Section 1: Matrix arithmetic

Exercise

1. Work out:

(i) $\begin{pmatrix} 1 & 2 \\ -3 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ (ii) $\begin{pmatrix} 4 & 2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 0 & -2 \end{pmatrix}$

(iii) $\begin{pmatrix} 4 & 1 \\ 5 & -2 \end{pmatrix}\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ (iv) $\begin{pmatrix} -2 & 5 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}$

(v) $\begin{pmatrix} 6 & 5 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ (vi) $\begin{pmatrix} 3 & 0 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -4 & 0 \end{pmatrix}$

(vii) $\begin{pmatrix} 8 & -6 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} -1 \\ -4 \end{pmatrix}$ (viii) $\begin{pmatrix} 0 & 5 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} -2 & 4 \\ 1 & -3 \end{pmatrix}$

2. If $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -2 & 1 \\ 3 & 0 \end{pmatrix}$, find

(i) $5\mathbf{A}$
(ii) $-2\mathbf{B}$
(iii) $\mathbf{AB}$
(iv) $\mathbf{BA}$

3. If $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ x & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 6 & 2 \\ 4 & y \end{pmatrix}$ find the values of x and y given that $\mathbf{AB} = \mathbf{BA}$.

4. If $\mathbf{P} = \begin{pmatrix} 3 & a \\ b & 2 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 2 & c \\ -1 & d \end{pmatrix}$ find the values of a, b, c and d given that $\mathbf{PQ} = \mathbf{I}$.

5. $\mathbf{A} = \begin{pmatrix} 1+\sqrt{3} & 0 \\ 1 & \sqrt{3} \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 1-\sqrt{3} & 1 \\ 1 & \sqrt{3} \end{pmatrix}$

(i) Work out $\mathbf{AB}$.
(ii) Find a matrix $\mathbf{C}$ such that $\mathbf{AC}$ consists of four non-zero whole numbers.

6. $\mathbf{M} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(i) Work out $\mathbf{M}^2$.
$\mathbf{M}^3 = \mathbf{M}^2\mathbf{M}$, $\mathbf{M}^4 = \mathbf{M}^3\mathbf{M}$ etc
(ii) Work out $\mathbf{M}^3$.
(iii) Write down, with reasoning, $\mathbf{M}^{10}$.

Section 1: Matrix arithmetic

Solutions to Exercise

1. (i) $\begin{pmatrix} 1 & 2 \\ -3 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$

(ii) $\begin{pmatrix} 4 & 2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 4 & 16 \\ -1 & -11 \end{pmatrix}$

(iii) $\begin{pmatrix} 4 & 1 \\ 5 & -2 \end{pmatrix}\begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ 16 \end{pmatrix}$

(iv) $\begin{pmatrix} -2 & 5 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} -16 & 7 \\ -11 & 5 \end{pmatrix}$

(v) $\begin{pmatrix} 6 & 5 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$

(vi) $\begin{pmatrix} 3 & 0 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -4 & 0 \end{pmatrix} = \begin{pmatrix} 6 & 3 \\ 12 & 4 \end{pmatrix}$

(vii) $\begin{pmatrix} 8 & -6 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} -1 \\ -4 \end{pmatrix} = \begin{pmatrix} 16 \\ 5 \end{pmatrix}$

(viii) $\begin{pmatrix} 0 & 5 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} -2 & 4 \\ 1 & -3 \end{pmatrix} = \begin{pmatrix} 5 & -15 \\ -4 & 6 \end{pmatrix}$

2. (i) $5A = 5\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$

$= \begin{pmatrix} 5 & 0 \\ 10 & -5 \end{pmatrix}$

(ii) $-2BA = -2\begin{pmatrix} -2 & 1 \\ 3 & 0 \end{pmatrix}$

$= \begin{pmatrix} 4 & -2 \\ -6 & 0 \end{pmatrix}$

(iii) $AB = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ 3 & 0 \end{pmatrix}$

$= \begin{pmatrix} -2 & 1 \\ -7 & 2 \end{pmatrix}$

(iv) $\mathbf{BA} = \begin{pmatrix} -2 & 1 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$

$= \begin{pmatrix} 0 & -1 \\ 3 & 0 \end{pmatrix}$

3. $\mathbf{AB} = \begin{pmatrix} 3 & 1 \\ x & 2 \end{pmatrix}\begin{pmatrix} 6 & 2 \\ 4 & y \end{pmatrix} = \begin{pmatrix} 22 & 6+y \\ 6x+8 & 2x+2y \end{pmatrix}$

$\mathbf{BA} = \begin{pmatrix} 6 & 2 \\ 4 & y \end{pmatrix}\begin{pmatrix} 3 & 1 \\ x & 2 \end{pmatrix} = \begin{pmatrix} 18+2x & 10 \\ 12+xy & 4+2y \end{pmatrix}$

