Exercise

1. A linear sequence starts

   $$250 \quad 246 \quad 242 \quad 238$$

   Which term is the first to have a negative value?
2. A sequence has nth term $\frac{5n+2}{2n}$. Show that the limiting value of the sequence S, as $n \to \infty$ is 2.5
3. (a) The nth term of a sequence is $4n - 10$.

   (a)(i) Show that the $(n+1)$th term can be written as $4n - 6$.

   (a)(ii) Prove that the sum of any two consecutive terms of the sequence is divisible by 8.

   (b) The nth term of a different sequence is $\frac{3n}{n+5}$

   (b)(i) Explain why 1 is **not** a term in this sequence.

   (b)(ii) Work out the limiting value of the sequence as $n \to \infty$.
4. Here is a linear sequence:

   $$4 \quad 11 \quad 18 \quad 25 \quad \ldots$$

   (a) Work out an expression for the nth term.

   (b) How many terms are less than 150?
5. The nth term of sequence X is $an + b$. The nth term of sequence Y is $bn + a$.

   (a) Show that the sequences have the same first term.

   (b) The 2nd term of sequence X is equal to the 3rd term of sequence Y. Show that $a = 2b$.

   (c) Prove that:

   $$\frac{nth\ term\ of\ sequence\ X}{nth\ term\ of\ sequence\ Y} = \frac{2n+1}{n+2}$$
6. (a) $f(n) = n^2 + n$ Show that

   $$f(n+1) - f(n) = 2n + 2$$

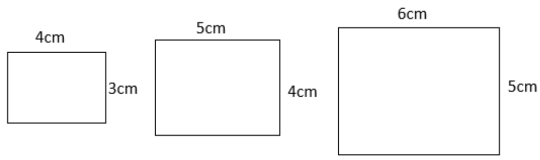
   (b) The nth term of a sequence is $n^2 + n$

   Two consecutive terms in the sequence have a difference of 32. Work out the two terms.
7. The nth term of the linear sequence 2, 7, 12, 17, … is $5n - 3$. A new sequence is formed by squaring each term of the linear sequence and adding 1. Prove algebraically that **all** the terms in the new sequence are multiples of 5.
8. (a) Write down the nth term of the linear sequence

   $$4 \quad 7 \quad 10 \quad 13 \quad \ldots$$

   (b) Hence, write down the nth term of the quadratic sequence.

   $$16 \quad 49 \quad 100 \quad 169 \quad \ldots$$

   (c) For the sequence in (b), show that the 30th term is equal to the product of the 2nd and 4th.
9. 

   This pattern of rectangles continues. Show that the sequence of numbers formed by the areas of these rectangles has nth term.

   $$n^2 + 5n + 6$$
10. A linear sequence starts

   $$a+b \quad a+3b \quad a+5b \quad a+7b$$

   The 5th and 8th terms have values 35 and 59.

   (a) Work out a and b.

   (b) Work out the nth term of the sequence.
11. A sequence has nth term $\frac{3n+1}{n}$.

   (a) Show that the difference between the nth and $(n+1)$th term is $\frac{1}{n(n+1)}$

   (b) Which are the first two consecutive terms with a difference less than 0.01?

   (c) Write down the limiting value of the sequence as $n \to \infty$.

Solutions

Exercise 1

1 A linear sequence starts

250 246 242 238

Which term is the first to have a negative value?

nth term is $254 - 4n < 0 \quad \rightarrow 64th$

2 A sequence has nth term $\frac{5n+2}{2n}$. Show that the limiting value of the sequence S, as $n \rightarrow \infty$ is 2.5

As n becomes large $\frac{5n+2}{2n} \rightarrow \frac{5n}{2n} = \frac{5}{2}$

3 (a) The nth term of a sequence is $4n - 10$.

(a)(i) Show that the $(n+1)$th term can be written as $4n - 6$. **$4(n+1) - 10 = 4n - 6$**

(a)(ii) Prove that the sum of any two consecutive terms of the sequence is divisible by 8.

$4n - 10 + 4n - 6 = 8n - 16 = 8(n-2)$

(b) The nth term of a different sequence is $\frac{3n}{n+5}$

(b)(i) Explain why 1 is **not** a term in this sequence. **$\frac{3n}{n+5} = 1 \quad \rightarrow \quad n = 2.5$ which is not a whole number.**

(b)(ii) Work out the limiting value of the sequence as $n \rightarrow \infty$. **3**

4 Here is a linear sequence:

4 11 18 25 ...

(a) Work out an expression for the nth term. **$7n - 3$**

(b) How many terms are less than 150?

$7n - 3 < 150$

$n = 21$

5 The nth term of sequence X is $an + b$.

The nth term of sequence Y is $bn + a$.

