Section 1: Trigonometric functions and identities

Exercise level 1

Do not use a calculator in this exercise.

1. Triangle ABC is right angled at B. AB = 10 cm and AC = 26 cm.
   (i) Calculate the length of BC.
   (ii) Write down the values of sin A, cos A, and tan A leaving your answers as fractions.
   (iii) Write down the values of sin C, cos C, and tan C leaving your answers as fractions.
   (iv) Write down three separate equations connecting the trig ratios for angle A to those for angle C.
   (v) In general, what conclusions can you draw from your answers to (iv)?

2. (i) Sketch the curve of $y = \tan x$ for angles between 0° and 360°.
   (ii) Add the line $y = 1$ to your sketch and mark the points where the graphs intersect. Find the values of x between 0° and 360° for which $\tan x = 1$.
   (iii) Without using a calculator, find the values of x in the interval 0° to 360° for which $\tan x = -1$.

3. Using a sketch of $y = \sin x$, write down all of the angles between 90° and 540°
   (i) that have the same sine as 40°;
   (ii) that have the same sine as 160°.

4. Find all of the values of x between 0° to 360° such that
   (i) $\cos x = \cos 25°$
   (ii) $\sin x = \sin 50°$
   (iii) $\tan x = \tan 120°$
   (iv) $\sin x = -\sin 60°$
   (v) $\cos x = -\cos 20°$

Section 1: Trigonometric functions and identities

Exercise level 2

Do not use a calculator in this exercise.

1. Write the following as fractions or using square roots. You should not need your calculator.
   (i) $\sin 120°$
   (ii) $\cos(-120°)$
   (iii) $\tan 135°$
   (iv) $\sin 300°$
   (v) $\cos 270°$

2. In the following give your answers as fractions
   (i) θ is acute and $\sin\theta = \frac{12}{13}$. Write down the value of $\cos\theta$.
   (ii) θ is obtuse and $\sin\theta = \frac{7}{25}$. write down the values of $\cos\theta$ and $\tan\theta$.
   (iii) θ is obtuse and $\tan\theta = -\frac{8}{15}$. Write down the values of $\sin\theta$ and $\cos\theta$.

3. Using the identities $\sin^2 x + \cos^2 x \equiv 1$ and/or $\tan x \equiv \dfrac{\sin x}{\cos x}$, simplify
   (i) $\dfrac{\sqrt{1-\cos^2 x}}{\tan x}$ (ii) $\dfrac{\sin x}{\sqrt{1-\sin^2 x}}$ (iii) $\dfrac{\cos^2 x}{1+\sin x}$

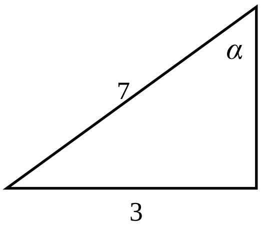
4. Find exactly:
   (i) $\sin 120° - \sin 150°$
   (ii) $\tan 225° + \cos(-30°)$
   (iii) $\dfrac{\cos 45°}{\sin 135°}$
   (iv) $2\tan 60° - 2\tan(-60°)$
   (v) $\dfrac{\sin 50°}{\sqrt{1-\cos^2 50°}}$

Section 1: Trigonometric functions and identities

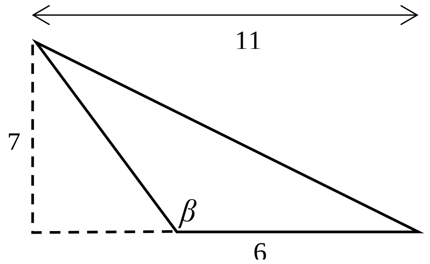
Exercise level 3 (Extension)

1. In the following diagrams, find the sine, cosine and tangent of the marked angles α, β and δ.

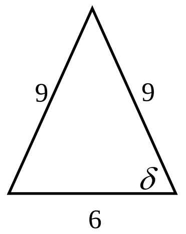
(i)



(ii)



(iii)



2. [Make sure you use degree mode on your calculator throughout this question.]

An engineer is testing a new design of spring component to be fitted in a sports car, in order to find its ability to withstand vibration. The component is fixed vertically so that the end A of the spring is at a point where $y = 0$.

(i) Initially, the end A of the spring is forced to oscillate according to a function $y = 3\sin(10\theta)° - 1$, where θ is measured in seconds, and y is measured in millimetres. Sketch the graph of the position of end A during the first 50 seconds of the test.

(ii) Find the times during the first 50 seconds of the test when the end A is displaced by exactly 1 mm from the original point where $y = 0$.

(iii) In a second test, the engineer forces end A to oscillate according to the function $y = 2\sin^2(10\theta)°$. Again, sketch the graph of the position of end A during the first 50 seconds of the test.

(iv) Find the times during the first 50 seconds of each test when the position of end A is exactly the same for both tests.

OCR AS Mathematics Trigonometry

Section 3: The sine and cosine rules

Exercise level 1

1. Solve the triangle ABC in which A = 66°, B = 42° and c = 12 cm.

2. Find two possible values of c in triangle ABC given that a = 16 cm, b = 10 cm, and B = 30°.

3. Solve the triangle ABC in which a = 6 cm, b = 9 cm and C = 97°.

4. Solve the triangle PQR in which p = 8 cm, q = 9 cm and r = 10 cm.

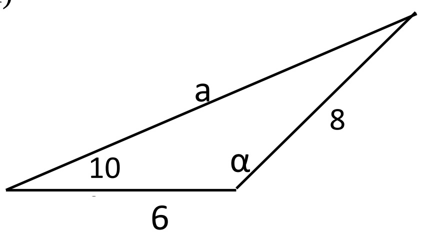
5. In triangle XYZ, X = 100°, Y = 30° and XY = 10 cm. Calculate the area of the triangle.

