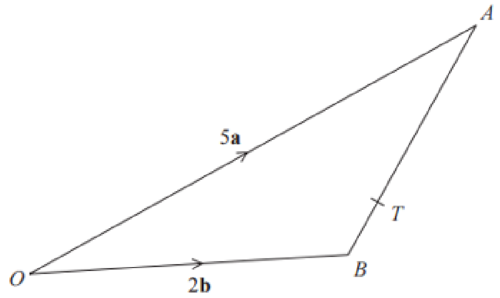
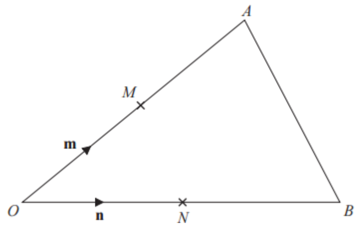
Exercise 3 (Proofs)

1. [Nov 2016 2H Q23]



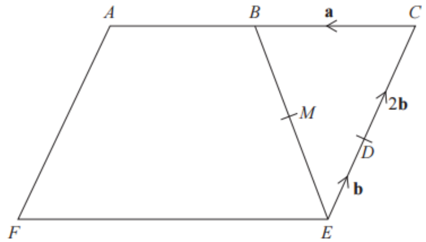
T is the point on AB such that $AT:TB = 5:1$. Show that OT is parallel to the vector $\boldsymbol{a} + 2\boldsymbol{b}$.

2. [June 2014 1H Q24]



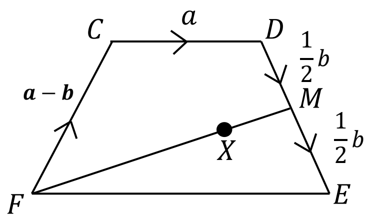
M is the midpoint of OA. N is the midpoint of OB. Prove that AB is parallel to OB.

3. [November 2015 2H Q20]



$ACEF$ is a parallelogram. B is the midpoint of AC. M is the midpoint of BE. Show that AMD is a straight line.

4.



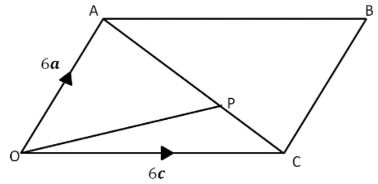
$\overrightarrow{CD} = \boldsymbol{a}$, $\overrightarrow{DE} = \boldsymbol{b}$ and $\overrightarrow{FC} = \boldsymbol{a} - \boldsymbol{b}$

i) Express $\overrightarrow{CE}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

ii) Prove that $\overrightarrow{FE}$ is parallel to $\overrightarrow{CD}$.

iii) X is the point on FM such that such that $FX:XM = 4:1$. Prove that C, X and E lie on the same straight line.

5.

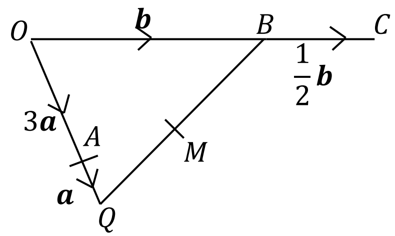


$OABC$ is a parallelogram. P is the point on AC such that $AP = \frac{2}{3}AC$.

i) Find the vector $\overrightarrow{OP}$. Give your answer in terms of $\boldsymbol{a}$ and $\boldsymbol{c}$.

ii) Given that the midpoint of CB is M, prove that OPM is a straight line.

6. $\overrightarrow{OA} = 3\boldsymbol{a}$ and $\overrightarrow{AQ} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$ and $\overrightarrow{BC} = \frac{1}{2}\boldsymbol{b}$. $M$is the midpoint of QB. Prove that AMC is a straight line.

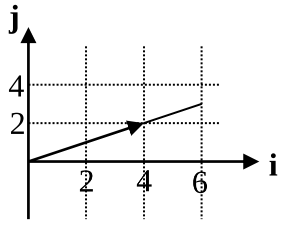


Section 1: Introduction to vectors

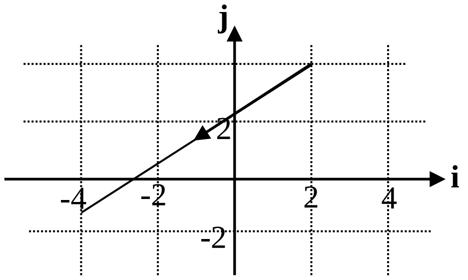
Exercise level 1

1. Write the following vectors in component form.

(i)



