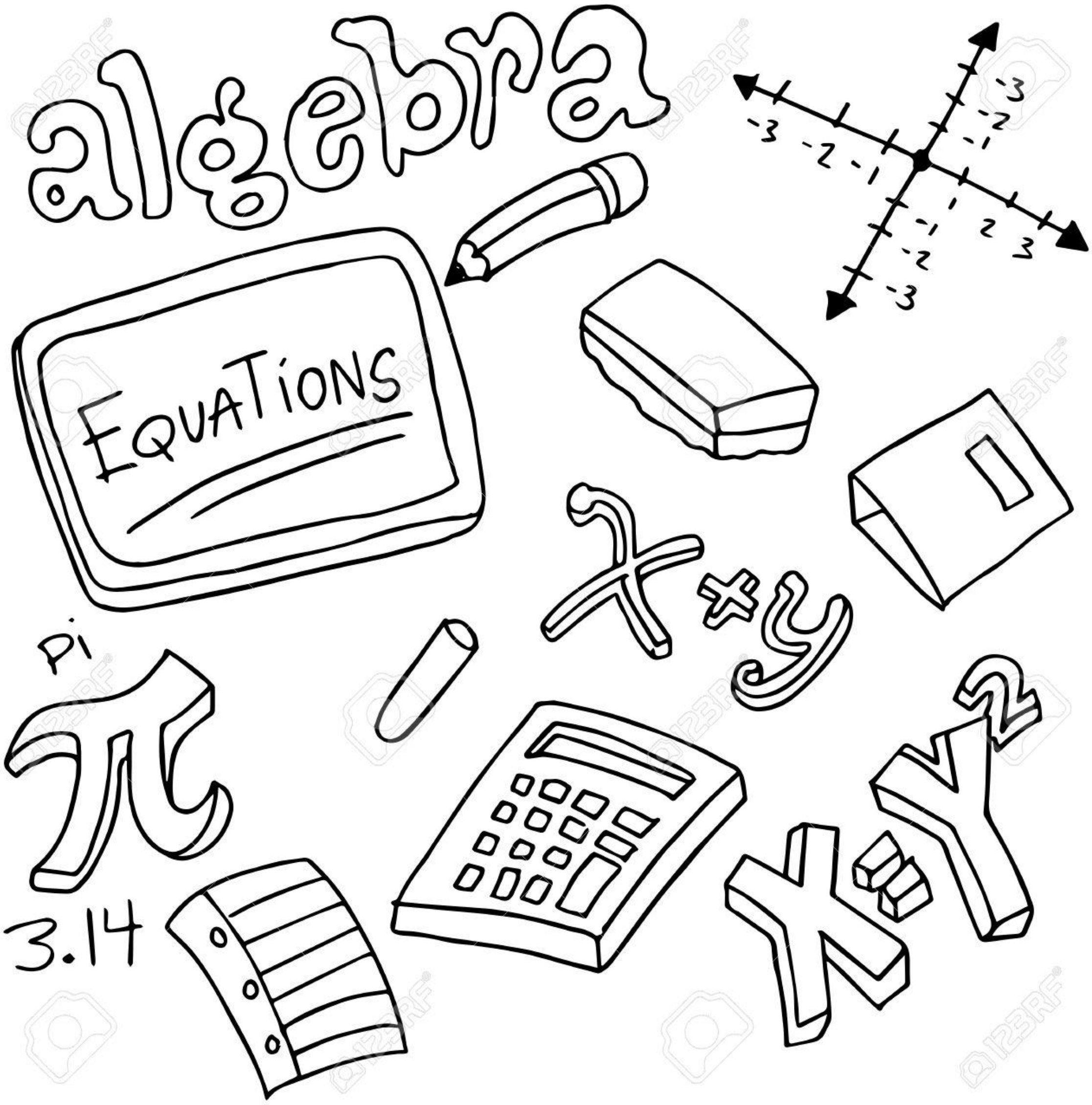
A-Level Further Mathematics



Bridging Course – Week 3

The following work requires a lot of time in order to ensure techniques are properly embedded and ready to be developed and built upon. Remember to **take regular breaks**, complete as much or as little additional practise as you need to successfully complete the section test for each section and **try your best**. There are links to instructional videos included, as well as notes and examples to help you with each topic area. Your maths teacher will review each of these topics during year 12.

Vector Geometry

During the third week of this bridging course you will be developing skills in **vector geometry** to include;

- Vector proof including the triangle law for vector addition
- Parallel vectors
- Collinear points
- The magnitude of a vector including unit vectors

You may wish to approach the work in the following way:

1. Read the notes and example pages and watch the video tutorials, making your own notes to file and keep
2. Complete as many of the questions from the two question sheets as you need to feel confident with the concepts
3. Complete and mark the review questions to assess your understanding
4. Return to step 1 if necessary

Helpful video tutorials for this topic:

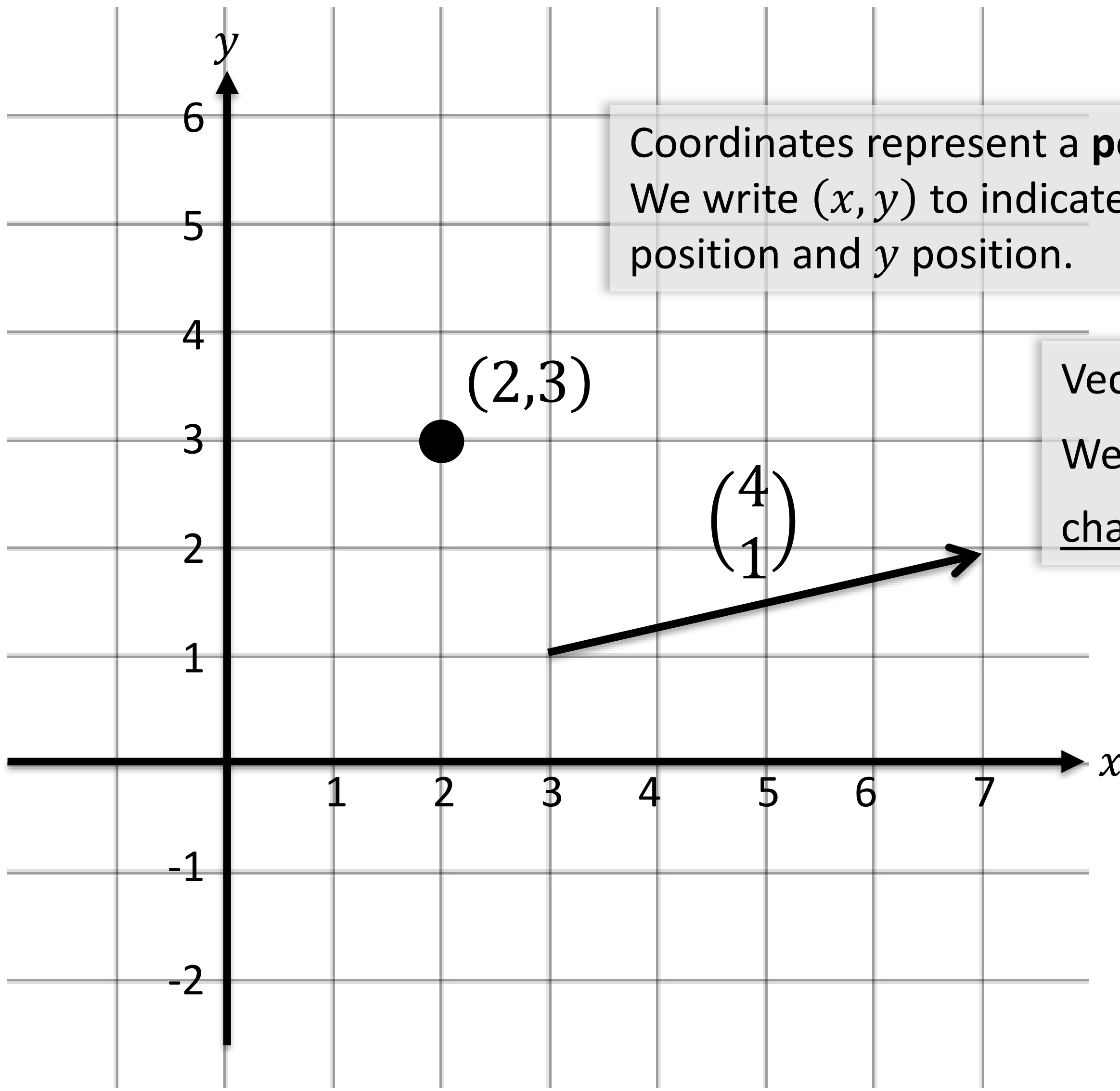
Hegarty Maths: Videos and quizzes: 622-636
Corbett Maths : https://corbettmaths.com/2016/04/25/vectors/
TL Maths This link has further videos for this section: https://sites.google.com/site/tlmaths314/home/a-level-maths-2017/full-a-level/j-vectors/01-introducing-vectors J1-01 TO J1-04

