# A-Level Further Mathematics
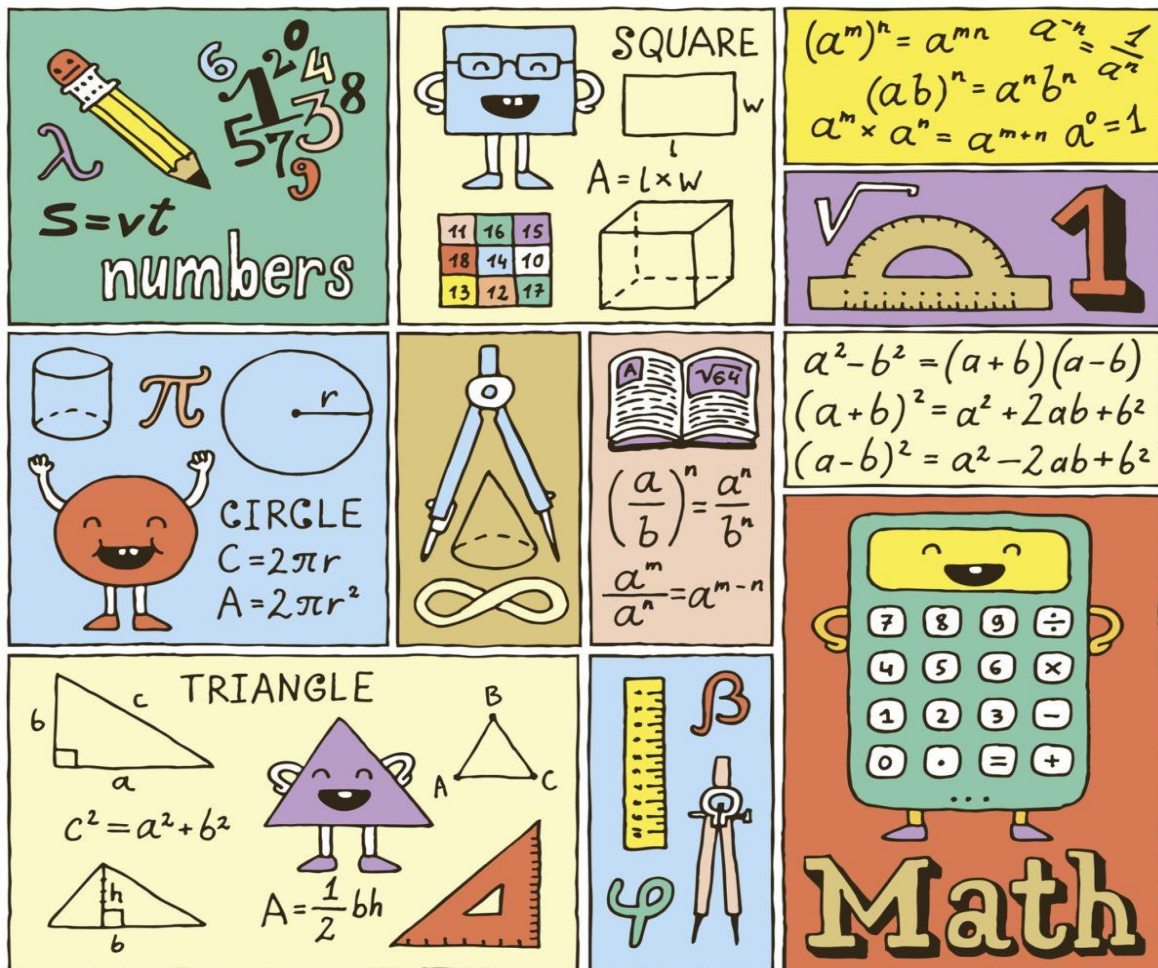


# Subject Knowledge Assessment
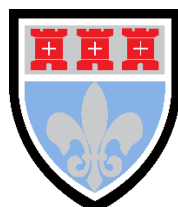
## Vectors

**1.**



Fig. 2

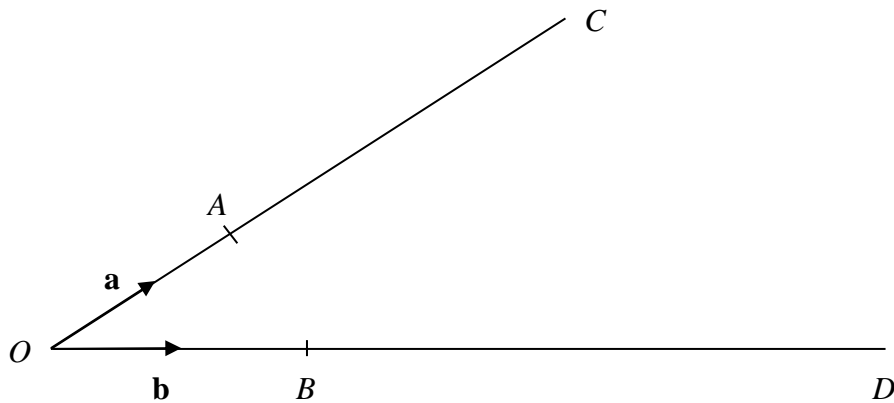In Fig. 2, *OB* is produced to *D* so that *BD* = 2*OB* and *OA* is produced to *C* so that *AC* = 2*OA*. Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$,

(*a*)  express in terms of **a** and **b**

(i)   $\overrightarrow{OD}$, (ii) $\overrightarrow{BC}$, (iii) $\overrightarrow{AB}$, (iv) $\overrightarrow{CD}$,

(*b*)  state two geometrical relationships between *AB* and *CD*.
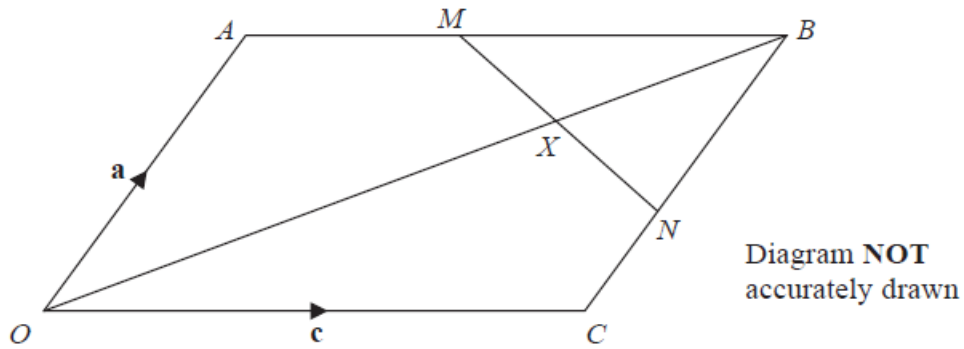
**(6 marks)**

**2.**



**Figure 3**

Figure 3 shows the parallelogram $OABC$

$$\overrightarrow{OA} = \mathbf{a} \qquad \overrightarrow{OC} = \mathbf{c}$$

The midpoint of $AB$ is $M$ and the midpoint of $BC$ is $N$.

The line $OB$ intersects $MN$ at the point $X$.

($a$)    Find in terms of $\mathbf{a}$ and $\mathbf{c}$,

(i)    $\overrightarrow{OB}$

(ii)    $\overrightarrow{MN}$

**(2)**

Given $\overrightarrow{MX} = \lambda\overrightarrow{MN}$ and that $\overrightarrow{OX} = \mu\overrightarrow{OB}$,

($b$)    use a vector method to find the value of $\lambda$ and the value of $\mu$.