6. The area of a triangle is 12 cm^2. Two of the sides are of lengths 6 cm and 7 cm. Calculate possible lengths for the third side.

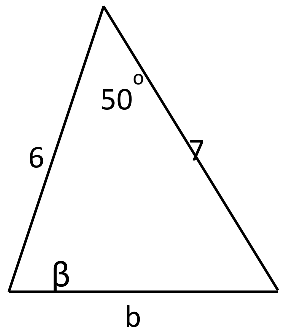
7. A ship S is 6.8 km from a lighthouse on a bearing of 310°. A second ship T is 8.4 km from the lighthouse on a bearing 075°. Calculate ST and the bearing of T from S correct to the nearest degree.

8. Find all the lettered edges and angles in the figures in the following diagrams:

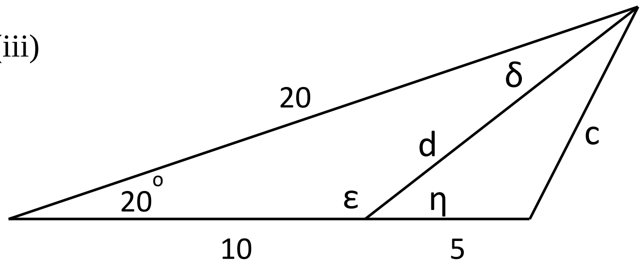
(i)



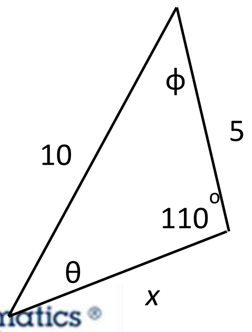
(ii)



(iii)



(iv)

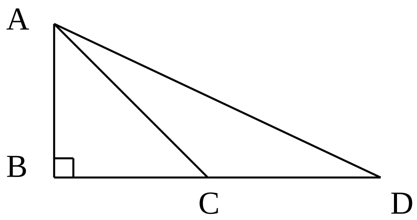


Section 3: The sine and cosine rules

Exercise level 2

1. A golfer hits a ball B a distance of 170 m on a hole that measures 195 m from tee to hole. If his shot is directed 10° away from the direct line to the hole, find how far his ball is from the hole.

2. Calculate AB in the diagram below given that CD is 15 m, angle BCA = 50° and angle BDA = 20°.

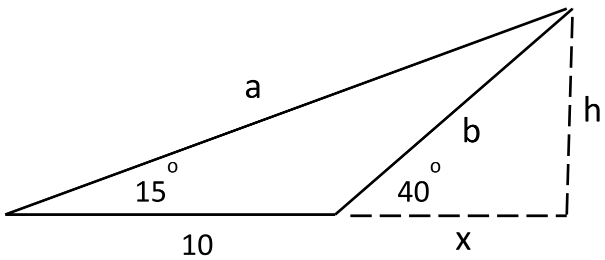


3. A tower stands on a slope inclined at 18° to the horizontal. From a point lower down the slope and 150 m from the base of the tower, the angle of elevation of the top of the tower is 27.5°, measured from the horizontal. Find the height of the tower.

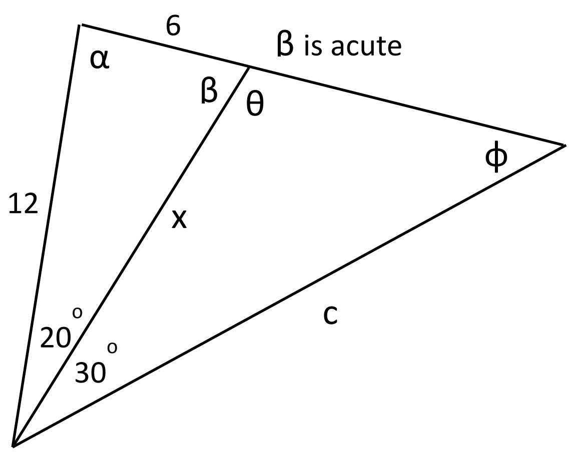
4. A barge is moving at a constant speed along a straight canal. The angle of elevation of a bridge is 10°. After 10 minutes the angle of elevation is 15°. After how much longer does the barge reach the bridge? Give your answer to the nearest second.

5. Find all the lettered edges and angles in the figures in the following diagrams:

(i)



(ii)



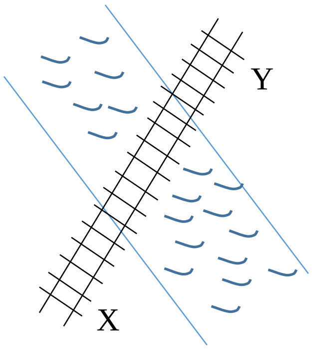
OCR AS Mathematics
Trigonometry
Section 3: The sine and cosine rules

Exercise level 3 (Extension)

1. A surveyor walks 40 metres from the base of a vertical radio mast PQ across horizontal ground to a point A. She then measures that the foot of the mast is on a bearing of 030°, and the angle of elevation of the top of the mast is 42°. She then walks due East to point B, where she measures the new angle of elevation as 31°.

