A-Level Mathematics

Bridging Course – Week 2

Surds and Indices

During the second week of this bridging course you will be reviewing your understanding and ability to apply knowledge and skills of simplifying expressions involving Indices and Surds.

You may wish to approach each section in one of two ways:

1. Systematically work through the topics;

- Read the notes and examples pages and watch video tutorials of any areas you need further instruction on, making your own notes on lined paper to file and keep.
- Complete as many of the questions in exercises 1 and 2 of the additional document as you need to feel confident with the concepts.
- Complete and mark the review test at the end of each section to assess your understanding.
- Repeat any sections as needed.

2. Assess before beginning;

- Complete and mark the end of section test.
- Review your work and identify areas of weakness.
- For these areas: watch the video tutorials and read the notes and examples pages, making your own notes on lined paper to file and keep.
- Complete as many of the questions in exercises 1 and 2 of the additional document as you need to help secure your understanding of that concept.
- Re-try the questions previously answered incorrectly from the end of section test.

For each section we have included "challenge" exercises, which you can attempt if you are confident with a topic and want to test your ability to apply skills to more complex problems.

Section A:

Surds

1. Notes and examples on the following 6 pages of this document.

2. Helpful video tutorials for this topic:

Hegarty Maths Video Numbers Multiplication, Division and Simplifying Surds: 113-117 Rationalising Denominators: 118-119
Corbett Maths Video Numbers Multiplication, Division and Simplifying Surds: 305, 306, 308 Rationalising Denominators: 307
TL Maths https://sites.google.com/site/tlmaths314/home/gcse-to-a-level-maths-bridging-the-gap This link has all bridging the gap videos on – for this section: • 1 video on surds towards the bottom

3. Pages 1 - 3 of the additional Surds and Indices booklet:
   - Exercises 1 and 2 provide opportunities to practise these skills.
   - Exercise 3 is a challenge exercise.
   - Answers for review are at the back of that booklet.

4. Additional sources of support:
   - CGP guide – "Head Start to A Level Maths"

5. Section test on page **10** of this booklet, after the notes and examples (worked solutions for review at the end of booklet).

Section 1: Surds

Notes and Examples

These notes contain subsections on

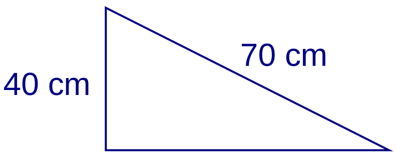
- **Rational and irrational numbers**
- **Writing a square root in terms of a simpler square root**
- **Adding and subtracting surds**
- **Multiplying surds**
- **Rationalising the denominator**

Remember, an **irrational number** is a number which cannot be expressed as one whole number divided by another whole number.

Rational and irrational numbers

The square root of any number which is not itself a perfect square is an **irrational number**. So $\sqrt{2}$ and $\sqrt{3}$ are irrational numbers, but $\sqrt{4}$ is not as it is equal to 2, which is a rational number. A number which is partly rational and partly square root (or cube root etc.) is called a **surd**. (There are of course other irrational numbers which do not involve a root, such as π.

In this section you will learn to manipulate and simplify expressions involving surds. This is an important skill in many areas of mathematics. For example, suppose you have a triangular paving slab like this:



You can use Pythagoras' theorem to work out that the length of the third side is $\sqrt{3300}$. You could use a calculator to work this out and give the answer to two or three decimal places, but this would no longer be exact. Suppose you wanted to find the area and perimeter of the slab, or the total area of 100 slabs, or find out how many slabs you could make from a certain volume of concrete? It is much better to use the exact answer in these calculations, and then the results will also be exact.

Writing a square root in terms of a simpler square root

Square roots like the one in the example above look quite daunting and can be difficult to work with. However, many square roots can be written in terms of a simpler square root like $\sqrt{2}$ or $\sqrt{3}$ (and the same applies to cube roots and so on). Example 1 shows how to do this.

OCR AS Maths Surds 1 Notes and Examples

Example 1

Write these numbers in terms of the simplest possible square roots.