(ii)



2. Find the magnitude of each of these vectors.
   (i) $3\mathbf{i} + 4\mathbf{j}$
   (ii) $3\mathbf{i} - 6\mathbf{j}$
   (iii) $-\mathbf{i} - \mathbf{j}$

3. The points A, B and C have coordinates (4, -1), (3, 7) and (-2, 3) respectively.
   Find
   (i) $\overrightarrow{AB}$
   (ii) $\overrightarrow{BA}$
   (iii) $\overrightarrow{AC}$
   (iv) $\overrightarrow{CB}$

4. The vectors **a**, **b** and **c** are given by $\mathbf{a} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$

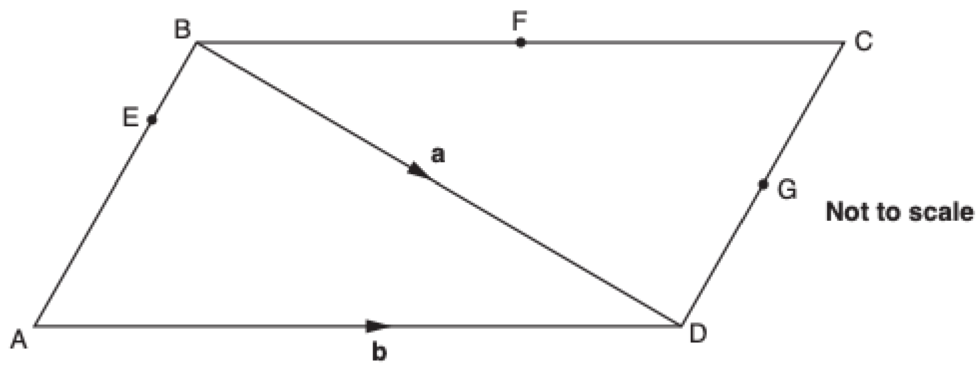
   Find the vectors
   (i) $\mathbf{b} + 2\mathbf{a}$
   (ii) $2\mathbf{c} - \mathbf{b}$
   (iii) $\mathbf{a} - \mathbf{b} + 3\mathbf{c}$

Vector Geometry review

Total Marks: 13

Question 1

$ABCD$ is a parallelogram.



$\overrightarrow{BD} = \mathbf{a}$ and $\overrightarrow{AD} = \mathbf{b}$.

F is the midpoint of BC.G is the midpoint of DC.$AE = 3EB$.

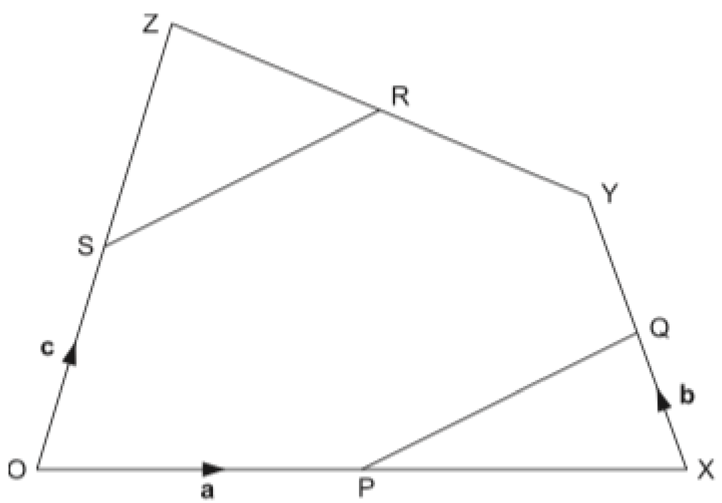
Prove that $\overrightarrow{EF}$ and $\overrightarrow{AG}$ are parallel.

$\overrightarrow{AG} =$ …………………….. $\overrightarrow{EF}$ ∴ they are parallel

(3 marks)

Question 2

P, Q, R and S are the midpoints of OX, XY, YZ and OZ respectively.



$\overrightarrow{OP} = \mathbf{a}$, $\overrightarrow{XQ} = \mathbf{b}$ and $\overrightarrow{OS} = \mathbf{c}$

Show that PQ is parallel to SR.