   (i) Draw a diagram to show the configuration.
   (ii) How far has she walked from A to B?
   (iii) What is the bearing of the foot of the mast from her at point B?

2. A railway bridge is to be built at an angle across a canal as in the diagram. The railway runs in a straight line in a direction 040°, and the ends of the final support columns of the bridge are to be built at X and Y, each 10 metres along the railway from the banks of the canal. A surveyor walks 40 metres due South from point X to point Z, and the bearing of point Y is now 022°.

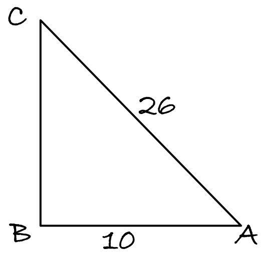


   (i) What is the length of the bridge from X to Y?
   (ii) The canal flows in the direction 155°, and where the bridge crosses it, the banks are straight, and parallel. What is the width of the canal?
   (iii) The highest point of the bridge structure is above H, exactly half-way between X and Y. What is the bearing of that point from the surveyor at Z?

Section 1: Trigonometric functions and identities

Solutions to Exercise level 1

1.



(i) $BC^2 = AC^2 - AB^2 = 26^2 - 10^2 = 576$

$BC = 24$ cm

(ii) $\sin A = \frac{24}{26} = \frac{12}{13}$

$\cos A = \frac{10}{26} = \frac{5}{13}$

$\tan A = \frac{24}{10} = \frac{12}{5}$

(iii) $\sin C = \frac{10}{26} = \frac{5}{13}$

$\cos C = \frac{24}{26} = \frac{12}{13}$

$\tan C = \frac{10}{24} = \frac{5}{12}$

(iv) $\sin A = \cos C$

$\cos A = \sin C$

$\tan A = \frac{1}{\tan C}$

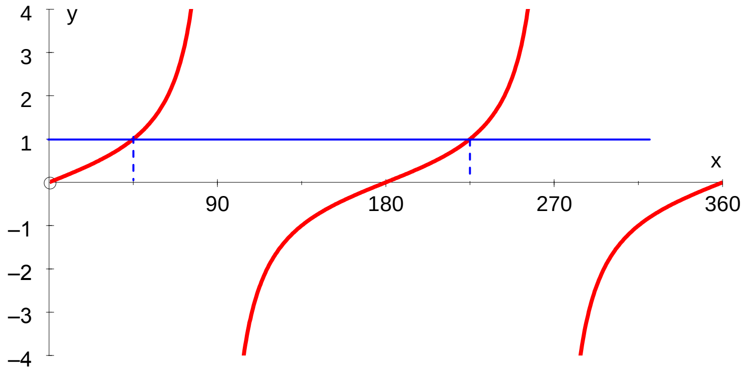
(v) Since $C = 90° - A$, this can be generalised to

$\sin x = \cos(90° - x)$

$\cos x = \sin(90° - x)$

$\tan x = \frac{1}{\tan(90° - x)}$

2. (i)



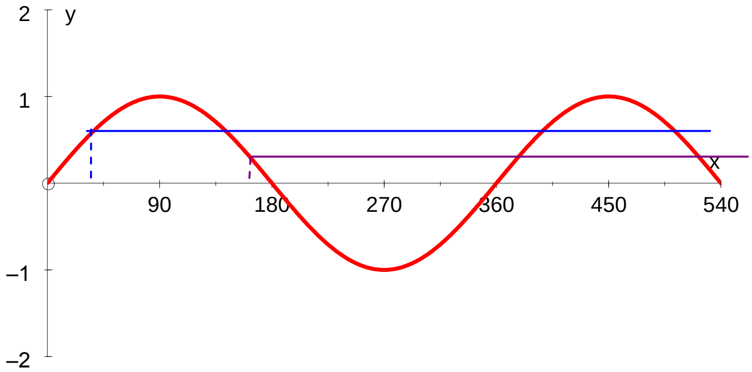
(ii) $\tan x = 1$

$x = 45^\circ$ or $180^\circ + 45^\circ$

$x = 45^\circ$ or 225°

(iii) By symmetry, angles are $180^\circ - 45^\circ = 135^\circ$ and $360^\circ - 45^\circ = 315^\circ$

3.



(i) $180^\circ - 40^\circ = 140^\circ$
$360^\circ + 40^\circ = 400^\circ$
$540^\circ - 40^\circ = 500^\circ$

(ii) $360^\circ + 20^\circ = 380^\circ$
$540^\circ - 20^\circ = 520^\circ$

4. (i) $x = 360^\circ - 25^\circ = 335^\circ$
(ii) $x = 180^\circ - 50^\circ = 130^\circ$
(iii) $x = 180^\circ + 120^\circ = 300^\circ$
(iv) $x = 180^\circ + 60^\circ = 240^\circ$ and $x = 360^\circ - 60^\circ = 300^\circ$
(v) $x = 180^\circ - 20^\circ = 160^\circ$ and $x = 180^\circ + 20^\circ = 200^\circ$

Section 1: Trigonometric functions and identities

Solutions to Exercise level 2

1. (i) $\sin 120° = \sin 60° = \frac{\sqrt{3}}{2}$

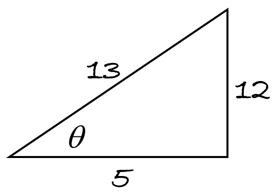
   (ii) $\cos(-120°) = \cos 120° = -\cos 60° = -\frac{1}{2}$

   (iii) $\tan 135° = -\tan 45° = -1$

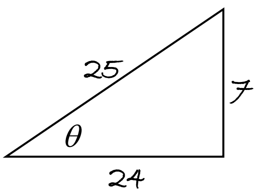
   (iv) $\sin 300° = -\sin 60° = -\frac{\sqrt{3}}{2}$

   (v) $\cos 270° = -\cos 90° = 0$

2. (i) $\cos\theta = \frac{5}{13}$



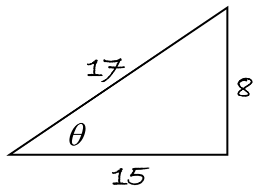
   (ii) Since θ is in the second quadrant, cos θ and tan θ are both negative.