(i) $\sqrt{12}$ (ii) $\sqrt{72}$ (iii) $\sqrt{150}$

Solution

(i) $12 = 4 \times 3$

$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

Look for any factors which are perfect squares

(ii) $72 = 36 \times 2$

$\sqrt{72} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$

(iii) $150 = 25 \times 6$

$\sqrt{150} = \sqrt{25} \times \sqrt{6} = 5\sqrt{6}$

Adding and subtracting surds

Adding and subtracting surds is rather like adding or subtracting algebraic expressions, in that you have to collect "like terms". You should collect together any rational numbers, and collect together any terms involving roots of the same number. You cannot collect together terms involving roots of different numbers, such as $\sqrt{2}$ and $\sqrt{3}$.

Example 2

Simplify

(i) $(2+\sqrt{2})+(3-2\sqrt{2})$

(ii) $(4-\sqrt{3})-(1-2\sqrt{2}+3\sqrt{3})$

(iii) $\sqrt{32}-\sqrt{18}$

Solution

(i) $(2+\sqrt{2})+(3-2\sqrt{2}) = 2+3+\sqrt{2}-2\sqrt{2}$

$= 5-\sqrt{2}$

(ii) $(4-\sqrt{3})-(1-2\sqrt{2}+3\sqrt{3}) = 4-\sqrt{3}-1+2\sqrt{2}-3\sqrt{3}$

$= 4-1-\sqrt{3}-3\sqrt{3}+2\sqrt{2}$

$= 3-4\sqrt{3}+2\sqrt{2}$

(iii) $\sqrt{32}-\sqrt{18}$

$\sqrt{32} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$

$\sqrt{18} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$

$\sqrt{32}-\sqrt{18} = 4\sqrt{2}-3\sqrt{2}$

$= \sqrt{2}$

At first sight this looks like it cannot be simplified. However, both surds can be written in terms of simpler surds (as in Example 1)

For further practice in examples like the ones above, use the ***Simplifying surds skill pack***.

Multiplying surds

Multiplying two or more square roots is quite simple – you just multiply the numbers. You may then be able to write the result as a simpler surd.

Example 3

Simplify

(i) $\sqrt{2}\times\sqrt{6}$

(ii) $2\sqrt{15}\times\sqrt{6}\times3\sqrt{10}$

Solution

(i) $$\begin{aligned}\sqrt{2}\times\sqrt{6} &= \sqrt{12}\\ &= \sqrt{4}\times\sqrt{3}\\ &= 2\sqrt{3}\end{aligned}$$

The approach used in (i) works well for small numbers, but for (ii) you would get the square root of a large number to simplify. An easier way for this example is to split each surd into simpler ones and then look for any pairs.

(ii) $$\begin{aligned}2\sqrt{15}\times\sqrt{6}\times3\sqrt{10} &= (2\times\sqrt{3}\times\sqrt{5})\times(\sqrt{2}\times\sqrt{3})\times(3\times\sqrt{5}\times\sqrt{2})\\ &= 2\times3\times\sqrt{3}\times\sqrt{3}\times\sqrt{5}\times\sqrt{5}\times\sqrt{2}\times\sqrt{2}\\ &= 6\times3\times5\times2\\ &= 180\end{aligned}$$

The next example deals with multiplying expressions involving a mixture of rational numbers and roots. You have to use brackets for this, and it is very similar to multiplying out two brackets in algebra – each term in the first bracket needs to be multiplied by each term in the second bracket. You can use FOIL (First, Outer, Inner, Last) if it helps you.

Example 4

Multiply out and simplify

(i) $(2+\sqrt{3})(1-2\sqrt{3})$

(ii) $(3-\sqrt{2})^2$

(iii) $(\sqrt{5}-2)(\sqrt{5}+2)$

Solution

(i) $$(2+\sqrt{3})(1-2\sqrt{3}) = 2-4\sqrt{3}+\sqrt{3}-2\sqrt{3}\times\sqrt{3}$$
$$= 2-3\sqrt{3}-6$$
$$= -4-3\sqrt{3}$$

(ii) $$(3-\sqrt{2})^2 = (3-\sqrt{2})(3-\sqrt{2})$$
$$= 9-3\sqrt{2}-3\sqrt{2}+\sqrt{2}\times\sqrt{2}$$
$$= 9-6\sqrt{2}+2$$
$$= 11-6\sqrt{2}$$

(iii) $$(\sqrt{5}-2)(\sqrt{5}+2) = \sqrt{5}\times\sqrt{5}+2\sqrt{5}-2\sqrt{5}-2\times 2$$
$$= 5-4$$
$$= 1$$

For further practice in examples like the one above, try the ***Multiplying surds skill pack***.

