons.
- Activities should be differentiated to suit the needs of the pupils; more able pupils should be encouraged to further deepen their conceptual understanding rather than learning new content.
- Opportunities to work within mixed ability groups should be employed where applicable.
- Solving problems should be an integral and daily part of the maths curriculum.
- Pupils should develop into confident, independent mathematicians through rich mathematical experiences including engaging, challenging mathematical investigations
- Pupils should be encouraged to take risks, make mistakes, and learn from their experiences.
- Teachers should explore misconceptions with pupils in order to deepen their understanding.
- Teachers should use their professional judgement to decide if children are ready to move onto the next phase in calculation.

## Further Calculation guidance

*(Ideas taken from National Centre for Excellence in the Teaching of Mathematics)*

**Develop children's fluency with basic number facts**
Fluent computational skills are dependent on accurate and rapid recall of basic number bonds to 20 and times-tables facts. Therefore, each class should spend 10 minutes every day on these basic facts as this will quickly lead to improved fluency. This can be done in a variety of ways including interactive games, use of a counting stick and using simple whole class chorus chanting. This is an important step in developing conceptual understanding through identifying patterns and relationships between the tables (for example, that the products in the 6× table are double the products in the 3× table). Children will consequently develop a strong sense of number relationships, an important prerequisite for procedural fluency.

Children at East Boldon Junior School should learn their multiplication tables in this order to provide opportunities to make connections:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ×10 | ×5 | ×2 | ×4 | ×8 | ×3 | ×6 | ×9 | ×7 |

**Develop children's fluency in mental calculation**
Efficiency in calculation requires having a variety of mental strategies. One mental strategy involves the importance of 10 and partitioning numbers to bridge through 10 ('magic 10').
For example: **9 + 6 = 9 + 1 + 5 = 10 + 5 = 15.**
It is helpful to make a 10 here, and in many others, as this makes the calculation easier.

**Develop fluency in the use of formal written methods**
Teaching column methods for calculation provides the opportunity to develop both procedural and conceptual fluency. Children must understand the structure of the mathematics presented in the algorithms, with a particular emphasis on place value. Base ten apparatus should be used to support the development of fluency and understanding.

**Develop children's understanding of the = symbol**
The symbol = is an assertion of equivalence. If we write: **3 + 4 = 6 + 1** then we are saying that what is on the left of the = symbol is necessarily equivalent to what is on the right of the symbol. But many children interpret = as being simply an instruction to evaluate a calculation, as a result of always seeing it used thus:
**3 + 4 =**
**5 × 7 =**
**16 – 9 =**
If children only think of = as meaning "work out the answer to this calculation" then they are likely to get confused by empty box questions such as 3 + □ = 8

Later they are very likely to struggle with even simple algebraic equations, such as:
**3y = 18**
One way to model equivalence such as 2 + 3 = 5 is to use balance scales.

Children from Year 3 should be given calculations with variation in the position of the = symbol and include empty box problems to deepen children's understanding of the = symbol.

**Look for pattern and make connections**
At East Boldon Juniors we use many visual representations of mathematics and also concrete resources. Understanding, however, does not happen automatically, children need to reason by and with themselves and make their own connections. Children should be getting into good habits from Year 3 in terms of reasoning and looking for pattern and connections in the mathematics. The question "What's the same, what's different?" should be used frequently to make comparisons. For example: "What's the same, what's different between the three times table and the six times table?"

**Use intelligent practice**
Children should engage in a significant amount of practice of mathematics through class and homework exercises. However, in designing these exercises, teachers should avoid mechanical repetition and instead engage in activities which provide the opportunity to develop both procedural and conceptual fluency. Children in these activities are required to reason and make connections between calculations. The connections made improve their fluency.
e.g.

| | | |
|---|---|---|
| 2 × 3 = | 6 × 7 = | 9 × 8 = |
| 2 × 30 = | 6 × 70 = | 9 × 80 = |
| 2 × 300 = | 6 × 700 = | 9 × 800 = |
| 20 × 3 = | 60 × 7 = | 90 × 8 = |
| 200 × 3 = | 600 × 7 = | 900 × 8 = |

**Use empty box problems**
Empty box problems are a powerful way to help children develop a strong sense of number through intelligent practice. They provide the opportunity for reasoning and finding easy ways to calculate. They enable children to practise procedures, whilst at the same time thinking about conceptual connections.

A sequence of examples such as:

**3 + □ = 8**
**3 + □ = 9**
**3 + □ = 10**
**3 + □ = 11**

helps children develop their understanding that the = symbol is an assertion of equivalence, and invites children to spot the pattern and use this to work out the answers.

This sequence of examples does the same at a deeper level:

**3 × □ + 2 = 20**

$$3 \times \square + 2 = 23$$
$$3 \times \square + 2 = 26$$
$$3 \times \square + 2 = 29$$
$$3 \times \square + 2 = 35$$

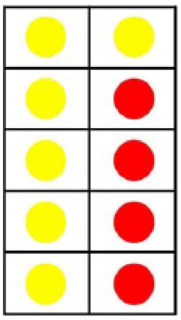
Children should also be given examples where the empty box represents the operation, for example:

$$4 \times 5 = 10 \ \square \ 10$$
$$6 \ \square \ 5 = 15 + 15$$
$$6 \ \square \ 5 = 20 \ \square \ 10$$
$$8 \ \square \ 5 = 20 \ \square \ 20$$
$$8 \ \square \ 5 = 60 \ \square \ 20$$

These examples also illustrate the careful use of variation to help children develop both procedural and conceptual fluency.

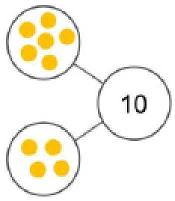
**Expose mathematical structure and work systematically**

Using structured models such as tens frames, part whole models or bar models can help children to reason about mathematical relationships.



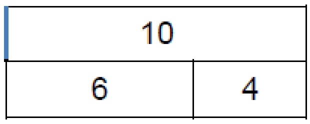
6 + 4 = 10
4 + 6 = 10
10 – 4 = 6
10 – 6 = 4

Tens Frame




6 + 4 = 10
4 + 6 = 10
10 – 4 = 6
10 – 6 = 4

Part Whole Model




6 + 4 = 10
4 + 6 = 10
10 – 4 = 6
10 – 6 = 4

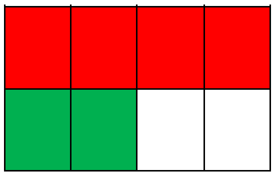
Bar Model

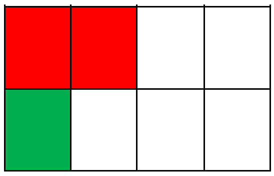
Connections between these models should be made, so that children understand the same mathematics is represented in different ways. Asking the question "What's the same what's different?" has the potential for children to draw out the connections.