(a) Show that the sequences have the same first term. **$1a + b = 1b + a$**

(b) The 2nd term of sequence X is equal to the 3rd term of sequence Y. Show that $a = 2b$. **$2a + b = 3b + a \quad \rightarrow a = 2b$**

(c) Prove that:

$$\frac{nth\ term\ of\ sequence\ X}{nth\ term\ of\ sequence\ Y} = \frac{2n+1}{n+2}$$

$$\boldsymbol{\frac{an+b}{bn+a} = \frac{2bn+b}{bn+2b} = \frac{2n+1}{n+2}}$$

6

(a) $f(n) = n^2 + n$ Show that

$f(n+1) - f(n) = 2n + 2$

$$(\boldsymbol{n}+\mathbf{1})^{\mathbf{2}} + (\boldsymbol{n}+\mathbf{1}) - \boldsymbol{n}^{\mathbf{2}} - \boldsymbol{n}$$

$$= \mathbf{2}\boldsymbol{n} + \mathbf{2}$$

(b) The nth term of a sequence is $n^2 + n$

Two consecutive terms in the sequence have a difference of 32. Work out the two terms.

$$\mathbf{2}\boldsymbol{n} + \mathbf{2} = \mathbf{32} \quad \rightarrow \quad \boldsymbol{n} = \mathbf{15}$$

$$\mathbf{15}^{\mathbf{2}} + \mathbf{15} = \mathbf{240}$$

$$\mathbf{16}^{\mathbf{2}} + \mathbf{16} = \mathbf{272}$$

7

The nth term of the linear sequence 2, 7, 12, 17, … is $5n - 3$. A new sequence is formed by squaring each term of the linear sequence and adding 1. Prove algebraically that **all** the terms in the new sequence are multiples of 5.

$$(\mathbf{5}\boldsymbol{n} - \mathbf{3})^{\mathbf{2}} + \mathbf{1} = \mathbf{25}\boldsymbol{n}^{\mathbf{2}} - \mathbf{30}\boldsymbol{n} + \mathbf{9} + \mathbf{1}$$

$$= \mathbf{25}\boldsymbol{n}^{\mathbf{2}} - \mathbf{30}\boldsymbol{n} + \mathbf{10}$$

$$= \mathbf{5}(\mathbf{5}\boldsymbol{n}^{\mathbf{2}} - \mathbf{6}\boldsymbol{n} + \mathbf{2})$$

8

(a) Write down the nth term of the linear sequence

$$4 \quad 7 \quad 10 \quad 13 \quad \ldots$$

$$\mathbf{3}\boldsymbol{n} + \mathbf{1}$$

(b) Hence, write down the nth term of the quadratic sequence.

$$16 \quad 49 \quad 100 \quad 169 \quad \ldots$$

$$(\mathbf{3}\boldsymbol{n} + \mathbf{1})^{\mathbf{2}} = \mathbf{9}\boldsymbol{n}^{\mathbf{2}} + \mathbf{6}\boldsymbol{n} + \mathbf{1}$$

(c) For the sequence in (b), show that the 30th term is equal to the product of the 2nd and 4th.

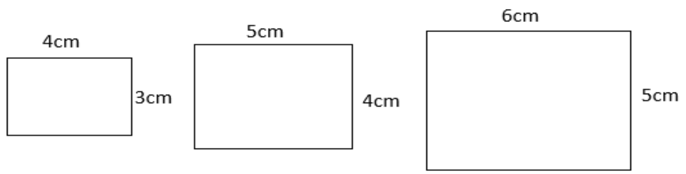
2nd term: 49, 4th term: 169

30th term: $\mathbf{91}^{\mathbf{2}} = \mathbf{8281}$

$$\mathbf{49} \times \mathbf{169} = \mathbf{8281}$$

9 This pattern of rectangles continues. Show that the sequence of numbers formed by the areas of these rectangles has nth term.

$$n^2 + 5n + 6$$



10 A linear sequence starts

$$a + b \quad a + 3b \quad a + 5b \quad a + 7b$$

The 5th and 8th terms have values 35 and 59.

(a) Work out a and b. $\boldsymbol{a = -1, b = 4}$

(b) Work out the nth term of the sequence. $\boldsymbol{8n - 5}$

11 A sequence has nth term $\frac{3n+1}{n}$.

(a) Show that the difference between the nth and $(n+1)$th term is $\frac{1}{n(n+1)}$

$$\boldsymbol{\frac{3(n+1)+1}{n+1} - \frac{3n+1}{n} = \cdots = \frac{1}{n(n+1)}}$$

(b) Which are the first two consecutive terms with a difference less than 0.01?

$$\boldsymbol{\frac{1}{n(n+1)} < \frac{1}{100}}$$

$$\boldsymbol{n(n+1) > 100}$$

$\boldsymbol{n = 10}$ **and 11**

(c) Write down the limiting value of the sequence as $n \to \infty$.