**(8)**

($c$)    Hence find, in its simplest form, the ratio

Area of quadrilateral $OXNC$ : Area of parallelogram $OABC$.

**(3)**

**(13 marks)**

# Matrices

1.

Under the transformation represented by $\begin{pmatrix} -1 & -3 \\ 2 & 4 \end{pmatrix}$,

the image of point $P(a, 2)$ is point $Q$.

Can point $Q$ be the same as point $P$?
You **must** show your working.

**[4 marks]**

2.

Shape $A$ maps to shape $B$ by an enlargement, scale factor 3, centre the origin.
Shape $B$ maps to shape $C$ by a rotation through 180°, centre the origin.

Shape $A$ can be mapped to shape $C$ by a **single** transformation.

Use matrices to show that the single transformation is an enlargement, centre the origin.
State the scale factor of the enlargement.

**[5 marks]**

3.

Use **matrix multiplication** to show that, in the $x$–$y$ plane,

- a reflection in the line $y = -x$,  followed by
- a rotation, 90° anticlockwise about the origin,  followed by
- a reflection in the $x$-axis

is equivalent to a transformation by the identity matrix.

**[5 marks]**

## Sequences

1.

The first four terms of a quadratic sequence are     10    33    64    103    ...

Work out an expression for the $n$th term.

**[4 marks]**

2.