Part (iii) of Example 4 illustrates a very important and useful idea. Multiplying out any expression of the form $(a+b)(a-b)$ gives the result a^2-b^2. The "outer" and "inner" products, $-ab$ and ab, cancel each other out. When either or both of a and b are surds, the result a^2-b^2 is a rational number.

Rationalising the denominator

Surds in the denominator of a fraction can be a real nuisance! However, you can get rid of them from the denominator by a process called *rationalising the denominator*, which uses the idea above. If the denominator of a fraction is $a+b$, where either or both of a and b are surds, then you can multiply both top and bottom of the fraction by $a-b$. The denominator is then $(a+b)(a-b)$, which works out to be rational. *Since you multiply the top and bottom of the fraction by the same amount, its value is unchanged*. The numerator still involves surds, but this is not quite so difficult to work with.

This technique is shown in Example 5.

Example 5

Simplify the following by rationalising the denominator.

(i) $\dfrac{1}{\sqrt{3}}$ (ii) $\dfrac{1}{\sqrt{2}-1}$ (iii) $\dfrac{2+\sqrt{3}}{1-\sqrt{3}}$

Solution

(i) $\dfrac{1}{\sqrt{3}} = \dfrac{1}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{\sqrt{3}}{3}$

Multiply top and bottom by $\sqrt{3}$

(ii) $\dfrac{1}{\sqrt{2}-1} = \dfrac{1}{\sqrt{2}-1} \times \dfrac{\sqrt{2}+1}{\sqrt{2}+1}$

$= \dfrac{\sqrt{2}+1}{(\sqrt{2}-1)(\sqrt{2}+1)}$

$= \dfrac{\sqrt{2}+1}{2-1}$

$= \sqrt{2}+1$

Multiply top and bottom by $\sqrt{2}+1$

(iii) $\dfrac{2+\sqrt{3}}{1-\sqrt{3}} = \dfrac{2+\sqrt{3}}{1-\sqrt{3}} \times \dfrac{1+\sqrt{3}}{1+\sqrt{3}}$

$= \dfrac{(2+\sqrt{3})(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})}$

$= \dfrac{2+2\sqrt{3}+\sqrt{3}+3}{1-3}$

$= -\tfrac{1}{2}(5+3\sqrt{3})$

Multiply top and bottom by $1+\sqrt{3}$

For further practice in examples like the one above, try the ***Rationalising the denominator skill pack***.

Section 1: Surds

Section test

Do not use a calculator for this test.

1) Which of the following is a rational number?

(a) π (b) $\sqrt{48}$

(c) $\sqrt{3}$ (d) $\sqrt{36}$

2) Write $\sqrt{540}$ in terms of the simplest possible surd.

3) Which of the following is a correct simplification of $\sqrt{2}+1-2\sqrt{3}+4\sqrt{2}-3$

(a) $3\sqrt{2}-2$ (b) $5\sqrt{2}-2\sqrt{3}-2$

(c) $3\sqrt{7}-2$ (d) $\sqrt{7}$

4) Simplify $\sqrt{75}-\sqrt{27}$

5) Simplify $\sqrt{12}\times\sqrt{8}\times\sqrt{98}$

6) Multiply out $\left(2-\sqrt{3}\right)\left(1+2\sqrt{3}\right)$ and simplify as far as possible

7) Which of the following expression are equal to $\frac{\sqrt{20}}{\sqrt{5}+1}$? Choose as many as apply.

(a) $\frac{4\sqrt{5}}{\sqrt{5}+1}$ (b) $\frac{5+\sqrt{5}}{2}$

(c) $\frac{5-\sqrt{5}}{2}$ (d) $\frac{10}{5+\sqrt{5}}$

8) Write $\frac{2}{3\sqrt{2}}$ in the form $\frac{a}{b}\sqrt{2}$.