$\boldsymbol{3}$

Section 6: Sequences and proof

Exercise

1. n is a positive integer. Prove that $n^3 - n^2$ is always even.

2. Prove that the product of three consecutive integers is always be a multiple of 6.

3. Prove that the square of any odd number is always 1 more than a multiple of 8.

4. Write down the first four terms of each sequence defined below, starting with $n = 1$ in each case.
   (i) $3n-1$
   (ii) n^2-1
   (iii) $3n^2-2n+1$

5. Find a formula for the nth term of each the linear sequences below.
   (i) 2, 5, 8, 11, ...
   (ii) 10, 8, 6, 4, ...

6. Find a formula for the nth term of each the quadratic sequences below.
   (i) 3, 9, 17, 27, 39, ...
   (ii) -2, 4, 14, 28, 46, …
   (iii) 7, 12, 15, 16, 15, …

7. For each of the following sequences, find the 1st term, the 5th term, the 100th term, and the limit of the sequence as $n \to \infty$.
   (i) nth term $= \dfrac{2n+5}{4n-1}$
   (ii) nth term $= \dfrac{1-6n}{2n+3}$

8. The nth term of a sequence is given by the formula n^2+2n-5.
   Prove that 1000 cannot be a term of the sequence.

9. A sequence has all its terms positive.
   As $n \to \infty$, the nth term of the sequence approaches 3.

   (a) Give a possible formula for the nth term of an increasing sequence with the properties above.

   (b) Give a possible formula for the nth term of a decreasing sequence with the properties above.

Section 6: Sequences and proof

Solutions to Exercise

1. If n is odd, n^3 and n^2 are both odd, so $n^3 - n^2$ is even.
   If n is even, n^3 and n^2 are both even, so $n^3 - n^2$ is even.
   Therefore for all positive integers n, $n^3 - n^2$ is always even.

2. For three consecutive integers, at least one will be even, and one will be a multiple of 3. Therefore the product of the three integers is both a multiple of 2 and a multiple of 3, and so is a multiple of 6.

3. Any odd number can be written as $2n + 1$, where n is an integer.
   So the square of an odd number can be written as $(2n+1)^2 = 4n^2 + 4n + 1$
   $= 4n(n+1)+1$
   One of n and $n + 1$ is even, so $n(n+1)$ is a multiple of 2 and therefore $4n(n+1)$ is a multiple of 8.
   So $4n(n+1)+1$ is one more than a multiple of 8. So the square of any odd number is always 1 more than a multiple of 8.

4. (i) nth term $= 3n - 1$
   1st term $= 3\times1-1=2$
   2nd term $= 3\times2-1=5$
   3rd term $= 3\times3-1=8$
   4th term $= 3\times4-1=11$
   Sequence is 2, 5, 8, 11, ...

   (ii) nth term $= n^2 - 1$
   1st term $= 1^2-1=0$
   2nd term $= 2^2-1=4-1=3$
   3rd term $= 3^2-1=9-1=8$
   4th term $= 4^2-1=16-1=15$
   Sequence is 0, 3, 8, 18, ...

   (iii) nth term $= 3n^2 - 2n + 1$
   1st term $= 3\times1^2-2\times1+1=3-2+1=2$
   2nd term $= 3\times2^2-2\times2+1=12-4+1=9$
   3rd term $= 3\times3^2-2\times3+1=27-6+1=22$
   4th term $= 3\times4^2-2\times4+1=48-8+1=41$
   Sequence is 2, 9, 22, 41, ...

5. (i) Each term increases by 3, so the general term must involve 3n.
   nth term $= 3n - 1$.

   (ii) Each term decreases by 2, so the general term must involve -2n.
   nth term $= 12 - 2n$

6. (i) The sequence has nth term $an^2 + bn + c$.

Terms	3		9		17		27		39
Differences		6		8		10		12	
Second differences			2		2		2		

So $a = 1$

Terms	3	9	17	27	39
an^2	1	4	9	16	25
$bn + c$	2	5	8	11	14

The values of $bn + c$ go up by 3 each time, so $b = 3$, and $c = -1$

The nth term of the sequence is $n^2 + 3n - 1$.

(ii) The sequence has nth term $an^2 + bn + c$.

Terms	-2		4		14		28		46
Differences		6		10		14		18	
Second differences			4		4		4		

So $a = 2$

Terms	-2	4	14	28	46
an^2	2	8	18	32	50
$bn + c$	-4	-4	-4	-4	-4

The values of $bn + c$ are all the same, so $b = 0$, and $c = -4$

The nth term of the sequence is $2n^2 - 4$.

