

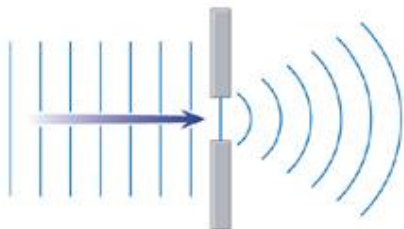
Volume  
Two

**ST MARY'S SCIENCE  
DEPARTMENT:  
PHYSICS**

**A LEVEL PHYSICS YEAR 1  
STUDENT CLASS BOOK  
WAVES**

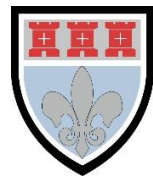
**3.3.2: REFRACTION, DIFFRACTION AND  
INTERFERENCE**

<b>NAME</b>	
<b>PHYSICS CLASS</b>	
<b>MODULE TEACHER</b>	
<b>ALPS GRADE</b>	



**A-LEVEL PHYSICS  
TOPIC 3  
CLASS BOOK 2**

**THIS MUST  
BE BROUGHT  
TO ALL  
PHYSICS  
LESSONS.**



## Contents

### 3.3.2.1 Interference

### 3.3.2.2 Diffraction

### 3.3.2.3 Refraction at a Plane Surface

#### Overview

GCSE studies of wave phenomena are extended through a development of knowledge of the characteristics, properties, and applications of travelling waves and stationary waves.

Topics treated include refraction, diffraction, superposition and interference.

#### **IMPORTANT NOTE**

This book, along with the preparatory reading notes and independent work, must be brought to all Physics lessons with the appropriate teacher.

This book may be used as a learning resource in lessons, you are not fully equipped to learn if this is not used in lesson.

This book may also be used as a revision resource for intervention, internal assessments and external assessments.

**Please keep this in your student file.**

There are several activities in this book which may not be covered in lessons.

**It is advised that students complete these activities outside of lessons as revision aides.**



## Definition List

Definitions you must learn for this module.

Key Word	Symbol	Definition
<b>Coherent</b>		When two sources of waves having a constant phase difference and the same frequency.
<b>Critical Angle</b>	$\theta_c$	The angle of incidence of a light ray that must be exceeded for total internal reflection to occur. It is also the angle of incidence where the angle of refraction is zero.
<b>Diffraction</b>		The spreading of waves on passing through a gap or near an edge.
<b>Diffraction Grating</b>		A plate with many closely ruled parallel slits on it.
<b>Fringe Spacing</b>	$w$	The perpendicular distance between the centre of two fringes of the same type e.g. bright fringe and bright fringe (adjacent maxima or minima).
<b>Interference</b>		The formation of points of cancellation and reinforcement where two coherent waves pass through each other.
<b>Intensity</b>	$I$	The power per unit area of a wave.
<b>Laser</b>		a device which produces a parallel coherent beam of monochromatic light.
<b>Maxima</b>		Points produced when waves with a $n\lambda$ path difference (or $n2\pi$ phase difference) constructively interfere.
<b>Minima</b>		Points produced when waves with a $n/2\lambda$ path difference (or $n\pi$ phase difference) destructively interfere.
<b>Modal Dispersion</b>		The lengthening of a light pulse as it travels along an optical fibre, due to rays that repeatedly undergo total internal reflection having to travel a longer distance than the rays that undergo less total internal reflection.
<b>Monochromatic Light</b>		Light of single wavelength (and frequency) – the same colour.
<b>Optical Fibre</b>		A thin, flexible transparent fibre used to carry light pulses from one end to the other.
<b>Path Difference</b>		The difference in distances from two coherent sources to an interference fringe.
<b>Phase Difference</b>		The fraction of a cycle between the vibrations of two vibrating particles, measured either in degrees or radians.

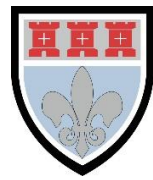


<b>Superposition</b>		The effect of two waves adding together when they meet. This can be either displacement or any vector property of a wave
<b>Refraction</b>		The change in direction of a wave when it crosses a boundary where its speed changes. In refraction, frequency must remain constant.
<b>Refractive Index</b>	<b>n</b>	The speed of light in free space compared to the speed of light in the substance.
<b>Total Internal Reflection</b>		Occurs when a light ray travelling in a substance is reflected at the boundary with a substance of lower refractive index, if the angle of incidence is greater than a certain value known as the critical angle.
<b>Young's Fringes</b>		The parallel bright and dark fringes observed when light from a narrow-slit passes through two closely spaced slit.

**IMPORTANT NOTE**

These definitions must be memorised by students.

You will be tested on your knowledge of these definitions.



## Equations

The equations below are used in this module.

Quantity/Concept	Equation(s)
<b>Intensity</b>	$I = P / A$ This is not given in your examination.
<b>Fringe Spacing</b>	$w = \frac{\lambda D}{s}$
<b>Distance Between Slits on a Diffraction Grating</b>	$d = n\lambda / \sin \theta$
<b>Maximum Number of Diffraction Orders</b>	$n = d / \lambda$ This is not given in your examination.
<b>Refractive Index</b>	$n = c/c_s$
<b>Relative Refractive Index</b>	${}_1n_2 = c_1 / c_2$ ${}_1n_2 = n_1 / n_2$  These two equations can be equated to each other.
<b>Snell's Law</b>	$n_1 \sin \theta_1 = n_2 \sin \theta_2$
<b>Critical Angle</b>	$\sin \theta_c = n_2 / n_1 = {}_1n_2$

### IMPORTANT NOTE

These equations must be memorised by students.

You will be tested on these equations.



## The Language of Measurement

The following subject specific vocabulary provides definitions of key terms used in the A-level Science specifications.

### Accuracy

A measurement result is considered accurate if it is judged to be close to the true value.

### Calibration

Marking a scale on a measuring instrument.

This involves establishing the relationship between indications of a measuring instrument and standard or reference quantity values, which must be applied.

For example, placing a thermometer in melting ice to see whether it reads 0 °C, to check if it has been calibrated correctly.

### Data

Information, either qualitative or quantitative, that has been collected.

### Errors

See also uncertainties.

### Measurement error

The difference between a measured value and the true value.

anomalies

These are values in a set of results which are judged not to be part of the variation caused by random uncertainty.

### Random error

These cause readings to be spread about the true value, due to results varying in an unpredictable way from one measurement to the next.

Random errors are present when any measurement is made, and cannot be corrected. The effect of random errors can be reduced by making more measurements and calculating a new mean.

### Systematic error

These cause readings to differ from the true value by a consistent amount each time a measurement is made.

Sources of systematic error can include the environment, methods of observation or instruments used.

Systematic errors cannot be dealt with by simple repeats. If a systematic error is suspected, the data collection should be repeated using a different technique or a different set of equipment, and the results compared.

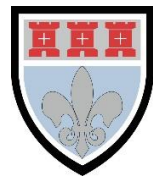
### Zero error

Any indication that a measuring system gives a false reading when the true value of a measured quantity is zero, e.g. the needle on an ammeter failing to return to zero when no current flows.

A zero error may result in a systematic uncertainty.

### Evidence

Data which has been shown to be valid.

**Fair test**

A fair test is one in which only the independent variable has been allowed to affect the dependent variable.

**Hypothesis**

A proposal intended to explain certain facts or observations.

**Interval**

The quantity between readings, e.g. a set of 11 readings equally spaced over a distance of 1 metre would give an interval of 10 centimetres.