9) Write $\frac{1}{\sqrt{5}-2}$ in the form $\frac{a\sqrt{5}+b}{c}$

10) The expression $\frac{2+\sqrt{3}}{1+\sqrt{2}}$ is equivalent to

(a) 5 (b) $2\sqrt{2}+\sqrt{6}-\sqrt{3}-2$

(c) $2+\sqrt{3}-2\sqrt{2}-\sqrt{6}$ (d) $\frac{2+\sqrt{3}}{5}$

Section B:

Indices

1. Notes and examples on the following 5 pages of this document.

2. Helpful video tutorials for this topic:

Hegarty Maths Video Numbers Index Rules: 104-109, 174-175
Corbett Maths Video Numbers Index Rules: 173-175
TL Maths https://sites.google.com/site/tlmaths314/home/gcse-to-a-level-maths-bridging-the-gap This link has all bridging the gap videos on – for this section: • Indices (x2) – about halfway down the page

3. Pages 4 - 7 of the additional Surds and Indices booklet:
   a. Exercises 1 and 2 provide opportunities to practise these skills.
   b. Exercise 3 is a challenge exercise.
   c. Answers for review are at the back of that booklet.

4. Additional sources of support:
   - CGP guide – "Head Start to A Level Maths"

5. Section test on page **17** of this booklet, after the notes and examples (worked solutions for review at the end of booklet).

Section 2: Indices

Notes and Examples

These notes contain subsections on

- Multiplying expressions
- The rules of indices
- Negative indices
- Fractional indices
- More difficult examples

Multiplying expressions

The example below illustrates multiplying expressions involving indices.

Example 1
Simplify the expression $2xy \times 3yz^2 \times 4x^2z$.

Solution

$$\begin{aligned} 2xy \times 3yz^2 \times 4x^2z &= 2 \times 3 \times 4 \times x \times x \times x^2 \times y \times y \times z^2 \times z \\ &= 24x^3y^2z^3 \end{aligned}$$

You may be happy to do this in your head, without writing out the intermediate line of working.

When you are multiplying expressions like the ones in Example 1, you are using one of the rules of indices.

The rules of indices

Three rules of indices are:

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$\left(a^m\right)^n = a^{mn}$$

You can investigate these rules and see why they work by trying them out with simple cases, writing the sums out in full:
E.g., to demonstrate rule 3:

$$\begin{aligned}(2^3)^2 &= (2\times2\times2)^2\\ &= (2\times2\times2)\times(2\times2\times2)\\ &= 2\times2\times2\times2\times2\times2\\ &= 2^6\\ &= 2^{3\times2}\end{aligned}$$

Try some for yourself.

The number being raised to a power (a in this case) is called the **base**.
Note:
You can only apply these rules to numbers involving the same base.
So, for example, you cannot apply the rules of indices to $2^3\times3^5$.

Can you explain why?

Example 2
Simplify
(i) $2^4\times2^7$ (ii) $3^9\div3^4$ (iii) $(5^3)^6$ (iv) $2^3\times4^3$

Solution

(i) $2^4\times2^7 = 2^{4+7}$
$= 2^{11}$

using the first rule

(ii) $3^9\div3^4 = 3^{9-4}$
$= 3^5$

using the second rule

(iii) $(5^3)^6 = 5^{3\times6}$
$= 5^{18}$

using the third rule

(iv) $2^3\times4^3 = 2^3\times(2^2)^3$
$= 2^3\times2^6$
$= 2^{3+6}$
$= 2^9$

At first sight this looks as if it cannot be simplified, as the bases are different. However, 4 can be written as a power of 2.

You can see and practise some similar examples using the ***Laws of indices skill pack***.

Negative indices

There are two more rules, which follow from the three already introduced:

$$a^{-n} = \frac{1}{a^n}$$
$$a^0 = 1$$

> Again, it's worth experimenting with numbers to get a feel for how and why these rules work.
> e.g.
>
> $$2^{-1} = \tfrac{1}{2}$$
> $$\Rightarrow 2^1 \times 2^{-1} = 2 \times \tfrac{1}{2} = 1$$
>
> And from rule 3,
>
> $$2^1 \times 2^{-1} = 2^{1-1} = 2^0 = 1$$
>
> Try some for yourself.

