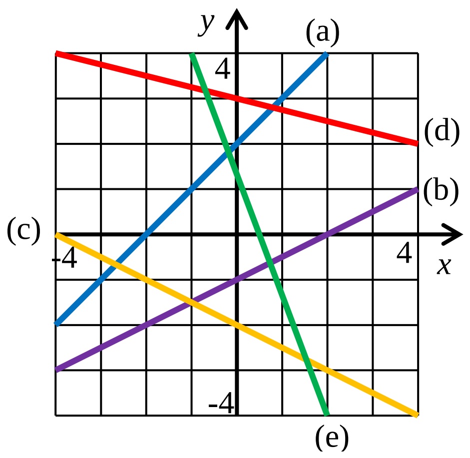
Section 1: Points and straight lines

Exercise level 1

1. (a) For the points A(3, 1) and B(7, 4) calculate
   (i) the gradient of AB
   (ii) the gradient of a line perpendicular to AB
   (iii) the midpoint of AB
   (iv) the distance AB
   (b) Repeat part (a) for the points A(-2, 9) and B(3, -1)

2. Sketch the following lines.
   (i) $y = x + 3$ (ii) $y = 2x - 1$ (iii) $x + y = 5$
   (iv) $4y = x + 12$ (v) $3y + x + 6 = 0$ (vi) $5y = 15 - 2x$

3. Find the equations of the lines (a)-(e) in the diagram below.



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4. A quadrilateral has vertices A(3, 5), B(9, 7), C(10, 4) and D(4, 2).
   Show that ABCD is a rectangle.

5. The following questions are about the coordinate geometry of the following points: A (2, 2), B (3, 1), C (4, 4), D (5, 5), E (6, -1), F (7, -3)
   (i) Find the lengths of the line segments AE and AB.
   (ii) Find the gradients of each of AB, AC, AE, DE, and CD.
   (iii) State which of the lines in part (ii) are parallel or perpendicular to each other.
   (iv) What is the angle between AC and BF?

Section 1: Points and straight lines

Exercise level 2

1. Given the points A(3, 1), B(6, y) and C(12, -2) find the value(s) of y for which
   (i) the line AB has gradient 2
   (ii) the distance AB is 5
   (iii) A, B and C are collinear
   (iv) AB is perpendicular to BC
   (v) the lengths AB and BC are equal

2. Find the equations of the following lines.
   (i) parallel to $y = 4x - 1$ and passing through (2, 3)
   (ii) perpendicular to $y = 2x + 7$ and passing through (1, 2)
   (iii) parallel to $3y + x = 10$ and passing through (4, -1)
   (iv) perpendicular to $3x + 4y = 12$ and passing through (-3, 0)
   (v) parallel to $x + 5y + 8 = 0$ and passing through (-1, -6)

3. Find the equation of the line AB in each of the following cases.
   (i) A(1, 6), B(3, 2)
   (ii) A(8, -1), B(-2, 3)
   (iii) A(-5, 2), B(7, -4)
   (iv) A(-3, -5), B(5, 1)

4. The point E is (2, -1), F is (1, 3), G is (3, 5) and H is (4, 1).
   Show, by calculation that EFGH is a parallelogram.
   Is EFGH also a rhombus? Explain your answer.

5. P is the point (2, 1), Q is (6, 9) and R is (10, 2).
   (i) Sketch the triangle PQR.
   (ii) Prove that triangle PQR is isosceles.
   (iii) Work out the area of triangle PQR.

6. Three points are A (-1, 5), B (1, 0), and C (11, 4).
   (i) Find the gradient of BA.
   (ii) Find the gradient of BC, and show that BA is perpendicular to BC.
   (iii) Find the equation of the line through C, parallel to BA.
   (iv) Find the equation of the line through A, parallel to BC.
   (v) Find the coordinates of point D, the remaining vertex of the rectangle ABCD.

Section 1: Points and straight lines

Exercise level 3 (Extension)

1. A triangle has vertices E(2, 5), F(4, 1) and G(-2, -3).
   (i) Find the midpoint of each side and hence find the equations of the three medians.
   (Medians are the lines from the midpoint of each side to the opposite vertex).
   (ii) Show that the point $\left(\frac{4}{3}, 1\right)$ lies on each median.

2. The sides of a triangle are formed by parts of the lines $y+3x=11$, $3y=x+3$ and $7y+x=37$.
   (i) Find the coordinates of the vertices of the triangle.
   (ii) Show that the triangle is right-angled.
   (iii) Work out the area of the triangle.

3. ABCD is a parallelogram. The equation of AB is $y=4x-3$ and the equation of BC is $y=2x+1$.
   (i) Find the coordinates of B.
   (ii) The coordinates of A are (3, 9). Find the equation of AD.
   (iii) The coordinates of C are (7, 15). Find the equation of CD.
   (iv) Find the coordinates of D.

4. The perpendicular bisector of AB, where A is (4, 2) and B is (10, 12), crosses the axes at points P and Q. Find the area of triangle OPQ.

5. Point A is (3, 1) and B is (8, 4). A line passes through B perpendicular to AB, and meets the axes at points P and Q. A second line through A perpendicular to AB meets the axes at R and S. Find the area of PQRS. What shape is it?

6. Point A is (5, 2), B is (1, 5), and C is (6, 6). Point D lies on AB, with CD perpendicular to AB. Find the coordinates of D.

7. Point A is (4, 5), B is (2, 1), C is (7, 1), and D is (-1, 5).
   (i) Find the midpoint of AB and CD.
   (ii) Find the gradients of AB and CD.
   (iii) What shape is the figure ACBD?
   (iv) Find the area of figure ACBD.

Section 2: Circles

Exercise level 1

1. Find, in the form $x^2 + y^2 + px + qy = c$, the equation for each of the following circles.
   (i) centre (0, 0), radius 6
   (ii) centre (3, 1), radius 5
   (iii) centre (-2, 5), radius 1
   (iv) centre (0, -4), radius 3

2. For each of these circles, write down the coordinates of the centre and the radius.
   (i) $x^2 + y^2 = 100$
   (ii) $(x-2)^2 + (y-7)^2 = 16$
   (iii) $(x+3)^2 + (y-4)^2 = 4$
   (iv) $(x+4)^2 + (y+5)^2 = 20$

3. For each of these circles, find the coordinates of the centre and the radius.
   (i) $x^2 + y^2 + 4x - 5 = 0$
   (ii) $x^2 + y^2 - 6x + 10y + 20 = 0$
   (iii) $x^2 + y^2 - 2x - 3y + 3 = 0$

4. The point C is (4, -2) and the point A is (6, 3).
   Find the equation of the circle centre C and radius CA.

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5. The points A (2, 0) and B (6, 4) form the diameter of a circle. Find the equation of the circle.