1. Read:
   Vector Geometry Notes 1 & 2 and Crucial Points

2. Complete
   Vector Geometry Exercises 1 & 2
   Answers for review are at the back of that booklet.

3. Complete Review Questions

Coordinates vs Vectors



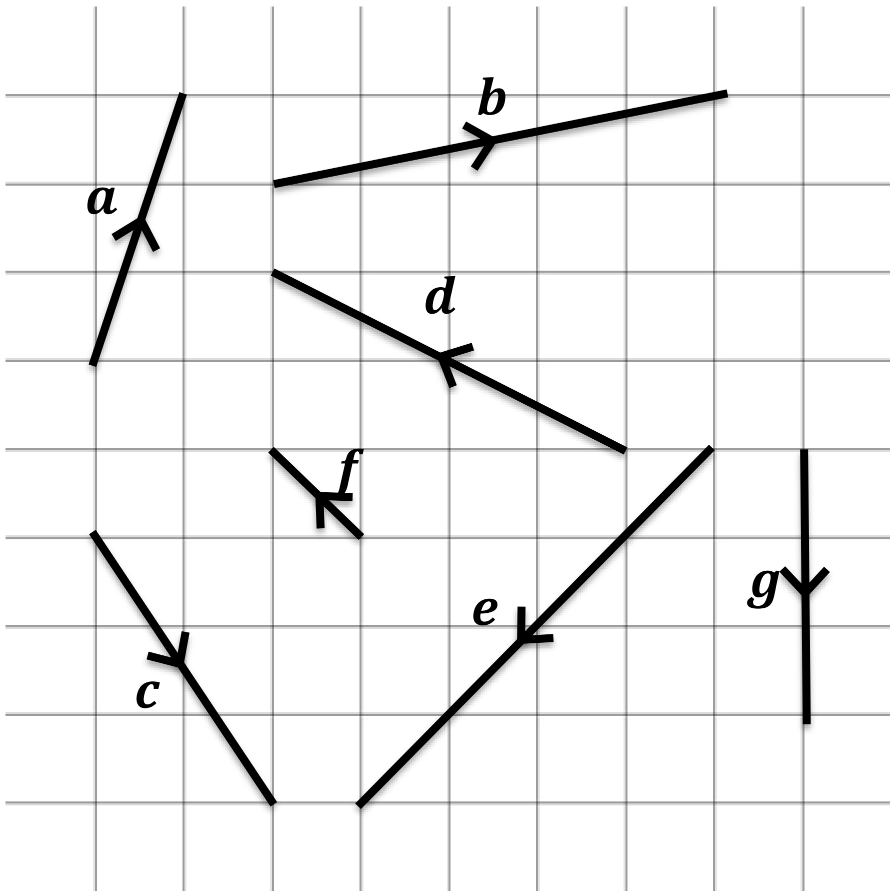
Coordinates represent a **position**. We write (x, y) to indicate the x position and y position.

Vectors represent a **movement**. We write $\begin{pmatrix}x\\y\end{pmatrix}$ to indicate the <u>change</u> in x and <u>change</u> in y.

Note we put the numbers vertically. Do **NOT** write $\left(\frac{4}{1}\right)$; there is no line and vectors are very different to fractions!

You may have seen vectors briefly before if you've done transformations; we can use them to describe the movement of a shape in a **translation**.

Vectors to represent movements



$$\boldsymbol{a} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad \boldsymbol{b} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

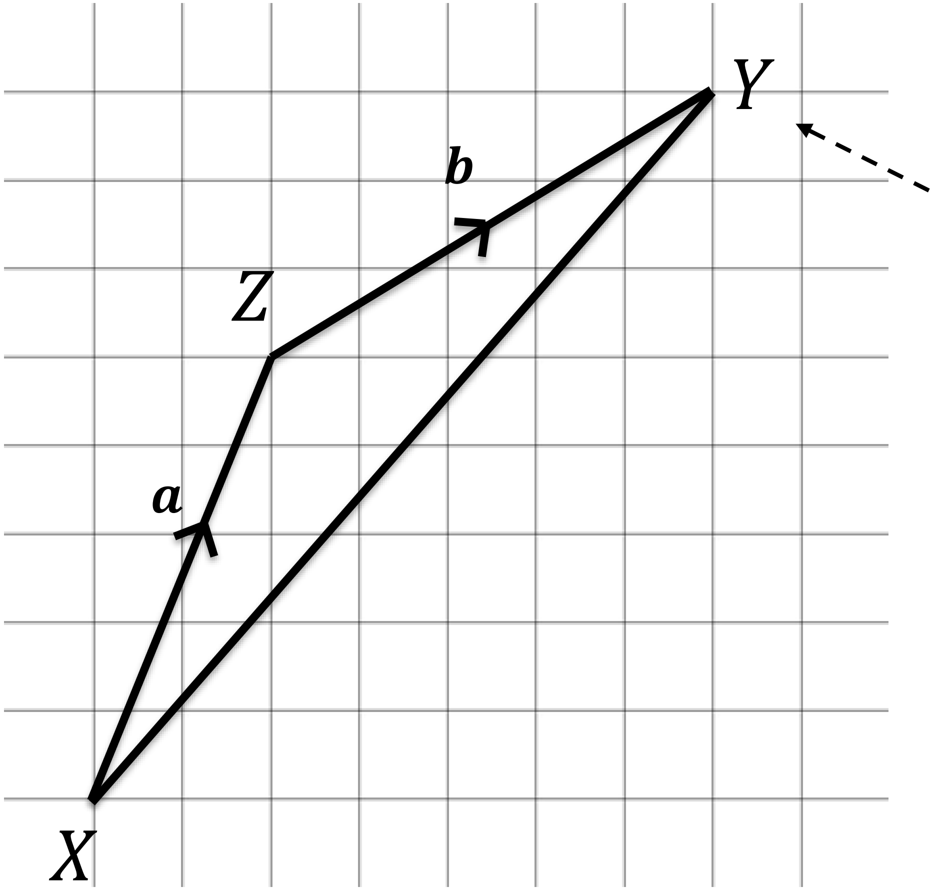
$$\boldsymbol{c} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \quad \boldsymbol{d} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

$$\boldsymbol{e} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} \quad \boldsymbol{f} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\boldsymbol{g} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$$

Writing Vectors

You're used to variables representing numbers in maths. They can also represent vectors!

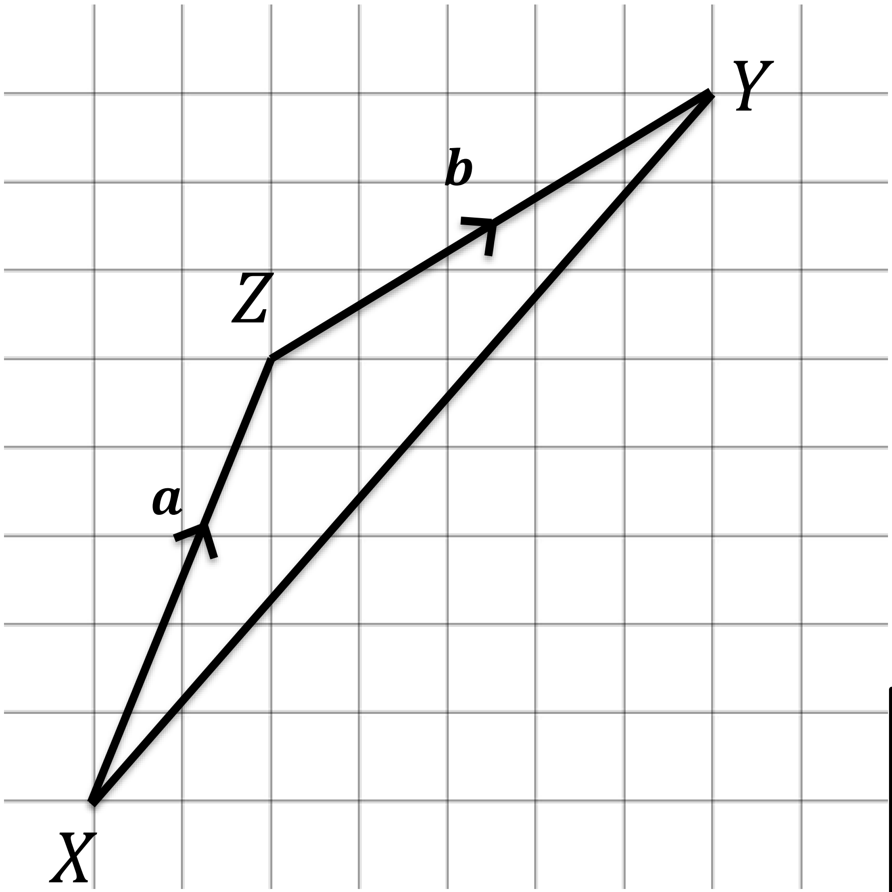


What can you say about how we use variables for vertices (points) vs variables for vectors?
*We use **capital letters** for vertices and **lower case letters** for vectors.*

There's 3 ways in which can represent the vector from point X to Z:

1. $\boldsymbol{a}$ (in **bold**)
2. $\underline{a}$ (with an 'underbar')
3. $\overrightarrow{XZ}$

Adding Vectors



$$\overrightarrow{XZ} = \boldsymbol{a} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$
$$\overrightarrow{ZY} = \boldsymbol{b} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$
$$\overrightarrow{XY} = \begin{pmatrix} 7 \\ 8 \end{pmatrix}$$

What do you notice about the numbers in $\begin{pmatrix} 7 \\ 8 \end{pmatrix}$ when compared to $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$?