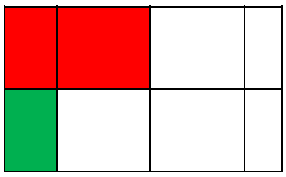
Illustrating that the same structure can be applied to any numbers helps children to generalise mathematical ideas and build from the simple to more complex numbers, recognising that the structure stays the same; it is only the numbers that change. For example:

| | | |
|---|---|---|
| 10 (6 \| 4) | 247 (173 \| 74) | 6.2 (3.4 \| 2.8) |
| 6 + 4 = 10 | 173 + 74 = 247 | 3.4 + 2.8 = 6.2 |
| 4 + 6 = 10 | 74 + 173 = 247 | 2.8 + 3.4 = 6.2 |
| 10 – 6 = 4 | 247 – 173 = 74 | 6.2 – 3.4 = 2.8 |
| 10 – 4 = 6 | 247 – 74 = 173 | 6.2 – 2.8 = 3.4 |

**Move between the concrete and the abstract**
Children's conceptual understanding and fluency is strengthened if they experience concrete, visual and abstract representations of a concept during a lesson. Moving between the concrete and the abstract helps children to connect abstract symbols with familiar contexts, thus providing the opportunity to make sense of, and develop fluency in the use of, abstract symbols.
For example, in a lesson about addition of fractions children could be asked to draw a picture to represent the sum $\frac{1}{4}$ + 1/8 = 3/8
Alternatively, or in a subsequent lesson, they could be asked to discuss which of three visual images correctly represents the sum, and to explain their reasoning:







**Contextualise the mathematics**
Mathematics should be contextualised and related to real life as much as possible. A lesson about addition and subtraction could start with this contextual story:
*"There are 11 people on a bus. At the next stop 4 people get on. At the next stop 6 people get off. How many are now on the bus?"*
This helps children develop their understanding of the concepts of addition and subtraction. But during the lesson the teacher should keep returning to the story. For example, if the children are thinking about this calculation: **14 – 8**
Then the teacher should ask the children:
"What does the 14 mean? What does the 8 mean?", expecting that children will answer:
"There were 14 people on the bus, and 8 is the number who got off."
Then asking the children to interpret the meaning of the terms in a sum such as 7 + 7 = 14 will give a good assessment of the depth of their conceptual understanding and their ability to link the concrete and abstract representations of mathematics

**Use questioning to develop mathematical reasoning**
In order to develop children's conceptual understanding and fluency there needs to be a strong and consistent focus on questioning throughout school that encourages and develops their mathematical reasoning.
This can be done simply by asking children to explain how they worked out a calculation or solved a problem, and to compare and contrast different methods that are described.

It has been found that children quickly come to expect that they need to explain and justify their mathematical reasoning, and they soon start to do so automatically – and enthusiastically. Some calculation strategies are more efficient and teachers should scaffold children's thinking to guide them to the most efficient methods, whilst at the same time valuing their own ideas.

Rich questioning strategies which should be used regularly include:

- **"What's the same, what's different?"**

*In this sequence of expressions, what stays the same each time and what's different?*
*23 + 10*
*23 + 20*
*23 + 30*
*23 + 40*
Discussion of the variation in these examples can help children to identify the relationship between the calculations and hence to use the pattern to calculate the answers.

- **"Odd one out"**

*Which is the odd one out in this list of numbers: 24, 15, 16 and 22?*
This encourages children to apply their existing conceptual understanding. Possible answers could be:
"15 is the odd one out because it's the only odd number in the list."
"16 is the odd one out because it's the only square number in the list."
"22 is the odd one out because it's the only number in the list with exactly four factors."
*Which is the odd one out in list of products:*
*24 × 3 36 × 4 13 × 5 32 × 2*
They might suggest:
"36 × 4 is the only product whose answer is greater than 100."
"13 × 5 is the only product whose answer is an odd number."

- **"Here's the answer. What could the question have been?"**

Children are asked to suggest possible questions that have a given answer. For example, in a lesson about addition of fractions, children could be asked to suggest possible ways to complete this sum:

$$\square + \square = \frac{3}{4}$$

- **True or False**

Children are given a series of equations are asked whether they are true or false:
4 × 6 = 23
4 × 6 = 6 × 4
12 ÷ 2 = 24 ÷ 4
12 × 2 = 24 × 4
Children are expected to reason about the relationships within the calculations rather than calculate.

- *Greater than, less than or equal to >, <, or =*

**3.4 × 1.2 ○ 3.4    5.76 ○ 5.76 ÷ 0.4    4.69 × 0.1 ○ 4.69 ÷ 10**

These types of questions are further examples of intelligent practice where conceptual understanding is developed alongside the development of procedural fluency. They also give pupils who are, to use Ofsted's phrase, 'rapid graspers' the opportunity to apply their understanding in more complex ways.

**Expect children to use correct mathematical terminology and to express their reasoning in complete sentences**

The quality of children's mathematical reasoning and conceptual understanding is significantly enhanced if they are consistently expected to use correct mathematical terminology (e.g. saying 'digit' rather than 'number') and to explain their mathematical thinking in complete sentences. Therefore, there should be a high expectation of children using the correct mathematical terminology.

***I say, you say, you say, you say, we all say***

This technique enables the teacher to provide a sentence stem for children to communicate their ideas with mathematical precision and clarity. These sentence structures often express key conceptual ideas or generalities and provide a framework to embed conceptual knowledge and build understanding. For example:
If the rectangle is the whole, the shaded part is one third of the whole.
There are 12 stars, 1/3 of the stars is equal to 4 stars
Children use the same sentence stem to express other relationships. For example:
There are 12 stars, $\frac{1}{4}$ of the stars is equal to 3 stars
There are 12 stars, $\frac{1}{2}$ of the stars is equal to 6 stars

Having modelled the sentence, the teacher then asks individual children to repeat this, before asking the whole class to chorus chant the sentence. This provides children with a valuable sentence for talking about fractions. Repeated use helps to embed key conceptual knowledge. This technique is also useful when reaffirming mathematical rules:
e.g. When adding 10 to a number, the ones digit stays the same

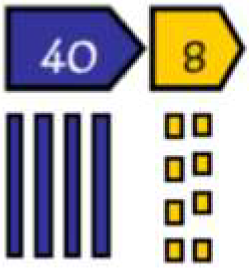
**Identify difficult points and misconceptions**

Difficult points need to be identified and anticipated when lessons are being designed and these need to be an explicit part of the teaching, rather than the teacher just responding to children's difficulties if they happen to arise in the lesson. The teacher should be actively seeking to uncover possible difficulties because if one child has a difficulty it is likely that others will too. Difficult points also give an opportunity to reinforce that we learn most by working on and through ideas with which we are not fully secure or confident. Discussion about difficult points can be stimulated by asking children to share thoughts about their own examples when these show errors arising from insufficient understanding. For example:

$$\frac{2}{14} - \frac{1}{7} = \frac{1}{7}$$

A visualiser is a valuable resource since it allows the teacher quickly to share a child's thinking with the whole class.