6. From the following equations, which represent the Cartesian equation of a circle? For each circle, find the coordinates of the centre, and find the radius.
   (i) $x^2 + y^2 - 4x + 6y = 51$
   (ii) $x^2 + 2y^2 - 3x = 11$
   (iii) $4x^2 + 4y^2 = 65$
   (iv) $8x^2 + 8y^2 - 48x - 16y = -104$

Section 2: Circles

Exercise level 2

1. (i) Show that the line $y = 4 - x$ is a tangent to the circle $x^2 + y^2 = 8$.
   (ii) Show that the line $4y = 3x - 25$ is a tangent to the circle $x^2 + y^2 = 25$.

2. A circle passes through the points Q(0, 3) and R(0, 9) and touches the x-axis. Work out two possible equations for the circle.

3. The line $2y + x = 10$ meets the circle $x^2 + y^2 = 65$ at P and Q. Calculate the length of PQ.

4. The line $y = x + 1$ does not intersect the circle $(x-1)^2 + (y+2)^2 = k$. Find the possible values for k.

5. The points P (-2, 6), Q (6, 0) and R (5, 7) all lie on a circle.
   (i) Show that PR is perpendicular to QR.
   (ii) Explain why the result from (i) shows that PQ is a diameter of the circle.
   (iii) Hence calculate the equation of the circle.

6. (i) Write down the equation of the circle centre (0, 0) and radius $\sqrt{17}$.
   (ii) Show that the point P(-4, -1) lies on the circle.
   (iii) Find the equation of the tangent at P.
   (iv) The line $x + y = 3$ meets the circle at two points, Q and R.
   Find the coordinates of Q and R.
   (v) Find the coordinates of the point, S, where the tangent at P intersects the line $x + y = 3$.

Section 2: Circles

Exercise level 3 (Extension)

1. Find k so that point P $\left(3, \sqrt{27}\right)$ lies on the circle $x^2 + y^2 = k^2$. If P, Q, and R lie on the circle, and triangle PQR is equilateral, write down the coordinates of the two vertices Q and R.

2. (i) P is point (2, 1) and Q is (10, 5). Find the midpoint M of PQ, and hence write down the equation of the circle with PQ as diameter.
   (ii) Line L_1 has equation $y = 3x - 15$. Find the points U, V where line L_1 intersects the circle. What is the angle PUQ?
   (iii) Line L_2 has equation $y + 2x = 5$. Point R lies on line L_2. Find angle RPQ.

3. A set of circles all pass through the points P (1, -3) and Q (5, 7). Show that all their centres lie on a straight line, and find its equation.

4. A gardener is planning an exhibition garden based on a design made up of circles and straight lines. She decides to create a plan, using coordinate geometry, where each unit on her graph represents a distance of 1 metre.
   (i) Write down the equation of a circle centre C (5, 0), with radius 5.
   (ii) On her plan, she draws two straight paths from point P (20, 0) to points Q and R on the circle. Point Q has coordinates (a, b). If she draws PQ so that CQ and PQ are at right angles, what is the length of the path PQ?
   (iii) Find the gradients of the lines CQ and QP in terms of a and b, and hence find the position of Q, and then R.
   (iv) Write down the shape of PQCR, and find its area.

Section 1: Points and straight lines

Solutions to Exercise level 1

1. (a) (i) Gradient of AB $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{1-4}{3-7} = \dfrac{-3}{-4} = \dfrac{3}{4}$.

(ii) Gradient m of perpendicular line satisfies $m \times \frac{3}{4} = -1$

so gradient of perpendicular line $= -\dfrac{4}{3}$.

(iii) Midpoint of AB $= \left(\dfrac{3+7}{2}, \dfrac{1+4}{2}\right) = (5, 2.5)$

(iv) Distance AB $= \sqrt{(3-7)^2 + (1-4)^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

$= 5$

(b) (i) Gradient of AB $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{9-(-1)}{-2-3} = \dfrac{10}{-5} = -2$.

(ii) Gradient m of perpendicular line satisfies $m \times -2 = -1$

so gradient of perpendicular line $= \dfrac{1}{2}$.

(iii) Midpoint of AB $= \left(\dfrac{-2+3}{2}, \dfrac{9+(-1)}{2}\right) = (0.5, 4)$