(iii) The sequence has nth term $an^2 + bn + c$.

Terms	7		12		15		16		15
Differences		5		3		1		-1	
Second differences			-2		-2		-2		

So $a = -1$

Terms	7	12	15	16	15
an^2	-1	-4	-9	-16	-25
$bn + c$	8	16	24	32	40

The values of $bn + c$ go up by 8 each time, so $b = 8$, and $c = 0$

The nth term of the sequence is $-n^2 + 8n$.

7. (i) nth term $= \dfrac{2n+5}{4n-1}$

1^{st} term $= \dfrac{2\times1+5}{4\times1-1} = \dfrac{2+5}{4-1} = \dfrac{7}{3}$

5^{th} term $= \dfrac{2\times5+5}{4\times5-1} = \dfrac{10+5}{20-1} = \dfrac{15}{19}$

100^{th} term $= \dfrac{2\times100+5}{4\times100-1} = \dfrac{200+5}{400-1} = \dfrac{205}{399}$

As $n\to\infty$, $2n+5\to 2n$, $4n-1\to 4n$

so $\dfrac{2n+5}{4n-1} \to \dfrac{2n}{4n} = \dfrac{1}{2}$

The limit of the sequence is $\frac{1}{2}$.

(ii) nth term $= \dfrac{1-6n}{2n+3}$

1^{st} term $= \dfrac{1-6\times1}{2\times1+3} = \dfrac{1-6}{2+3} = -\dfrac{5}{5} = -1$

5^{th} term $= \dfrac{1-6\times5}{2\times5+3} = \dfrac{1-30}{10+3} = -\dfrac{29}{13}$

100^{th} term $= \dfrac{1-6\times100}{2\times100+3} = \dfrac{1-600}{200+3} = -\dfrac{599}{203}$

As $n\to\infty$, $1-6n\to -6n$, $2n+3\to 2n$

so $\dfrac{1-6n}{2n+3} \to \dfrac{-6n}{2n} = -3$

8. Method 1

$n^2+2n-5=1000$

$n^2+2n=1005$

$n^2+2n+1=1006$

$(n+1)^2=1006$

$n+1$ is a whole number but 1006 is not a square number so 1000 cannot be a term in the sequence.

Method 2

$n^2 + 2n - 5 = 1000$

$n^2 + 2n = 1005$

$n(n+2) = 1005$

n and $n+2$ are consecutive even or consecutive odd numbers. To multiply to make 1005, they must both be odd.

1005 = 3 x 5 x 67

There are no consecutive odd numbers that multiply to make 1005 so 1000 cannot be a term in the sequence.

Method 3

$n^2 + 2n - 5 = 1000$

$n^2 + 2n - 1005 = 0$

Solve the quadratic equation.

n = 30.72 or -32.72

n is not an integer so 1000 is not a term in the sequence.

9. (a) One possible sequence is $\frac{3n}{n+1}$

(b) One possible sequence is $4 - \frac{n}{n+1}$

Section 5: Sequences and recurrence relations

Exercise

1. Write down the first four terms of each sequence defined below, starting with $k = 1$ in each case.

   (i) $a_k = 3k - 1$ (ii) $a_k = 2 \times 3^k$

   (iii) $a_k = k^2$ (iv) $a_k = (-1)^k 2^k$

   (v) $a_{k+1} = 2a_k + 1, a_1 = 2$ (vi) $a_{k+1} = 1 - a_k, a_1 = 3$

2. Write down the first four terms of each sequence defined below, starting with $n = 5$ in each case.

   (i) $u_n = n - 5$

   (ii) $u_n = \dfrac{1}{n^2}$

   (iii) $u_n = (-1)^n \left(\frac{1}{2}\right)^n$

   (iv) $u_n = u_{n-1} + u_{n-2}$, $u_1 = 1$, $u_2 = -2$

3. The value of a car decreases by 10% every year of its life. If its original value is $a_0 = 14{,}000$ write down recursive definition for the sequence a_k where a_k is the value of the car after k years.

4. The value of a rare toy train set increases by 2% each year. If the value of the train set is £150 now write down a formula in terms of k for a_k, the value of the train set in k years' time.