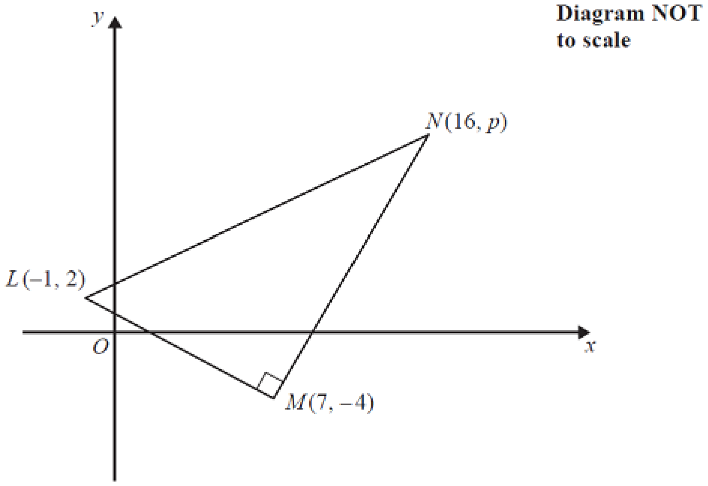
Input note: express $\overrightarrow{SR}$ in terms of $\overrightarrow{PQ}$.

$\overrightarrow{SR} =$ .... $\overrightarrow{PQ}$

(5 marks)

Question 3

The figure shows a right angled triangle LMN.
The points L and M have coordinates $(-1,2)$ and $(7,-4)$ respectively.

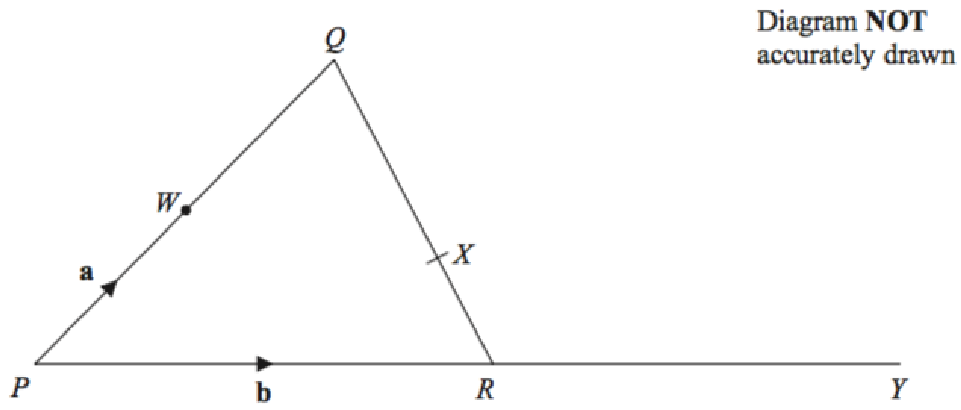


Find an equation for the straight line passing through the points L and M. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

....

(4 marks)

Question 4



PQR is a triangle.
The midpoint of *PQ* is *W*.
X is the point on *QR* such that *QX* : *XR* = 2 : 1
PRY is a straight line.

$\overrightarrow{PW} = \mathbf{a}$ and $\overrightarrow{PR} = \mathbf{b}$

Find $\overrightarrow{QR}$ in terms of $\mathbf{a}$and $\mathbf{b}$.

....

(1 mark)

Answers

Question 1

$\overrightarrow{AG} = 2$

$\overrightarrow{AG} = \frac{3}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}$ $\overrightarrow{AG} = 2\overrightarrow{EF}$ oe so are parallel.	**3**	**B2** for $\overrightarrow{AG} = \frac{3}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}$ or **M1** for $\mathbf{b} + \frac{1}{2}$(*their* part **(a)(i)**) oe

Question 2

$\overrightarrow{SR} = 1\,\overrightarrow{PQ}$

$\overrightarrow{ZY}$ = -2**c** + 2**a** + 2**b** $\overrightarrow{SR}$ = **c** + (-**c** + **a** + **b**) so $\overrightarrow{SR}$ = **a** + **b** $\overrightarrow{PQ}$ = **a** + **b** $\overrightarrow{SR}$ = $\overrightarrow{PQ}$ so they are parallel	**5** **1 AO1.3a** **2 AO2.2** **2 AO2.4b**	**M1** for $\overrightarrow{ZY}$ = -2**c** + 2**a** + 2**b** **M1** for $\overrightarrow{SR}$ = **c** + (-**c** + **a** + **b**) **M1** for $\overrightarrow{SR}$ = **a** + **b** **M1** for $\overrightarrow{PQ}$ = **a** + **b**