We've simply added the x values and y values to describe the combined movement.

i.e.

$$\boldsymbol{a} + \boldsymbol{b} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ 8 \end{pmatrix}$$
$$\overrightarrow{XZ} + \overrightarrow{ZY} = \overrightarrow{XY}$$

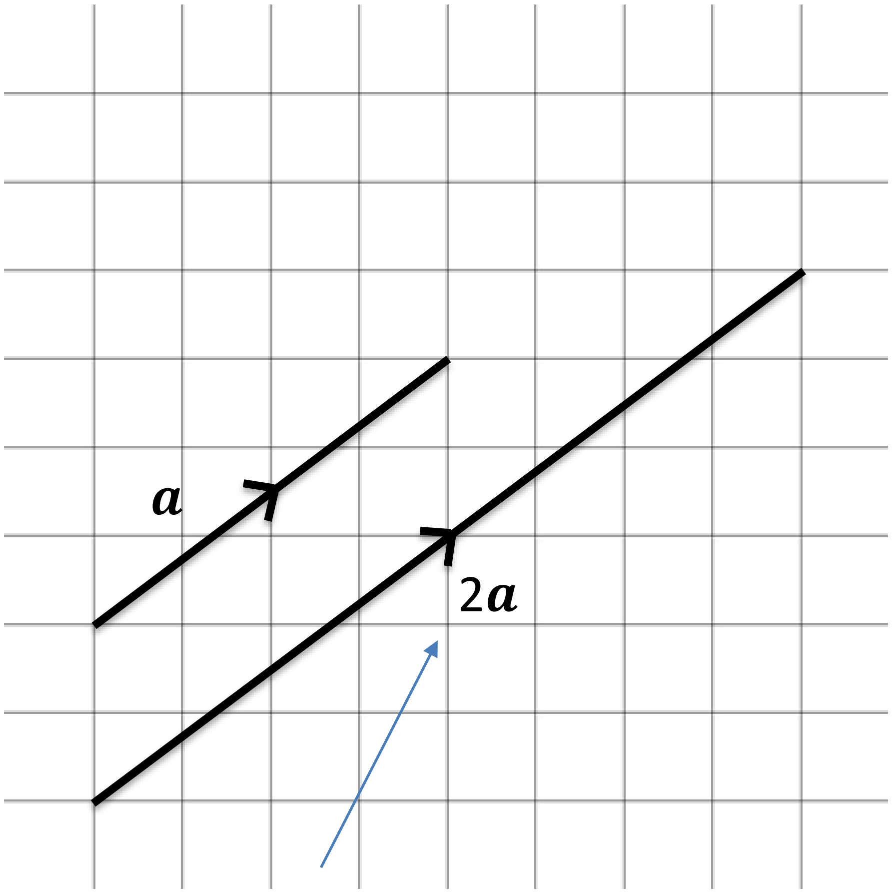
The point is that we can use <u>any route</u> to get from the start to finish, and the vector will always be the same.

- Route 1: We go from X to Y via Z.

$$\overrightarrow{XZ} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ 8 \end{pmatrix}$$

- Route 2: Use the direct line from X to Y: $\begin{pmatrix} 7 \\ 8 \end{pmatrix}$

Scaling Vectors



Note that vector letters are bold but scalars are not.

We can 'scale' a vector by multiplying it by a normal number, aptly known as a **scalar**.

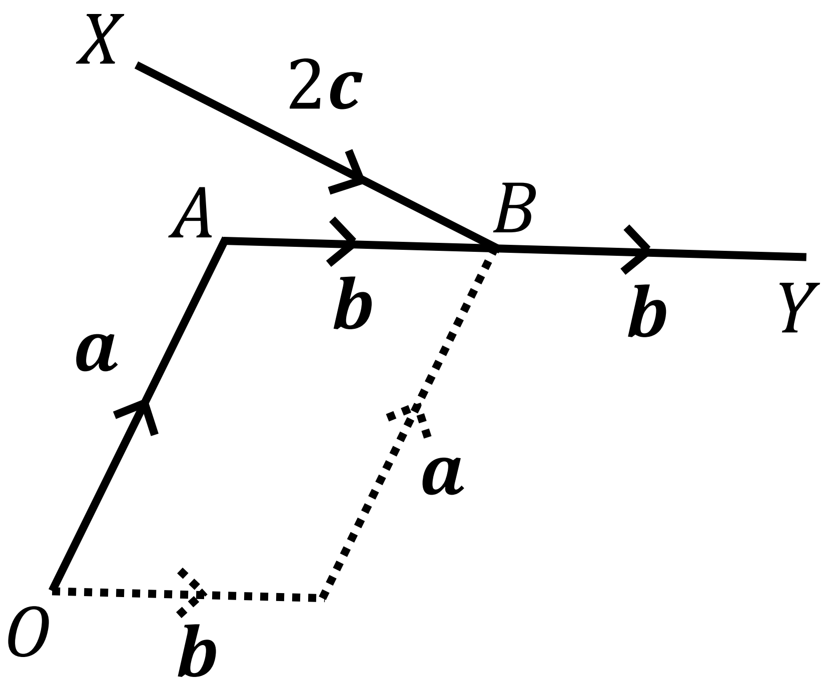
If $\boldsymbol{a} = \binom{4}{3}$, then

$$2\boldsymbol{a} = 2\binom{4}{3} = \binom{8}{6}$$

What is the same about $\boldsymbol{a}$ and $2\boldsymbol{a}$ and what is different?

Same: Same direction / Parallel
Different: The length of the vector, known as the **magnitude**, is longer.

More on Adding/Subtracting Vectors



If $\overrightarrow{OA} = \boldsymbol{a}$, $\overrightarrow{AB} = \boldsymbol{b}$ and $\overrightarrow{XB} = 2\boldsymbol{c}$, then find the following in terms of a, b and c:

$$\overrightarrow{OB} = \boldsymbol{a} + \boldsymbol{b}$$
$$\overrightarrow{OY} = \boldsymbol{a} + 2\boldsymbol{b}$$
$$\overrightarrow{AX} = \boldsymbol{b} - 2\boldsymbol{c}$$
$$\overrightarrow{XO} = 2\boldsymbol{c} - \boldsymbol{b} - \boldsymbol{a}$$
$$\overrightarrow{YX} = -\boldsymbol{b} - 2\boldsymbol{c}$$

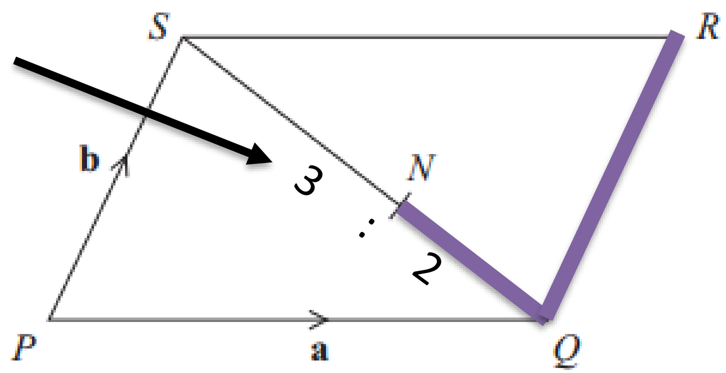
Note: Since $-\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix}$, subtracting a vector goes in the opposite direction.

The 'Two Parter' exam question

Many exams questions follow a two-part format:

a) Find a relatively easy vector

b) Find a harder vector that uses a fraction of your vector from part (a).



PQRS is a parallelogram.

N is the point on *SQ* such that $SN : NQ = 3 : 2$

$\overrightarrow{PQ} = \mathbf{a}$ $\quad \overrightarrow{PS} = \mathbf{b}$

(a) Write down, in terms of **a** and **b**, an expression for $\overrightarrow{SQ}$.

(b) Express $\overrightarrow{NR}$ in terms of **a** and **b**.

a

$$\overrightarrow{SQ} = -\boldsymbol{b} + \boldsymbol{a}$$

For (b), there's two possible paths to get from N to R: via S or via Q. But which is best?

In (a) we found $\boldsymbol{S}$ to $\boldsymbol{Q}$ rather than $\boldsymbol{Q}$ to $\boldsymbol{S}$, so it makes sense to go in this direction so that we can use our result in (a).

b

$$\overrightarrow{NR} = \frac{2}{5}\overrightarrow{SQ} + \boldsymbol{b}$$

$$= \frac{2}{5}(-\boldsymbol{b} + \boldsymbol{a}) + \boldsymbol{b}$$

$$= \frac{2}{5}\boldsymbol{a} + \frac{3}{5}\boldsymbol{b}$$

$\overrightarrow{QR}$ is also b because it is exactly the same movement as $\overrightarrow{PS}$.