Section 5: Sequences and recurrence relations

Exercise solutions

1. (i) $a_1 = 3\times 1 - 1 = 2$
$a_2 = 3\times 2 - 1 = 5$
$a_3 = 3\times 3 - 1 = 8$
$a_4 = 3\times 4 - 1 = 11$

(ii) $a_1 = 2\times 3^1 = 6$
$a_2 = 2\times 3^2 = 18$
$a_3 = 2\times 3^3 = 54$
$a_4 = 2\times 3^4 = 162$

(iii) $a_1 = 1^2 = 1$
$a_2 = 2^2 = 4$
$a_3 = 3^2 = 9$
$a_4 = 4^2 = 16$

(iv) $a_1 = (-1)^1 2^1 = -2$
$a_2 = (-1)^2 2^2 = 4$
$a_3 = (-1)^3 2^3 = -8$
$a_4 = (-1)^4 2^4 = 16$

(v) $a_1 = 2$
$a_2 = 2a_1 + 1 = 2\times 2 + 1 = 5$
$a_3 = 2a_2 + 1 = 2\times 5 + 1 = 11$
$a_4 = 2a_3 + 1 = 2\times 11 + 1 = 23$

(vi) $a_1 = 3$
$a_2 = 1 - a_1 = 1 - 3 = -2$
$a_3 = 1 - a_2 = 1 - (-2) = 3$
$a_4 = 1 - a_3 = 1 - 3 = -2$

2. (i) $u_5 = 0$, and then next terms are 1, 2, 3

(ii) $u_5 = \frac{1}{25}$, and then next terms are $\frac{1}{36}$, $\frac{1}{49}$, $\frac{1}{64}$

(iii) $u_5 = -\frac{1}{32}$, and then next terms are $+\frac{1}{64}$, $-\frac{1}{128}$, $+\frac{1}{256}$

(iv) $u_3 = -1$, $u_4 = -3$, and so $u_5 = -4$ and then next terms are -7, -11, -18 (this is an example of a Fibonacci sequence)

3. Since the value of the car reduces by 10% each year. The value each year is 0.9 times the value the previous year. Therefore $a_{k+1} = 0.9a_k$.

4. Since the value of the train set increases by 2% each year, if the original value is £150, the value one year later, a_1, is 150 x 1.02.
The value the next year a_2, is 150 x 1.02 x 1.02 = 150 x $(1.02)^2$.
This pattern continues and so the value at the end of the next year a_3 is 150 x $(1.02)^3$.
In general $a_k = 150 \times (1.02)^k$.

Thinking about sequences

	$a_n = 3^n$	$b_n = 2n+3$	$c_n =$	$d_n =$	$u_n =$
First few terms	$3, 9, 27, 81, \ldots$		$9, 5, 1, -3, \ldots$		
Inductive definition	$a_{n+1} = 3 \times a_n$, $a_1 = 3$			$d_{n+1} = \frac{1}{2} d_n$, $d_1 = 4$	$u_{n+1} = -3 \times u_n$, $u_3 = 18$
Specific term	$a_{20} = 3^{20}$	$b_{100} =$	$c_{50} =$	$d_{10} =$	$u_{21} =$
Associated terminology	Geometric Diverging				
Sum of terms	$\sum_{n=1}^{5} a_n = 363$	$\sum_{n=1}^{10} b_n =$	$\sum_{n=1}^{12} c_n =$	$\sum_{n=4}^{8} d_n =$	$\sum_{n=1}^{7} u_n - \sum_{n=1}^{6} u_n =$

Introduction to Sequences and Series

The notation u_r or x_r is used for the general term, or formula, of a sequence. So u_1 means the first term, and you work it out by putting $r=1$ into the formula; u_2 means the second term, and you work it out by putting $r=2$ into the formula etc.

A sequence is described as **oscillating** if the terms repeat

e.g. 1, -1, 1, -1, 1, -1,… or 7, 5, 3, 5, 7, 9, 7, 5, 3, 5, 7, 9, 7,…

A sequence is described as **convergent** if the terms are getting closer and closer to a fixed value. The fixed value is called the **limit**

e.g. 7, 5, 4, 3.5, 3.25, 3.125, … is convergent with a limit of 3

A sequence that is neither oscillating nor convergent is **divergent**. The terms may be getting further apart, or staying the same distance apart but not oscillating

e.g. 1, 2, 4, 8, 16, 32, … or -7, -5, -3, -1, 1, 3, 5, ….

Sometimes the formula for the sequence is given by a **recurrence relation**, where you are told the first term and then how to get each of the other terms using the first term

e.g. $u_{r+1}=2u_r, u_1=3$ tells you that the first term is 3 and that to get the next term you double the previous term. The sequence is 3, 6, 12, 24, 48

If a sequence is convergent and the formula is given by a recurrence relation then you can work out the limit by substituting x instead of both u_{r+1} and u_r or x_{n+1} and x_n in the formula and solving the equation ***but you must check that the sequence is convergent first***

Introduction to Sequences and Series – Examples

Example 1

Write out the first four terms of the sequences below and say whether they are convergent, divergent or oscillating. If a sequence is convergent state its limit.