(iv) Distance AB $= \sqrt{(-2-3)^2 + (9-(-1))^2}$

$= \sqrt{25+100}$

$= \sqrt{125}$

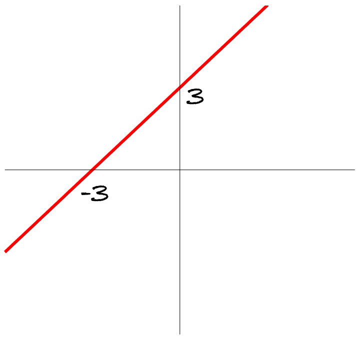
$= 5\sqrt{5}$

2. (i) $y = x + 3$

Gradient = 1

When $x = 0$, $y = 3$

When $y = 0$, $x + 3 = 0 \Rightarrow x = -3$

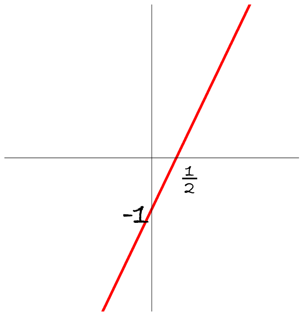


(ii) $y = 2x - 1$

Gradient = 2

When $x = 0$, $y = -1$

When $y = 0$, $2x - 1 = 0 \Rightarrow x = \frac{1}{2}$



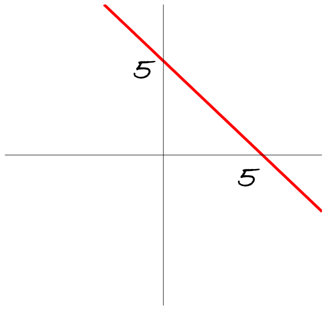
(iii) $x + y = 5$

$y = -x + 5$

Gradient = -1

When $x = 0$, $y = 5$

When $y = 0$, $x = 5$



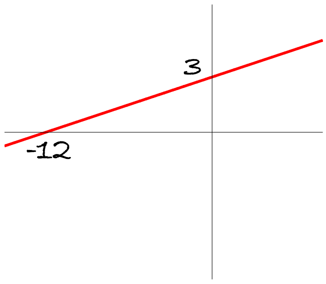
(iv) $4y = x + 12$

$y = \frac{1}{4}x + 3$

Gradient $= \frac{1}{4}$

When $x = 0$, $y = 3$

When $y = 0$, $x + 12 = 0 \Rightarrow x = -12$



(v) $3y + x + 6 = 0$

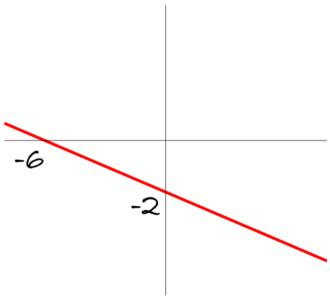
$3y = -x - 6$

$y = -\frac{1}{3}x - 2$

Gradient $= -\frac{1}{3}$

When $x = 0$, $y = -2$

When $y = 0$, $x + 6 = 0 \Rightarrow x = -6$



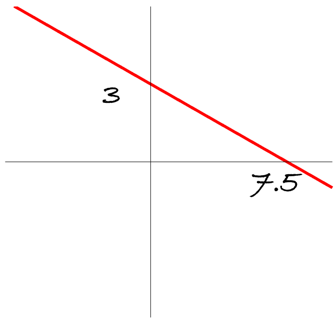
(vi) $5y = 15 - 2x$

$y = 3 - \frac{2}{5}x$

Gradient $= -\frac{2}{5}$

When $x = 0$, $5y = 15 \Rightarrow y = 3$

When $y = 0$, $15 - 2x = 0 \Rightarrow x = 7.5$



3. (a) Gradient = 1, y-intercept = 2

Equation of line is $y = x + 2$

(b) Gradient = $\frac{1}{2}$, y-intercept = -1

Equation of line is $y = \frac{1}{2}x - 1$

or $2y = x - 2$

(c) Gradient $= -\frac{1}{2}$, y-intercept = -2
Equation of line is $y = -\frac{1}{2}x - 2$
or $2y + x + 4 = 0$

(d) Gradient $= -\frac{1}{4}$, y- intercept = 3
Equation of line is $y = -\frac{1}{4}x + 3$
or $4y + x = 12$

(e) Gradient $= -\frac{8}{3}$, passes through (-1, 4)
Equation of line is $y - 4 = -\frac{8}{3}(x - (-1))$

$$3(y-4) = -8(x+1)$$
$$3y - 12 = -8x - 8$$
$$3y + 8x = 4$$

4. Gradient of AB $= \dfrac{7-5}{9-3} = \dfrac{2}{6} = \dfrac{1}{3}$

Gradient of BC $= \dfrac{4-7}{10-9} = \dfrac{-3}{1} = -3$

Gradient of CD $= \dfrac{2-4}{4-10} = \dfrac{-2}{-6} = \dfrac{1}{3}$

Gradient of AD $= \dfrac{2-5}{4-3} = \dfrac{-3}{1} = -3$

AB and CD are parallel, and BC and AD are parallel.
The gradients of AB and CD are $\frac{1}{3}$,
and the gradients of BC and AD are -3.
Since $\frac{1}{3} \times -3 = -1$, AB and CD are perpendicular to BC and AD
so ABCD is a rectangle.

5. (i) $|AE| = \sqrt{4^2 + 3^2} = 5$

$|AB| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

(ii) gradient AB $= \frac{-1}{1} = -1$ gradient AC $= \frac{2}{2} = 1$
gradient AE $= \frac{-3}{4} = -\frac{3}{4}$ gradient DE $= \frac{-6}{1} = -6$
gradient CD $= \frac{1}{1} = 1$

(iii) AC and CD are parallel to each other
AB is perpendicular to AC and to CD

(iv) gradient BF $= \frac{-4}{4} = -1$
so the angle between AC and BF is 90°.

Section 1: Points and straight lines

Solutions to Exercise level 2

1. (i) Gradient of AB $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{1-y}{3-6} = \dfrac{1-y}{-3}$

Gradient of AB $= 2 \Rightarrow \dfrac{1-y}{-3} = 2$

$\Rightarrow 1 - y = -6$

$y = 7$

(ii) Distance AB is 5

$\sqrt{(3-6)^2 + (1-y)^2} = 5$

$9 + (1-y)^2 = 25$

$(1-y)^2 = 16$

$1 - y = \pm 4$

$y = 1 - 4$ or $1 + 4$

$y = -3$ or 5

(iii) If A, B and C are collinear, gradient of AB = gradient of AC.

Gradient of AC $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{1-(-2)}{3-12} = \dfrac{3}{-9} = -\dfrac{1}{3}$

From (i), gradient of AB $= \dfrac{1-y}{-3}$

$\dfrac{1-y}{-3} = -\dfrac{1}{3}$

$1 - y = 1$

$y = 0$

(iv) If AB is perpendicular to BC, then grad AB × grad BC = -1

From (i), gradient of AB $= \dfrac{1-y}{-3}$

Gradient of BC $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{y-(-2)}{6-12} = \dfrac{y+2}{-6}$

$$\frac{1-y}{-3}\times\frac{y+2}{-6}=-1$$
$$(1-y)(y+2)=-18$$
$$2-y-y^2=-18$$
$$y^2+y-20=0$$
$$(y+5)(y-4)=0$$
$y=-5$ or $y=4$

(v) Length AB = length BC

$$\sqrt{(3-6)^2+(1-y)^2}=\sqrt{(6-12)^2+(y-(-2))^2}$$
$$9+(1-y)^2=36+(y+2)^2$$
$$1-2y+y^2=27+y^2+4y+4$$
$$0=6y+30$$
$$y=-5$$

2\. (i) Gradient of $y=4x-1$ is 4

Gradient of parallel line = 4

Equation of line is $y-3=4(x-2)$

$$y-3=4x-8$$
$$y=4x-5$$

(ii) Gradient of $y=2x+7$ is 2

Gradient of perpendicular line is $-\frac{1}{2}$

Equation of line is $y-2=-\frac{1}{2}(x-1)$

$$2(y-2)=-(x-1)$$
$$2y-4=-x+1$$
$$2y+x=5$$

(iii) $3y+x=10\Rightarrow y=-\frac{1}{3}x+\frac{10}{3}$

Gradient is $-\frac{1}{3}$

Gradient of parallel line is $-\frac{1}{3}$

Equation of line is $y-(-1)=-\frac{1}{3}(x-4)$

$$3(y+1)=-(x-4)$$
$$3y+3=-x+4$$
$$3y+x=1$$

(iv) $3x+4y=12\Rightarrow y=-\frac{3}{4}x+3$

Gradient is $-\frac{3}{4}$

Gradient of perpendicular line is $\frac{4}{3}$

Equation of line is $y - 0 = \frac{4}{3}(x - (-3))$

$$3y = 4(x+3)$$
$$3y = 4x + 12$$

(v) $x + 5y + 8 = 0 \Rightarrow y = -\frac{1}{5}x - \frac{8}{5}$

Gradient is $-\frac{1}{5}$

Gradient of parallel line is $-\frac{1}{5}$

Equation of line is $y - (-6) = -\frac{1}{5}(x - (-1))$

$$5(y+6) = -(x+1)$$
$$5y + 30 = -x - 1$$
$$5y + x + 31 = 0$$

3. (i) Gradient of AB $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{6-2}{1-3} = \dfrac{4}{-2} = -2$