Test Your Understanding

Edexcel June 2012

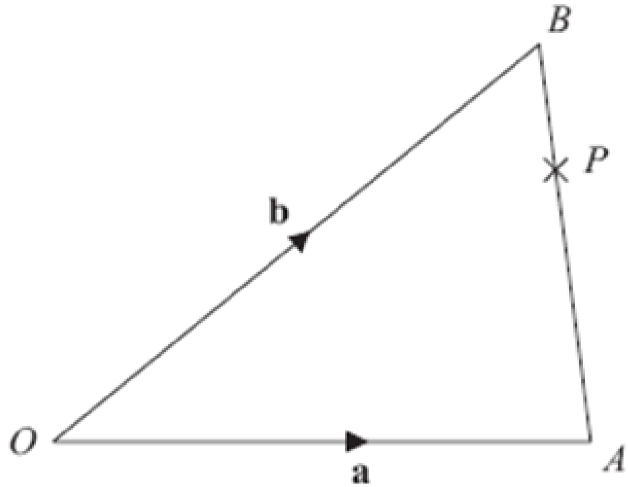


Diagram **NOT** accurately drawn

OAB is a triangle.

$\overrightarrow{OA} = \mathbf{a}$
$\overrightarrow{OB} = \mathbf{b}$

(a) Find $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

?

....

(1)

P is the point on AB such that $AP : PB = 3 : 1$

(b) Find $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
Give your answer in its simplest form.

?

You MUST expand and simplify.

....

Test Your Understanding

Edexcel June 2012

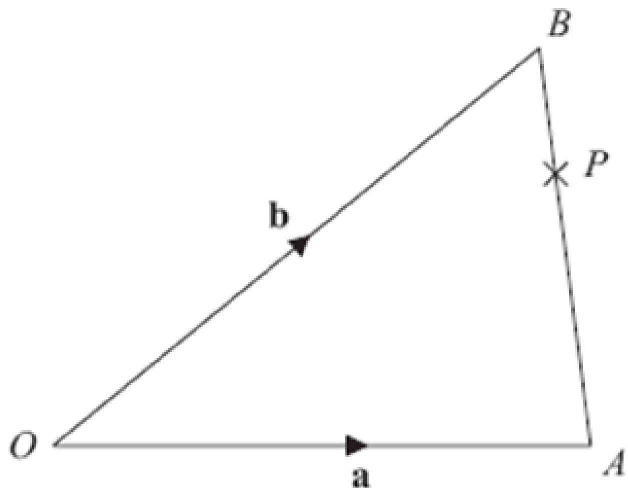


Diagram **NOT** accurately drawn

OAB is a triangle.

$\overrightarrow{OA} = \mathbf{a}$
$\overrightarrow{OB} = \mathbf{b}$

(a) Find $\overrightarrow{AB}$ in terms of **a** and **b**.

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$
$$= -\boldsymbol{a} + \boldsymbol{b}$$

(1)

P is the point on *AB* such that *AP* : *PB* = 3 : 1

(b) Find $\overrightarrow{OP}$ in terms of **a** and **b**.
Give your answer in its simplest form.

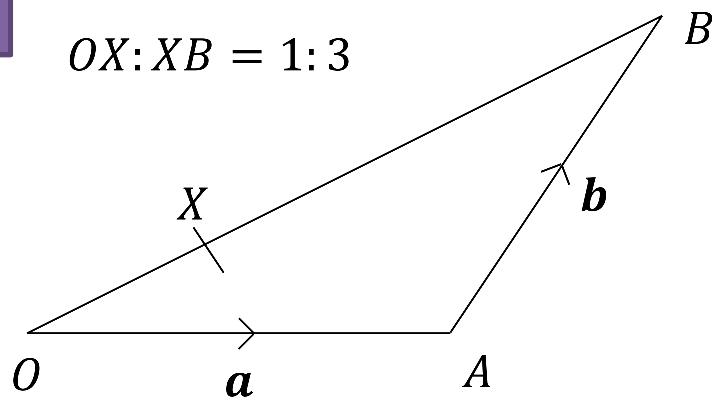
$$\overrightarrow{OP} = \boldsymbol{a} + \frac{3}{4}\overrightarrow{AB}$$
$$= \boldsymbol{a} + \frac{3}{4}(-\boldsymbol{a} + \boldsymbol{b})$$
$$= \frac{1}{4}\boldsymbol{a} + \frac{3}{4}\boldsymbol{b}$$

You MUST expand and simplify.

Further Test Your Understanding

A

$OX:XB = 1:3$



$\overrightarrow{AX} =$ First Step?

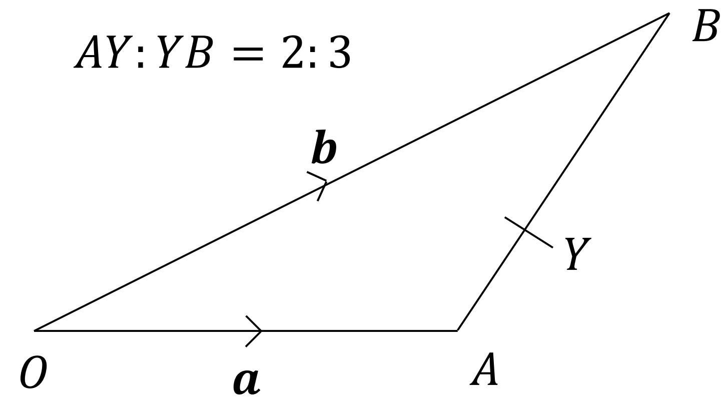
$=$?

$=$

$=$

B

$AY:YB = 2:3$



$\overrightarrow{OY} =$ First Step?

$=$?

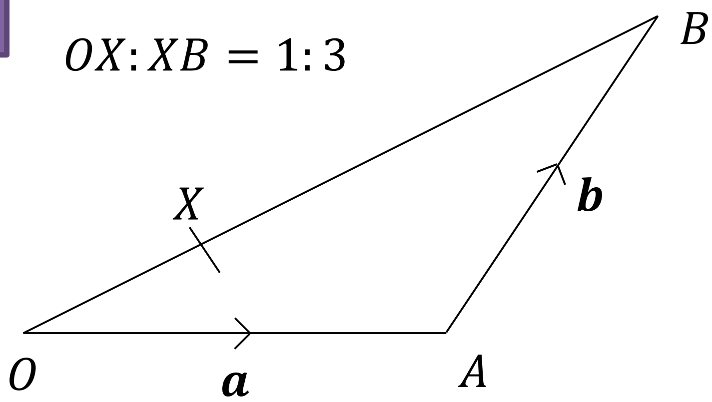
$=$

$=$

Further Test Your Understanding

A

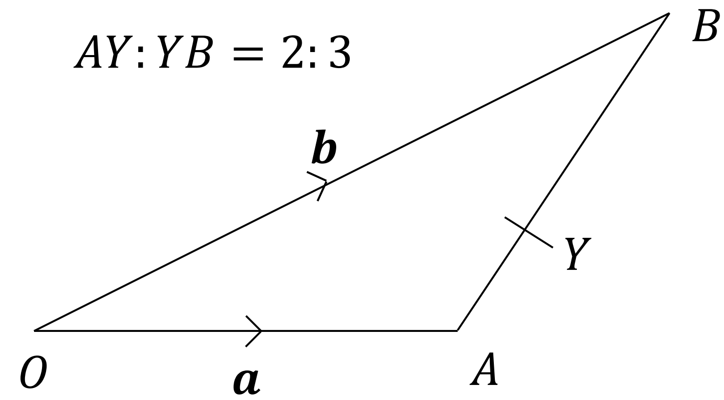
$OX:XB = 1:3$



$$\overrightarrow{AX} = -\boldsymbol{a} + \frac{1}{4}\overrightarrow{OB}$$
$$= -\boldsymbol{a} + \frac{1}{4}(\boldsymbol{a} + \boldsymbol{b})$$
$$= -\boldsymbol{a} + \frac{1}{4}\boldsymbol{a} + \frac{1}{4}\boldsymbol{b}$$
$$= -\frac{3}{4}\boldsymbol{a} + \frac{1}{4}\boldsymbol{b}$$

B

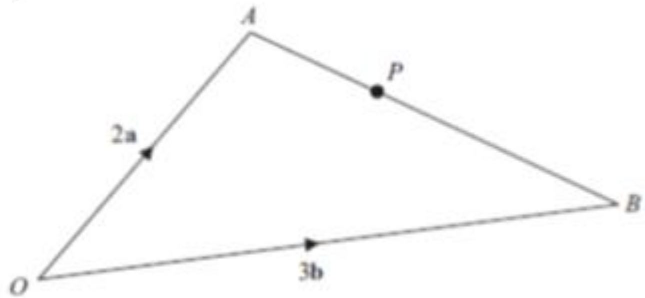
$AY:YB = 2:3$



$$\overrightarrow{OY} = \boldsymbol{a} + \frac{2}{5}\overrightarrow{AB}$$
$$= \boldsymbol{a} + \frac{2}{5}(-\boldsymbol{a} + \boldsymbol{b})$$
$$= \boldsymbol{a} - \frac{2}{5}\boldsymbol{a} + \frac{2}{5}\boldsymbol{b}$$
$$= \frac{3}{5}\boldsymbol{a} + \frac{2}{5}\boldsymbol{b}$$

Types of vector 'proof' questions

[June 2011 NonCalc] 26.