# Progression in Addition

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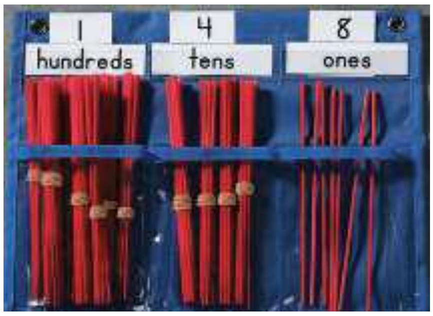
**Concrete resources:**
100 square
Number lines
Bead strings
Straws
Dienes
Place value cards
Place value dice
Place value counters
Numicon




| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

# Addition: Year 2

Year 2 statutory requirements :

✓ Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts to 100.

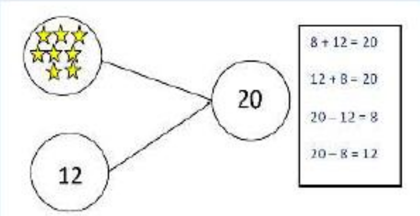
✓Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

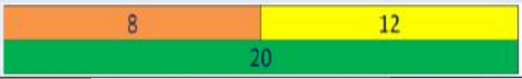
✓ Add numbers using concrete objects, pictorial representations, and mentally, including:

- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
- adding three one-digit numbers.

✓ Solve problems with addition including those involving numbers, quantities and measures.

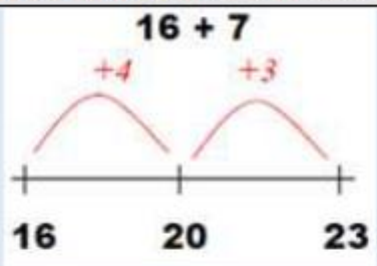
Memorise and reason with number facts to 20 in several forms.



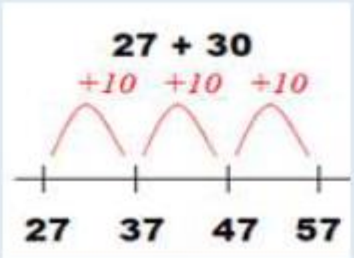




Partition two 2-digit numbers using a variety of models and images.

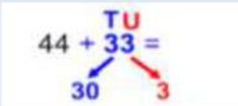

Add 2 digit number and ones

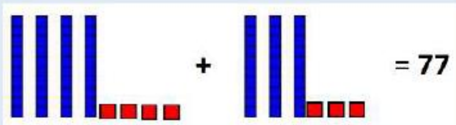


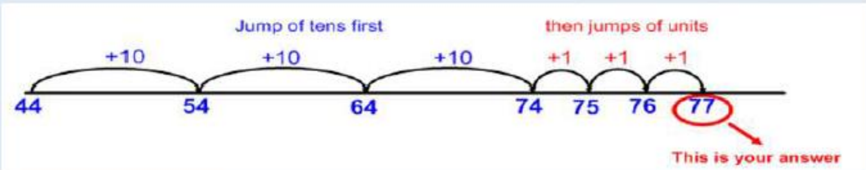

Add 2 digit number and tens



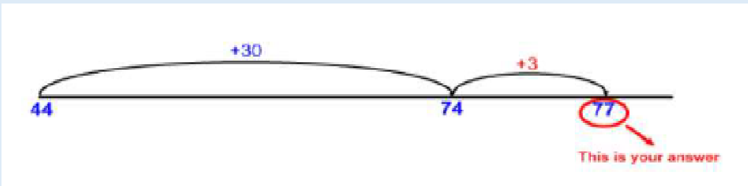

Use partitioning to add two 2-digit numbers using concrete resources and/or a numbered number line and then progressing to an empty number line.




As children gain confidence with adding on tens and ones, they should be taught to combine the jumps on an empty number line.



# Addition: Year 3

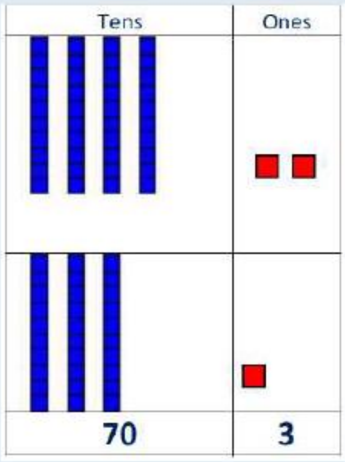
Year 3 statutory requirements :
- Find 10 or 100 more than a given number.
- Recognise the place value of each digit in a three-digit number (hundreds, tens, ones).
- Add numbers with up to three digits, using formal written methods of columnar addition.

Use expanded column method with place value resources to support the conceptual understanding of adding numbers up to three digits ***with no carrying***.

42 + 31 = 73

40 + 2
30 + 1
70 + 3

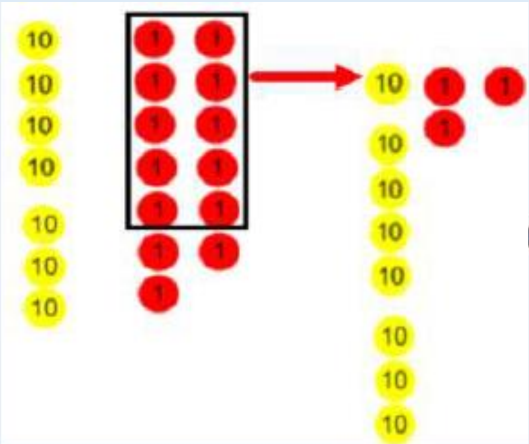

OR



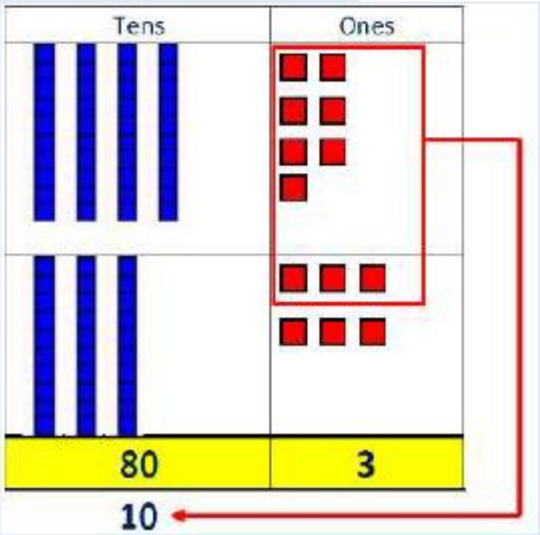

Progress to using the expanded column method with place value resources to support the conceptual understanding of adding numbers up to three digits ***with carrying***.

47 + 36 = 83

40 + 7
30 + 6
80 + 3
10



OR




Extend to using the expanded column method to add three digit numbers + three digit numbers ***with carrying***.

367 + 185 = 552

300 + 60 + 7
100 + 80 + 5
500 + 50 + 2
100 10

**Note:** The carried ten or carried hundred is just as important as any other number, therefore, it should be written as clear and as large as any other number, and placed at the **bottom** of the column in which it is to be added.