Equation of AB is $y - 6 = -2(x - 1)$

$$y - 6 = -2x + 2$$
$$y + 2x = 8$$

(ii) Gradient of AB $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{-1-3}{8-(-2)} = \dfrac{-4}{10} = -\dfrac{2}{5}$

Equation of AB is $y - (-1) = -\frac{2}{5}(x - 8)$

$$5(y+1) = -2(x-8)$$
$$5y + 5 = -2x + 16$$
$$5y + 2x = 11$$

(iii) Gradient of AB $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{2-(-4)}{-5-7} = \dfrac{6}{-12} = -\dfrac{1}{2}$

Equation of AB is $y - 2 = -\frac{1}{2}(x - (-5))$

$$2(y-2) = -(x+5)$$
$$2y - 4 = -x - 5$$
$$2y + x + 1 = 0$$

(iv) Gradient of AB $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{-5-1}{-3-5} = \dfrac{-6}{-8} = \dfrac{3}{4}$

Equation of AB is $y - (-5) = \frac{3}{4}(x - (-3))$

$$4(y+5) = 3(x+3)$$
$$4y + 20 = 3x + 9$$
$$4y = 3x - 11$$

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4. Gradient of EF $= \frac{3-(-1)}{1-2} = \frac{4}{-1} = -4$

Gradient of FG $= \frac{5-3}{3-1} = \frac{2}{2} = 1$

Gradient of GH $= \frac{1-5}{4-3} = \frac{-4}{1} = -4$

Gradient of EH $= \frac{1-(-1)}{4-2} = \frac{2}{2} = 1$

EF is parallel to GH and FG is parallel to EH
so EFGH is a parallelogram.

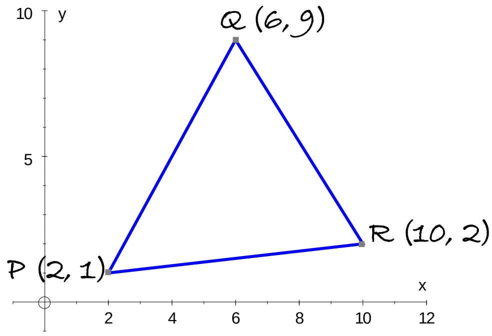
If EFGH were a rhombus, all the sides would be equal.

$EF^2 = (2-1)^2 + (-1-3)^2 = 1^2 + (-4)^2 = 17$

$FG^2 = (1-3)^2 + (3-5)^2 = (-2)^2 + (-2)^2 = 8$

The lengths of EF and FG are not equal, so EFGH is not a rhombus.

5. (i)



(ii) $PQ = \sqrt{(6-2)^2 + (9-1)^2} = \sqrt{16+64} = \sqrt{80}$

$PR = \sqrt{(10-2)^2 + (2-1)^2} = \sqrt{64+1} = \sqrt{65}$

$QR = \sqrt{(10-6)^2 + (2-9)^2} = \sqrt{16+49} = \sqrt{65}$

Since PR and QR are the same length, the triangle is isosceles.

(iii) Take the base of the triangle as PQ.
Let M be the midpoint of PQ

$M = \left(\frac{2+6}{2}, \frac{1+9}{2}\right) = (4, 5)$

Height of triangle is $MR = \sqrt{(10-4)^2 + (2-5)^2} = \sqrt{36+9} = \sqrt{45}$

$$\text{Area of triangle} = \tfrac{1}{2} \times PQ \times MR$$
$$= \tfrac{1}{2}\sqrt{80}\sqrt{45}$$
$$= \tfrac{1}{2}\sqrt{16\times5}\sqrt{9\times5}$$
$$= \tfrac{1}{2}\times4\sqrt{5}\times3\sqrt{5}$$
$$= 6\times5$$
$$= 30$$

6. (i) gradient BA $= \dfrac{5-0}{(-1)-1} = -\dfrac{5}{2}$

(ii) gradient BC $= \dfrac{4-0}{11-1} = \dfrac{2}{5}$

gradient BA × gradient BC $= -\dfrac{5}{2}\times\dfrac{2}{5} = -1$, so BA and BC are perpendicular to each other.

(iii) $y - 4 = -\tfrac{5}{2}(x - 11)$

$\Rightarrow 2y + 5x = 63$

(iv) $y - 5 = \tfrac{2}{5}(x + 1)$

$\Rightarrow 5y - 2x = 27$

(v) $\left.\begin{array}{ll} 2y + 5x = 63 & (1) \\ 5y - 2x = 27 & (2) \end{array}\right\}$

mult (1) x 5 $\quad 10y + 25x = 315$

mult (2) x 2 $\quad 10y - 4x = 54$

subtracting $\quad 29x = 261$

$\Rightarrow x = 9, y = 9$

so D is the point $(9, 9)$

Section 1: Points and straight lines

Solutions to Exercise level 3 (Extension)

1. (i) Midpoint of EF $= \left(\frac{2+4}{2}, \frac{5+1}{2}\right) = (3, 3)$

Midpoint of FG $= \left(\frac{4+(-2)}{2}, \frac{1+(-3)}{2}\right) = (1, -1)$

Midpoint of EG $= \left(\frac{2+(-2)}{2}, \frac{5+(-3)}{2}\right) = (0, 1)$

Median from midpoint of EF (3, 3) to G (-2, -3)

Gradient of median $= \frac{-3-3}{-2-3} = \frac{-6}{-5} = \frac{6}{5}$

Equation of median is $y - 3 = \frac{6}{5}(x-3)$

$5(y-3) = 6(x-3)$

$5y - 15 = 6x - 18$

$5y = 6x - 3$

Median from midpoint of FG (1, -1) to E (2, 5)

Gradient of median $= \frac{5-(-1)}{2-1} = \frac{6}{1} = 6$

Equation of median is $y - (-1) = 6(x-1)$

$y + 1 = 6x - 6$

$y = 6x - 7$

Median from midpoint of EG (0, 1) to F (4, 1)

Gradient of median $= \frac{1-1}{4-0} = \frac{0}{4} = 0$

Equation of median is $y = 1$

(ii) Equation of first median is $5y = 6x - 3$

Substituting $x = \frac{4}{3}$ gives $5y = 6 \times \frac{4}{3} - 3 = 8 - 3 = 5$

$y = 1$

so $\left(\frac{4}{3}, 1\right)$ lies on the median.