(a) $u_r = 3 + r^2$

(b) $x_r = 2\cos\left(180r^\circ\right)$

(c) $u_r = 2 - \dfrac{9}{3^r}$

Example 2

Write out the first four terms of the sequences given by the recurrence relations below. If the sequence is convergent, use the substitution method to find its limit.

(a) $u_{r+1} = u_r - 3 \qquad u_1 = 10$

(b) $x_{n+1} = \dfrac{x_n + 2}{4} \qquad x_1 = 14$

Sort it out

Problem

Sort these infinite sequences into groups.

- How will you choose to define the groups?
- Are there some sequences which belong to more than one group?

Once you have sorted the sequences into groups, write a further sequence for each group.

How else could you have sorted the sequences?

Ⓐ $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \ldots$	Ⓑ $3\sqrt{2}, 4\sqrt{2}, 5\sqrt{2}, 6\sqrt{2}, \ldots$
Ⓒ $2, 0, -2, -4, \ldots$	Ⓓ $\ln 1, \ln 2, \ln 3, \ln 4, \ldots$
Ⓔ $-1, -\sqrt{2}, -2, -2\sqrt{2}, \ldots$	Ⓕ $1, 1, 1, 1, \ldots$
Ⓖ $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$	Ⓗ $-1, 1, -1, 1, \ldots$
Ⓘ $0.1, 0.01, 0.001, 0.0001, \ldots$	Ⓙ $3, 2\frac{1}{3}, 1\frac{2}{3}, 1, \ldots$
Ⓚ $4, 8, 16, 32, \ldots$	Ⓛ $\cos\left(\frac{\pi}{4}\right), \cos\left(\frac{2\pi}{4}\right), \cos\left(\frac{3\pi}{4}\right), \cos\left(\frac{4\pi}{4}\right), \ldots$
Ⓜ $1, 1\frac{1}{2}, 1\frac{2}{3}, 1\frac{3}{4}, \ldots$	Ⓝ $100, 105, 110, 115, \ldots$
Ⓞ $3.75, 5.625, 8.4375, 12.65625, \ldots$	