# Addition: Year 4

Year 4 statutory requirements :

- Find 1000 more than a given number.
- Add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate.
- Solve addition two-step problems in contexts, deciding which operations and methods to use and why,

Build on learning from Year 3 and model how expanded method links to compact column addition method.

$$\begin{array}{l} 40 + 7 \\ \underline{30 + 6} \\ \underline{80 + 3} \\ 10 \end{array} \quad \Rightarrow \quad \begin{array}{r} 47 \\ \underline{+36} \\ \underline{83} \\ 1 \end{array}$$

$$\begin{array}{l} 300 + 60 + 7 \\ \underline{100 + 80 + 5} \\ \underline{500 + 50 + 2} \\ 100 \quad 10 \end{array} \quad \Rightarrow \quad \begin{array}{r} 367 \\ \underline{+185} \\ \underline{552} \\ 11 \end{array}$$

By the end of year 4, pupils should be adding numbers up to 4 digits using compact column addition method.

$$\begin{array}{r} 5\,2\,7\,1 \\ \underline{+\,2\,3\,5\,7} \\ \underline{7\,6\,2\,8} \\ 1\phantom{\,2\,8} \end{array}$$

Note: The carried ten or carried hundred is just as important as any other number, therefore, it should be written as clear and as large as any other number, and placed at the **bottom** of the column in which it is to be added.

# Addition: Year 5 & 6

Year 5 statutory requirements :

- Add whole numbers with more than 4 digits using formal written methods of columnar addition.
- Add numbers mentally, with increasingly large numbers.
- Solve addition multi-step problems in contexts, deciding which operations and methods to use and why.
- Solve problems involving numbers up to three decimal places

Year 6 statutory requirements :

- Pupils are expected to solve more complex addition and subtraction problems

In year 5 and 6 pupils should be adding numbers using compact column addition method. **Note:** The carried ten, hundred, thousand is just as important as any other number, therefore, it should be written as clear and as large as any other number, and placed at the **bottom** of the column in which it is to be added.

$$\begin{array}{r} 4\,6\,8\,9\,2 \\ +\ 3\,2\,7\,5\,8 \\ \hline 7\,9\,6\,5\,0 \\ \hline 1\,1\,1\ \ \ \end{array}$$

When adding decimals, it is essential that the decimal point does not move and kept in line.

Where necessary, a zero should be added as a ***place holder.***

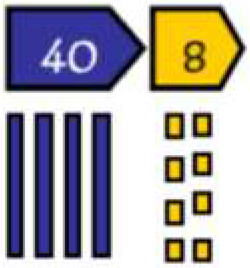
$12.5 + 23.7$

$$\begin{array}{r} 12.5 \\ +\ 23.7 \\ \hline 36.2 \\ \hline 1\ \ \ \ \ \end{array}$$

$34.5 + 27.43$

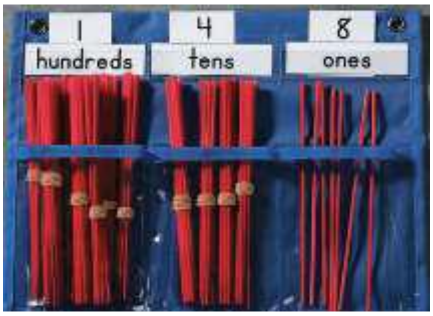
$$\begin{array}{r} 34.50 \\ +\ 27.43 \\ \hline 61.93 \\ \hline 1\ \ \ \ \ \ \ \end{array}$$

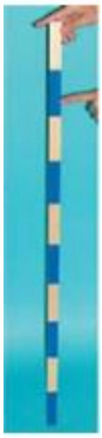
# Progression in Subtraction




**Concrete resources:**
100 square
Number lines
Bead strings
Straws
Dienes
Counting stick
Place value dice
Place value cards
Place value counters

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |



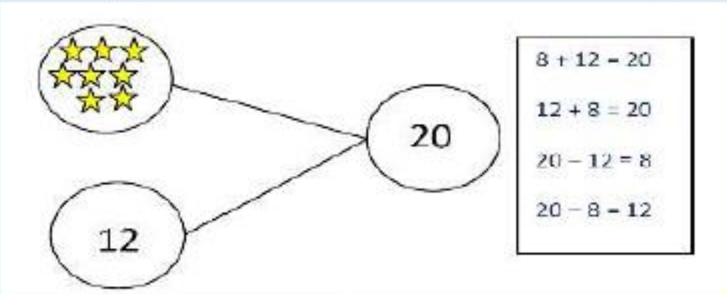



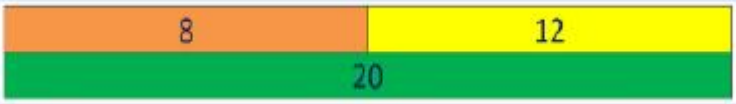
# Subtraction: Year 2

Year 2 statutory requirements:

- Recall and use subtraction facts to 20 fluently, and derive and use related facts to 100.
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.
- Subtract numbers using concrete objects, pictorial representations, and mentally, including:
    - a two-digit number and ones
    - a two-digit number and tens
    - two two-digit numbers
    - adding three one-digit numbers.

Memorise and reason with number facts to 20 in several forms.

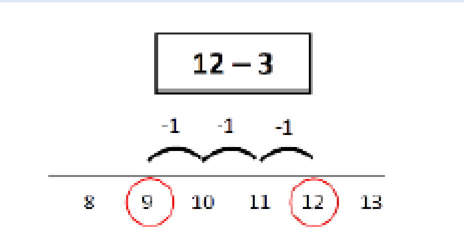


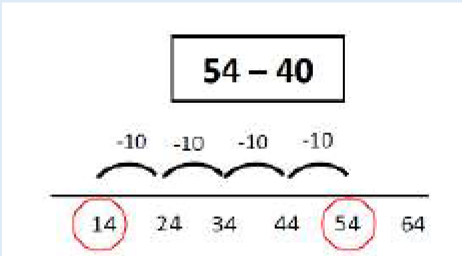


Partition two 2-digit numbers using a variety of models and images.