OAB is a triangle.

(*a*) Find *AB* in terms of **a** and **b**.

P is the point on *AB* such that $AP : PB = 2 : 3$

(*b*) Show that $\overrightarrow{OP}$ is parallel to the vector $\mathbf{a} + \mathbf{b}$.

"Show that … is parallel to …"

*(b) Prove that $\overrightarrow{OX} = \frac{2}{5}\overrightarrow{OY}$

"Prove these two vectors are equal."

[Nov 2012 NonCalc] 28.

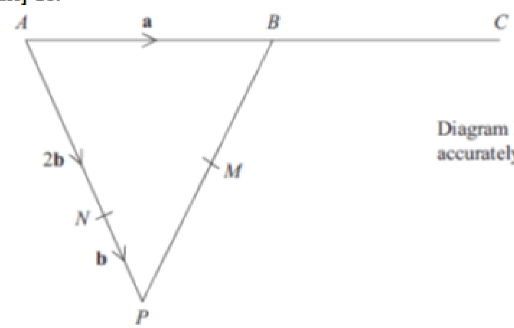


Diagram **NOT** accurately drawn

APB is a triangle.
N is a point on *AP*.

$\overrightarrow{AB} = \mathbf{a}$ $\overrightarrow{AN} = 2\mathbf{b}$ $\overrightarrow{NP} = \mathbf{b}$

(a) Find the vector $\overrightarrow{PB}$, in terms of **a** and **b**.

....

(1)

B is the midpoint of *AC*.
M is the midpoint of *PB*.

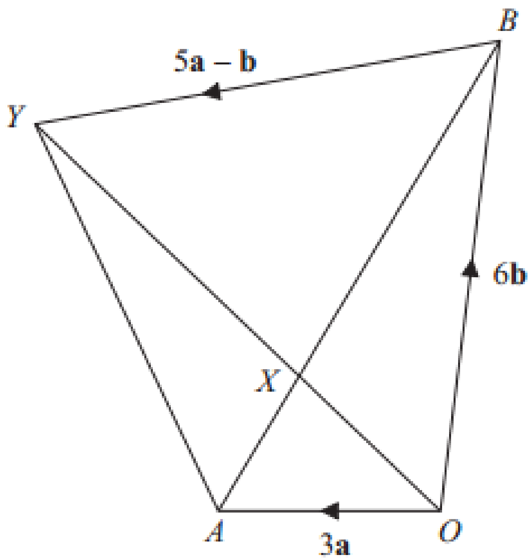
*(b) Show that *NMC* is a straight line.

(4)

(Total for Question 28 is 5 marks)

"Prove that … is a straight line."

Showing vectors are equal



(a) Express $\overrightarrow{AB}$ in terms of **a** and **b**.

Mark	Notes
1	B1 for 6**b** – 3**a** oe

X is the point on AB such that $AX : XB = 1 : 2$ and $\overrightarrow{BY} = 5\mathbf{a} - \mathbf{b}$

*(b) Prove that $\overrightarrow{OX} = \frac{2}{5}\overrightarrow{OY}$

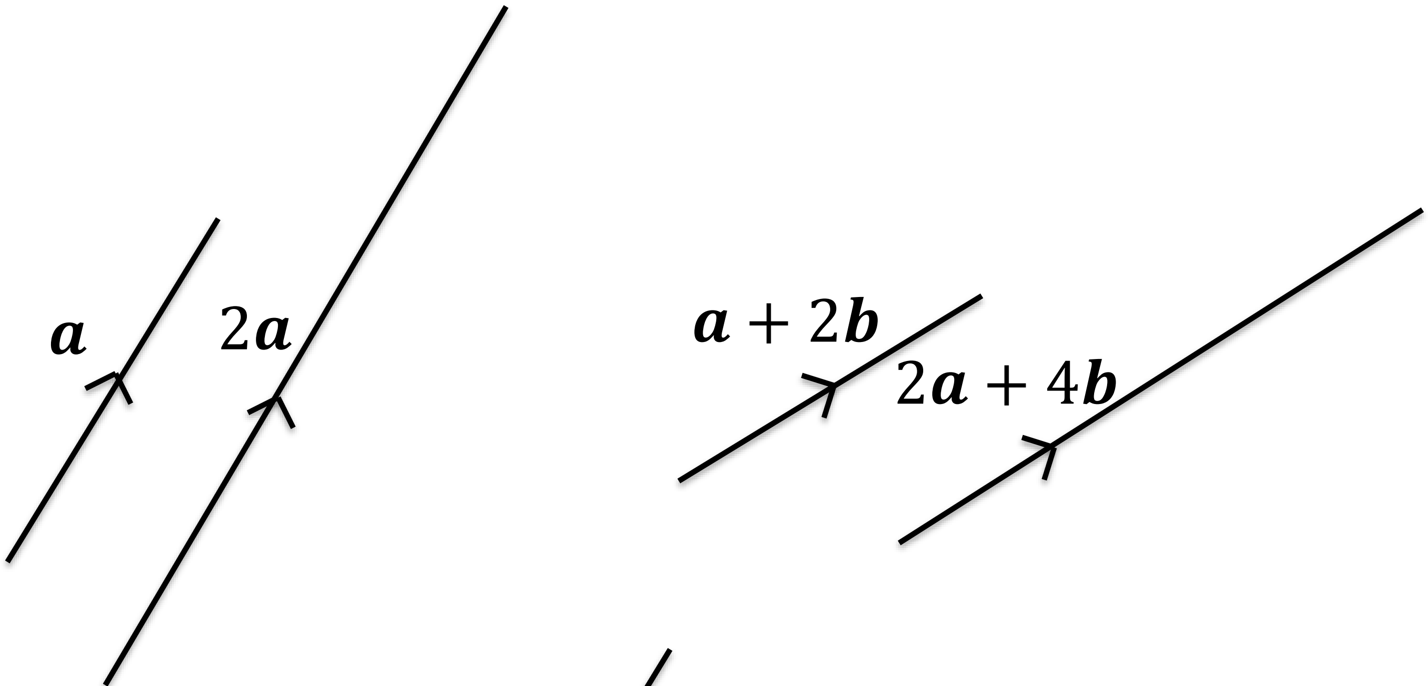
$$\frac{2}{5}\overrightarrow{OY} = \frac{2}{5}(6b + 5a - b)$$
$$= 2a + 2b$$

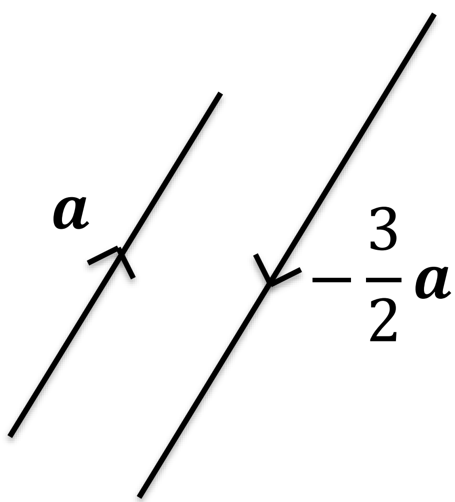
$$\overrightarrow{OX} = 3a + \frac{1}{3}(-3a + 6b)$$
$$= 2a + 2b$$

$$\therefore \overrightarrow{OX} = \frac{2}{5}\overrightarrow{OY}$$

With proof questions you should restate the thing you are trying to prove, as a 'conclusion'.

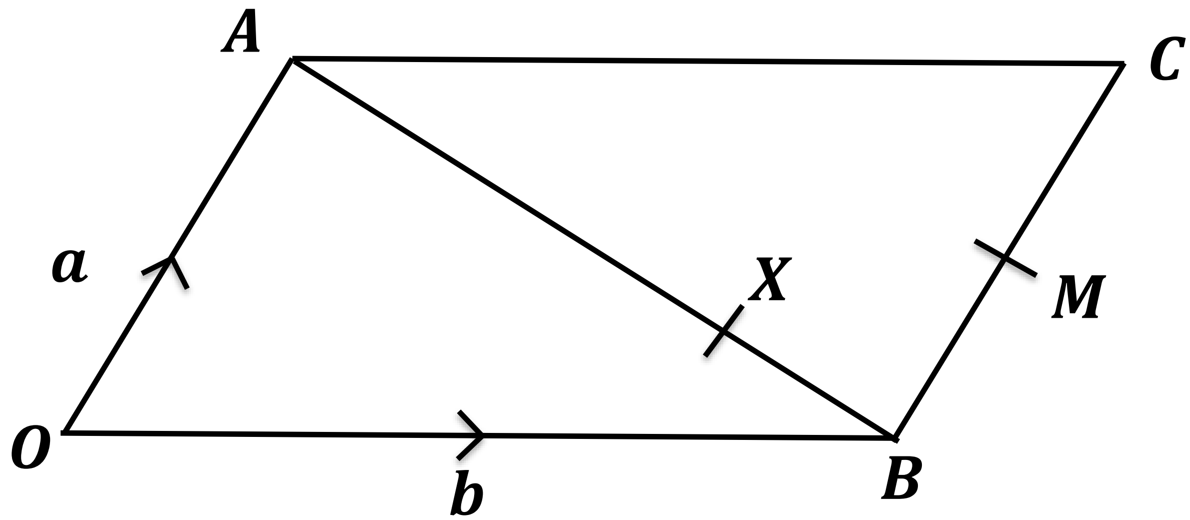
What do you notice?