Equation of second median is $y = 6x - 7$

Substituting $x = \frac{4}{3}$ gives $y = 6 \times \frac{4}{3} - 7 = 8 - 7 = 1$

so $\left(\frac{4}{3}, 1\right)$ lies on the median.

Equation of third median is $y = 1$, so $\left(\frac{4}{3}, 1\right)$ lies on the median.

OCR AS Maths Coordinate geometry 1 Exercise solutions

2. (i) Let the triangle be ABC.

Let A be the intersection point of $y+3x=11$ and $3y=x+3$.

$y+3x=11 \quad \Rightarrow y=11-3x$

Substituting into $3y=x+3$ gives $3(11-3x)=x+3$

$33-9x=x+3$

$30=10x$

$x=3$

When $x=3$, $y=11-3\times3=2$

The coordinates of A are (3, 2).

Let B be the intersection point of $3y=x+3$ and $7y+x=37$

$3y=x+3 \quad \Rightarrow x=3y-3$

Substituting into $7y+x=37$ gives $7y+3y-3=37$

$10y=40$

$y=4$

When $y=4$, $x=3\times4-3=9$

The coordinates of B are (9, 4).

Let C be the intersection point of $7y+x=37$ and $y+3x=11$

$y+3x=11 \quad \Rightarrow y=11-3x$

Substituting into $7y+x=37$ gives $7(11-3x)+x=37$

$77-21x+x=37$

$40=20x$

$x=2$

When $x=2$, $y=11-3\times2=5$

The coordinates of C are (2, 5).

The vertices of the triangle are (3, 2), (9, 4) and (2, 5).

(ii) AB is the line $y+3x=11 \quad \Rightarrow y=11-3x$

so the gradient of AB is -3.

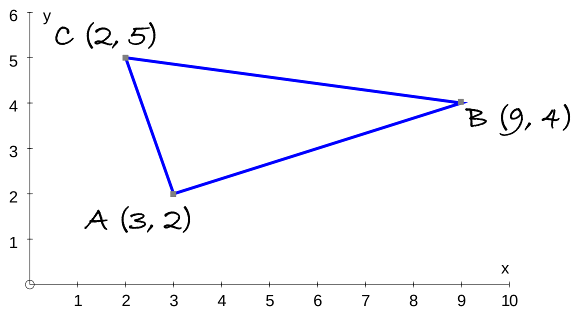
BC is the line $3y=x+3 \quad \Rightarrow y=\frac{1}{3}x+1$

so the gradient of BC is $\frac{1}{3}$.

Gradient of AB × gradient of AC $=-3\times\frac{1}{3}=-1$

so AB and AC are perpendicular, and therefore the triangle is right-angled.

(iii)



OCR AS Maths Coordinate geometry 1 Exercise solutions

$$AB = \sqrt{(3-9)^2 + (2-4)^2} = \sqrt{36+4} = \sqrt{40}$$

$$AC = \sqrt{(3-2)^2 + (2-5)^2} = \sqrt{1+9} = \sqrt{10}$$

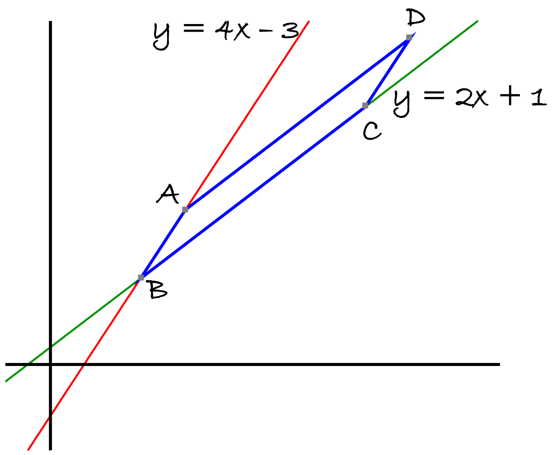
$$\begin{aligned}\text{Area of triangle } &= \tfrac{1}{2} \times AB \times AC \\ &= \tfrac{1}{2}\sqrt{40}\sqrt{10} \\ &= \tfrac{1}{2}\sqrt{4}\sqrt{10}\sqrt{10} \\ &= \tfrac{1}{2} \times 2 \times 10 \\ &= 10\end{aligned}$$

3. (i) B is the intersection point of $y = 4x - 3$ and $y = 2x + 1$.

$$4x - 3 = 2x + 1$$
$$2x = 4$$
$$x = 2$$

When $x = 2$, $y = 4 \times 2 - 3 = 5$

The coordinates of B are (2, 5).



(ii) AD is parallel to BC, so AD has gradient 2.
AD passes through the point (3, 9).
Equation of AD is $y - 9 = 2(x - 3)$

$$y - 9 = 2x - 6$$
$$y = 2x + 3$$

(iii) CD is parallel to AB, so CD has gradient 4.
CD passes through the point (7, 15).
Equation of CD is $y - 15 = 4(x - 7)$

$$y - 15 = 4x - 28$$
$$y = 4x - 13$$

(iv) D is the intersection point of AD and CD.

$$2x + 3 = 4x - 13$$
$$16 = 2x$$
$$x = 8$$

When $x = 8$, $y = 2 \times 8 + 3 = 19$

The coordinates of D are (8, 19).

4. Midpoint of AB $= \left(\dfrac{4+10}{2}, \dfrac{2+12}{2}\right) = (7, 7)$

Gradient of AB $= \dfrac{10}{6} = \dfrac{5}{3}$

Gradient of perpendicular to AB $= -\dfrac{3}{5}$

OCR AS Maths Coordinate geometry 1 Exercise solutions

Equation of perpendicular bisector is

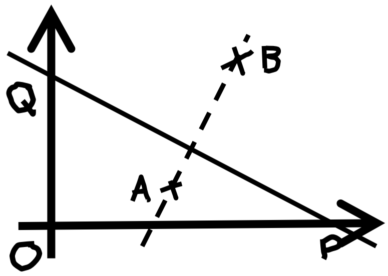
$$y-7=-\tfrac{3}{5}(x-7)$$

$$\Rightarrow 5y+3x=56$$

so P $=(\tfrac{56}{3},0)$ and Q $(0,\tfrac{56}{5})$

Area of OPQ $=\tfrac{1}{2}(\tfrac{56}{3})(\tfrac{56}{5})$

$$=104\tfrac{8}{15}\quad(\approx 104.53)$$



5. gradient AB $=\tfrac{3}{5}$

Equation of first line is $y-4=-\tfrac{5}{3}(x-8)$

$$\Rightarrow 3y=-5x+52$$

so P $=(\tfrac{52}{5},0)$ and Q $=(0,\tfrac{52}{3})$

Equation of second line is $y-1=-\tfrac{5}{3}(x-3)$

$$\Rightarrow 3y=-5x+18$$

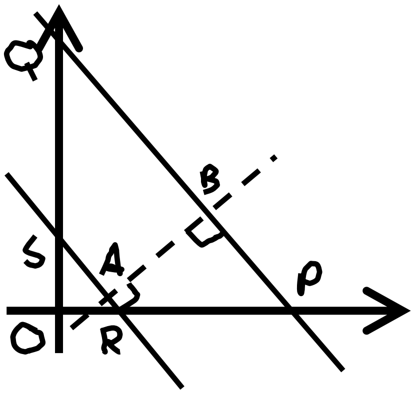
so R $=(\tfrac{18}{5},0)$ and S $=(0,6)$