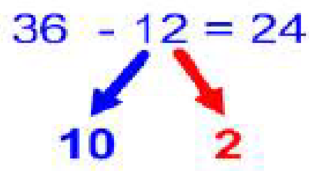

Subtract 2 digit and ones

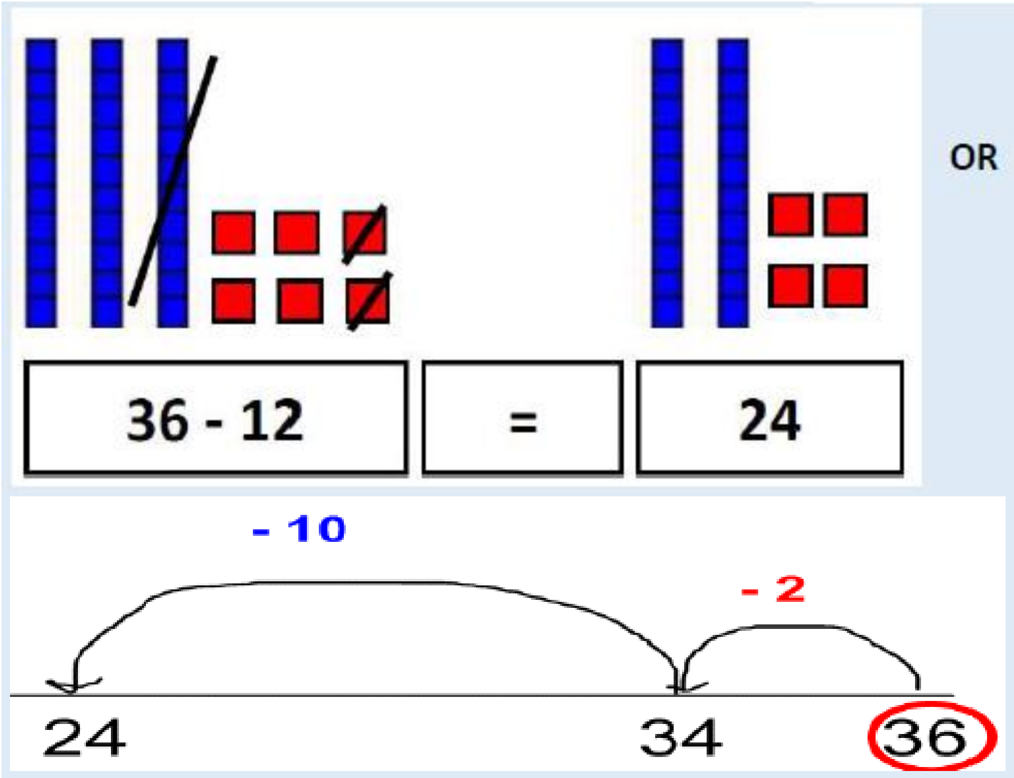



Subtract 2 digit and tens



Use partitioning to subtract two 2-digit numbers using concrete resources and/or a numbered number line and then progressing to an empty number line.



# Subtraction: Year 3

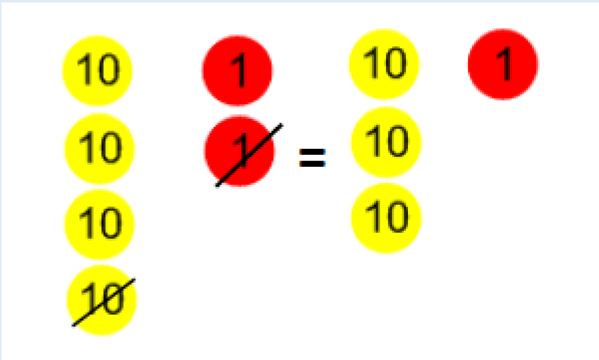
Year 3 statutory requirement:

- Find 10 or 100 less than a given number.
- Recognise the place value of each digit in a three-digit number (hundreds, tens, ones).
- Subtract numbers with up to three digits, using formal written methods of column subtraction.
- Subtract numbers mentally, including:
    - A three-digit number and ones
    - A three-digit number and tens
    - A three-digit number and hundreds.

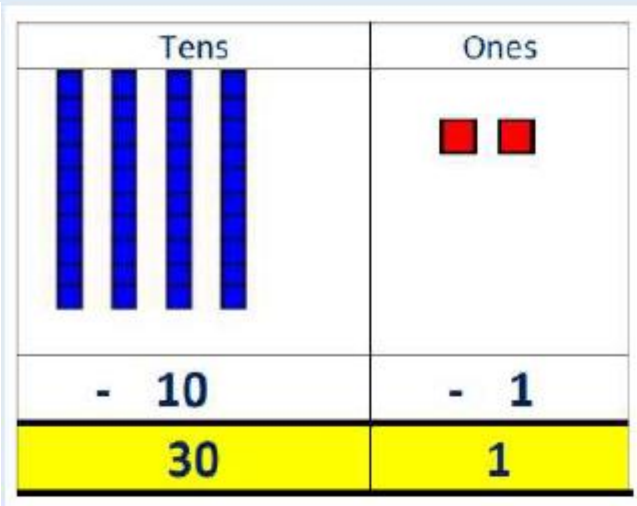
Use expanded column method with place value resources to support the conceptual understanding of subtracting numbers with up to three digits ***with no exchanging.***

**42 - 11 = 31**

$$\begin{array}{rr} & 40 + 2 \\ - & \underline{10 + 1} \\ & \underline{30 + 1} \end{array}$$



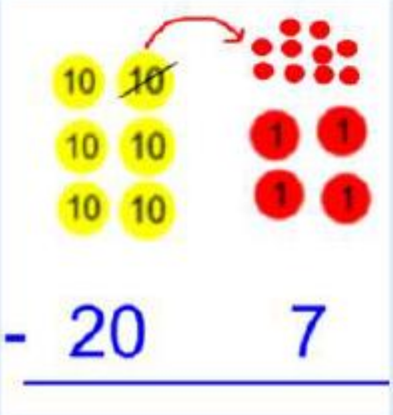

OR



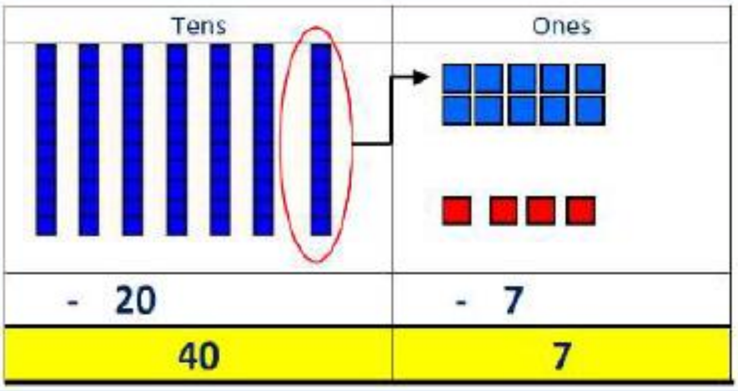
Progress to using the expanded column method with place value resources to support the conceptual understanding of subtracting numbers with up to three digits ***with exchanging tens and/or hundreds.***

**74 - 27 = 47**

$$
\begin{array}{rrcr}
 & 60 & + & 14 \\
 & \cancel{70} & + & \cancel{4} \\
- & 20 & + & 7 \\
\hline
 & 40 & + & 7 \\
\hline
\end{array}
$$




OR




In this example to subtract 7 ones from 4 ones we need to **exchange** a ten for ten ones. We now can subtract 7 ones from 14 ones.

Extend to using the expanded column method to subtract three digit numbers from three digit numbers.

**537 – 254 = 283**

$$
\begin{array}{rrcrcr}
 & 400 & + & 130 & & \\
 & \cancel{500} & + & \cancel{30} & + & 7 \\
- & 200 & + & 50 & + & 4 \\
\hline
 & 200 & + & 80 & + & 3 \\
\hline
\end{array}
$$

**Note:** The exchanged ten or hundred is just as important as any other number, therefore, it should be written as clear and as large as any other number, and placed at the **top** of the column which has been adjusted.