**Precision**

Precise measurements are ones in which there is very little spread about the mean value. Precision depends only on the extent of random errors – it gives no indication of how close results are to the true value.

**Prediction**

A prediction is a statement suggesting what will happen in the future, based on observation, experience or a hypothesis.

**Range**

The maximum and minimum values of the independent or dependent variables; important in ensuring that any pattern is detected.

For example, a range of distances may be quoted as either:

'From 10 cm to 50 cm'

or

'From 50 cm to 10 cm'

**Repeatable**

A measurement is repeatable if the original experimenter repeats the investigation using same method and equipment and obtains the same results.

**Reproducible**

A measurement is reproducible if the investigation is repeated by another person, or by using different equipment or techniques, and the same results are obtained.

**Resolution**

This is the smallest change in the quantity being measured (input) of a measuring instrument that gives a perceptible change in the reading.

**Sketch graph**

A line graph, not necessarily on a grid, that shows the general shape of the relationship between two variables. It will not have any points plotted and although the axes should be labelled they may not be scaled.

**True value**

This is the value that would be obtained in an ideal measurement.

**Uncertainty**

The interval within which the true value can be expected to lie, with a given level of confidence or probability, e.g. "the temperature is  $20\text{ }^{\circ}\text{C} \pm 2\text{ }^{\circ}\text{C}$ , at a level of confidence of 95%.

**Validity**

Suitability of the investigative procedure to answer the question being asked. For example, an investigation to find out if the rate of a chemical reaction depended upon the concentration of one of the reactants would not be a valid procedure if the temperature of the reactants was not controlled.

**Valid conclusion**

A conclusion supported by valid data, obtained from an appropriate experimental design and based on sound reasoning.

**Variables**

These are physical, chemical or biological quantities or characteristics.

**Categoric variables**

Categoric variables have values that are labels. E.g. names of plants or types of material.

**Continuous variables**

Continuous variables can have values (called a quantity) that can be given a magnitude either by counting (as in the case of the number of shrimp) or by measurement (e.g. light intensity, flow rate etc.).

**Control variables**

A control variable is one which may, in addition to the independent variable, affect the outcome of the investigation and therefore must be kept constant or at least monitored.

**Dependent variables**

The dependent variable is the variable of which the value is measured for each change in the independent variable.

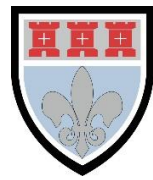
**Independent variables**

The independent variable is the variable for which values are changed or selected by the investigator.

**IMPORTANT NOTE**

These definitions must be memorised by students.

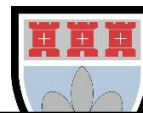
You will be tested on your knowledge of these definitions.



## TOPIC: 3.3.2.1 Interference

### SPEC CHECK

Specification	Completed?
Path difference. Coherence.	
Interference and diffraction using a laser as a source of monochromatic light.	
Young's double-slit experiment: the use of two coherent sources or the use of a single source with double slits to produce an interference pattern.	
Fringe spacing, $w = \frac{\lambda D}{s}$	
Production of interference pattern using white light.	
Show awareness of safety issues associated with using lasers.	
Describe and explain interference produced with sound and electromagnetic waves.	
Appreciation of how knowledge and understanding of nature of electromagnetic radiation has changed over time.	
Investigate two-source interference with sound, light and microwave radiation.	



These notes are brief.

More detailed notes are found in the student preparatory reading book.

Please read the preparatory reading notes.

## NOTES

### Interference

Interference is a special case of superposition where the waves that combine are coherent. Two waves are coherent if they have the same frequency and constant phase difference. The waves overlap and form a repeating interference pattern of maxima and minima areas. If the waves weren't coherent the interference pattern would change rapidly and continuously. This is not considered at A-Level Physics.

**Coherence:** Waves which are of the same frequency, wavelength, polarisation and amplitude and in a constant phase relationship. A laser is a coherent source but a light bulb is not.

**Constructive Interference:** The path difference between the waves is a whole number of wavelengths so the waves arrive in phase adding together to give a large wave. This can also be expressed in terms of phase difference; two waves constructively interfere if they have a phase difference of  $n2\pi$  radians.

**For example: 2 peaks overlap**

**Destructive Interference:** The path difference between the waves is a half number of wavelengths so the waves arrive out of phase cancelling out to give no wave at all. This can also be expressed in terms of phase difference; two waves destructively interfere if they have a phase difference of  $n\pi$  radians.

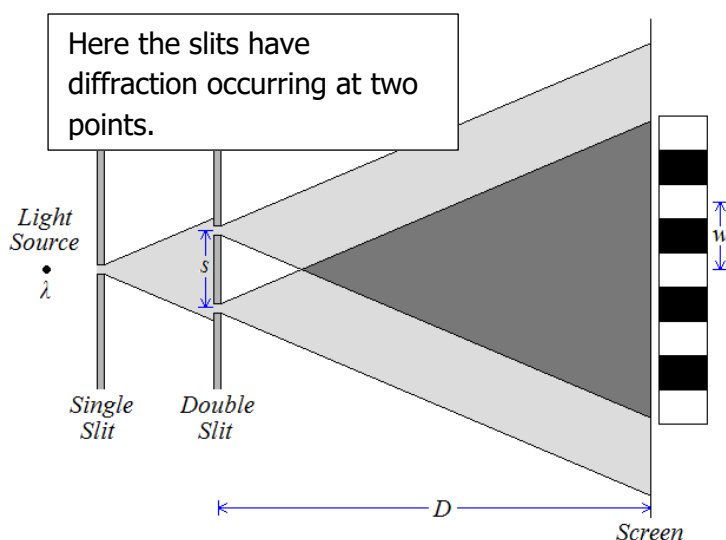
**For example: A peak and trough overlap**

### Young's Double Slit Experiment

In 1803 Thomas Young settled a debate started over 100 years earlier between Newton and Huygens by demonstrating the interference of light. Newton thought that light was made up of tiny particles called corpuscles and Huygens thought that light was a wave, Young's interference of light proves light is a wave.

Interference occurs where the light from the two slits overlaps. Constructive interference produces bright areas, while deconstructive interference produces dark areas. These areas are called interference fringes.

Here is Young's double slit set up, the two slits act as coherent sources of waves



The waves produced are coherent as they come from the same light source.

This means a diffraction pattern must form.

This was the first experimental proof that light must act like a wave.

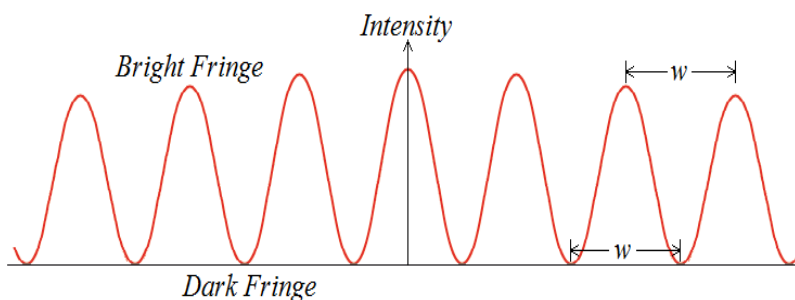
Diffraction and interference are wave-only properties.



## Fringes

There is a central bright fringe directly behind the midpoint between the slits with more fringes evenly spaced and parallel to the slits.