For sequence A, the $n$th term $= \dfrac{n}{14n+30}$

For sequence B, the $n$th term $= \dfrac{2}{n}$

The $k$th term of sequence A equals the $k$th term of sequence B.

Work out the value of $k$.

**[4 marks]**

3.

A sequence is given by the rule $u_{n+2} = 2u_n + u_{n+1}$ . Given that $u_6 = 75$ and $u_7 = 149$,
find;

(i)   $u_8$

(ii)   $u_5$

(iii)   $u_1$

**[ 5 marks]**

4.

The first 7 terms of a sequence are $a$, $b$, $c$, 12, $e$, $f$, 50. The sequence follows the same
rules as the Fibonacci sequence (i.e. the 3rd term is the sum of the 1st & 2nd, the 4th
term is the sum of the 2nd & 3rd, etc.). What are the values of $a$, $b$, $c$, $e$ & $f$ ?

**[5 marks]**

# Differentiation

1.

Work out the rate of change of $y$ with respect to $x$ at the point on the curve

$$y = x^2(x^2 - 9) \quad \text{where} \quad x = -2$$

You **must** show your working.

**[4 marks]**

2.

$$f(x) = 2x^3 - 12x^2 + 25x - 11$$

Find an expression for $f'(x)$ in the form $a(\ldots)^2 + b$ which would show that $f(x)$ is an increasing function for all values of $x$.

$$f'(x) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

**[ 5 marks ]**

3.

The curve $y = f(x)$, where $f(x) = x^3 - ax + 1$, has a stationary point at $x = -2$
Find the value of $a$

**[ 5 marks ]**

## Miscellaneous

**1.**

Solve the simultaneous equations

$$2a + b - c = 8$$
$$4a - 3b - 2c = -9$$
$$6a + 3b + c = 0$$

..........................

**[ 4 marks ]**

**2.**

Find the coefficient of $x^3$ in the expansion of $(3-2x)^5$.

**[ 4 marks ]**

**3.**

(i)   Write down the first 3 terms, in ascending powers of $x$, of the binomial expansion of :
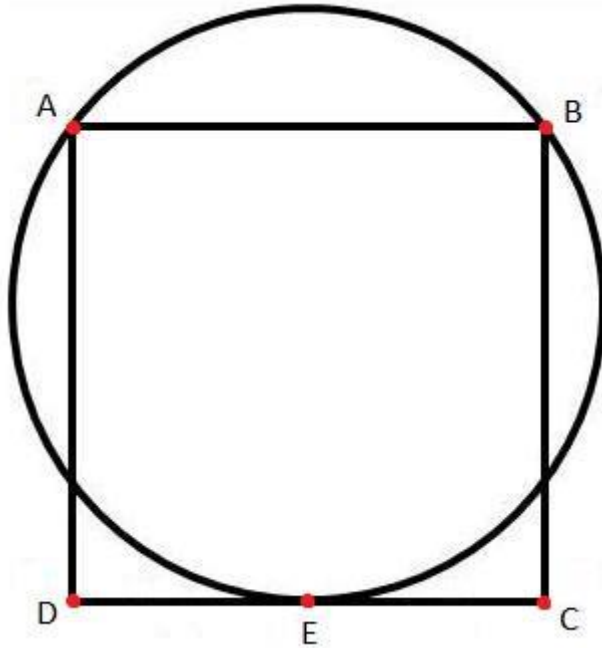
$(1+ax)^7$, where $a$ is a non-zero constant.

(ii)  Given that in the expansion of $(1+ax)^7$, the coefficient of $x^2$ is nine times the coefficient of $x$, find the value of $a$.

**[ 6 marks ]**

**4.**

ABCD is a square, side length 8 cm. ABE are points on a circle, CE = DE.
Find the radius of the circle.



**[ 6 marks ]**

**5**.

A circle with centre at the origin has radius $2\sqrt{5}$. A tangent to the circle has the equation $y = 2x + k$. Find the possible values of $k$.

**[ 5 marks ]**

**6.**

Solve the equation $(2 + \sqrt{5})x = 6\sqrt{5}$, giving $x$ in the form $a + b\sqrt{5}$ where $a$ and $b$ are integers.

**[4 marks]**

**7.**

The square $ABCD$ has sides of length 105. The point $M$ is the midpoint of side $BC$. The point $N$ is the midpoint of $BM$. The lines $BD$ and $AM$ meet at the point $P$. The lines $BD$ and $AN$ meet at the point $Q$.
What is the area of triangle $APQ$?

**[ 6 marks ]**

**Well Done**

☺