# Subtraction: Year 4

Year 4 statutory requirements:

- Find 1000 less than a given number.
- Subtract numbers with up to four digits, using formal written methods of columnar subtraction where appropriate.
- Solve subtraction two-step problems in contexts, deciding which operations and methods to use and why.

Build on learning from Year 3 and model how expanded method links to compact column subtraction method.

$$\begin{array}{rrcr} & 60 & + & 14 \\ & \cancel{70} & + & \cancel{4} \\ - & 20 & + & 7 \\ \hline & 40 & + & 7 \\ \hline \end{array} \rightarrow \begin{array}{rr} 6 & \\ \cancel{7} & {}^{1}4 \\ 2 & 7 \\ \hline 4 & 7 \\ \hline \end{array}$$

$$\begin{array}{rrcrcr} & 400 & + & 130 & & \\ & \cancel{500} & + & \cancel{30} & + & 7 \\ - & 200 & + & 50 & + & 4 \\ \hline & 200 & + & 80 & + & 3 \\ \hline \end{array} \rightarrow \begin{array}{rrrr} & 4 & & \\ & \cancel{5} & {}^{1}\cancel{3} & 7 \\ - & 2 & 5 & 4 \\ \hline & 2 & 8 & 3 \\ \hline \end{array}$$

By the end of year 4, pupils should be subtracting numbers up to 4 digits using compact column subtraction method.

$$\begin{array}{rrrrr} & & & 3 & \\ & 7 & 8 & \cancel{4} & {}^{1}2 \\ - & 1 & 8 & 2 & 9 \\ \hline & 6 & 0 & 1 & 3 \\ \hline \end{array}$$

**Note:** The exchanged ten or hundred is just as important as any other number, therefore, it should be written as clear and as large as any other number, and placed at the **top** of the column which has been adjusted.

# Subtraction: Year 5 & 6

Year 5 statutory requirements :

- Subtract whole numbers with more than 4 digits using formal written methods of columnar subtraction.
- Subtract numbers mentally, with increasingly large numbers.
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
- Solve problems involving numbers up to three decimal places.

Year 6 statutory requirements: pupils are expected to solve more complex addition and subtraction problems

In year 5 and 6 pupils should be subtracting numbers using compact column subtraction method. Note: The exchanged ten or hundred is just as important as any other number. Therefore, it should be written as clear and as large as any other number, and placed at the **top** of the column which has been adjusted.

| | | | | | |
|---|---|---|---|---|---|
| | 8 | | | 7 | |
| | ~~9~~ | $^{1}$6 | 7 | ~~8~~ | $^{1}$3 |
| - | 5 | 8 | 7 | 3 | 5 |
| | 3 | 8 | 0 | 4 | 8 |

When subtracting decimals, it is essential that the decimal point does not move and kept in line.

Where necessary, a zero should be added as a *place holder*.

| | | | | |
|---|---|---|---|---|
| | 4 | | | |
| | ~~5~~ | . | $^{1}$3 | 7 |
| - | 2 | . | 5 | 4 |
| | 2 | . | 8 | 3 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $^{0}$~~1~~ | ~~10~~ | $^{1}$5 | . | $^{3}$~~4~~ | $^{1}$1 | 9 | kg |
| − | | 3 | 6 | . | 0 | 8 | 0 | kg |
| | | 6 | 9 | . | 3 | 3 | 9 | kg |

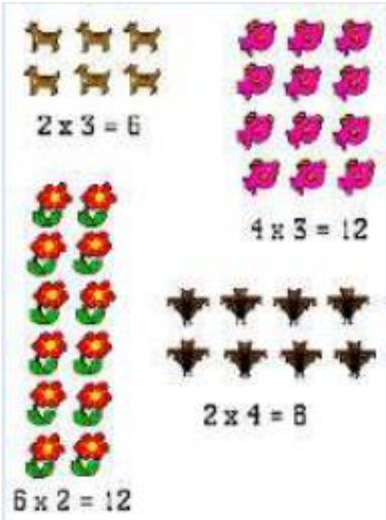
# Progression in Multiplication

# xxxxxx

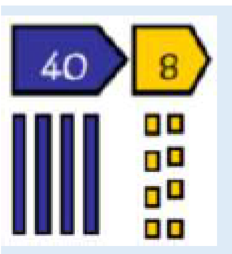
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

**Concrete resources**

Place value counters
Dienes
Place value charts
Arrays
Multiplication squares
100 square
Number lines
Blank number lines
Counting stick




| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
| 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |



# Multiplication: Year 2

Year 2 statutory requirement:

✓Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.

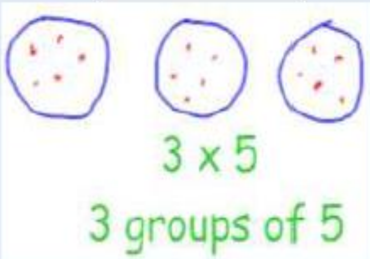
✓Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs.

✓Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.

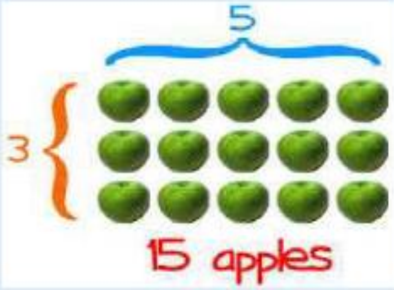
✓Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

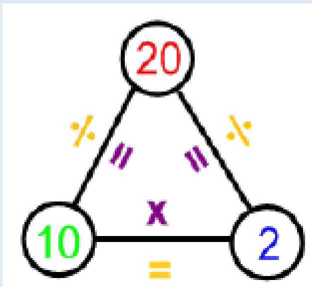
Further develop understanding multiplication as repeated addition.


Use pictorial representations



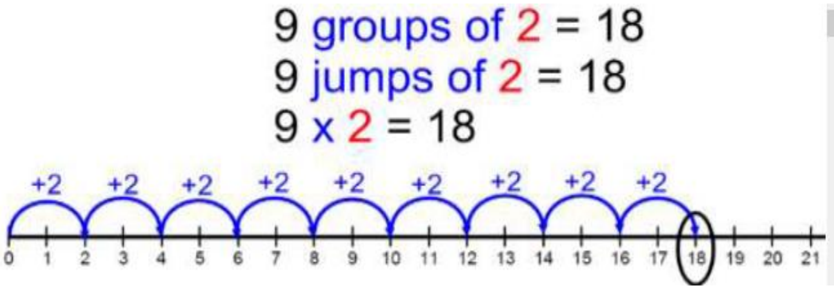

Use arrays




*Recall multiplication and division facts for 2, 5, 10*

Model and bridge link from repeated addition to solving multiplication problems using a number line.