$\cos\theta = -\frac{24}{25}$

$\tan\theta = -\frac{7}{24}$

   (iii) Since θ is in the second quadrant, sin θ is positive and cos θ is negative.



$\sin\theta = \frac{8}{17}$

$\cos\theta = -\frac{15}{17}$

OCR AS Maths Trigonometry 1 Exercise solutions

3. (i) $$\frac{\sqrt{1-\cos^2 x}}{\tan x} = \frac{\sqrt{\sin^2 x}}{\tan x}$$
$$= \sin x \times \frac{\cos x}{\sin x}$$
$$= \cos x$$

(ii) $$\frac{\sin x}{\sqrt{1-\sin^2 x}} = \frac{\sin x}{\sqrt{\cos x}}$$
$$= \frac{\sin x}{\cos x}$$
$$= \tan x$$

(iii) $$\frac{\cos^2 x}{1+\sin x} = \frac{1-\sin^2 x}{1+\sin x}$$
$$= \frac{(1+\sin x)(1-\sin x)}{1+\sin x}$$
$$= 1-\sin x$$

4. (i) $$\sin 120° - \sin 150° = +\tfrac{\sqrt{3}}{2} - \tfrac{1}{2}$$
$$= \tfrac{1}{2}\left(\sqrt{3}-1\right)$$

(ii) $$\tan 225° + \cos(-30°) = 1 + \tfrac{\sqrt{3}}{2}$$
$$= \tfrac{1}{2}\left(2+\sqrt{3}\right)$$

(iii) $$\frac{\cos 45°}{\sin 135°} = \frac{\left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{1}{\sqrt{2}}\right)}$$
$$= 1$$

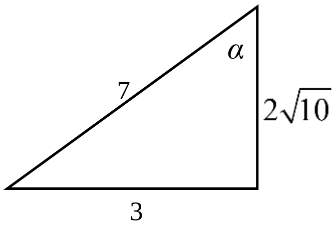
(iv) $$2\tan 60° - 2\tan(-60°) = 2\sqrt{3} - 2\left(-\sqrt{3}\right)$$
$$= 4\sqrt{3}$$

(v) $$\frac{\sin 50°}{\sqrt{1-\cos^2 50°}} = \frac{\sin 50°}{\sin 50°}$$
$$= 1$$

Section 1: Trigonometric functions and identities

Solutions to Exercise level 3

1. (i) The third side of the triangle is $\sqrt{40} = 2\sqrt{10}$

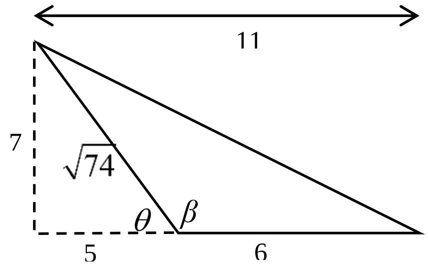


So $\sin\alpha = \frac{3}{7}$

$\cos\alpha = \frac{2\sqrt{10}}{7}$

$\tan\alpha = \frac{3}{2\sqrt{10}}$

(ii)

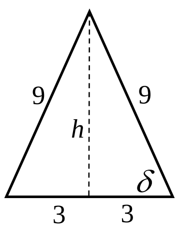


$\sin\beta = \sin(180-\theta) = \sin\theta = \frac{7}{\sqrt{74}}$

$\cos\beta = \cos(180-\theta) = -\cos\theta = -\frac{5}{\sqrt{74}}$

$\tan\beta = \tan(180-\theta) = -\tan\theta = -\frac{7}{5}$

(iii) The triangle is isosceles and $h = \sqrt{81-9} = \sqrt{72} = 6\sqrt{2}$



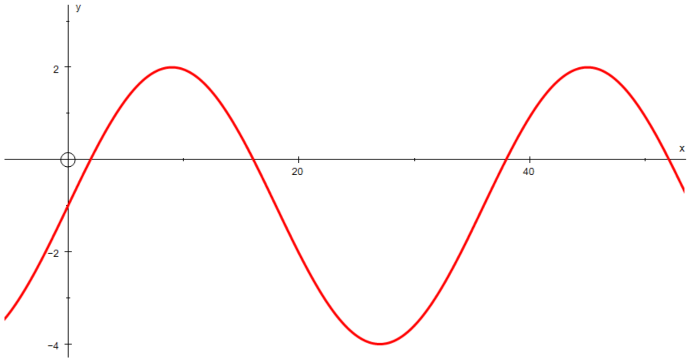
$\sin\delta = \frac{6\sqrt{2}}{9} = \frac{2}{3}\sqrt{2}$