Note that it might seem strange that $a^0 = 1$ for any value of a, but if this were not so, the other rules would be inconsistent. If you consider a graph of $y = a^x$, for different values of a, you will see that it is perfectly natural that $a^0 = 1$. Try this on your graphical calculator.

Example 3
Find, as fractions or whole numbers,
(i) 2^{-4} (ii) 5^{-2} (iii) 3^0

Solution
(i) $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

(ii) $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

(iii) $3^0 = 1$

Fractional indices

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

Although these are equivalent, it is usually easier to use the first form, working out the root first so that you are dealing with smaller numbers.

As before, try experimenting with numbers to get a feel for how an why these rules work.

Example 4

Find, as fractions or whole numbers,

(i) $8^{\frac{1}{3}}$ (ii) $9^{\frac{3}{2}}$ (iii) $25^{-\frac{1}{2}}$ (iv) $4^{-\frac{5}{2}}$

Solution

(i) $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

(ii) $9^{\frac{3}{2}} = (\sqrt{9})^3 = 3^3 = 27$

(iii) $25^{-\frac{1}{2}} = \dfrac{1}{25^{\frac{1}{2}}} = \dfrac{1}{\sqrt{25}} = \dfrac{1}{5}$

(iv) $4^{-\frac{5}{2}} = \dfrac{1}{4^{\frac{5}{2}}} = \dfrac{1}{(4^{\frac{1}{2}})^5} = \dfrac{1}{(\sqrt{4})^5} = \dfrac{1}{2^5} = \dfrac{1}{32}$

You can see and practise more examples like the ones above using the ***Negative, fractional and zero indices skill pack***. You can also look at the ***Indices video***.

More difficult examples

The next example shows how you can sometimes simplify quite complicated looking expressions involving different bases by splitting them up into their factors.

Example 5

Simplify $6^{5/2} \times \dfrac{1}{\sqrt{12}} \times 18^{-3/2}$

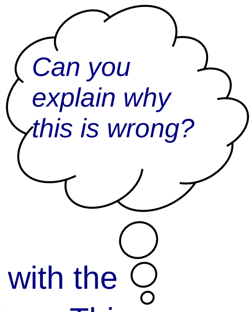
Solution

$6^{5/2} = (2 \times 3)^{5/2} = 2^{5/2} \times 3^{5/2}$

$\dfrac{1}{\sqrt{12}} = \dfrac{1}{\sqrt{4}} \times \dfrac{1}{\sqrt{3}} = \dfrac{1}{2} \times \dfrac{1}{3^{1/2}} = 2^{-1} \times 3^{-1/2}$

$$18^{-3/2} = 9^{-3/2} \times 2^{-3/2} = (9^{1/2})^{-3} \times 2^{-3/2} = 3^{-3} \times 2^{-3/2}$$

$$\begin{aligned} 6^{5/2} \times \frac{1}{\sqrt{12}} \times 18^{-3/2} &= 2^{5/2} \times 3^{5/2} \times 2^{-1} \times 3^{-1/2} \times 3^{-3} \times 2^{-3/2} \\ &= (2^{5/2} \times 2^{-1} \times 2^{-3/2}) \times (3^{5/2} \times 3^{-1/2} \times 3^{-3}) \\ &= 2^{0} \times 3^{-1} \\ &= 1 \times 3^{-1} \\ &= \frac{1}{3} \end{aligned}$$



A common mistake when dealing with indices is to try to add terms with the same base but a different index, such as $2^3 + 2^5$, by adding the indices. This is wrong, but you can sometimes simplify expressions like this by taking out a common factor. This is shown in the example below.

Example 6
Simplify $2^{5/2} + 2^{1/2}$.

Solution

$$\begin{aligned} 2^{5/2} + 2^{1/2} &= 2^{2} \times 2^{1/2} + 2^{1/2} \\ &= 4 \times 2^{1/2} + 2^{1/2} \\ &= 5 \times 2^{1/2} \\ &= 5\sqrt{2} \end{aligned}$$

OCR AS Mathematics Surds and indices

Section 2: Indices

Section test

Do not use a calculator in this test.