So area PQSR = area of OPQ – area of ORS

$$=\tfrac{1}{2}(\tfrac{52}{5})(\tfrac{52}{3})-\tfrac{1}{2}(\tfrac{18}{5})(6)$$

$$=79\tfrac{1}{3}\quad(\approx \tfrac{238}{3})$$

The shape is a trapezium (since PQ and RS are parallel)



6. gradient AB $=-\tfrac{3}{4}$

$\Rightarrow$ eqtn of AB is $y-2=-\tfrac{3}{4}(x-5)$

$$\Rightarrow 4y=-3x+23\quad(1)$$

so gradient CD $=\tfrac{4}{3}$

$\Rightarrow$ eqtn of CD is $y-6=\tfrac{4}{3}(x-6)$

$$\Rightarrow 3y=4x-6\quad(2)$$

(1) x 4 $\Rightarrow 12y=16x-24$

(2) x 3 $\Rightarrow 12y=-9x+69$

subtracting $\Rightarrow 0=25x-93$

$$\Rightarrow x=\tfrac{93}{25},\ y=\tfrac{74}{25}\ \text{at point D}$$

7. (i) Midpoint of AB $=\left(\dfrac{4+2}{2},\dfrac{5+1}{2}\right)=(3,3)$

Midpoint of CD $=\left(\dfrac{7+-1}{2},\dfrac{1+5}{2}\right)=(3,3)$

(ii) Gradient of AB $=\dfrac{5-1}{4-2}=2$

$$\text{Gradient of CD} = \frac{1-5}{7-(-1)} = -\frac{1}{2}$$

(iii) AB and CD cross at right-angles at their midpoints, so ACBD is a rhombus.

(iv) $\text{Length AB} = \sqrt{(4-2)^2 + (5-1)^2} = \sqrt{4+16} = \sqrt{20}$

$\text{Length CD} = \sqrt{(7-(-1))^2 + (1-5)^2} = \sqrt{64+16} = \sqrt{80}$

$\text{Area} = \frac{1}{2}\sqrt{20}\sqrt{80} = 20$

Section 2: Circles

Solutions to Exercise level 1

1. (i) $(x-0)^2+(y-0)^2=6^2$

$x^2+y^2=36$

(ii) $(x-3)^2+(y-1)^2=5^2$

$x^2-6x+9+y^2-2y+1=25$

$x^2+y^2-6x-2y=15$

(iii) $(x+2)^2+(y-5)^2=1^2$

$x^2+4x+4+y^2-10y+25=1$

$x^2+y^2+4x-10y=-28$

(iv) $(x-0)^2+(y+4)^2=3^2$

$x^2+y^2+8y+16=9$

$x^2+y^2+8y=-7$

2. (i) $x^2+y^2=100=10^2$

Centre = (0, 0), radius = 10.

(ii) $(x-2)^2+(y-7)^2=16=4^2$

Centre = (2, 7), radius = 4

(iii) $(x+3)^2+(y-4)^2=4=2^2$

Centre = (-3, 4), radius = 2

(iv) $(x+4)^2+(y+5)^2=20$

Centre = (-4, -5), radius = $\sqrt{20}$

3. (i) $x^2+y^2+4x-5=0$

$x^2+4x+y^2-5=0$

$(x+2)^2-4+y^2-5=0$

$(x+2)^2+y^2=9$

Centre = (-2, 0), radius = 3.

OCR AS Maths Coordinate geometry 2 Exercise solutions

(ii) $x^2 + y^2 - 6x + 10y + 20 = 0$

$x^2 - 6x + y^2 + 10y + 20 = 0$

$(x-3)^2 - 9 + (y+5)^2 - 25 + 20 = 0$

$(x-3)^2 + (y+5)^2 = 14$

Centre is (3, -5) and radius $= \sqrt{14}$

(iii) $x^2 + y^2 - 2x - 3y + 3 = 0$

$x^2 - 2x + y^2 - 3y + 3 = 0$

$(x-1)^2 - 1 + (y - \tfrac{3}{2})^2 - \tfrac{9}{4} + 3 = 0$

$(x-1)^2 + (y - \tfrac{3}{2})^2 = 1 + \tfrac{9}{4} - 3$

$(x-1)^2 + (y - \tfrac{3}{2})^2 = \tfrac{1}{4}$

Centre is $(1, \tfrac{3}{2})$ and radius $= \tfrac{1}{2}$.

4. Radius of circle $= \sqrt{(6-4)^2 + (3-(-2))^2} = \sqrt{4+25} = \sqrt{29}$

Equation of circle is $(x-4)^2 + (y+2)^2 = 29$

$x^2 - 8x + 16 + y^2 + 4y + 4 = 29$

$x^2 + y^2 - 8x + 4y = 9$

5. Centre of circle C is the midpoint of AB.

$C = \left(\frac{2+6}{2}, \frac{0+4}{2}\right) = (4, 2)$

Radius of circle is distance AC $= \sqrt{(2-4)^2 + (0-2)^2} = \sqrt{4+4} = \sqrt{8}$

Equation of circle is $(x-4)^2 + (y-2)^2 = 8$

$x^2 - 8x + 16 + y^2 - 4y + 4 = 8$

$x^2 + y^2 - 8x - 4y + 12 = 0$

6. (i) $x^2 + y^2 - 4x + 6y = 51$

$\Rightarrow (x-2)^2 + (y+3)^2 = 64$

$\Rightarrow$ centre is (2, -3), radius = 8

(ii) $x^2 + 2y^2 - 3x = 11$

The coefficients of x^2 and y^2 are different, so this is not a circle.