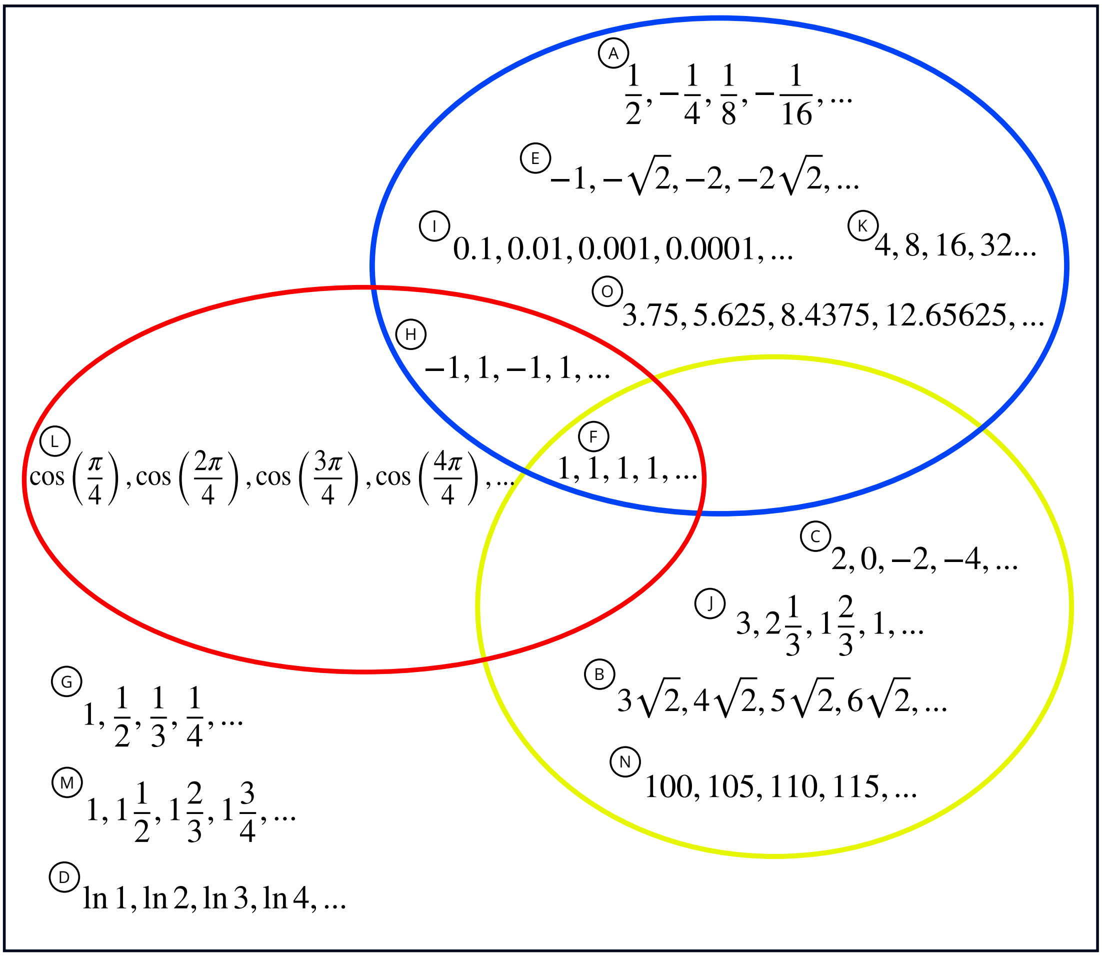
Sort it out

Possible groupings

Sort these infinite sequences into groups.

- How will you choose to define the groups?
- Are there some sequences which belong to more than one group?

Here is one possible grouping. How do you think this grouping was constructed?



Possible rules

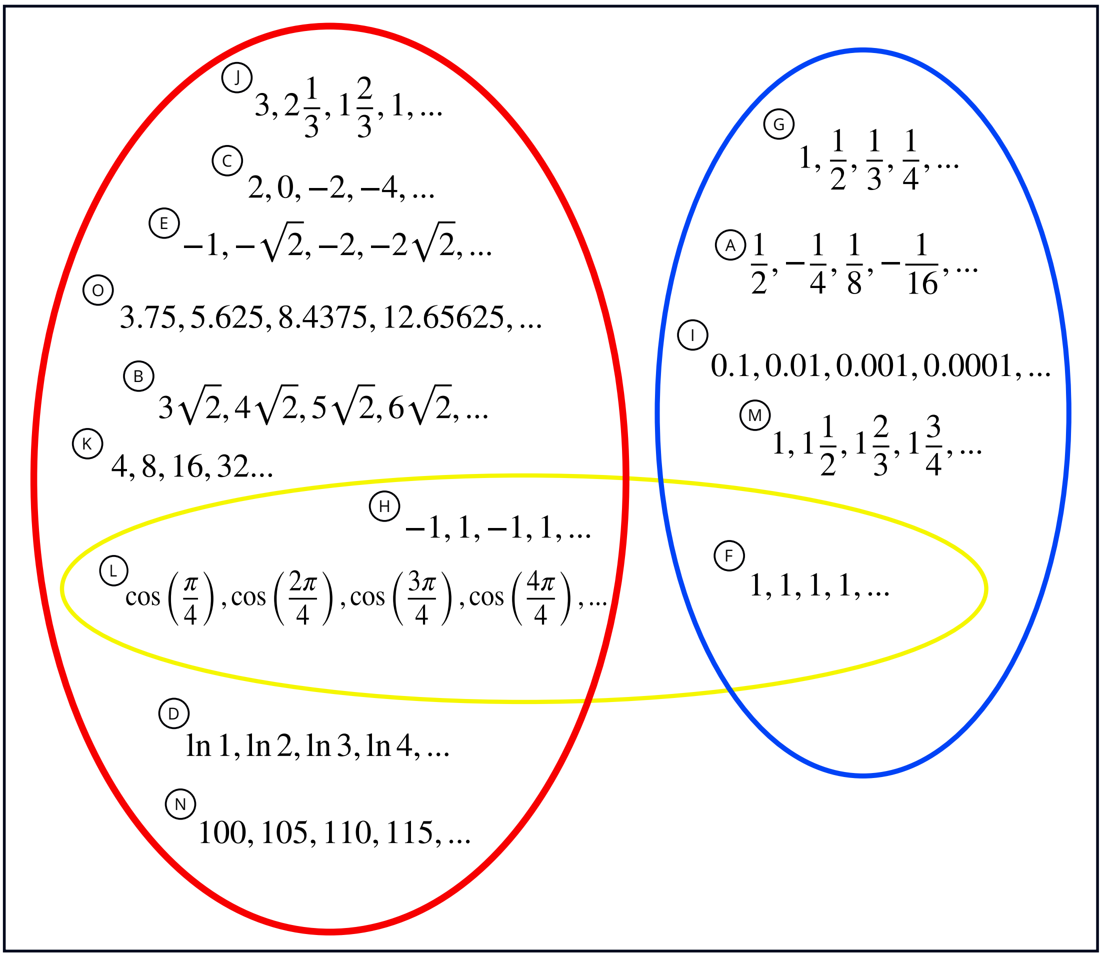
Periodic sequences; arithmetic sequences; geometric sequences

Some didn't seem to fit in any of these groups.