Question 3

$3x + 4y - 5 = 0$

Method 1: $gradient = \frac{y_1 - y_2}{x_1 - x_2} = \frac{2-(-4)}{-1-7}, = -\frac{3}{4}$	Method 2: $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$, so $\frac{y - y_1}{6} = \frac{x - x_1}{-8}$	M1, A1	
$y - 2 = -\frac{3}{4}(x+1)$ or $y + 4 = -\frac{3}{4}(x-7)$ or $y = their\ '-\frac{3}{4}'x + c$		M1	
$\Rightarrow \pm(4y + 3x - 5) = 0$		A1	(4)
Method 3: Substitute $x = -1$, $y = 2$ and $x = 7$, $y = -4$ into $ax + by + c = 0$		M1	
$-a + 2b + c = 0$ and $7a - 4b + c = 0$		A1	
Solve to obtain $a = 3$, $b = 4$ and $c = -5$ or multiple of these numbers		M1 A1	(4)

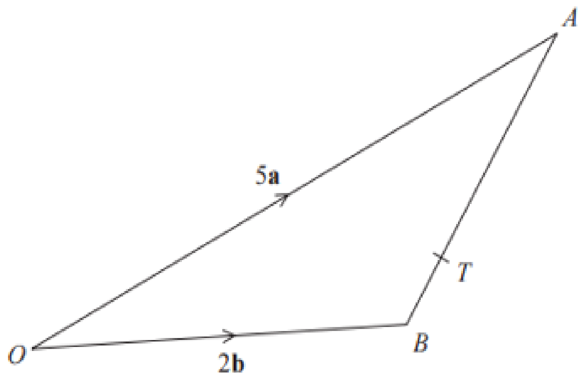
Question 4

$b - 2a$

b – 2**a**		B1

Exercise 3

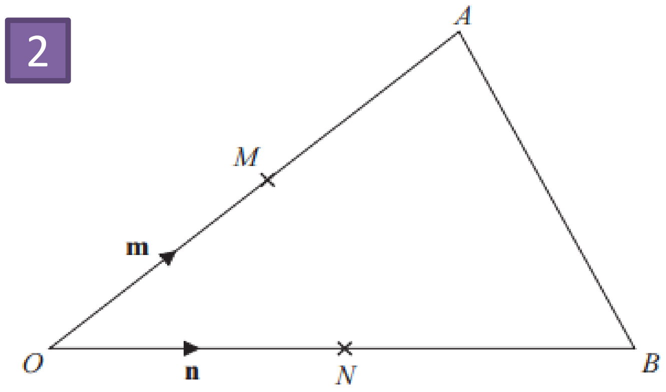
1



T is the point on AB such that $AT:TB = 5:1$. Show that OT is parallel to the vector $\boldsymbol{a} + 2\boldsymbol{b}$.

$$\overrightarrow{OT} = 2\boldsymbol{b} + \frac{1}{6}(-2\boldsymbol{b} + 5\boldsymbol{a})$$
$$= 2\boldsymbol{b} - \frac{1}{3}\boldsymbol{b} + \frac{5}{6}\boldsymbol{a}$$
$$= \frac{5}{6}\boldsymbol{a} + \frac{5}{3}\boldsymbol{b}$$
$$= \frac{5}{6}(\boldsymbol{a} + 2\boldsymbol{b})$$

2



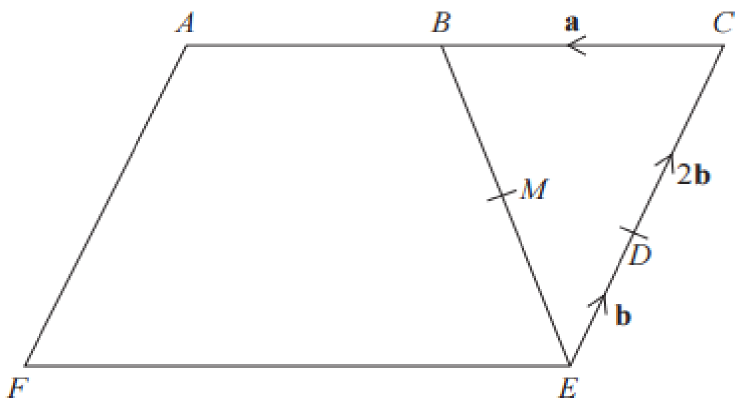
M is the midpoint of OA. N is the midpoint of OB. Prove that AB is parallel to MN.