(iii) $4x^2 + 4y^2 = 65$

$\Rightarrow x^2 + y^2 = \frac{65}{4}$

$\Rightarrow$ centre is (0, 0), radius $= \frac{\sqrt{5}.\sqrt{13}}{2}$

(iv) $8x^2 + 8y^2 - 48x - 16y = -104$

$\Rightarrow x^2 + y^2 - 6x - 2y = -13$

$\Rightarrow (x-3)^2 + (y-1)^2 = -3$

so not a circle, as no (real) radius.

(There are no real values (x, y) which satisfy the equation)

Section 2: Circles

Solutions to Exercise level 2

1. (i) $x^2 + y^2 = 8$

Substituting in $y = 4 - x$ gives $x^2 + (4-x)^2 = 8$

$$x^2 + 16 - 8x + x^2 = 8$$
$$2x^2 - 8x + 8 = 0$$
$$x^2 - 4x + 4 = 0$$
$$(x-2)^2 = 0$$

The line meets the circle at just one point, so the line touches the circle and is therefore a tangent.

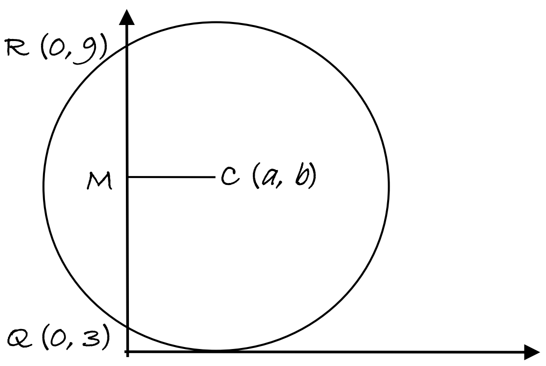
(ii) $x^2 + y^2 = 25$

Substituting in $4y = 3x - 25 \Rightarrow y = \frac{3x-25}{4}$

gives
$$x^2 + y^2 = 25$$
$$x^2 + \left(\frac{3x-25}{4}\right)^2 = 25$$
$$x^2 + \frac{(3x-25)^2}{16} = 25$$
$$16x^2 + 9x^2 - 150x + 625 = 400$$
$$25x^2 - 150x + 225 = 0$$
$$x^2 - 6x + 9 = 0$$
$$(x-3)^2 = 0$$

The line meets the circle at just one point, so the line touches the circle and is therefore a tangent.

2.



The midpoint M of QR is (0, 6).

Since a diameter which passes through M is perpendicular to QR, then the line CM must be horizontal, and therefore $b = 6$.
Since the circle touches the x-axis, the radius of the circle must be b, i.e. 6.
The equation of the circle is therefore $(x-a)^2+(y-6)^2=6^2$
The circle passes through (0, 3), so $(0-a)^2+(3-6)^2=6^2$

$$a^2+9=36$$
$$a^2=27$$
$$a=\pm\sqrt{27}=\pm 3\sqrt{3}$$

The equation of the circle is either $(x-3\sqrt{3})^2+(y-6)^2=36$
or $(x+3\sqrt{3})^2+(y-6)^2=36$.

3. $x^2+y^2=65$
$2y+x=10 \quad \Rightarrow x=10-2y$
Substituting in: $(10-2y)^2+y^2=65$

$$100-40y+4y^2+y^2=65$$
$$5y^2-40y+35=0$$
$$y^2-8y+7=0$$
$$(y-1)(y-7)=0$$
$$y=1 \text{ or } y=7$$

When $y = 1$, $x=10-2\times 1=8$
When $y = 7$, $x=10-2\times 7=-4$
so P is (8, 1) and Q is (-4, 7)
Length $PQ=\sqrt{(8-(-4))^2+(1-7)^2}=\sqrt{144+36}=\sqrt{180}$

4. Substituting $y=x+1$ into $(x-1)^2+(y+2)^2=k$:

$$(x-1)^2+(x+1+2)^2=k$$
$$(x-1)^2+(x+3)^2=k$$
$$x^2-2x+1+x^2+6x+9=k$$
$$2x^2+4x+10-k=0$$

If there are no intersections, then $b^2-4ac<0$

$$a=2, b=4, c=10-k$$
$$4^2-4\times 2(10-k)<0$$
$$16-8(10-k)<0$$
$$2-(10-k)<0$$
$$2-10+k<0$$
$$k<8$$

OCR AS Maths Coordinate geometry 2 Exercise solutions

5. (i) Gradient of PR $= \frac{7-6}{5-(-2)} = \frac{1}{7}$

Gradient of QR $= \frac{7-0}{5-6} = \frac{7}{-1} = -7$

Gradient of PR × gradient of QR $= \frac{1}{7} \times -7 = -1$

so PR and QR are perpendicular.

(ii) The angle in a semicircle is 90°, so PQ must be a diameter.

(iii) Since PQ is a diameter, the centre C of the circle is the midpoint of PQ.

$$C = \left(\frac{-2+6}{2}, \frac{6+0}{2}\right) = (2,3)$$

Radius of circle = length CQ $= \sqrt{(6-2)^2 + (0-3)^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$

Equation of circle is $(x-2)^2 + (y-3)^2 = 25$.

6. (i) $x^2 + y^2 = 17$

(ii) Substituting x = -4 and y = -1: $x^2 + y^2 = (-4)^2 + (-1)^2 = 16 + 1 = 17$

(iii) Gradient of radius OP $= \frac{-1-0}{-4-0} = \frac{1}{4}$

Tangent to circle at P is perpendicular to radius OP

so gradient of tangent = -4

Equation of tangent is $y-(-1) = -4(x-(-4))$

$$y + 1 = -4(x+4)$$
$$y + 1 = -4x - 16$$
$$y + 4x + 17 = 0$$

(iv) $x + y = 3 \quad \Rightarrow y = 3 - x$

Substituting into equation of circle:

$$x^2 + (3-x)^2 = 17$$
$$x^2 + 9 - 6x + x^2 = 17$$
$$2x^2 - 6x - 8 = 0$$
$$x^2 - 3x - 4 = 0$$
$$(x-4)(x+1) = 0$$
$$x = 4 \text{ or } x = -1$$

When x = 4, $y = 3 - 4 = -1$

When x = -1, $y = 3 - (-1) = 4$

Coordinates of Q and R are (4, -1) and (-1, 4).