**As we move away from the centre of the screen we see the intensity of the bright fringes decreases.**



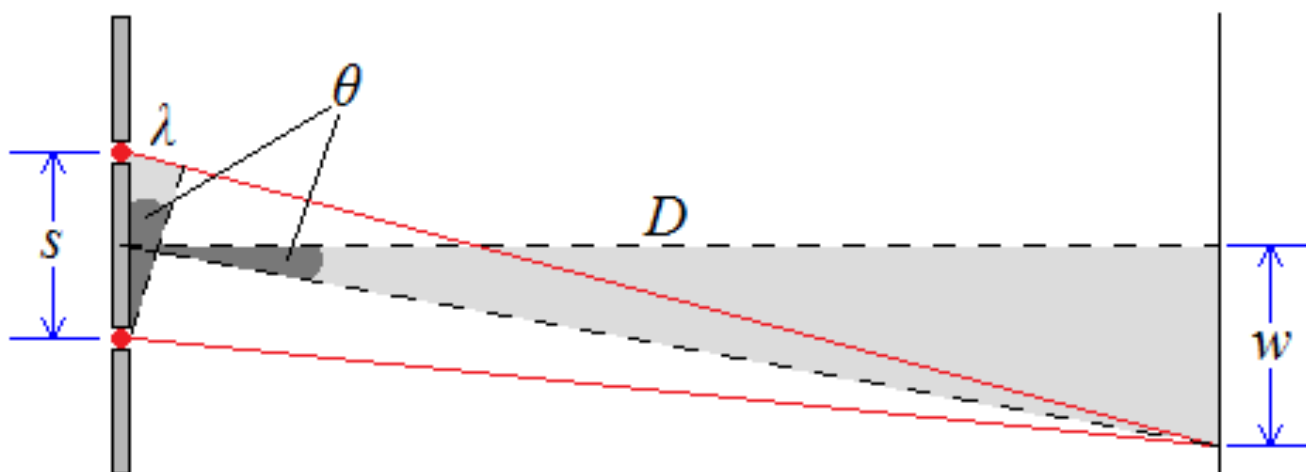
## Double Source Experiment

The interference of sound is easy to demonstrate with two speakers connected to the same signal generator. Waves of the same frequency (coherent) interfere with each other.

Constructive interference produces loud fringes, while deconstructive interference produces quiet fringes.

## Derivation

We can calculate the separation of the fringes ( $w$ ) if we consider the diagram to the right which shows the first bright fringe below the central fringe. The path difference between the two waves is equal to one whole wavelength ( $\lambda$ ) for constructive interference.



If the distance to the screen ( $D$ ) is massive compared to the separation of the sources ( $s$ ) the angle ( $\theta$ ) in the large triangle can be assumed the same as the angle in the smaller triangle.

$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

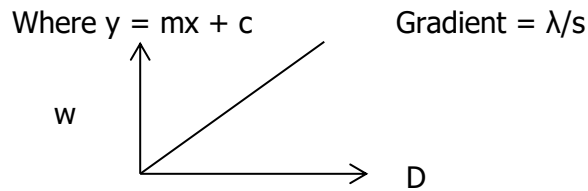
$$\text{For the small triangle: } \sin \theta = \frac{\lambda}{s} \quad \text{For the large triangle: } \sin \theta = \frac{w}{D}$$



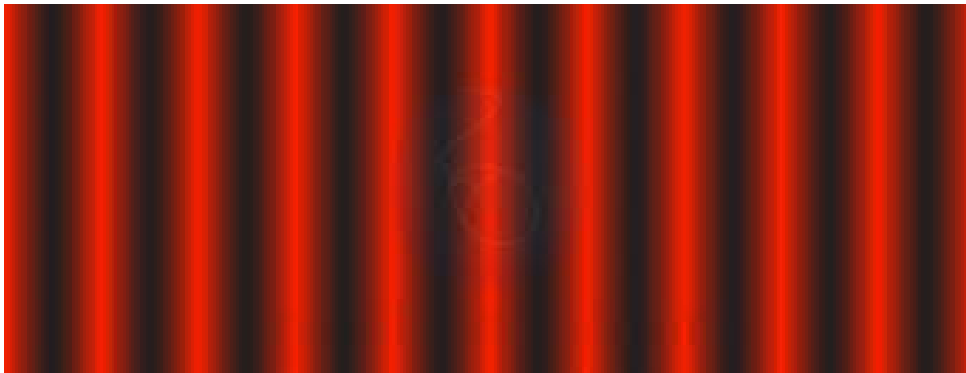
Since the angles are the same we can write  $\frac{w}{D} = \sin \theta = \frac{\lambda}{s}$  or  $\frac{w}{D} = \frac{\lambda}{s}$  which rearranges to:

$$w = \frac{\lambda D}{s}$$

We can draw a graph of this data...



**Fringe Separation, Source Separation, Distance to Screen and Wavelength are measured in metres, m**



Here is an example of an interference pattern produced by Young's slits.

Beware the fringe separation or spacing is measured from the centre of one fringe to the centre of the next corresponding fringe.

The more values which you measure over, the lower the percentage uncertainty of the value calculated.

However, be aware you are measuring the fringe spacing not the number of fringes produced.

#### **Experimental Note**

The fringes produced due to Young's slits are quite dim and blurry – this is because the wave is only passing through 2 slits.

This makes Young's slits a difficult investigation to gain valid results for.



## REVISION SHEET

Highlight or underline the key information on the revision sheet to consolidate your understanding.

### Superposition Happens When Two or More Waves Pass Through Each Other

- 1) At the **instant** the waves **cross**, the **displacements** due to each wave **combine**. Then **each wave** goes on its merry way. You can **see** this if **two pulses** are sent **simultaneously** from each end of a rope.
- 2) The **principle of superposition** says that when two or more **waves cross**, the **resultant** displacement equals the **vector sum** of the **individual** displacements.



"**Superposition**" means "one thing on top of another thing". You can use the same idea in **reverse** — a **complex wave** can be separated out mathematically into **several simple** sine waves of various sizes.

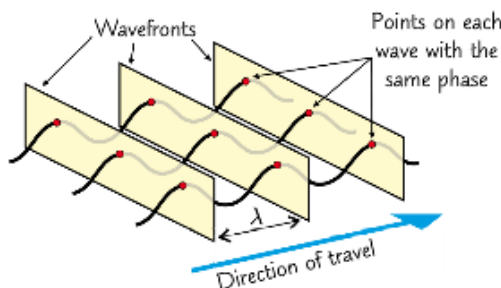
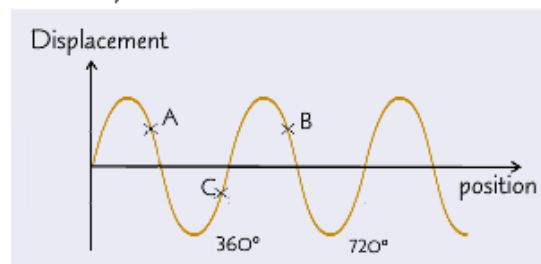
### Interference can be Constructive or Destructive

- 1) When two or more waves **superpose** with each other, the effect is called **interference**.
- 2) A **crest** plus a **crest** gives a **bigger crest**. A **trough** plus a **trough** gives a **bigger trough**. These are both examples of **constructive interference**.
- 3) A **crest** plus a **trough** of **equal size** gives... **nothing**. The two displacements **cancel each other out** completely. This is called **destructive interference**.
- 4) If the **crest** and the **trough** aren't the **same size**, then the destructive interference **isn't total**. For the interference to be **noticeable**, the two **amplitudes** should be **nearly equal**.

Graphically, you can superimpose waves by adding the individual displacements at each point along the x-axis, and then plotting them.