$\mathbf{AB} = \mathbf{BA} \Rightarrow \begin{pmatrix} 22 & 6+y \\ 6x+8 & 2x+2y \end{pmatrix} = \begin{pmatrix} 18+2x & 10 \\ 12+xy & 4+2y \end{pmatrix}$

$22 = 18 + 2x \quad \Rightarrow x = 2$

$6 + y = 10 \quad \Rightarrow y = 4$

Check: $6x + 8 = 12 + 8 = 20$ $\quad 12 + xy = 12 + 8 = 20$

$2x + 2y = 4 + 8 = 12$ $\quad 4 + 2y = 4 + 8 = 12$

4. $\begin{pmatrix} 3 & a \\ b & 2 \end{pmatrix}\begin{pmatrix} 2 & c \\ -1 & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 6-a & 3c+ad \\ 2b-2 & bc+2d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$6 - a = 1 \quad \Rightarrow a = 5$

$2b - 2 = 0 \quad \Rightarrow b = 1$

$3c + ad = 0 \quad \Rightarrow 3c + 5d = 0 \quad (1)$

$bc + 2d = 1 \quad \Rightarrow c + 2d = 1 \quad \Rightarrow c = 1 - 2d \quad (2)$

Substituting (2) into (1) gives $3(1 - 2d) + 5d = 0$

$3 - 6d + 5d = 0$

$d = 3$

$c = -5$

So $a = 5$, $b = 1$, $c = -5$, $d = 3$.

5. (i) $\mathbf{AB} = \begin{pmatrix} 1+\sqrt{3} & 0 \\ 1 & \sqrt{3} \end{pmatrix}\begin{pmatrix} 1-\sqrt{3} & 1 \\ 1 & \sqrt{3} \end{pmatrix} = \begin{pmatrix} -2 & 1+\sqrt{3} \\ 1 & 4 \end{pmatrix}$

(ii) One possible matrix is $\begin{pmatrix} 1-\sqrt{3} & 1-\sqrt{3} \\ 1 & 1 \end{pmatrix}$

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6. (i) $M^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

(ii) $M^3 = IM = M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(iii) $M^{10} = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ because all even powers of M will be the identity.

Matrices

Exercise – Matrix Multiplication

Question 1

Work out

(a) $\begin{pmatrix} 4 & 2 \\ -3 & 5 \end{pmatrix}\begin{pmatrix} 7 \\ 1 \end{pmatrix}$ **(b)** $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} -3 \\ -4 \end{pmatrix}$ **(c)** $2\begin{pmatrix} 5 & -2 \\ 6 & -3 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ **(e)** $6\begin{pmatrix} -4 & 7 \\ -1 & -3 \end{pmatrix}$ **(f)** $\begin{pmatrix} 8 & 4 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} -3 \\ 6 \end{pmatrix}$

Question 2

$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 7 & 4 \\ 5 & 3 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix}$

Work out

(a) **AB** **(b)** **BC** **(c)** 3**A**

(d) **BA** **(e)** –**C** **(f)** $\mathbf{B}\begin{pmatrix} 1 & -4 \\ -5 & 7 \end{pmatrix}$

Question 3

$\mathbf{P} = \begin{pmatrix} -2 & 0 \\ 5 & 1 \end{pmatrix}$ $\mathbf{Q} = \begin{pmatrix} -4 & 1 \\ 3 & -2 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

Work out

(a) $\mathbf{P}^2$ **(b)** **QP** **(c)** 5**Q**

(d) **PC** **(e)** **IQ** **(f)** 3**I**

Question 4

Work out

(a) $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 0 & 3 \\ 1 & -4 \end{pmatrix}$ **(b)** $\begin{pmatrix} -3 & -2 \\ -1 & 5 \end{pmatrix}\begin{pmatrix} -2 & 4 \\ 3 & 4 \end{pmatrix}$ **(c)** $\begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}\begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix}$

(d) $\begin{pmatrix} 10 & -7 \\ 9 & 8 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ -2 & 3 \end{pmatrix}$ **(e)** $\begin{pmatrix} 1 & -2 \\ 3 & -5 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ **(f)** $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 3 & -5 \end{pmatrix}$

Question 5 (Non-calculator)

Work out, giving your answers as simply as possible.