$$\overrightarrow{AB} = -2\boldsymbol{m} + 2\boldsymbol{n}$$
$$= 2(-\boldsymbol{m} + \boldsymbol{n})$$
$$\overrightarrow{MN} = -\boldsymbol{m} + \boldsymbol{n}$$

$\overrightarrow{AB}$ is a multiple of $\overrightarrow{MN}$ $\therefore$ parallel.

Exercise 3

3

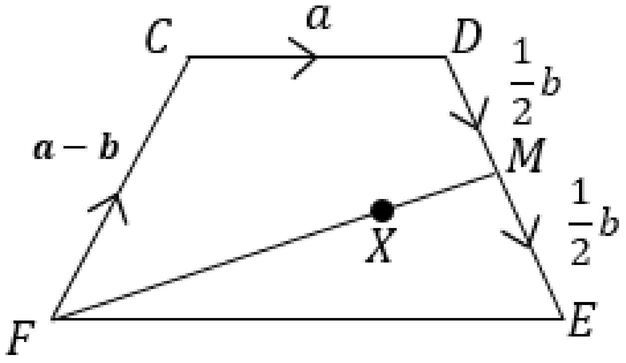


$ACEF$ is a parallelogram. B is the midpoint of AC. M is the midpoint of BE. Show that AMD is a straight line.

$$\overrightarrow{DM} = -\boldsymbol{b} + \frac{1}{2}(3\boldsymbol{b} + \boldsymbol{a})$$
$$= -\boldsymbol{b} + \frac{3}{2}\boldsymbol{b} + \frac{1}{2}\boldsymbol{a}$$
$$= \frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b} = \frac{1}{2}(\boldsymbol{a} + \boldsymbol{b})$$
$$\overrightarrow{DA} = 2\boldsymbol{b} + 2\boldsymbol{a} = 2(\boldsymbol{a} + \boldsymbol{b})$$

$\overrightarrow{DA}$ is a multiple of $\overrightarrow{DM}$ and D is a common point, so AMD is a straight line.

4



$\overrightarrow{CD} = \boldsymbol{a}$, $\overrightarrow{DE} = \boldsymbol{b}$ and $\overrightarrow{FC} = \boldsymbol{a} - \boldsymbol{b}$

i) Express $\overrightarrow{CE}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

$\boldsymbol{a} + \boldsymbol{b}$

ii) Prove that $\overrightarrow{FE}$ is parallel to $\overrightarrow{CD}$.

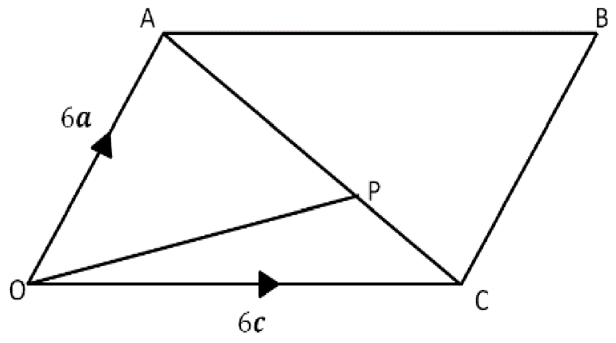
$\overrightarrow{FE} = a - b + a + b = 2\boldsymbol{a}$ which is a multiple of $\overrightarrow{CD}$

iii) X is the point on FM such that such that $FX:XM = 4:1$. Prove that C, X and E lie on the same straight line.

$$\overrightarrow{CX} = -\boldsymbol{a} + \boldsymbol{b} + \frac{4}{5}\left(2\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}\right) = \frac{3}{5}\boldsymbol{a} + \frac{3}{5}\boldsymbol{b}$$
$$= \frac{3}{5}(\boldsymbol{a} + \boldsymbol{b})$$
$$\overrightarrow{CE} = \boldsymbol{a} + \boldsymbol{b}$$

Exercise 3

5



$OABC$ is a parallelogram. P is the point on AC such that $AP = \frac{2}{3}AC$.

i) Find the vector $\overrightarrow{OP}$. Give your answer in terms of $\boldsymbol{a}$ and $\boldsymbol{c}$.