### In Phase Means In Step — Two Points In Phase Interfere Constructively

- 1) Two points on a wave are **in phase** if they are both at the **same point** in the **wave cycle**. Points in phase have the **same displacement** and **velocity**.
- 2) On the graph on the right, points **A** and **B** are **in phase**; points **A** and **C** are **out of phase**.
- 3) It's mathematically **handy** to show one **complete cycle** of a wave as an **angle of  $360^\circ$  ( $2\pi$  radians)**.
- 4) **Two points** with a **phase difference** of **zero** or a **multiple of  $360^\circ$**  are **in phase**.
- 5) **Points** with a **phase difference** of **odd-number multiples of  $180^\circ$  ( $\pi$  radians)** are **exactly out of phase**.



- 6) You can also talk about two **different waves** being **in phase**. **In practice** this usually happens because **both** waves came from the **same oscillator**. In **other** situations there will nearly always be a **phase difference** between two waves.
- 7) Two or more waves that are **coherent** (see below), **in phase** and travelling in the **same direction** will have **wavefronts**. These are imaginary planes that cut **across** all the waves, joining up all the **points** that are **in phase** with each other. The **distance** between each wavefront is equal to one **wavelength**, i.e. each wavefront is at the same point in the cycle.

### To Get Interference Patterns the Two Sources Must Be Coherent

Interference **still happens** when you're observing waves of **different wavelength** and **frequency** — but it happens in a **jumble**. In order to get clear **interference patterns**, the two or more sources must be **coherent**.

**Coherent** sources — they have the **same wavelength** and **frequency** and a **fixed phase difference** between them.



### **Constructive or Destructive Interference Depends on the Path Difference**

- 1) Whether you get **constructive** or **destructive** interference at a **point** depends on how **much further one wave** has travelled than the **other wave** to get to that point (assuming the sources are coherent and in phase).
- 2) The **amount** by which the path travelled by one wave is **longer** than the path travelled by the other wave is called the **path difference**.
- 3) At **any point an equal distance** from both sources you will get **constructive interference**. You also get constructive interference at any point where the **path difference** is a **whole number of wavelengths**, because the waves arrive at the same point **in phase**. At points where the path difference is an odd number of **half wavelengths**, the waves arrive **out of phase** and you get **destructive interference**.

**Constructive interference**  
occurs when:

$$\text{path difference} = n\lambda \quad (\text{where } n \text{ is an integer})$$

**Destructive interference**  
occurs when:

$$\text{path difference} = \frac{(2n+1)\lambda}{2} = (n + \frac{1}{2})\lambda$$

If the sources are not in phase but are coherent, there will be constructive and destructive interference, but they will not occur at these path differences.



**Additional Note Space**



**Additional Note Space**



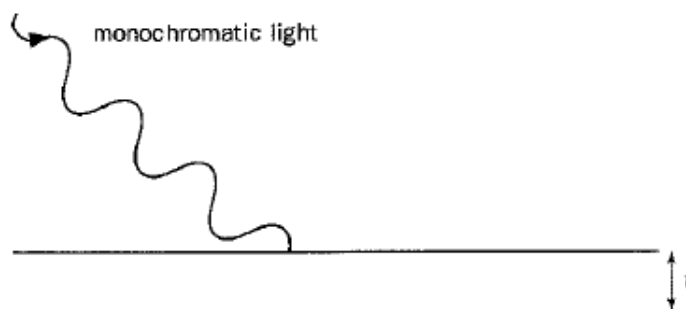
## PUZZLES QUESTIONS

To improve your understanding, answer the following puzzles.

The answers are overleaf.

- What causes the colour of an oil film on water?
  - What causes the colour of a CD?
  - What causes the colour of a pearl shell?
- When two waves interfere destructively with each other, where does the wave energy go?

- Complete the diagram on the right to show how interference can occur. What must be the path difference (in wavelengths) for there to be destructive interference?

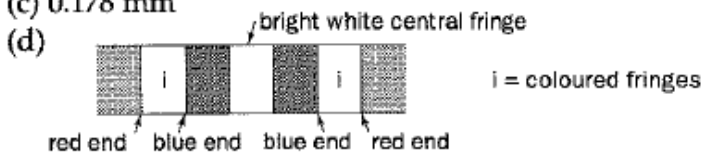


- You set up a neon laser ( $\lambda = 700 \text{ nm}$ ) in a Young's double-slit experiment. You measure the distance from the first to the tenth bright fringe and find it to be  $8.00 \text{ mm}$ . If the slit separation is  $0.60 \text{ mm}$ , find the distance from the slits to the screen.
- You use a light of wavelength  $540 \text{ nm}$  to produce fringes on a screen  $0.5 \text{ m}$  away from the slits. If the slit separation is  $0.40 \text{ mm}$ , calculate the fringe separation.
  - Why does the intensity of the fringe decrease on either side of the central bright fringe?
- Yellow light of wavelength  $600 \text{ nm}$  is used to illuminate a double-slit and fringes are observed  $0.8 \text{ m}$  from the slits. The distance from the first to the tenth fringe is  $4.8 \text{ mm}$ . Calculate the slit separation.
  - The yellow light is replaced with blue light of wavelength  $400 \text{ nm}$ . What is the fringe separation now?
  - You move the screen to  $0.4 \text{ m}$  from the slits. What is the spacing of the blue fringes now?
  - You now replace the blue light with white light (range  $400 \text{ nm}$ – $700 \text{ nm}$ ). Sketch what you would observe.



## ANSWERS

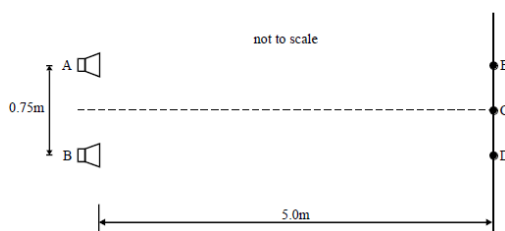
- 1 (a) The oil forms a layer only a few molecules thick on the surface of the water. Some of the incident light is reflected off the top surface and undergoes a phase change of  $\pi$ . The rays, which are reflected off the bottom of the oil surface, undergo no phase change on reflection. The two emerging rays interfere with each other and, depending on the thickness of the oil film, certain wavelengths are missing from the reflected beam.
  - (b) The reflected light from the CD has certain wavelengths missing because of interference.
  - (c) Pearl shells are made up of thin layers of translucent material. Light, which is reflected from the surfaces, produces interference patterns in the same way oil films do.
- 2 Wave energy from region of destructive interference is redistributed to regions of constructive interference.
- 3 The diagram should show reflection from the upper and lower surfaces. A phase change occurs at the top surface only, so destructive interference takes place when the path difference is a whole number of wavelengths.
- 4 0.762 m
- 5 (a) 0.675 mm
  - (b) Students should mention the inverse square law.
- 6 (a) 0.900 mm
  - (b) 0.356 mm
  - (c) 0.178 mm





## SAMPLE QUESTION

**S1.** The diagram shows two identical loudspeakers, A and B, placed 0.75 m apart. Each loudspeaker emits sound of frequency 2000 Hz.



Point C is on a line midway between the speakers and 5.0 m away from the line joining the speakers. A listener at C hears a maximum intensity of sound. If the listener then moves from C to E or D, the sound intensity heard decreases to a minimum.

Further movement in the same direction results in the repeated increase and decrease in the sound intensity.