$\cos\delta = \frac{3}{9} = \frac{1}{3}$

$\tan\delta = \frac{6\sqrt{2}}{3} = 2\sqrt{2}$

OCR AS Maths Trigonometry 1 Exercise solutions

2. (i)



(ii) $3\sin(10\theta) - 1 = 1$

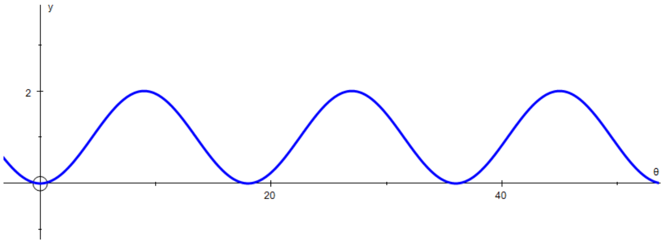
$\Rightarrow \sin(10\theta) = \frac{2}{3} \approx \sin 41.8°$

$\Rightarrow 10\theta \approx 41.8°,\ 138.2°,\ 401.8°$

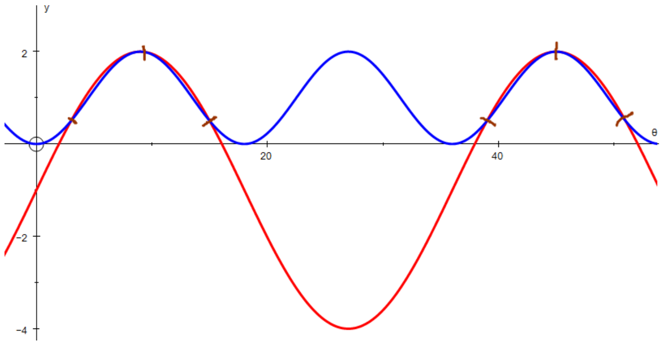
$\Rightarrow \theta \approx 4.18°,\ 13.82°,\ 40.18°$

so end A is 1 mm from the origin after 4.1 sec, 13.8 sec, 40.2 sec.

(iii)



(iv)



OCR AS Maths Trigonometry 1 Exercise solutions

$3\sin 10\theta - 1 = 2\sin^2 10\theta$

$\Rightarrow 2\sin^2 10\theta - 3\sin 10\theta + 1 = 0$

$\Rightarrow (2\sin 10\theta - 1)(\sin 10\theta - 1) = 0$

$\Rightarrow \sin 10\theta = \frac{1}{2}$ or $\sin 10\theta = 1$

$= \sin 30°$ $= \sin 90°$

$\Rightarrow 10\theta = 30°,\ 150°,\ 390°,\ 510°, \ldots\ldots$ or $90°,\ 450°,\ \ldots\ldots$

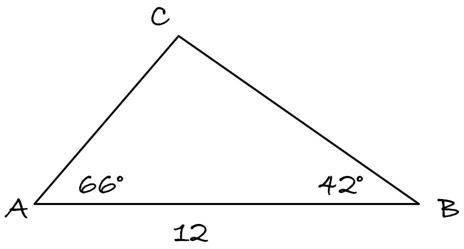
$\Rightarrow \theta = 3°,\ 9°,\ 15°,\ 35°,\ 39°,\ 45°$

so the positions are identical after 3, 9, 15, 35, 39, and 45 seconds.

Section 3: The sine and cosine rules

Solutions to Exercise level 1

1.



Angle $C = 180° - 66° - 42° = 72°$

Using the sine rule:
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 66°} = \frac{12}{\sin 72°}$$
$$a = \frac{12\sin 66°}{\sin 72°} = 11.53 \text{ cm}$$

Using the sine rule:
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 42°} = \frac{12}{\sin 72°}$$
$$b = \frac{12\sin 42°}{\sin 72°} = 8.44 \text{ cm}$$

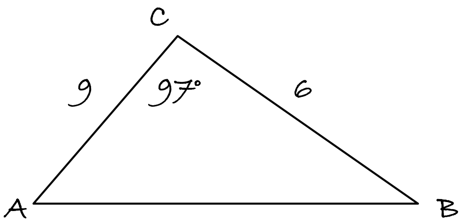
2. Using the sine rule:
$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin A}{16} = \frac{\sin 30°}{10}$$
$$\sin A = \frac{16\sin 30°}{10} = 0.8$$
$$A = 53.1° \text{ or } 126.9°$$

$C = 180° - 30° - A = 96.1°$ or $23.1°$

Using the sine rule:
$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{c}{\sin C} = \frac{10}{\sin 30°}$$
$$c = \frac{10\sin C}{\sin 30°} = 19.9 \text{ cm or } 7.9 \text{ cm}$$

OCR AS Maths Trigonometry 3 Exercise solutions

3.



Using the cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

$= 9^2 + 6^2 - 2\times 9\times 6\cos 97^\circ$

$c = 11.4$ cm

Using the sine rule: $\frac{\sin A}{a} = \frac{\sin C}{c}$

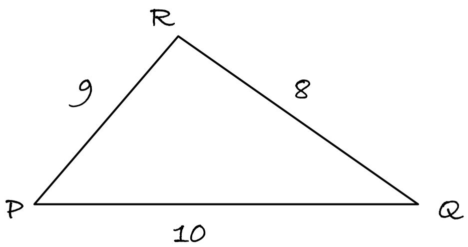
$\frac{\sin A}{6} = \frac{\sin 97}{11.4}$

$\sin A = \frac{6\sin 97}{11.4}$

$A = 31.5^\circ$

$B = 180^\circ - 97^\circ - 31.5^\circ = 51.5^\circ$.