✏ Vectors are parallel if one <u>is a multiple of</u> another.

How to show two vectors are parallel



X is a point on AB such that $AX:XB = 3:1$. M is the midpoint of BC.
Show that $\overrightarrow{XM}$ is parallel to $\overrightarrow{OC}$.

$\overrightarrow{OC} = a + b$

$\overrightarrow{XM} = \frac{1}{4}(-a+b) + \frac{1}{2}a = \frac{1}{4}a + \frac{1}{4}b$

$= \frac{1}{4}(a+b)$

$\overrightarrow{XM}$ **<u>is a multiple of</u>** $\overrightarrow{OC}$ $\therefore$ parallel.

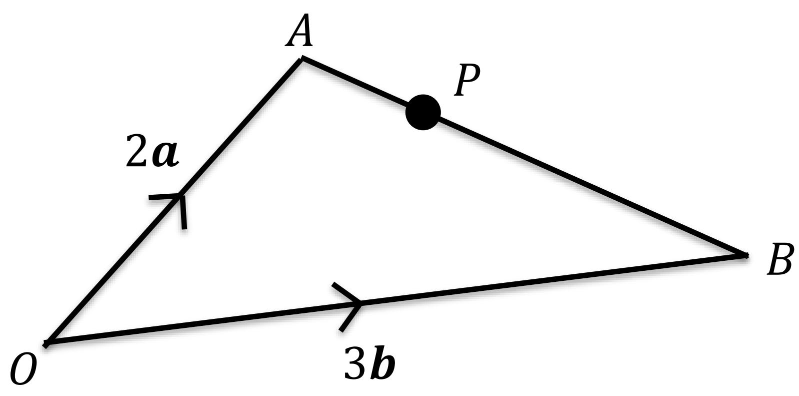
For any proof question always find the vectors involved first, in this case $\overrightarrow{XM}$ and $\overrightarrow{OC}$.

The key is to factor out a scalar such that we see the same vector.

The magic words here are "*is a multiple of*".

Test Your Understanding

Edexcel June 2011 Q26



a) Find $\overrightarrow{AB}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

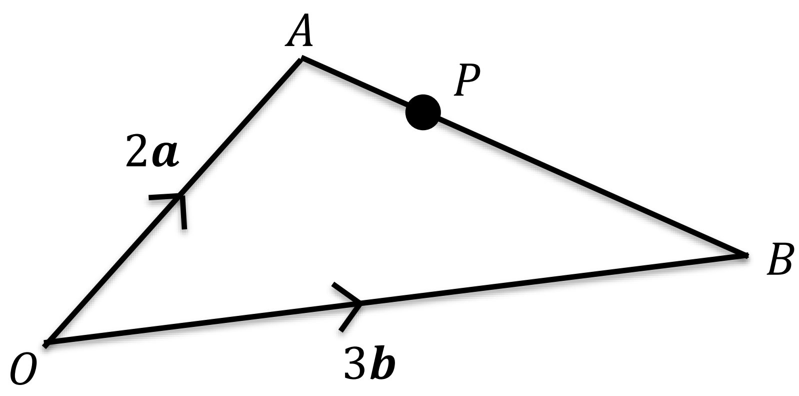
?

b) P is the point on AB such that $AP : PB = 2 : 3$.
Show that $\overrightarrow{OP}$ is parallel to the vector $\boldsymbol{a} + \boldsymbol{b}$.

?

Test Your Understanding

Edexcel June 2011 Q26



a) Find $\overrightarrow{AB}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

$$-2\boldsymbol{a} + 3\boldsymbol{b}$$

b) P is the point on AB such that $AP:PB = 2:3$.
Show that $\overrightarrow{OP}$ is parallel to the vector $\boldsymbol{a} + \boldsymbol{b}$.

M1 for $2\mathbf{a} \pm \frac{2}{5}('3\mathbf{b} - 2\mathbf{a}')$ OR $3\mathbf{b} \pm \frac{3}{5}('2\mathbf{a} - 3\mathbf{b}')$

A1 for $\frac{6}{5}\mathbf{a} + \frac{6}{5}\mathbf{b}$ oe

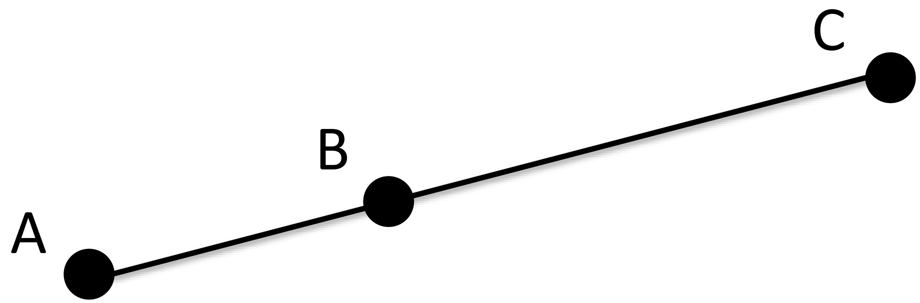
A1 for $\frac{6}{5}(\mathbf{a} + \mathbf{b})$ is parallel to $\mathbf{a} + \mathbf{b}$ oe

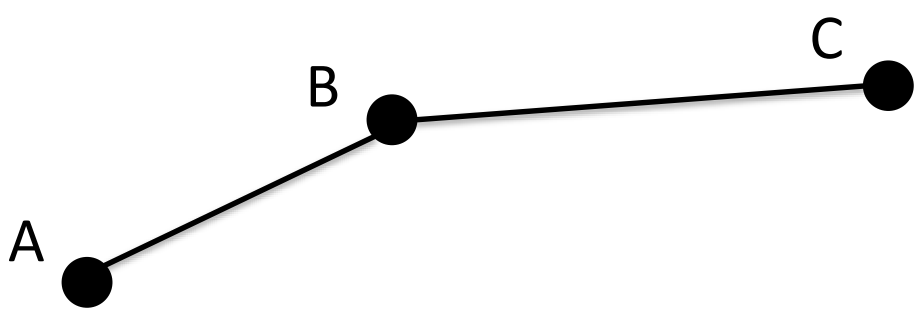
Proving three points form a straight line

Points A, B and C form a straight line if:

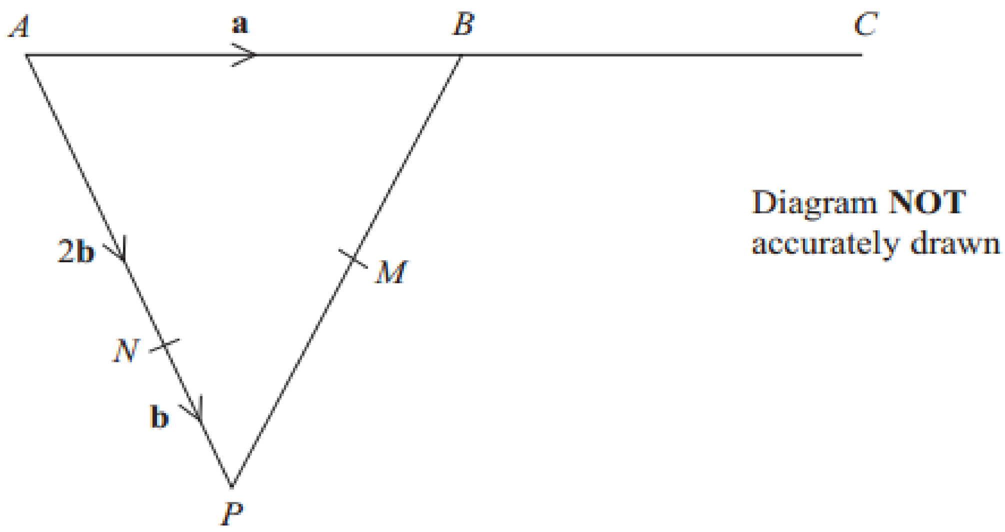
$\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel (and B is a common point).

Alternatively, we could show $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel. This tends to be easier.