Why can we think of the sequence $1, 1, 1, 1, \ldots$ as both an arithmetic sequence and a geometric sequence? Are there any other sequences that are both arithmetic and geometric?

Can we find a sequence that fits in the empty parts of this grouping diagram?

Here is another possible grouping. How do you think this one was constructed?



Possible rules

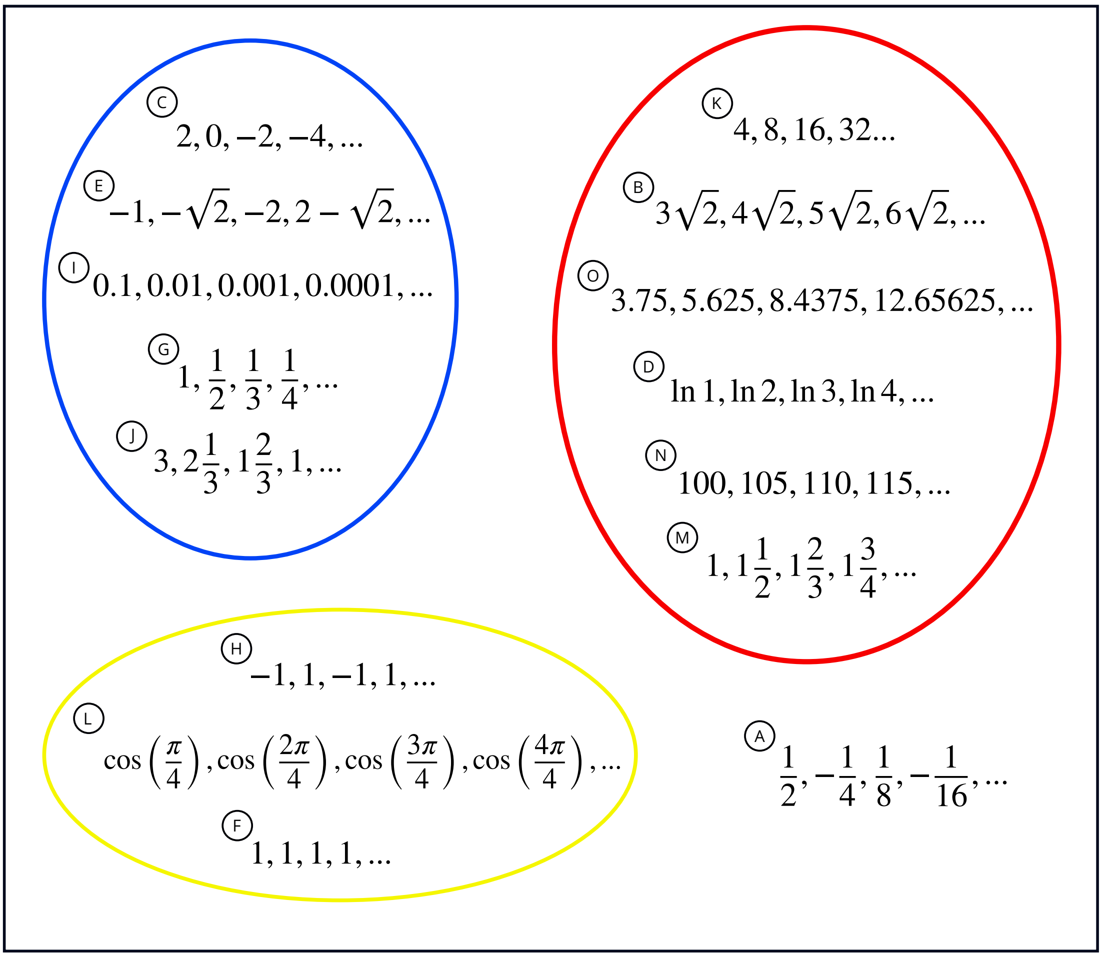
Periodic sequences; divergent sequences; convergent sequences

If we plotted the graphs of the convergent sequences, what would they have in common?

If we plotted the graphs of the divergent sequences, what would they have in common?

Can we find a sequence that fits in the empty parts of this grouping diagram?

And another… how do you think this one was constructed?



Possible rules

Strictly increasing sequences; strictly decreasing sequences; periodic sequences

One didn't seem to fit in any of these groups.

Why do you think our groupings are labelled *strictly* increasing and *strictly* decreasing sequences? (It might help to think about increasing and decreasing functions.)

How would the grouping diagram have changed if we had not used the word *strictly* in our labelling of our groupings?