4.



Using the cosine rule: $\cos P = \frac{q^2 + r^2 - p^2}{2qr} = \frac{9^2 + 10^2 - 8^2}{2\times 9\times 10}$

$P = 49.5^\circ$

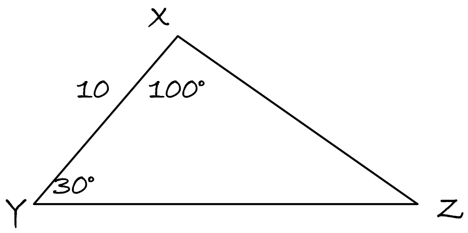
Using the cosine rule: $\cos Q = \frac{p^2 + r^2 - q^2}{2pr} = \frac{8^2 + 10^2 - 9^2}{2\times 8\times 10}$

$Q = 58.8^\circ$

$R = 180^\circ - 49.46^\circ - 58.75^\circ = 71.8^\circ$

OCR AS Maths Trigonometry 3 Exercise solutions

5.



Angle Z = 180° - 100° - 30° = 50°

Using the sine rule: $\frac{x}{\sin X} = \frac{z}{\sin z}$

$\frac{x}{\sin 100^\circ} = \frac{10}{\sin 50^\circ}$

$x = \frac{10\sin 100^\circ}{\sin 50^\circ} = 12.86$

Area of triangle $= \frac{1}{2} xz \sin Y$

$= \frac{1}{2} \times 12.86 \times 10 \sin 30^\circ$

$= 32.1 \text{ cm}^2$

6. Let $a = 6$ and $b = 7$

Area of triangle $= \frac{1}{2} ab \sin C$

$12 = \frac{1}{2} \times 6 \times 7 \sin C$

$C = 34.85^\circ$ or 145.15°

Using the cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

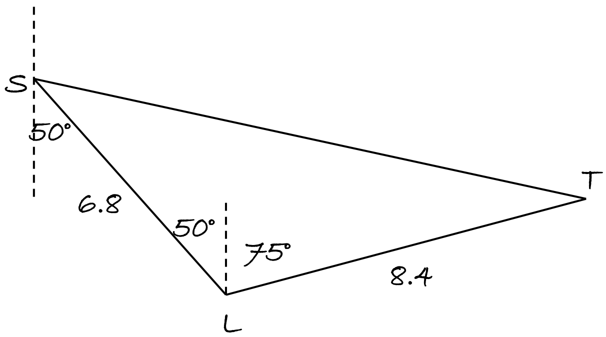
$= 6^2 + 7^2 - 2 \times 6 \times 7 \cos C$

$= 85 - 84 \cos C$

If $C = 34.85°$, $c = 4.01$ cm

If $C = 145.14°$, $c = 12.41$ cm

7.



Using cosine rule: $ST^2 = 6.8^2 + 8.4^2 - 2 \times 6.8 \times 8.4 \cos 125^\circ$

$ST = 13.5$ km

Using sine rule: $\dfrac{\sin S}{8.4} = \dfrac{\sin 125°}{13.5}$

$$\sin S = \frac{8.4 \sin 125°}{13.5}$$

$$S = 30.6°$$

Bearing of T from S = 180° - 50° - 30.6° = 099.4°

8. (i) $\dfrac{\sin 10}{8} = \dfrac{\sin \beta}{6}$

$\Rightarrow \sin \beta = 0.13$

$\Rightarrow \beta \approx 7.48°$

$\Rightarrow \alpha = 180° - 10° - 7.48° \approx 162.52°$

$\Rightarrow a^2 = 6^2 + 8^2 - 2(6)(8)\cos 162.52° \approx 191.57$

$\Rightarrow a \approx 13.8$

(ii) $b^2 = 6^2 + 7^2 - 2(6)(7)\cos 50° \approx 31.009$

$\Rightarrow b \approx 5.57$

$\dfrac{\sin \beta}{7} = \dfrac{\sin 50°}{5.57} \Rightarrow \sin \beta \approx 0.933$

$\Rightarrow \beta \approx 74.3°$

(iii) $c^2 = 20^2 + 15^2 - 2(15)(20)\cos 20° \approx 61.18$

$\Rightarrow c \approx 7.82$

$d^2 = 20^2 + 10^2 - 2(10)(20)\cos 20° \approx 124.12$

$\Rightarrow d \approx 11.1$

$\dfrac{\sin \delta}{10} = \dfrac{\sin 20°}{11.1} \Rightarrow \sin \delta \approx 0.308$

$\Rightarrow \delta \approx 17.9°$

$\Rightarrow \varepsilon \approx 180° - 20° - 17.9° = 142.1°$

$\Rightarrow \eta \approx 180° - 142.1° = 38.9°$

(iv) $\dfrac{\sin \theta}{5} = \dfrac{\sin 110°}{10} \Rightarrow \sin \theta \approx 0.470$

$\Rightarrow \theta \approx 28.0°$

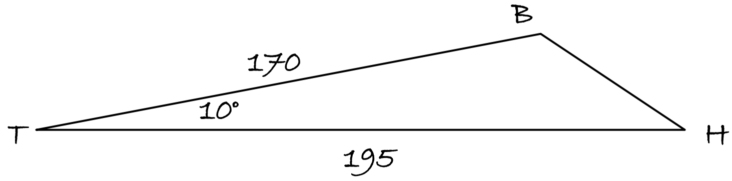
$\Rightarrow \phi \approx 42.0°$

$\dfrac{x}{\sin \phi} = \dfrac{5}{\sin \theta} \Rightarrow x \approx 7.13$

Section 3: The sine and cosine rules

Solutions to Exercise level 2

1.