Straight Line Example



$$\overrightarrow{AN} = 2\boldsymbol{b}, \qquad \overrightarrow{NP} = \boldsymbol{b}$$

B is the midpoint of AC. M is the midpoint of PB.

a) Find $\overrightarrow{PB}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

b) Show that NMC is a straight line.

a

$$-3\boldsymbol{b} + \boldsymbol{a}$$

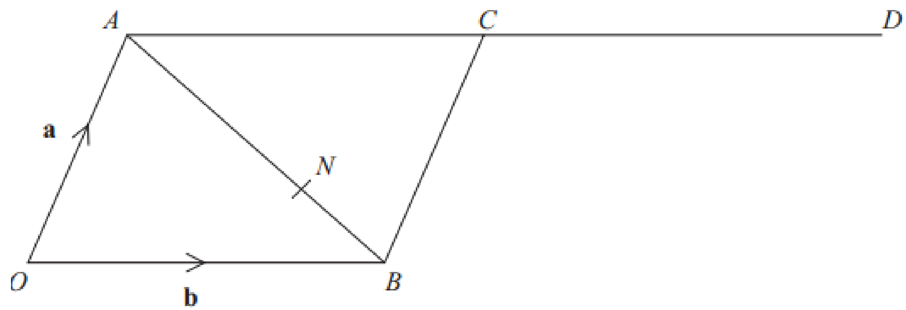
b

$$\overrightarrow{NM} = b + \frac{1}{2}(a - 3b)$$
$$= b + \frac{1}{2}a - \frac{3}{2}b$$
$$= \frac{1}{2}a - \frac{1}{2}b$$
$$= \frac{\mathbf{1}}{\mathbf{2}}(\boldsymbol{a} - \boldsymbol{b})$$
$$\overrightarrow{NC} = -2b + 2a$$
$$= \mathbf{2}(\boldsymbol{a} - \boldsymbol{b})$$

$\overrightarrow{NC}$ is a multiple of $\overrightarrow{NM}$ ∴ $\overrightarrow{NC}$ is parallel to $\overrightarrow{NM}$.
N is a common point.
∴ NMC is a straight line.

Test Your Understanding

November 2013 1H Q24



$\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$

D is the point such that $\overrightarrow{AC} = \overrightarrow{CD}$

The point N divides AB in the ratio $2:1$.

(a) Write an expression for $\overrightarrow{ON}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

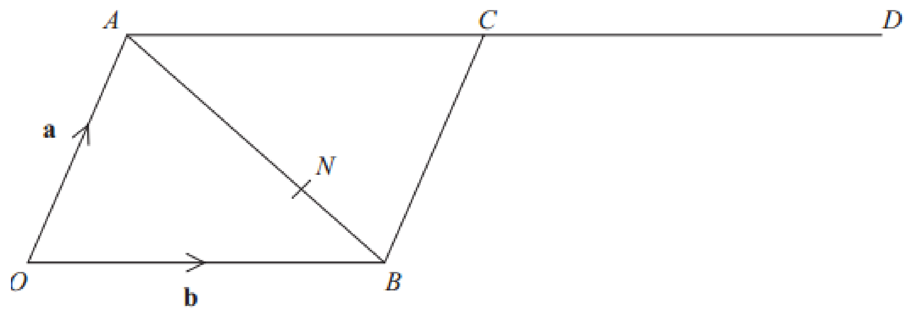
?

(b) Prove that OND is a straight line.

?

Test Your Understanding

November 2013 1H Q24



$\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$

D is the point such that $\overrightarrow{AC} = \overrightarrow{CD}$

The point N divides AB in the ratio $2:1$.

(a) Write an expression for $\overrightarrow{ON}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

$$\overrightarrow{\boldsymbol{ON}} = \boldsymbol{b} + \frac{\mathbf{1}}{\mathbf{3}}(-\boldsymbol{b} + \boldsymbol{a}) = \frac{\mathbf{1}}{\mathbf{3}}\boldsymbol{a} + \frac{\mathbf{2}}{\mathbf{3}}\boldsymbol{b}$$

(b) Prove that OND is a straight line.

$$\overrightarrow{\boldsymbol{OD}} = \boldsymbol{a} + \mathbf{2}\boldsymbol{b}$$

$$\overrightarrow{\boldsymbol{ON}} = \frac{\mathbf{1}}{\mathbf{3}}(\boldsymbol{a} + \mathbf{2}\boldsymbol{b})$$

$\overrightarrow{\boldsymbol{ON}}$ is a multiple of $\overrightarrow{\boldsymbol{OD}}$ and $\boldsymbol{O}$ is a common point.

$\therefore \boldsymbol{OND}$ is a straight line.

Section 1: Introduction to vectors

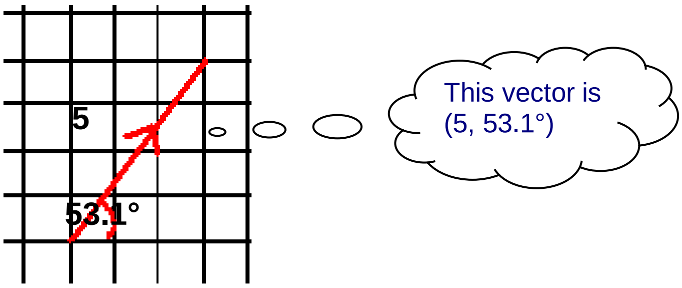
Notes and Examples

These notes contain subsections on

Vectors in magnitude-direction form or component form

A **vector** quantity has both **magnitude** (size) and **direction**. A **scalar** quantity has magnitude only.
Vectors may be written in bold, **a**, or underlined, a, or with an arrow above, $\vec{a}$.
Two vectors are **equal** if they have the same magnitude and direction.
You need to be able to write down a vector in two different ways:

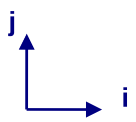
- Magnitude-direction form (r, θ)

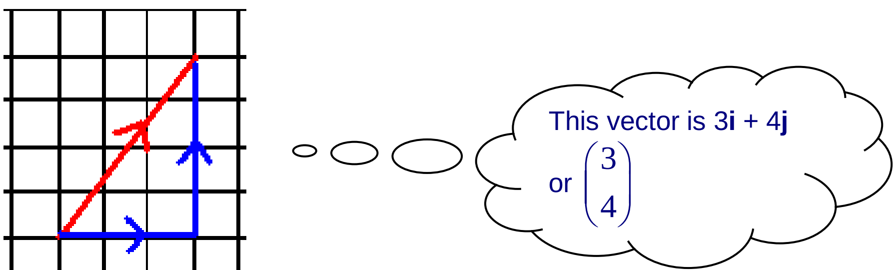


The angle, θ is measured in an **anticlockwise** direction from the **positive *x* axis**.

A unit vector has a magnitude of 1.

- Component form
  The vector is expressed using the unit vectors **i** and **j**.
  i is a unit vector in the x direction.
  j is a unit vector in the y direction.





OCR AS Maths Vectors 1 Notes and Examples

The magnitude of a vector given in component form is found using Pythagoras's theorem.
So the vector $\mathbf{c} = a\mathbf{i} + b\mathbf{j}$ has magnitude:

$$|\mathbf{c}| = \sqrt{a^2 + b^2}$$

The magnitude of a vector is sometimes called the **modulus**.

A vector given in magnitude-direction form can be written in component form using the rule:

$$\mathbf{a} = (r, \theta) \Rightarrow \mathbf{a} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j}$$

The following two examples show you how to convert between the two forms.

Example 1
Write the vectors:
(i) (10, 70°) (ii) (5, 230º)
in component form.

Solution

(i) Using the formula $\mathbf{a} = (r, \theta) \Rightarrow \mathbf{a} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j}$

$$(10, 70°) = 10\cos 70°\mathbf{i} + 10\sin 70°\mathbf{j}$$
$$= 3.42\mathbf{i} + 9.40\mathbf{j}$$

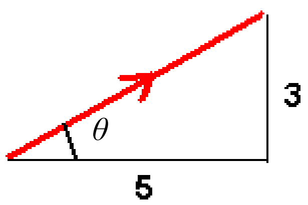
(ii) $$(5, 230°) = 5\cos 230°\mathbf{i} + 5\sin 230°\mathbf{j}$$
$$= -3.21\mathbf{i} - 3.83\mathbf{j}$$

Example 2
Write the vector:
(i) $5\mathbf{i} + 3\mathbf{j}$ (ii) $-2\mathbf{i} - 4\mathbf{j}$
in magnitude-direction form.

Solution

(i) The magnitude of the vector $5\mathbf{i} + 3\mathbf{j}$ is $\sqrt{5^2 + 3^2} = \sqrt{25 + 9} = \sqrt{34}$

Use a sketch to help you find the direction:



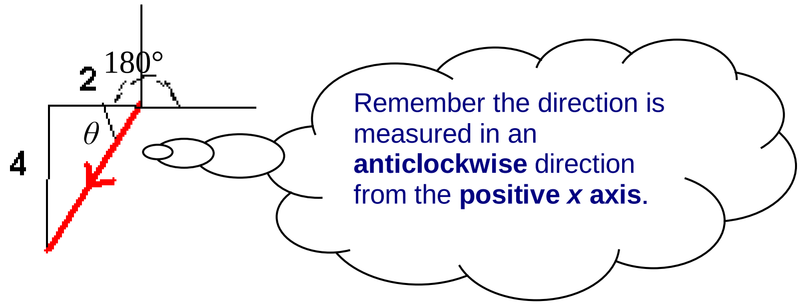
The angle θ gives the direction of the vector.