# Multiplication: Year 3

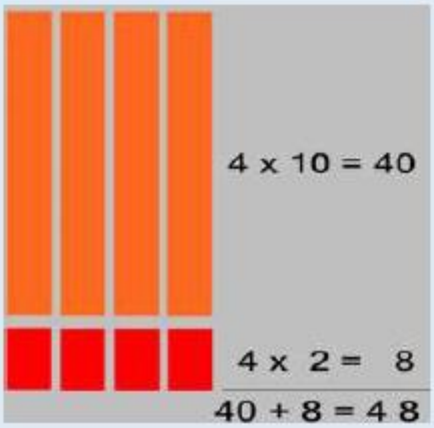
Year 3 statutory requirements:
✓Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.
✓Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
✓Solve problems, including missing number problems, involving multiplication including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Develop recall of multiplication facts (alongside inverse of the corresponding division facts).

| x | 3 | 4 | 8 |
|---|---|---|---|
| 5 | | | |
| 6 | | | |
| 4 | | | |

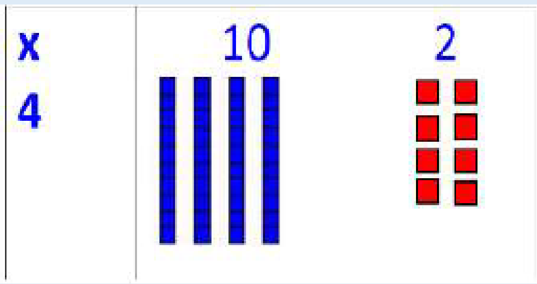
| x | 4 | ? | ? |
|---|---|---|---|
| ? | 8 | 6 | 10 |
| 6 | 24 | 18 | 30 |
| ? | 32 | 24 | 40 |

12 x 4 = 48




OR

Use concrete resources to develop conceptual understanding of the compact method introduced in Year 4.




| x | 10 | 2 |
|---|---|---|
| 4 | 40 | 8 |

$$\begin{array}{r} 10 + 2 \\ \times \quad\quad 4 \\ \hline 8 \\ 40 \\ \hline 48 \end{array}$$


The yellow ribbon is 4 times as long as the red ribbon. What is it's length?

# Multiplication: Year 4

Year 4 statutory requirement:
✓ Recall multiplication and division facts for multiplication tables up to 12 × 12
✓Use place value, known and derived facts to multiply and divide mentally, including: multiply two-digit and three-digit numbers by a one-digit number using formal written layout.
✓Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 |
| 10 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| 11 | 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 |
| 12 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |

Develop recall of multiplication facts (alongside the inverse of the corresponding division facts).

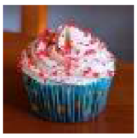
Build on learning from Year 3 and model how grid method and/or expanded method links to compact short multiplication.

| x | 30 | 6 |
|---|---|---|
| 4 | 10 10 10<br>10 10 10<br>10 10 10<br>10 10 10 | 1 1 1 1 1 1<br>1 1 1 1 1 1<br>1 1 1 1 1 1<br>1 1 1 1 1 1 |

$$\begin{array}{r} 30 + 6 \\ \times\ 4 \\ \hline 24 \\ +\ 120 \\ \hline 144 \end{array} \qquad \begin{array}{r} \overset{2}{3}6 \\ \times\ 4 \\ \hline 144 \\ \hline \end{array}$$

Use knowledge of times tables to solve scaling problems.

*Susie wants to bake 12 cupcakes people.*
*The ingredients given are for four cupcakes.*
*How much flour will she need?*



2 eggs
150g flour
180g sugar

| **Cupcakes** | **Flour** |
|---|---|
| 4 | 150g |
| 12 | 900g |

x3 x3

# Multiplication: Year 5

Year 5 statutory requirements:

✓Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.

✓Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

Build on learning from Year 4 and use concrete resources if needed to multiply numbers up to 4 digits by **one digit** using compact short multiplication.

643 × 8

```
 3 2
 643
 × 8
5144
```

To multiply by 10, 100, 1000 children should use place value charts to show that the digit moves a column (s) to the left .The value of the digit is increasing by 10, 100 or 1000 times.


| x | 600 | 40 | 3 |
|---|---|---|---|
| 50 | 30,000 | 2,000 | 150 |
| 4 | 2,400 | 160 | 12 |

Reinforce the connection between the grid method to multiply numbers up to 4 digits by **two digit** using long multiplication.

```
    2 1
    1 1
    643
  x  54
   1
   2572
 +32150
  34722
```

# Multiplication: Year 6

Year 6 statutory requirements:

✓ Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.

✓Multiply one-digit numbers with up to two decimal places by whole numbers.

Build on learning from Year 5 multiplying numbers using compact short multiplication and long division.

643 x 8

3 2
643
x 8
5144

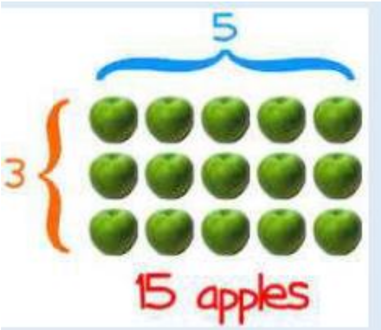
2 1
1 1
643
x 54
1
2572
+32150
34722

Use compact short multiplication to multiply decimal number by whole number.

2 3
7.6 8
x 4
30. 7 2

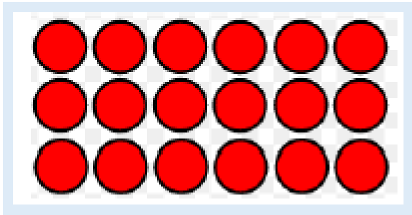
# Progression in Division

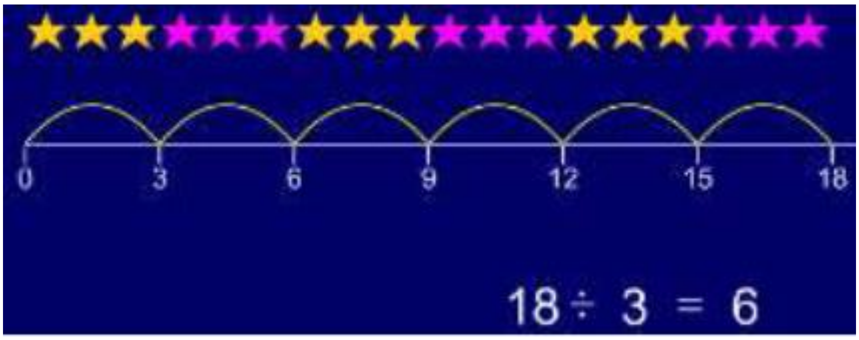
÷÷÷÷÷÷÷÷



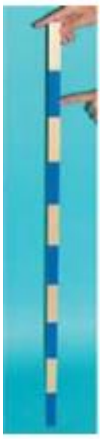

**Concrete resources:**

- Arrays
- Multiplication squares
- 100 square
- Number lines
- Blank number lines
- Counting stick
- Place value apparatus






| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

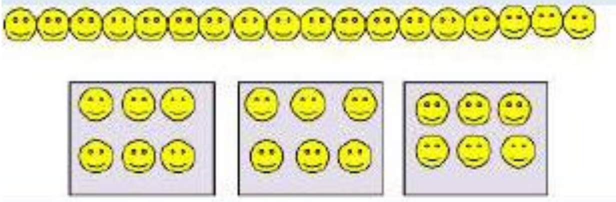


| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
| **2** | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| **3** | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
| **4** | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| **5** | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| **6** | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| **7** | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 |
| **8** | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |
| **9** | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 |
| **10** | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