(a) $\begin{pmatrix} \sqrt{2} & 1 \\ -1 & 3\sqrt{2} \end{pmatrix}\begin{pmatrix} \sqrt{2} & 0 \\ -3 & -2\sqrt{2} \end{pmatrix}$ **(b)** $\begin{pmatrix} -\frac{1}{2} & -1 \\ \frac{3}{2} & 5 \end{pmatrix}\begin{pmatrix} -2 & 4 \\ -\frac{1}{2} & 3 \end{pmatrix}$ **(c)** $\begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}^2$

(d) $\begin{pmatrix} 3\sqrt{3} & -4 \\ 2 & 3\sqrt{3} \end{pmatrix}\begin{pmatrix} \sqrt{3} & 1 \\ -4 & 0 \end{pmatrix}$ **(e)** $\begin{pmatrix} \frac{1}{3} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{4} \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ **(f)** $\begin{pmatrix} \sqrt{2} & 2 \\ 7 & \sqrt{3} \end{pmatrix}^2$

Question 6

Work out, giving your answers as simply as possible.

(a) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} p \\ p+1 \end{pmatrix}$ **(b)** $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ **(c)** $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} m \\ 2m \end{pmatrix}$

(d) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -a & 0 \\ 0 & a \end{pmatrix}$ **(e)** $\begin{pmatrix} 4t & 0 \\ 0 & 4t \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ **(f)** $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

Question 7

Work out, giving your answers as simply as possible.

(a) $\begin{pmatrix} 2x & -3 \\ -5 & 4x \end{pmatrix}\begin{pmatrix} x & 3x \\ -3 & 0 \end{pmatrix}$ **(b)** $\begin{pmatrix} a & 3a \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 7 & 8 \\ -10 & 11 \end{pmatrix}$ **(c)** $\begin{pmatrix} x & 0 \\ 1 & x \end{pmatrix}^2$

(d) $\begin{pmatrix} y & y \\ -3 & x \end{pmatrix}\begin{pmatrix} 2 & 3y \\ 0 & 1 \end{pmatrix}$ **(e)** $\begin{pmatrix} a+1 & a \\ a+2 & a+1 \end{pmatrix}\begin{pmatrix} a+1 & -a \\ -a-2 & a+1 \end{pmatrix}$ **(f)** $\begin{pmatrix} 3x & -3 \\ -9 & x+1 \end{pmatrix}^2$

Exercise 1b

Question 1

$$\begin{pmatrix} -2 & a \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 3 \\ 7 \end{pmatrix} = \begin{pmatrix} 22 \\ 9 \end{pmatrix}$$

Work out the value of a.

Question 2

a) Work out $\begin{pmatrix} 2 & -1 \\ \frac{1}{3} & 0 \end{pmatrix}\begin{pmatrix} 0 & b \\ a & c \end{pmatrix}$

Give your answer in terms of a, b, c.

b) **If** $\begin{pmatrix} 2 & -1 \\ \frac{1}{3} & 0 \end{pmatrix}\begin{pmatrix} 0 & b \\ a & c \end{pmatrix} = \boldsymbol{I}$ where $\boldsymbol{I}$ is the identity matrix, work out the values of a, b, c.

Question 3

Matrix $\mathbf{P} = \begin{pmatrix} 2 & 3 \\ a & b \end{pmatrix}$ Matrix $\mathbf{Q} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

You are given that $\mathbf{PQ} = \mathbf{QP}$

Work out the values of a and b.

Question 4

$$\begin{pmatrix} 2 & a \\ 1 & -3 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Work out **all** possible pairs of values of a and b.

Review Questions

1 (a) $3\begin{pmatrix} 2 & 1 \\ -5 & 0 \end{pmatrix}\begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$

Work out the values of a and b.

[3 marks]

$a =$ _____ $b =$ _____

1 (b) Work out $\begin{pmatrix} 5 & -3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix}$

[2 marks]

2 $\mathbf{A} = \begin{pmatrix} 4 & -1 \\ 3 & -2 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

2 (a) Work out $\mathbf{A}^2$

[2 marks]

Answer _____

2 (b) $k\mathbf{B} = \begin{pmatrix} 11-3k \\ 11-6k \end{pmatrix}$ where k is a constant.

Work out the value of k.

[2 marks]

Review Questions

1 (a) $3\begin{pmatrix} 2 & 1 \\ -5 & 0 \end{pmatrix}\begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$

Work out the values of a and b.

[3 marks]

$a =$ _____ $b =$ _____

1 (b) Work out $\begin{pmatrix} 5 & -3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix}$

[2 marks]

Solution

1a a = 15, b = -30

1b $\begin{pmatrix} 13 & -30 \\ 0 & 7 \end{pmatrix}$

2 $\mathbf{A} = \begin{pmatrix} 4 & -1 \\ 3 & -2 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

2 (a) Work out $\mathbf{A}^2$

[2 marks]

Answer _____

2 (b) $k\mathbf{B} = \begin{pmatrix} 11-3k \\ 11-6k \end{pmatrix}$ where k is a constant.

Work out the value of k.

[2 marks]

Solution

2a $\begin{pmatrix} 13 & -2 \\ 6 & 1 \end{pmatrix}$

2b k = 11/8