(v) Tangent is $y + 4x + 17 = 0$

Substituting in $y = 3 - x$ gives $(3 - x) + 4x + 17 = 0$

$$20 + 3x = 0$$

$$x = -\tfrac{20}{3}$$

When $x = -\frac{20}{3}$, $y = 3 + \frac{20}{3} = \frac{29}{3}$

Coordinates of S are $\left(-\frac{20}{3}, \frac{29}{3}\right)$

7. $\left.\begin{aligned} y &= x^2 + 8 \\ y &= 2x^2 + x + 6 \end{aligned}\right\}$

$\Rightarrow x^2 + 8 = 2x^2 + x + 6$

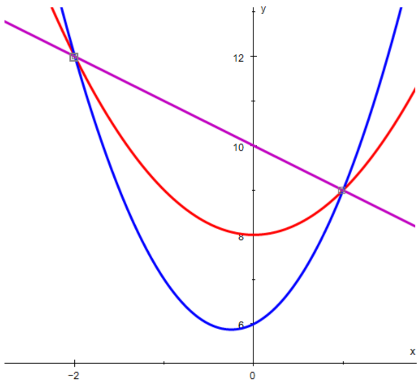
$\Rightarrow \quad x^2 + x - 2 = 0$

$\Rightarrow (x - 1)(x + 2) = 0$

$\Rightarrow x = 1, y = 9$ or $x = -2, y = 12$

So length PQ $= \sqrt{3^2 + 3^2}$

$= 3\sqrt{2}$



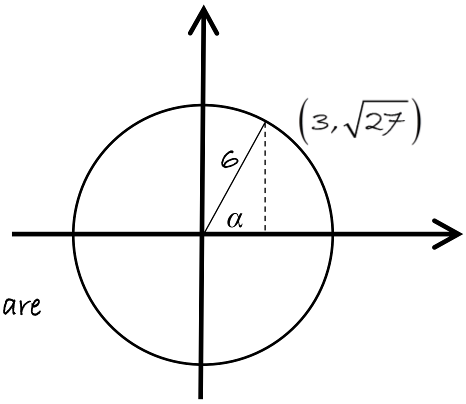
Section 2: Circles

Solutions to Exercise level 3 (Extension)

1. $3^2 + \left(\sqrt{27}\right)^2 = k^2 \Rightarrow k^2 = 36$

$$\Rightarrow k = 6$$

In the diagram, $\alpha = 60°$.
The three points are spaced out equally around the circle, so the angles at the centre are 120°.
So for an equilateral triangle the vertices Q and R are $\left(3, -\sqrt{27}\right)$ and (-6, 0).



2. (i) M is (6, 3), and the circle has radius $\sqrt{4^2 + 2^2} = \sqrt{20}$.
So the equation is $(x-6)^2 + (y-3)^2 = 20$

$$\Rightarrow x^2 + y^2 - 12x - 6y + 25 = 0$$

(ii)
$$\left.\begin{array}{r} y = 3x - 15 \\ x^2 + y^2 - 12x - 6y + 25 = 0 \end{array}\right\}$$

$$\Rightarrow x^2 + (3x-15)^2 - 12x - 6(3x-15) + 25 = 0$$
$$\Rightarrow x^2 - 12x + 34 = 0$$
$$\Rightarrow x = \frac{12 \pm \sqrt{144 - 136}}{2}$$
$$= 6 \pm \sqrt{2}$$

So U is (7.41, 7.24) and V is (4.59, -1.24)
$\angle PUQ$ is an angle in a semicircle, so $\angle PUQ = 90°$

(iii)
$$\left.\begin{array}{r} y + 2x = 5 \\ x^2 + y^2 - 12x - 6y + 25 = 0 \end{array}\right\}$$

$$\Rightarrow x^2 + (-2x+5)^2 - 12x - 6(-2x+5) + 25 = 0$$
$$\Rightarrow x^2 - 4x + 4 = 0$$
$$\Rightarrow (x-2)^2 = 0$$
$$\Rightarrow x = 2 \text{ (twice!)}, \ y = 1$$

So L_2 is a tangent to the circle at P, and therefore $\angle RPQ = 90°$.

3. For all the circles passing through P and Q, the line segment PQ is a chord. For each circle, the diameter through the midpoint of PQ is a perpendicular bisector. Therefore all centres lie along this line.

Midpoint of PQ is (3, 2)

Gradient of PQ $= \frac{10}{4} = \frac{5}{2}$

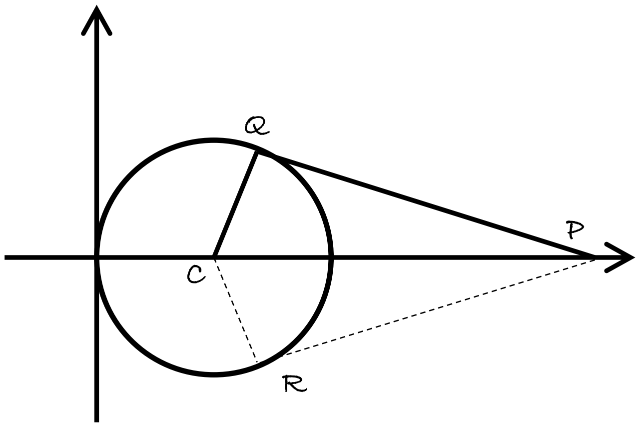
so the gradient of the perpendicular bisector $= -\frac{2}{5}$

So the equation of the line of centres is $y - 2 = -\frac{2}{5}(x-3)$

$$\Rightarrow 2x + 5y = 16$$

4. (i) $(x-5)^2 + y^2 = 5^2$

$$\Rightarrow x^2 + y^2 - 10x = 0$$



(ii) $PQ = \sqrt{15^2 - 5^2}$

$$= \sqrt{200} = 10\sqrt{2} \text{ m}$$

(iii) Since Q = (a, b)

gradient CQ $= \dfrac{b}{a-5}$

gradient QP $= \dfrac{-b}{20-a}$

$$\Rightarrow \frac{-b}{20-a} = -\frac{a-5}{b}$$

$$\Rightarrow -b^2 = (5-a)(20-a)$$

$$\Rightarrow -b^2 = 100 - 25a + a^2$$

But Q lies on the circle, so $a^2 + b^2 - 10a = 0$

$$\Rightarrow a^2 - 10a = 100 - 25a + a^2$$

$$\Rightarrow 15a = 100$$

$$\Rightarrow a = \tfrac{20}{3}$$

$$\Rightarrow b^2 = \tfrac{200}{3} - \tfrac{400}{9} = \tfrac{200}{9}$$

$$\Rightarrow b = \tfrac{10\sqrt{2}}{3}$$

So $Q = \left(\frac{20}{3}, \frac{10\sqrt{2}}{3}\right)$ and $R = \left(\frac{20}{3}, -\frac{10\sqrt{2}}{3}\right)$

(iv) Shape PQCR is a kite.

Area $= \frac{1}{2}(QR)(CP)$

$$= \tfrac{1}{2}\left(\tfrac{20\sqrt{2}}{3}\right)(15)$$

$$= 50\sqrt{2} \text{ m}^2$$