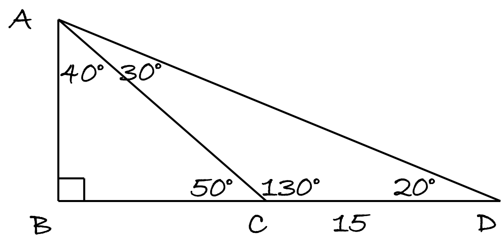


Using the cosine rule: $t^2 = 170^2 + 195^2 - 2 \times 170 \times 195 \cos 10°$

$t = 40.4$

It is 40.4 m from the hole.

2.



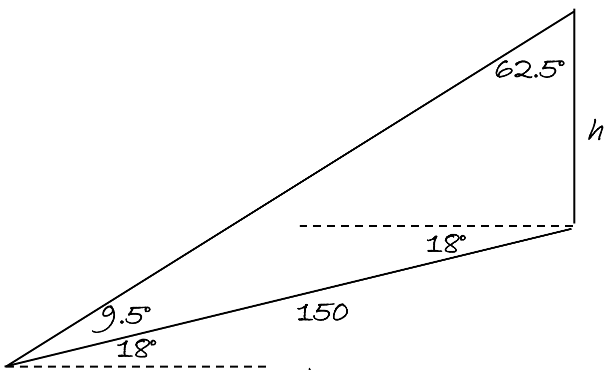
Using the sine rule on triangle ACD: $\frac{c}{\sin C} = \frac{a}{\sin A}$

$$\frac{c}{\sin 130°} = \frac{15}{\sin 30°}$$

$$c = \frac{15 \sin 130°}{\sin 30°} = 22.98$$

For triangle ABD: $AB = AD \cos 70° = 22.98 \cos 70° = 7.86$ m (3 s.f.)

3.

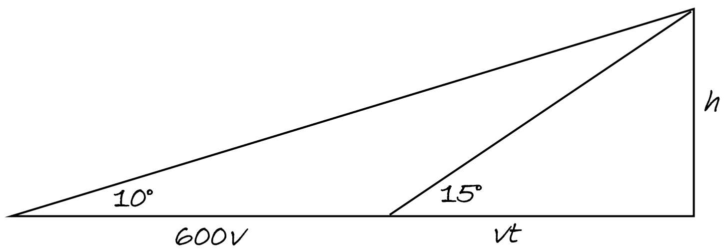


Using the sine rule: $\frac{h}{\sin 9.5°} = \frac{150}{\sin 62.5°}$

$$h = \frac{150 \sin 9.5°}{\sin 62.5°} = 27.9 \text{ m}$$

OCR AS Maths Trigonometry 3 Exercise solutions

4.



$$\tan 10° = \frac{h}{(600+t)v} \quad \Rightarrow h = v(600+t)\tan 10°$$

$$\tan 15° = \frac{h}{vt} \quad \Rightarrow h = vt \tan 15°$$

$$v(600+t)\tan 10° = vt \tan 15°$$

$$600 \tan 10° + t \tan 10° = t \tan 15°$$

$$t = \frac{600 \tan 10°}{\tan 15° - \tan 10°} = 1155 \text{ seconds}$$

Time taken = 19 mins 15 seconds

5. (i) $\left.\begin{aligned} h &= (10+x)\tan 15° \\ h &= x \tan 40° \end{aligned}\right\}$

$$\Rightarrow (10+x)\tan 15° = x \tan 40°$$

$$\Rightarrow 10 \tan 15° + x \tan 15° = x \tan 40°$$

$$\Rightarrow x = \frac{10 \tan 15°}{\tan 40° - \tan 15°}$$

$$\approx 4.69$$

$$\Rightarrow h = x \tan 40°$$

$$\approx 3.94$$

(ii) $\dfrac{\sin 20°}{6} = \dfrac{\sin \beta}{12} \quad \Rightarrow \sin \beta = 0.684...$