1) Write $3^4 \times 3^2$ in the form 3^a.

2) Write $5^{10} \div 5^2$ in the form 5^k.

3) Write $(2^4)^3$ in the form 2^b.

4) The expression $3a^2b \times (2ab^{-2})^3 \div 4ab^2$ can be simplified to give the expression

(a) $\dfrac{6a^4}{b^7}$

(b) $\dfrac{6a^4}{b^{5/2}}$

(c) $\dfrac{20a^4}{b^7}$

(d) $\dfrac{20a^4}{b^{5/2}}$

5) $3^{-4} =$

(a) $\dfrac{1}{81}$

(b) $-\dfrac{1}{81}$

(c) $\sqrt[4]{3}$

(d) $-\sqrt[4]{3}$

6) $16^{-1/4} =$

(a) $-\dfrac{1}{2}$

(b) $\dfrac{1}{2}$

(c) 2

(d) -2

7) $27^{2/3} =$

(a) $\dfrac{1}{18}$

(b) 18

(c) $\dfrac{1}{9}$

(d) 9

8) Evaluate $\left(\dfrac{4}{25}\right)^{-3/2}$

9) Write $8^3 \times 6^{1/2} \div 32^{3/2}$ in the form $a\sqrt{b}$.

10) Simplify $\dfrac{9^{1/3} \times 12^{-1/2}}{3^{1/6} \times 2^0}$

(a) $\dfrac{1}{4}$

(b) $\dfrac{1}{2}$

(c) 2

(d) $\sqrt{2}$

SOLUTIONS TO SECTION REVIEW TESTS

OCR AS Maths Surds 1 section test solutions

Solutions to section test

1) $\sqrt{36}$ is a rational number as it is equal to 6.
The other numbers are irrational numbers as they cannot be expressed in the form $\frac{p}{q}$ where p and q are integers.

2)
$$\begin{aligned}\sqrt{540} &= \sqrt{54\times 10}\\ &= \sqrt{6\times 9\times 5\times 2}\\ &= \sqrt{3\times 2\times 3^2\times 5\times 2}\\ &= \sqrt{3\times 2^2\times 3^2\times 5}\\ &= 2\times 3\sqrt{3\times 5}\\ &= 6\sqrt{15}\end{aligned}$$

3)
$$\begin{aligned}\sqrt{2}+1-2\sqrt{3}+4\sqrt{2}-3 &= \left(\sqrt{2}+4\sqrt{2}\right)-2\sqrt{3}+1-3\\ &= 5\sqrt{2}-2\sqrt{3}-2\end{aligned}$$

4)
$$\begin{aligned}\sqrt{75}-\sqrt{27} &= \sqrt{25\times 3}-\sqrt{9\times 3}\\ &= 5\sqrt{3}-3\sqrt{3}\\ &= 2\sqrt{3}\end{aligned}$$

5)
$$\begin{aligned}\sqrt{12}\times\sqrt{8}\times\sqrt{98} &= \sqrt{3\times 4}\times\sqrt{2\times 4}\times\sqrt{2\times 49}\\ &= 2\sqrt{3}\times 2\sqrt{2}\times 7\sqrt{2}\\ &= 28\times 2\sqrt{3}\\ &= 56\sqrt{3}\end{aligned}$$

6)
$$\begin{aligned}\left(2-\sqrt{3}\right)\left(1+2\sqrt{3}\right) &= 2+4\sqrt{3}-\sqrt{3}-2\sqrt{3}\sqrt{3}\\ &= 2+3\sqrt{3}-6\\ &= -4+3\sqrt{3}\end{aligned}$$

OCR AS Maths Surds 1 section test solutions

7) $\frac{\sqrt{20}}{\sqrt{5}+1} = \frac{\sqrt{4\times5}}{\sqrt{5}+1} = \frac{2\sqrt{5}}{\sqrt{5}+1}$