$\tan\theta = \frac{3}{5} \Rightarrow \theta = 31.0°$

So $5\mathbf{i} + 3\mathbf{j} = (\sqrt{34}, 31.0°)$

(ii) The magnitude of the vector $-2\mathbf{i} - 4\mathbf{j}$ is $\sqrt{(-2)^2 + (-4)^2} = \sqrt{4+16} = \sqrt{20}$

Use a sketch to help you find the direction:



The angle $\theta + 180°$ gives the direction of the vector.

$\tan\theta = \frac{4}{2} \Rightarrow \theta = 63.4°$ so the direction is 63.4° + 180°= 243.4°

So $-2\mathbf{i} - 4\mathbf{j} = (\sqrt{20}, 243.4°)$

Multiplying a vector by a scalar

To multiply a vector by a scalar (number) simply multiply each of the components by the scalar.

Note:

- when the scalar is positive the direction of the vector remains the same but the length (or magnitude) of the vector is multiplied by the same factor.
- when the scalar is negative the direction of the vector is reversed and the length (or magnitude) of the vector is multiplied by the magnitude of the scale factor.

Example 3

$\mathbf{a} = 2\mathbf{i} - 3\mathbf{j}$

(i) Find $4\mathbf{a}$

(ii) Find the value of $|\mathbf{a}|$

(iii) Write down the value of $|4\mathbf{a}|$

Solution

(i) $4\mathbf{a} = 4(2\mathbf{i} - 3\mathbf{j}) = 8\mathbf{i} - 12\mathbf{j}$

(ii) $|\mathbf{a}| = \sqrt{2^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$

(iii) $|4\mathbf{a}| = 4|\mathbf{a}| = 4\sqrt{13}$

OCR AS Maths Vectors 1 Notes and Examples

Example 4

$\mathbf{a} = 5\mathbf{i} - 7\mathbf{j}$

Find $-\mathbf{a}$.

Solution

$\mathbf{a} = 5\mathbf{i} - 7\mathbf{j}$

So $-\mathbf{a} = -5\mathbf{i} + 7\mathbf{j}$

This is the same as multiplying by -1.
Just reverse the signs!

Adding and subtracting vectors

To add/subtract vectors simply add/subtract the **i** components and then the **j** components.
Adding two or more vectors is called finding the **resultant**.

Example 5

(i) Find the resultant of $(5\mathbf{i} - 7\mathbf{j})$ and $(-3\mathbf{i} + 2\mathbf{j})$

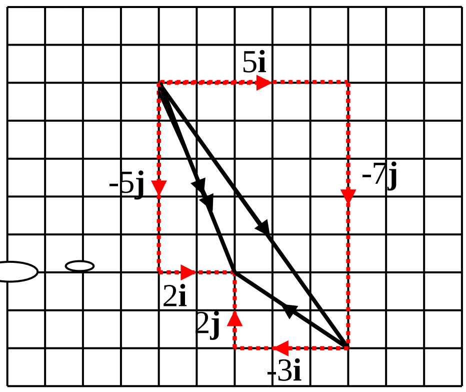
(ii) Work out $\begin{pmatrix} 9 \\ -8 \end{pmatrix} - \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

Solution

(i) To find the resultant you need to add the vectors.

$(5\mathbf{i} - 7\mathbf{j}) + (-3\mathbf{i} + 2\mathbf{j}) = 2\mathbf{i} - 5\mathbf{j}$

You can see this more clearly in this diagram:



The resultant is shown by a double headed arrow.

(ii) $\begin{pmatrix} 9 \\ -8 \end{pmatrix} - \begin{pmatrix} 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

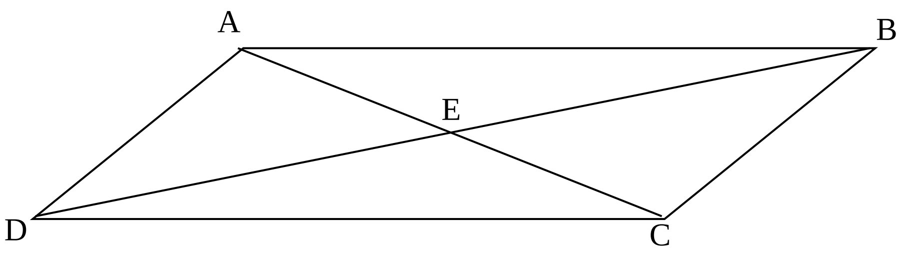
The Explore resource ***Adding and subtracting vectors*** demonstrates the geometrical interpretation of vector addition and subtraction.

Equal vectors and position vectors

Two vectors are **equal** if they have the same magnitude and direction.
They do not have to be in the same place!

Example 6

The diagram shows a parallelogram ABCD.



$\overrightarrow{DA} = \mathbf{a}$

$\overrightarrow{AE} = \mathbf{b}$

$\overrightarrow{AB} = \mathbf{c}$

(a) Find in terms of **a**, **b** and **c** the vectors:

(i) $\overrightarrow{CB}$ (ii) $\overrightarrow{BC}$ (iii) $\overrightarrow{BD}$.

(b) Find two equivalent expressions for $\overrightarrow{AC}$.

Solution

(a) (i) $\overrightarrow{CB} = \overrightarrow{DA} = \mathbf{a}$

(ii) $\overrightarrow{BC} = -\overrightarrow{CB} = -\mathbf{a}$

(iii) $\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD}$

$\overrightarrow{BD} = -\mathbf{c} - \mathbf{a}$

(b) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$\overrightarrow{AC} = \mathbf{c} - \mathbf{a}$

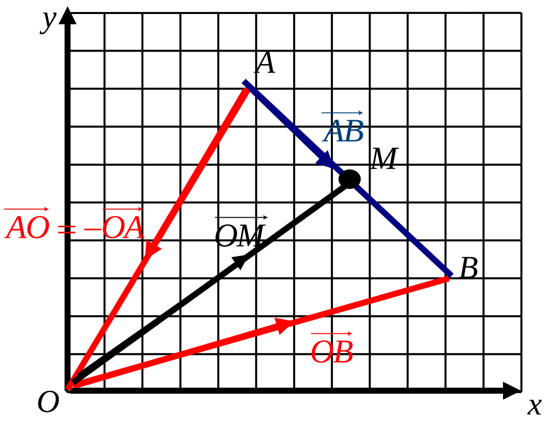
Also $\overrightarrow{AC} = 2\overrightarrow{AE} = 2\mathbf{b}$

A **position vector** is a vector which starts at the origin.
So if two position vectors are equal they will be in the same place!
For example the point A (5, -3) has the position vector $\overrightarrow{OA} = 5\mathbf{i} - 3\mathbf{j}$.

You need to know that

- $\overrightarrow{AO} = -\overrightarrow{OA}$
- $\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$
  So $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
- The mid-point, M, has position vector:
  $\overrightarrow{OM} = \overrightarrow{OA} + \tfrac{1}{2}\overrightarrow{AB}$

You can see the reason for these results more clearly in this diagram:



Example 7

The points A and B have coordinates (2, 4) and (5, -1) respectively.

(i) Write down the position vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

(ii) Find the vector $\overrightarrow{AB}$.

(iii) Find the position vector of the mid-point, M of AB.

Solution

(i) $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$\overrightarrow{OB} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

(ii) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$

(iii) $\overrightarrow{OM} = \overrightarrow{OA} + \tfrac{1}{2}\overrightarrow{AB}$

$$= \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 1\frac{1}{2} \\ -2\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 3\frac{1}{2} \\ 1\frac{1}{2} \end{pmatrix}$$

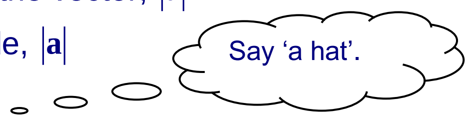
Unit vectors

A **unit vector** has a magnitude of 1.
i and **j** are examples of unit vectors.

You need to be able to find a unit vector which has the same direction as a given vector, **a**.
You do this by:

- Finding the magnitude of the vector, $|\mathbf{a}|$
- Dividing **a** by its magnitude, $|\mathbf{a}|$

The unit vector of **a** is written $\hat{\mathbf{a}}$.



Example 8

Find the unit vector in the direction of $\mathbf{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

Solution

$|\mathbf{a}| = \sqrt{2^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$

$\hat{\mathbf{a}} = \dfrac{2}{\sqrt{13}}\mathbf{i} - \dfrac{3}{\sqrt{13}}\mathbf{j}$

Section 1: Introduction to vectors

Crucial points

1. **Use vector notation correctly**
   Remember that in handwriting you should underline vectors, or in the case of a vector joining two points, use an arrow above, e.g. $\overrightarrow{AB}$.

2. **Make sure you know how to find the resultant of two vectors**
   To find the resultant of two or more vectors simply add them together.

3. **Make sure you know how to find the vector joining two points**
   The vector $\overrightarrow{AB}$ is found by $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

4. **Make sure that you know how to find a unit vector**
   To find a unit vector in the same direction as a given vector, $\mathbf{a}$, you divide by the magnitude, $|\mathbf{a}|$.