$$\Rightarrow \beta = 43.16...°$$

$$\Rightarrow \alpha = 180° - 43.16...° - 20° = 116.84°$$

$$\theta = 180° - \beta = 136.84°, \quad \phi = 180° - 30° - \theta = 13.16°$$

$$\Rightarrow x^2 = 6^2 + 12^2 - 2(6)(12)\cos 116.84°$$

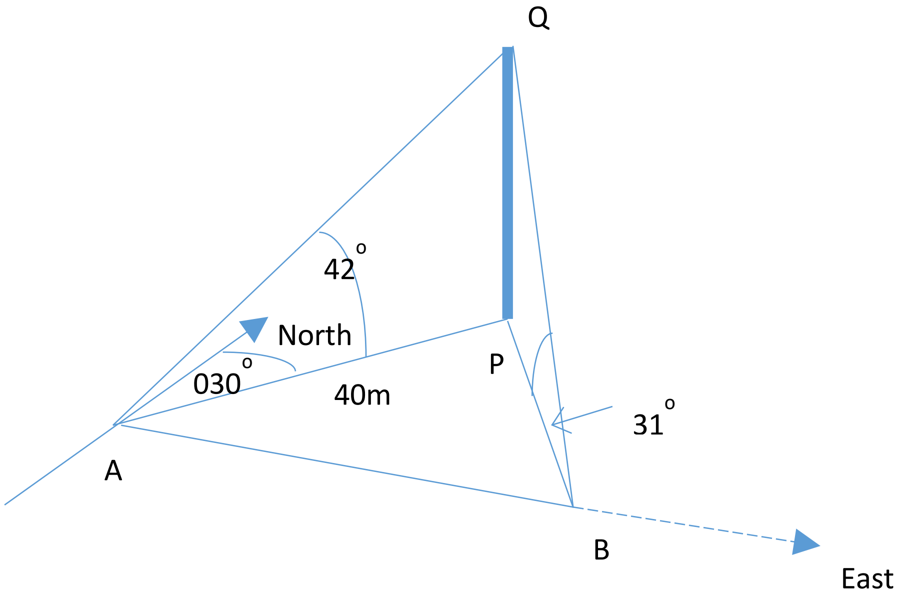
$$\Rightarrow x = 15.65$$

$$\frac{c}{\sin \theta} = \frac{x}{\sin \phi} \Rightarrow c = 47.02$$

Section 3: The sine and cosine rules

Solutions to Exercise level 3

1. (i)



(ii) $PQ = 40\tan 42° \approx 36.02$

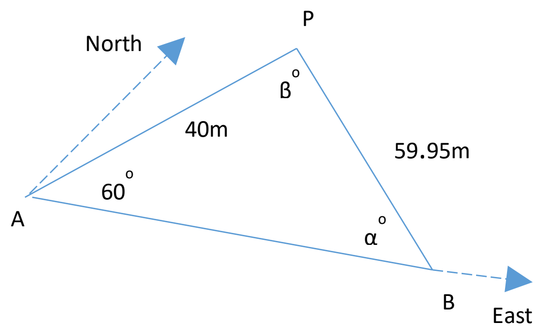
$\Rightarrow PB \approx \dfrac{36.02}{\tan 31°} = 59.95$

$\angle PAB = 60°$

so in ΔPAB, $\dfrac{\sin\alpha}{40} = \dfrac{\sin 60°}{59.95}$

$\Rightarrow \sin\alpha \approx 0.578$

$\Rightarrow \alpha \approx 35.3°$, and so $\beta \approx 84.7°$



$AB^2 = 40^2 + (59.95)^2 - 2(40)(59.95)\cos 84.7°$

≈ 4750.99

$\Rightarrow AB \approx 68.9$

so she has walked approximately 68.9 metres from A to B

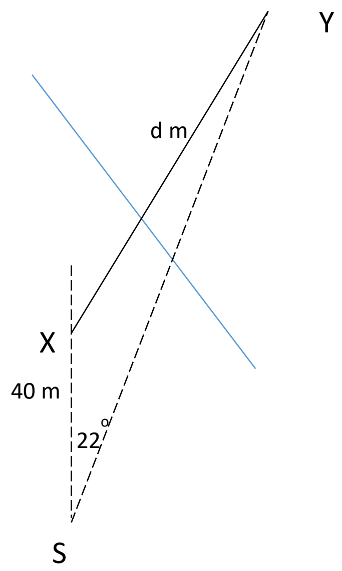
(iii) The bearing of the mast from B is approximately 324.7°.

2. (i) $\angle YXS = 140° \Rightarrow \angle XYS = 18°$

$\Rightarrow \dfrac{d}{\sin 22°} = \dfrac{40}{\sin 18°}$

$\Rightarrow d \approx 48.49$

so the bridge is approximately 48.5m long.



(ii) $PQ = 48.49 - 20 = 28.49$

$\angle QPD = \angle QPE - \angle DPE$

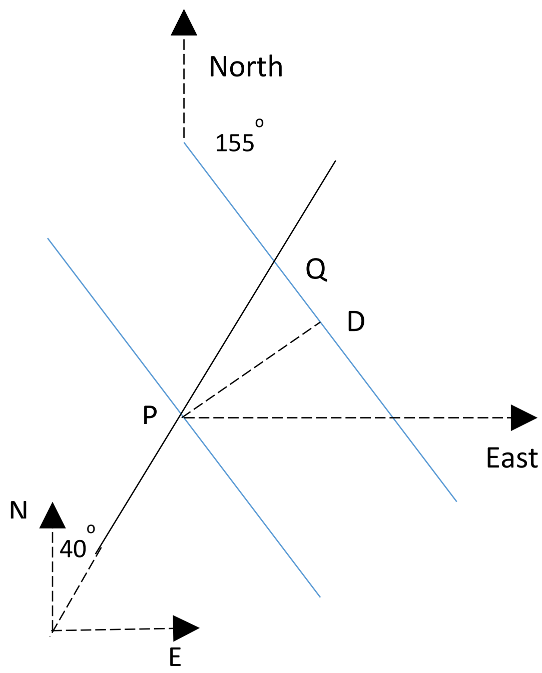
$= 50° - 25°$

$= 25°$

$\Rightarrow PD = PQ \cos 25°$

≈ 25.8

so the canal is 25.8 m wide.



(iii) $XH = \frac{1}{2}XY = 24.25$

$HR = (24.25)\sin 40°$

≈ 15.59

$RX = (24.25)\cos 40°$

≈ 18.58

$\tan\alpha = \dfrac{15.59}{40 + 18.58}$

≈ 0.266

$\Rightarrow \alpha \approx 14.9°$

so the bearing of H from the surveyor is 015°.