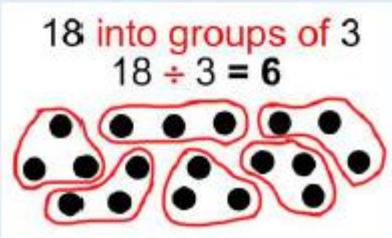
# Division: Year 2

Year 2 statutory requirement:
✓Recall and use division facts for 2, 5 and 10 multiplication tables.
✓Calculate mathematical statements for multiplication and division within the multiplication tables and write then using the multiplication (x), division ( ) and equals (=) signs.
✓Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.
✓***Find 1/3; 1/4; 2/4; ¾ of a length, shape, set of objects or quantity***

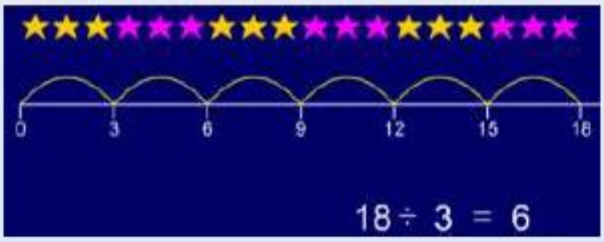
Further develop understanding of difference between **sharing and grouping** using concrete resources.



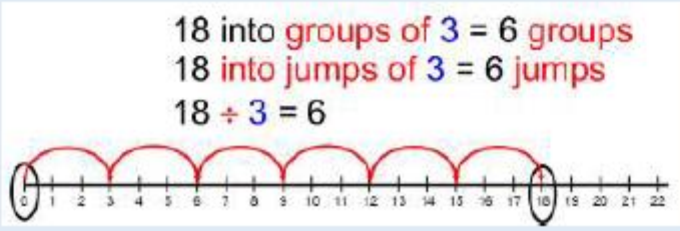
18 smiley faces shared between 3 classes.




Model division as grouping on a number line (ITP 'Grouping')

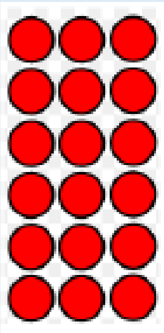



Children use numbered number lines to divide using grouping.




Reinforce division

through the use of arrays.



18 ÷3 = 6

18 ÷6 = 3

**Remember** to develop connections between fractions and division and rephrase this calculation as 1/3 of 18 is the same as 18 ÷ 3 = 6.

# Division: Year 3 & 4

Year 3 statutory requirement:
✓Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
✓Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
✓ Solve problems, including missing number problems, involving division including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.
Year 4 statutory requirement: Note - there isn't a statutory objective for division. However, Y4 statutory multiplication objectives are to (1) recall multiplication and division facts for multiplication tables up to 12 × 12 and (2) multiply two-digit and three-digit numbers by a one-digit number using formal written layout so we will build on the connections between multiplication and division.

$$\begin{array}{r} 3\;\;2 \\ 3\overline{)\,9\;\;6} \end{array}$$

Limit numbers to NO remainders in the answer OR carried (each digit must be a multiple of the divisor).

$$\begin{array}{r} 1\;\;8 \\ 4\overline{)\,7\;\;{}^{3}2} \end{array}$$

Limit numbers to NO remainders in the final answer, but with remainders occurring within the calculation.

$$\begin{array}{r} 2\;\;1\;\;8 \\ 4\overline{)\,8\;\;7\;\;{}^{3}2} \end{array}$$

$$\begin{array}{r} 0\;\;3\;\;7 \\ 5\overline{)\,1\;\;8\;\;{}^{3}5} \end{array}$$

Extend to 3-digit number first where the divisor can go into the first number and then progress to when the divisor cannot go into the first number.

**Remember** to develop connections between fractions and division and rephrase these calculations as 1/3 of 96; ¼ of 72, ¼ of 872 and 1/5 of 185. Note: Year 3 fraction objective - *Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators;* Year 4 fraction objective: *solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number.*

# Division: Year 5

Year 5 statutory requirement:
✓ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Further secure pupils' understanding of compact short division.

$218 \div 8 =$

$$\begin{array}{r|l} & 2\ \ 7\ r\ 2 \\ \hline 8 & \cancel{2}\,{}^{2}1\,{}^{5}8 \end{array}$$

Extend to expressing results in different ways according to the context, including with remainders as fractions, as decimals or by rounding. For example:

- Whole number remainder = 27 r 2
- Fraction remainder = $27\frac{2}{8} = 27\frac{1}{4}$
- Decimal remainder = $27\frac{1}{4} = 27\frac{25}{100} = 27.25$

# Division: Year 6

Year 6 statutory requirement:
✓ divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

Continue to use compact short division to divide numbers up to 4 digits by a 1-digit whole number.

$218 \div 8 =$

$$\begin{array}{r} 2\;\,7\;\,\text{r}\,2 \\ 8\,\overline{)\,2\,{}^{2}1\,{}^{5}8} \end{array}$$

- Whole number remainder = $27 \text{ r } 2$
- Fraction remainder = $27\frac{2}{8} = 27\frac{1}{4}$
- Decimal remainder = $27\frac{1}{4} = 27\frac{25}{100} = 27.25$

Use long division to divide numbers up to 4 digits by a 2-digit whole number.

$$\begin{array}{rrrrl} & 0 & 2 & 4 & \text{r } 1\;2 \\ 2\;4 \,\big) & 5 & 8 & 8 & \\ - & 4 & 8 & & \\ \hline & 1 & 0 & 8 & \\ - & & 9 & 6 & \\ \hline & & 1 & 2 & \end{array}$$

# Glossary

- **Array** – numbers or objects arranged in rows and columns.
- **Commutative law** – numbers can be added or multiplied in any order and the answer will be the same.
- **Decomposition** – a method of subtraction using borrowing from the larger digit.
- **Denominator** – the number on the bottom of a fraction. It represents how many parts the whole has been split into.
- **Denominator families** – denominators which are multiples of the same number (E.g. One half, One quarter, One eighth).
- **Dividend** – the number (of objects) that you begin with when dividing.
- **Divisor** - the number you are dividing by in a division calculation.
- **Find the difference** – subtraction method where children cover the larger number with the smaller number and then name the uncovered number.
- **Manipulatives/concrete situations** – practical resources to support the children when they are calculating.
- **Non-unit fractions** – A fraction where the numerator is more than one. E.g. 2/3 of a pizza.
- **Numerator** – the number on the top of a fraction. This represents how many parts of the whole we are looking for.
- **Partition** – to split numbers into hundreds, tens and units, to make them easier to work with.
- **Quotient** – the answer in a division calculation.
- **Remainder** – the number (of objects) left over when you do not have enough left to share equally.
- **Unit fraction** – a unit fraction is just one part of a whole. The numerator of the fraction is just one.
- **Whole** – when working with fractions, the whole is the complete shape, the starting number or amount of objects.

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