Speed of sound in air = 330 m s<sup>-1</sup>

Explain why the sound intensity

**1.1** is a maximum at C,

[3 Marks]

**superposition takes place (1 mark)**  
**between waves in phase (1 mark)**  
**gives constructive interference (1 mark)**

**1.2** is a minimum at D or E.

[2 Marks]

**at D or E the waves are out of phase (1 mark)**  
**so destructive interference (1 mark)**

Calculate

**1.3** the wavelength of the sound,

[1 Mark]

$$\lambda = \frac{330}{2 \times 10^3} = 0.165\text{m (1 mark)}$$



**1.4** the distance CE.

**[2 Marks]**

$$\text{Separation between maxima} = \frac{\lambda D}{s} \quad (1 \text{ mark})$$

$$\left( = \frac{0.165 \times 5}{0.75} \right) = 1.10(\text{m}) \quad (1 \text{ mark})$$

$$\text{Distance CE} \left( = \frac{1}{2} \times \text{separation} \right) = 0.55 \text{ m} \quad (1 \text{ mark})$$



# SELF ASSESSMENT

**A1.** A laser emits light of wavelength  $6.3 \times 10^{-7}$  m and is used to illuminate a double slit which has a separation of  $2.4 \times 10^{-4}$  m. Interference fringes are observed 4.2 m from the slits.

**1.1** Calculate the fringe separation.

[2 Marks]

.....

.....

.....

.....

**1.2** The double slit acts as a pair of *coherent* sources. Explain what is meant by **coherent** sources.

[2 Marks]

.....

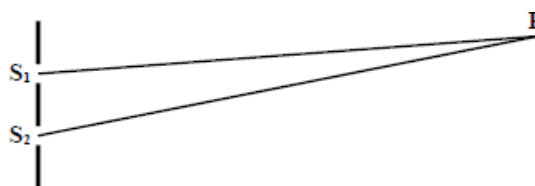
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**1.3** The diagram shows the light from the slits,  $S_1$  and  $S_2$ , meeting at **P** where the first dark fringe is observed.



Explain why a dark fringe is observed at **P**.

[3 Marks]

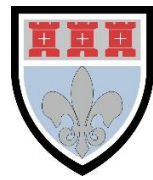
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**Reference:** AQA A Level Physics Legacy B Examinations



**A2.** Short pulses of sound are reflected from the wall of a building 18 m away from the sound source. The reflected pulses return to the source after 0.11 s.

**2.1** Calculate the speed of sound.

**[3 Marks]**

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Speed of Sound = .....

**2.2** The sound source now emits a continuous tone at a constant frequency. An observer, walking at a constant speed from the source to the wall, hears a regular rise and fall in the intensity of the sound. Explain how the **minima** of intensity occur.

**[3 Marks]**

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**Reference:** AQA A Level Physics Legacy B Examinations



**A3.** An interference pattern is produced using monochromatic light from two coherent sources. The separation of the two sources is 0.25 mm and the fringe separation is 7.8 mm. The interference pattern is observed on a screen that is 3.5 m from the sources.

**3.1** Calculate the wavelength of the light used to produce the interference pattern.

**[3 Marks]**

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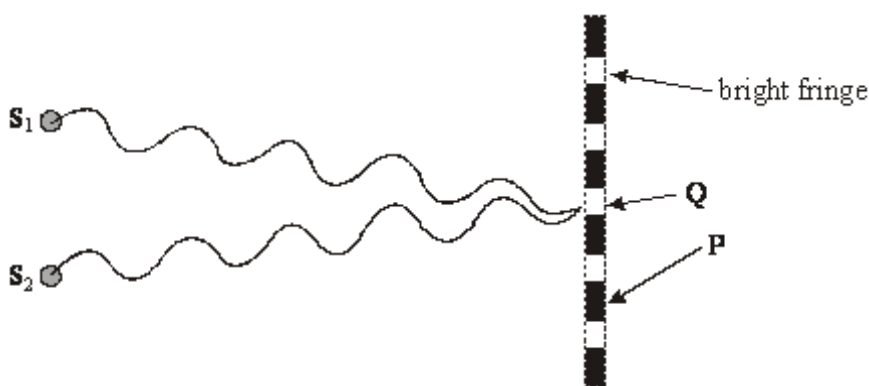
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wavelength .....

**3.2** The figure below shows light from two coherent sources,  $S_1$  and  $S_2$ , superimposing to create a bright fringe at point  $Q$ .  $Q$  is equidistant from  $S_1$  and  $S_2$ . The diagram is not to scale.



Explain how the dark fringe at the point  $P$  is caused.

**[3 Marks]**

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**Reference:** AQA A Level Physics Legacy B Examinations



## TOPIC: 3.3.2.2 Diffraction

### SPEC CHECK

Specification	Completed?
Appearance of the diffraction pattern from a single slit using monochromatic and white light.	
Qualitative treatment of the variation of the width of the central diffraction maximum with wavelength and slit width.	
Plane transmission diffraction grating at normal incidence.	
Derivation of $d\sin\theta = n\lambda$	
Applications of diffraction gratings.	



## NOTES

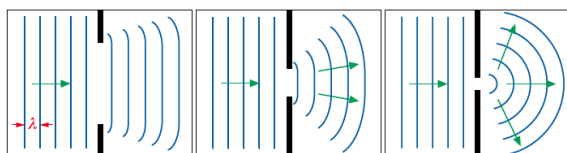
These notes are brief.

More detailed notes are found in the student preparatory reading book.

Please read the preparatory reading notes.

## Diffraction

When waves pass through a gap they spread out, this is called diffraction. The amount of diffraction depends on the size of the wavelength compared to the size of the gap.



In the first diagram the gap is several times wider than the wavelength so the wave only spreads out a little.

**When gap size > wavelength there is little diffraction**

In the second diagram the gap is closer to the wavelength so it begins to spread out more.

In the third diagram the gap is now roughly the same size as the wavelength so it spreads out the most.

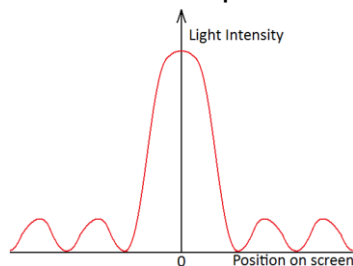
**When gap size  $\approx$  wavelength there is a lot of diffraction**

## Diffraction Patterns

Here is the diffraction pattern from light being shone through a single slit.

There is a central maximum that is twice as wide as the others and by far the brightest.

The outer fringes are dimmer and of equal width.



The smaller the slit compared to the wavelength, the greater the diffraction.

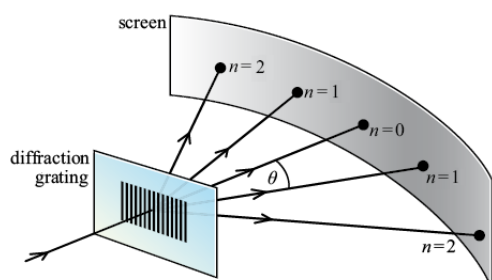
This means the central maximum would be wider but less intense.

If we use three, four or more slits the interference maxima become brighter, narrower and further apart.

## Diffraction Grating

A diffraction grating is a series of narrow, parallel slits. They usually have around 500 slits per mm.

When light shines on the diffraction grating several bright sharp lines can be seen as shown in the diagram below.



We call the different lines made by diffraction – orders of diffraction.

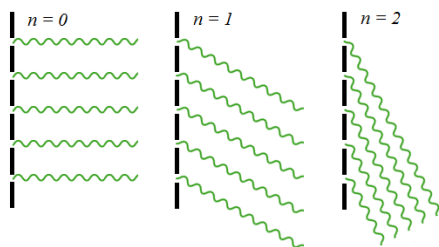
We label the middle one, the zeroth order and every order out from that is +1 order.