$\frac{2\sqrt{5}}{\sqrt{5}+1} = \frac{2\sqrt{5}}{\sqrt{5}+1} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10}{5+\sqrt{5}}$

$$\begin{aligned}\frac{2\sqrt{5}}{\sqrt{5}+1} &= \frac{2\sqrt{5}}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1} \\ &= \frac{2\sqrt{5}\left(\sqrt{5}-1\right)}{\left(\sqrt{5}+1\right)\left(\sqrt{5}-1\right)} \\ &= \frac{10-2\sqrt{5}}{5-\sqrt{5}+\sqrt{5}-1} \\ &= \frac{10-2\sqrt{5}}{4} \\ &= \frac{5-\sqrt{5}}{2}\end{aligned}$$

so $\frac{\sqrt{20}}{\sqrt{5}+1}$ is equal to $\frac{5-\sqrt{5}}{2}$ and $\frac{10}{5+\sqrt{5}}$.

8) $\frac{2}{3\sqrt{2}} = \frac{2}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{6} = \frac{\sqrt{2}}{3}$

9)
$$\begin{aligned}\frac{1}{\sqrt{5}-2} &= \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} \\ &= \frac{\sqrt{5}+2}{\left(\sqrt{5}-2\right)\left(\sqrt{5}+2\right)} \\ &= \frac{\sqrt{5}+2}{5+2\sqrt{5}-2\sqrt{5}-4} \\ &= \frac{\sqrt{5}+2}{1} = \sqrt{5}+2\end{aligned}$$

10)
$$\begin{aligned}\frac{2+\sqrt{3}}{1+\sqrt{2}} &= \frac{\left(2+\sqrt{3}\right)\left(1-\sqrt{2}\right)}{\left(1+\sqrt{2}\right)\left(1-\sqrt{2}\right)} \\ &= \frac{2-2\sqrt{2}+\sqrt{3}-\sqrt{6}}{1-\sqrt{2}+\sqrt{2}-2} \\ &= \frac{2-2\sqrt{2}+\sqrt{3}-\sqrt{6}}{-1} \\ &= 2\sqrt{2}-\sqrt{3}+\sqrt{6}-2\end{aligned}$$

OCR AS Maths Surds and indices 2 section test solutions

Solutions to section test

1) $3^4 \times 3^2 = 3^{4+2} = 3^6$

2) $5^{10} \div 5^2 = 5^{10-2} = 5^8$

3) $(2^4)^3 = 2^{4\times 3} = 2^{12}$

4) $$\begin{aligned} 3a^2b \times (2ab^{-2})^3 \div 4ab^2 &= \frac{3a^2b \times 8a^3b^{-6}}{4ab^2} \\ &= \frac{24a^5b^{-5}}{4ab^2} \\ &= 6a^4b^{-7} \\ &= \frac{6a^4}{b^7} \end{aligned}$$

5) $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

6) $16^{-1/4} = \frac{1}{\sqrt[4]{16}} = \frac{1}{2}$

7) $27^{2/3} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$

8) $\left(\frac{4}{25}\right)^{-3/2} = \left(\frac{25}{4}\right)^{3/2} = \left(\sqrt{\frac{25}{4}}\right)^3 = \left(\frac{5}{2}\right)^3 = \frac{125}{8}$

9) $$\begin{aligned} 8^3 \times 6^{1/2} \div 32^{3/2} &= (2^3)^3 \times (2\times 3)^{1/2} \div (2^5)^{3/2} \\ &= 2^9 \times 2^{1/2} \times 3^{1/2} \div 2^{15/2} \\ &= 2^{9+\frac{1}{2}-\frac{15}{2}} \times 3^{1/2} \\ &= 2^2 \times 3^{1/2} \\ &= 4\sqrt{3} \end{aligned}$$

OCR AS Maths Surds and indices 2 section test solutions

10) $$\frac{9^{1/3}\times 12^{-1/2}}{3^{1/6}\times 2^{0}}=\frac{\left(3^{2}\right)^{1/3}\times\left(3\times 2^{2}\right)^{-1/2}}{3^{1/6}\times 1}$$

$$=\frac{3^{2/3}\times 3^{-1/2}\times 2^{-1}}{3^{1/6}}$$

$$=3^{\frac{2}{3}-\frac{1}{2}-\frac{1}{6}}\times 2^{-1}$$

$$=3^{0}\times 2^{-1}$$

$$=1\times\tfrac{1}{2}=\tfrac{1}{2}$$