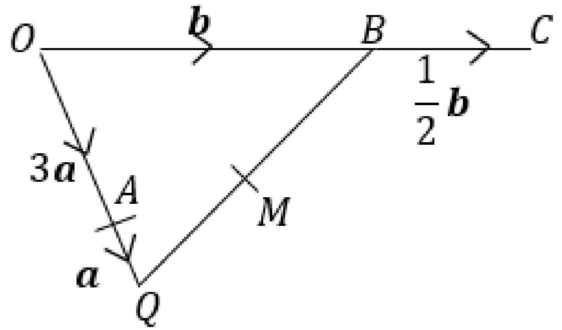
$$\overrightarrow{OP} = 6\boldsymbol{a} + \frac{2}{3}(-6\boldsymbol{a} + 6\boldsymbol{c})$$
$$= 2\boldsymbol{a} + 4\boldsymbol{c} = 2(\boldsymbol{a} + 2\boldsymbol{c})$$

ii) Given that the midpoint of CB is M, prove that OPM is a straight line.

$$\overrightarrow{OM} = 3\boldsymbol{a} + 6\boldsymbol{c} = 3(\boldsymbol{a} + 2\boldsymbol{c})$$

$\overrightarrow{OM}$ is a multiple of $\overrightarrow{OP}$ and O is a common point, therefore OPM is a straight line.

6



$\overrightarrow{OA} = 3\boldsymbol{a}$ and $\overrightarrow{AQ} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$ and $\overrightarrow{BC} = \frac{1}{2}\boldsymbol{b}$. $M$is the midpoint of QB. Prove that AMC is a straight line.

$$\overrightarrow{AM} = \boldsymbol{a} + \frac{1}{2}(-4\boldsymbol{a} + \boldsymbol{b}) = -\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$$
$$\overrightarrow{AC} = -3\boldsymbol{a} + \frac{3}{2}\boldsymbol{b} = 3\left(-\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}\right)$$

$\overrightarrow{AC}$ is a multiple of $\overrightarrow{AM}$ and A is a common point, therefore AMC is a straight line.

OCR AS Mathematics Vectors

Section 1: Introduction to vectors

Solutions to Exercise level 1

1. (i) $6\underset{\sim}{i} + 3\underset{\sim}{j}$ or $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$

   (ii) $-6\underset{\sim}{i} - 5\underset{\sim}{j}$ or $\begin{pmatrix} -6 \\ -5 \end{pmatrix}$

2. Find the magnitude of each of these vectors.

   (i) $\left|3\underset{\sim}{i} + 4\underset{\sim}{j}\right| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

   (ii) $\left|3\underset{\sim}{i} - 6\underset{\sim}{j}\right| = \sqrt{3^2 + (-6)^2} = \sqrt{45} = 3\sqrt{5}$

   (iii) $\left|-\underset{\sim}{i} - \underset{\sim}{j}\right| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$

3. (i) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 3 \\ 7 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 8 \end{pmatrix}$

   (ii) $\overrightarrow{BA} = -\overrightarrow{AB} = -\begin{pmatrix} -1 \\ 8 \end{pmatrix} = \begin{pmatrix} 1 \\ -8 \end{pmatrix}$

   (iii) $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \end{pmatrix}$

   (iv) $\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC} = \begin{pmatrix} 3 \\ 7 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

4. (i) $\underset{\sim}{b} + 2\underset{\sim}{a} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + 2\begin{pmatrix} 3 \\ -4 \end{pmatrix}$

   $= \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} 6 \\ -8 \end{pmatrix}$

   $= \begin{pmatrix} 8 \\ -3 \end{pmatrix}$

(ii) $$2\underset{\sim}{c} - \underset{\sim}{b} = 2\begin{pmatrix} -1 \\ -3 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$
$$= \begin{pmatrix} -2 \\ -6 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$
$$= \begin{pmatrix} -4 \\ -11 \end{pmatrix}$$

(iii) $$\underset{\sim}{a} - \underset{\sim}{b} + 3c = \begin{pmatrix} 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \end{pmatrix} + 3\begin{pmatrix} -1 \\ -3 \end{pmatrix}$$
$$= \begin{pmatrix} 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} -3 \\ -9 \end{pmatrix}$$
$$= \begin{pmatrix} -2 \\ -18 \end{pmatrix}$$