For example; 0<sup>th</sup>, 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup>.



The first bright line (or interference maximum) lies directly behind where the light shines on the grating. We call this the zero-order maximum.

At an angle of  $\theta$  from this lies the next bright line called the first-order maximum and so forth.



A diffraction grating gives a sharper, more intense fringe pattern than Young's slits since there is more light getting through as there are more slits.

This makes results easier to measure and the conclusion is more valid.

### The zero-order maximum ( $n=0$ )

There is no path difference between neighbouring waves. They arrive in phase and interfere constructively.

**The order number refers to the number of wavelengths difference there are in the wave paths.**

### The first-order maximum ( $n=1$ )

There is a path difference of 1 wavelength between neighbouring waves. They arrive in phase and interfere constructively.

### The second-order maximum ( $n=2$ )

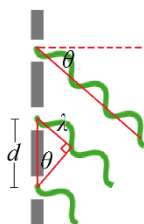
There is a path difference of 2 wavelengths between neighbouring waves. They arrive in phase and interfere constructively.

### Between the maxima

The path difference is not a whole number of wavelengths so the waves arrive out of phase and interfere destructively.

### Derivation

The angle to the maxima depends on the wavelength of the light and the separation of the slits. We can derive an equation that links them by taking a closer look at two neighbouring waves going to the first-order maximum.



Here

$d$  = Grating Spacing

The distance between the slits.

The distance to the screen is so much bigger than the distance between two slits that emerging waves appear to be parallel and can be treated that way.

Consider the triangle to the right.

$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}} \quad \rightarrow \quad \sin \theta = \frac{\lambda}{d} \quad \rightarrow \quad d \sin \theta = \lambda$$

For the  $n$ th order the opposite side of the triangle becomes  $n\lambda$ , making the equation:

$$d \sin \theta = n\lambda$$

The highest possible angle is  $90^\circ$ , so to find the highest possible order to be produced by a diffraction grating, you must substitute  $\theta = 90^\circ$  into the equation.



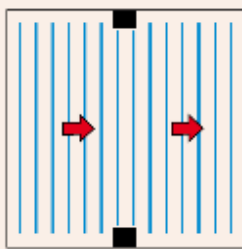
## REVISION SHEET

Highlight or underline the key information on the revision sheet to consolidate your understanding.

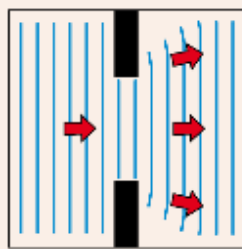
The way that **waves spread out** as they come through a **narrow gap** or go round obstacles is called **diffraction**. **All** waves diffract, but it's not always easy to observe. The amount of diffraction depends on the **size of the gap** in comparison to the **wavelength** of the wave.

### *You Can Use a Ripple Tank to Investigate Diffraction*

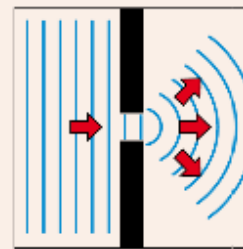
- 1) **Ripple tanks** are shallow tanks of water that you can generate a wave in.
- 2) This is done by an **oscillating paddle**, which continually dips into the water and creates regular waves with straight, parallel wavefronts.
- 3) Objects are then placed into the ripple tank to create a **barrier** with a **gap** in the middle of it.
- 4) This gap can be varied to see the effects this has on how the waves spread through the tank.



When the gap is **a lot bigger** than the **wavelength**, diffraction is **unnoticeable**.



You get **noticeable diffraction** through a gap **several** wavelengths wide.



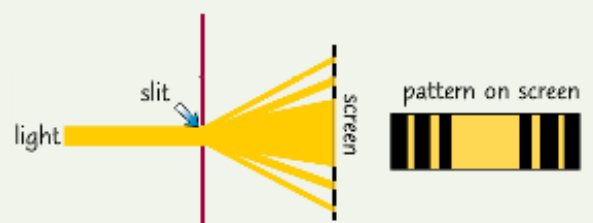
You get the **most** diffraction when the gap is **the same** size as the **wavelength**.

As the gap decreases, the diffraction becomes more noticeable until the gap becomes too small and the water waves cannot pass through it anymore. The waves are then **reflected** back on themselves.

When **sound** passes through a **doorway**, the **size of gap** and the **wavelength** are usually roughly **equal**, so **a lot** of **diffraction** occurs. That's why you have no trouble **hearing** someone through an **open door** to the next room, even if the other person is out of your **line of sight**. The reason that you can't **see** him or her is that when **light** passes through the doorway, it is passing through a **gap** around a **hundred million times bigger** than its wavelength — the amount of diffraction is **tiny**.

### *With Light Waves you get a Pattern of Light and Dark Fringes*

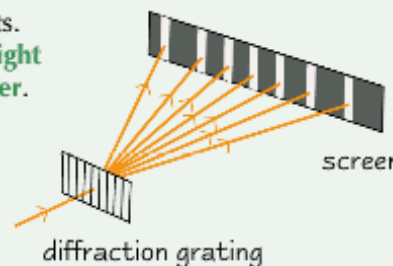
- 1) If the **wavelength** of a **light wave** is roughly similar to the size of the **aperture**, you get a **diffraction pattern** of light and dark fringes.
- 2) The pattern has a **bright central fringe** with alternating **dark and bright fringes** on either side of it.
- 3) The **spread** of the diffraction pattern depends on the **relative sizes** of the wavelength and the slit width. The **longer** the wavelength is **compared** to the **width** of the slit, the **wider** the diffraction pattern.



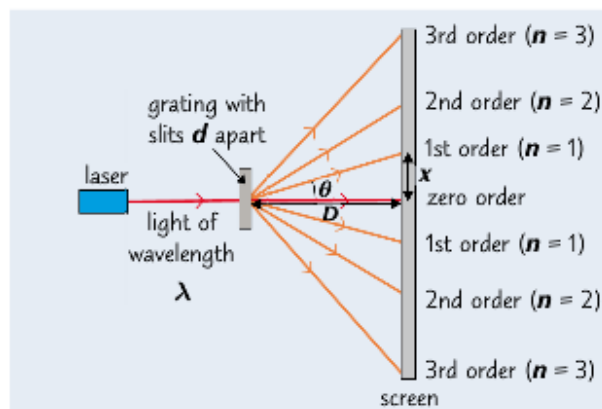


## Interference Patterns Get Sharper When You Diffract Through More Slits

- 1) You saw on pages 74-75 that **two sources** that produce waves that are **coherent** and **in phase** will result in an **interference pattern** (alternating bands of constructive and destructive interference). If you are using **light waves**, you can pass one **monochromatic** (one wavelength) **beam of light** through **two slits**. The slits are cut into the **same** piece of material, so that the wave passes through them at the **same time**. The light **diffracts** at both slits, producing two coherent sources of light. The interference pattern is made up of alternating **dark bands** and **light bands**.
- 2) You can repeat this experiment with **more than two equally spaced** slits. You get basically the **same shaped** pattern as for two slits — but the **bright bands** are **brighter** and **narrower** and the **dark areas** between are **darker**.
- 3) When **monochromatic light** is passed through a **grating**, which has **hundreds** of slits per millimetre, the interference pattern is **really sharp** because there are so **many beams reinforcing the pattern**.
- 4) Sharper fringes make for more **precise** measurements as they are easier to tell apart and so are **easier** to measure.



## Measurements Can be Made from Interference Patterns



- 1) For monochromatic light, all of the maxima are sharp lines. (It's different for white light — see the next page).
- 2) This means the distance between the maxima can be easily measured (**fringe width**).
- 3) There's a line of **maximum brightness** at the centre called the **zero order** line.
- 4) The lines just **either side** of the central one are called **first order lines**. The **next pair out** are called **second order lines** and so on.

If the grating has  $N$  slits per metre, then the slit spacing,  $d$ , is just  $1/N$  metres.

### Measuring the Wavelength of Light using a Diffraction Grating

- 1) Position a **laser** (or other monochromatic light source) in front of a **diffraction grating** so that the light travels through the grating and creates an interference pattern on a **flat wall** or **screen** a few metres away.
- 2) Measure the **distance,  $D$** , between the diffraction grating and the wall.
- 3) Measure the **distance,  $x$** , between the **zero order maximum** and the **1st order maximum** for both sides and take an **average** of the two readings.
- 4) Using the **fringe width,  $x$** , and the distance to the wall,  **$D$** , the angle the 1st order fringe makes with the zero order line can be calculated using **small angle approximations**.

Don't forget to set your calculator to radians when using this equation.

$$\tan \theta \approx \theta \text{ and } \tan \theta = \frac{x}{D}, \text{ so } \theta \approx \frac{x}{D}$$

Make sure the light is travelling at right angles to the diffraction grating and wall.

The value for  $d$  is usually given on the grating. Don't forget that  $n = 1$  for the zero order line.

- 5) You can then use the following **equation** to **calculate** the **wavelength** of light:  $d \sin \theta = n\lambda$
- 6) **Repeat** the measurements for more **order lines** to find an **average** for the wavelength.
- 7) Repeat the experiment for a diffraction grating that has a **different distance** between the **slits**.

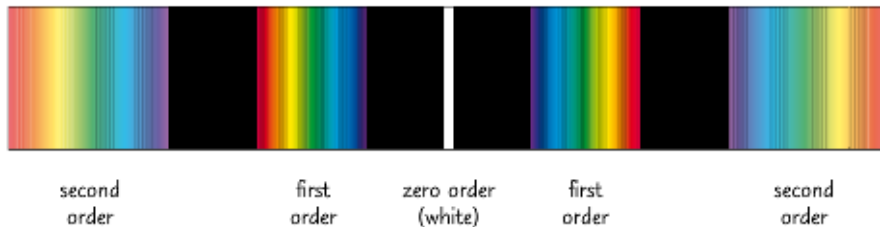


### You can Draw **General Conclusions** from $d \sin \theta = n\lambda$

- 1) If  $\lambda$  is **bigger**,  $\sin \theta$  is **bigger**, and so  $\theta$  is **bigger**. This means that the larger the **wavelength**, the more the pattern will **spread out**.
- 2) If  $d$  is **bigger**,  $\sin \theta$  is **smaller**. This means that the **coarser** the **grating**, the **less** the pattern will **spread out**.
- 3) Values of  $\sin \theta$  greater than **1** are **impossible**. So if for a certain  $n$  you get a result of **more than 1** for  $\sin \theta$  you know that that order **doesn't exist**.

### Shining **White Light** Through a **Diffraction Grating** Produces **Spectra**

- 1) **White light** is really a **mixture** of **colours**. If you **diffract** white light through a **grating** then the patterns due to **different wavelengths** within the white light are **spread out** by **different** amounts.
- 2) Each **order** in the pattern becomes a **spectrum**, with **red** on the **outside** and **violet** on the **inside**. The **zero order maximum** stays **white** because all the wavelengths just pass straight through.



- 3) **Astronomers** and **chemists** often need to study spectra to help identify elements. They use diffraction gratings rather than prisms because they're **more accurate**.
- 4) Another example of white light being **split** into a spectrum due to diffraction can be seen on **CDs** and **DVDs**. There are **grooves** etched into the **reflective surface**, which causes the light to **diffract**. Constructive interference occurs at different points for light with different wavelengths, so you end up seeing a **rainbow pattern**.



**Additional Note Space**



**Additional Note Space**



## PUZZLES QUESTIONS

To improve your understanding, answer the following puzzles.

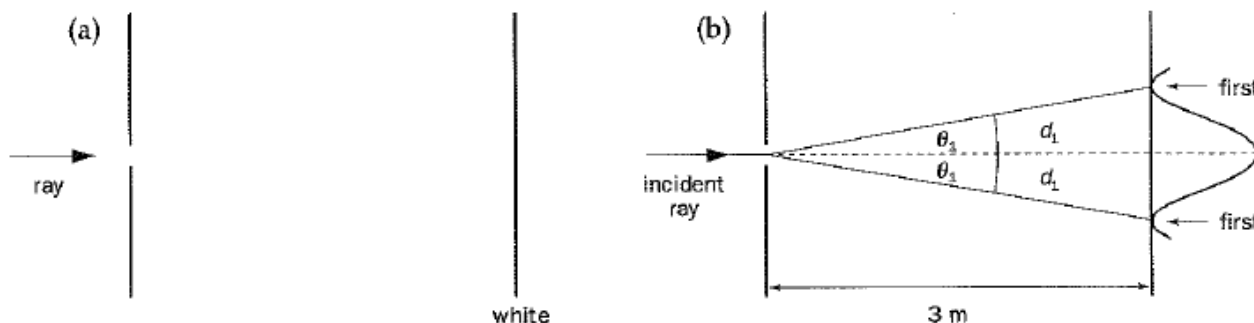
The answers are overleaf.

- A diffraction grating has 600 lines per millimetre. Monochromatic light of wavelength 650 nm is incident normally on it.

  - What do the words 'incident normally' mean?
  - What is the spacing  $d$  of the slits?
  - How many diffracted beams would you observe?
  - At what angle is each maximum formed?
- A student shines a laser of wavelength 600 nm on to a grating, which has 300 lines per millimetre. Sketch what the student sees and label the angles between the maxima. She now replaces the laser with a source of white light (wavelength range 400–750 nm). How many orders would she see now? What would be the spread of the first order?
- Monochromatic light falls on a grating, which has 500 lines per millimetre. The second-order line was observed at  $30.6^\circ$  to the central line. What is the wavelength of the light and how many orders are observed? If dark blue light had been used would more or fewer orders have been seen?
- A teacher sets up a single-slit diffraction pattern to show his pupils. The sodium light used has a wavelength of  $5.9 \times 10^{-7}$  m, and it shines on to a slit of width 0.10 mm. The students observe the diffraction pattern on a white board 3.0 m away. Complete diagram (a) to show how the observed brightness varies across the board.

Calculate the distance between the centre of the central maximum and the first minimum. Use diagram (b) to help you, and make the assumption that for small angles  $\theta = \sin \theta$ .

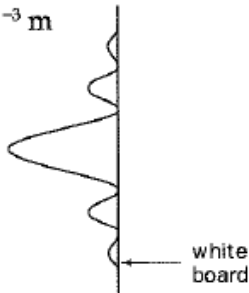
*Note:* The width of the central maximum is twice that of the other maxima.





## ANSWERS

- 1** (b)  $1.67 \times 10^{-6}$  m  
(c) 2 orders  
(d) central maximum is at  $0^\circ$  to the incident beam, first order is  $22.9^\circ$  to the incident beam and second order is  $51.1^\circ$  to the incident beam. The first and second orders appear each side of the central maximum.
- 2** First order =  $10.4^\circ$  second order =  $21.1^\circ$  third order =  $32.7^\circ$   
fourth order =  $46.1^\circ$  fifth order =  $64.2^\circ$   
Only five orders are seen with white light,  $6.8^\circ$ – $13.0^\circ$ .
- 3** 509 nm, two orders are observed and, when dark blue light is used, more orders are seen, since it has a shorter wavelength.
- 4**  $17.7 \times 10^{-3}$  m





## SAMPLE QUESTION

**S1.** A double slit interference experiment is set up in a laboratory using a source of yellow monochromatic light of wavelength  $5.86 \times 10^{-7}$  m. The separation of the two **vertical** parallel slits is 0.36 mm and the distance from the slits to the plane where the fringes are observed is 1.80 m.

**1.1** Describe the appearance of the fringes.

[2 Marks]

**They are vertical or parallel (1 mark)**

**They are equally spaced (1 mark)**

**They are black and yellow [or dark and light] bands (1 mark)**

**1.2** Calculate the fringe separation, and also the angle between the middle of the central fringe and the middle of the second bright fringe.

[2 Marks]

$$w \left( = \frac{\lambda D}{s} \right) = \frac{5.86 \times 10^{-7} \times 1.8}{0.36 \times 10^{-3}} \text{ (1)}$$

**=  $2.9 \times 10^{-3}$  m (1 mark)**

$$\tan \theta = \frac{2 \times 2.9 \times 10^{-3}}{1.8} \text{ (1 mark) gives } \theta = 0.18^\circ \text{ (1 mark)}$$

**1.3** Explain why more fringes will be seen if each of the slits is made narrower, assuming that no other changes are made.

[2 Marks]

**narrower slits give more diffraction (1 mark)**

**more overlap (so more fringes) (1 mark)**

**fringes same width (1 mark)**

Light of wavelength  $5.86 \times 10^{-7}$  m falls at right angles on a diffraction grating which has 400 lines per mm.

**1.4** Calculate the angle between the straight through image and the first order image.

[2 Marks]

$$d = \frac{1}{400 \times 10^3} \text{ (1)}$$

$$\frac{1}{400 \times 10^3} \times \sin \theta = 5.86 \times 10^{-7} \text{ (1 mark)}$$

**$\theta = 13.6^\circ$  (1 mark)**



**1.5** Determine the highest order image which can be seen with this arrangement.

[2 Marks]

**$\theta = 90^\circ$  and correctly used (1 mark)**

$$n = \frac{1}{400 \times 10^3 \times 5.86 \times 10^{-7}} = 4.3 \therefore \text{4th order (1 mark)}$$

**1.6** Give **two** reasons why the diffraction grating arrangement is more suitable for the accurate measurement of the wavelength of light than the two-slit interference arrangement.

[2 Marks]

**brighter images (1 mark)**

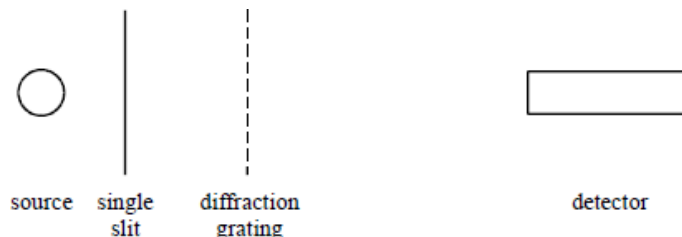
**large angles (1 mark)**

**sharper (or narrower) lines (1 mark)**



# SELF ASSESSMENT

**A1.** The diagram below is an arrangement for analysing the light emitted by a source.



**1.1** Suggest a light source that would emit a continuous spectrum.

**[1 Mark]**

.....

The light source emits a range of wavelengths from 500 nm to 700 nm. The light is incident on a diffraction grating that has 10 000 lines per metre.

**1.2** Calculate the angle from the straight through direction at which the first order maximum for the 500 nm wavelength is formed.

**[3 Marks]**

.....  
 .....  
 .....

Angle = .....

**1.3** Calculate the angular width of the first order spectrum.

**[1 Mark]**

.....  
 .....

Angular Width .....

**1.4** The detector is positioned 2.0 m from the grating. Calculate the distance between the extreme ends of the first order spectrum in this position.

**[1 Mark]**

.....  
 .....

Distance = .....



**1.5** The single slit is initially illuminated by light from a point source that is 0.02 m from the slit.

State and explain how the intensity of light incident on the single slit changes when the light source is moved to a position 0.05 m from the slit.

**[4 Marks]**

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**Reference:** AQA A Level Physics Legacy B Examinations

**A2.** A white-light source illuminates a diffraction grating that has  $6.30 \times 10^5$  lines per metre. The light is incident normally on the grating.

**2.1** Show that adjacent lines in the grating are separated by a distance of about 0.0016 mm.

**[1 Mark]**

.....

.....

.....

The table below shows the diffracting angles measured from the normal for the visible spectral orders using this grating. The angles are given for the red and blue ends of each spectrum.

	<b>First order</b>	<b>Second order</b>	<b>Third order</b>
<b>red</b>	25.4°	59.0°	not possible
<b>blue</b>	15.0°	31.1°	50.0°

**2.2** Use the value for the first order diffracting angle to calculate the wavelength of the red light.

**[3 Marks]**

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Wavelength of the Red Light .....



**2.3** Describe carefully the appearance of the complete diffraction pattern on the screen. You may draw a sketch of the pattern to help your explanation if you choose.

**[4 Marks]**

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**Reference:** AQA A Level Physics Legacy B Examinations

**A3.** Light from a laser has a wavelength of  $6.30 \times 10^{-7}$  m. When the laser light is incident normally on a diffraction grating the first order maximum is produced at an angle of  $12^\circ$ .

**3.1** Calculate the spacing between the lines on the grating.

**[2 Marks]**

.....

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Spacing of lines .....

**3.2** Calculate the number of positions of maximum light intensity that are produced when the laser light is incident on the grating.  
Show your reasoning clearly.

**[3 Marks]**

.....

.....

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.....

.....

Number of positions .....

**Reference:** AQA A Level Physics Legacy B Examinations



## TOPIC: 3.3.2.3 Refraction at a Plane Surface

### SPEC CHECK

Specification	Completed?
Refractive index of a substance, $n = \frac{c}{c_s}$	
Students should recall that the refractive index of air is approximately 1.	
Snell's law of refraction for a boundary $n_1 \sin \phi_1 = n_2 \sin \phi_2$	
Total internal reflection $\sin \phi_c = \frac{n_2}{n_1}$	
Simple treatment of fibre optics including the function of the cladding.	
Material and modal dispersion.	
Understand the principles and consequences of pulse broadening and absorption.	



## NOTES

These notes are brief.

More detailed notes are found in the student preparatory reading book.

Please read the preparatory reading notes.

### Refractive Index

The refractive index of a material is a measure of how the wave speed of a wave changes as it moves through different material. The refractive index of material,  $s$ , can be calculated using:

$$n = \frac{c}{c_s}$$

The speed of a wave can change as the wavelength of the wave changes, in slower mediums, the wavelength decreases.

Throughout refraction, frequency remains constant.

This equation is only valid if you consider the starting medium of the wave to be either a vacuum or air.

This is because the refractive index of air/vacuum is 1.00

If the starting medium is not air or a vacuum a more detailed equation must be used.

Where  $n$  is the refractive index,  $c$  is the speed of light in a vacuum and  $c_s$  is the speed of light in material  $s$ .

#### Refractive Index, $n$ , has no units

If light can travel at  $c$  in material  $x$  then the refractive index is:  $n = \frac{c}{c_x} \rightarrow n = \frac{c}{c} \rightarrow n = 1$

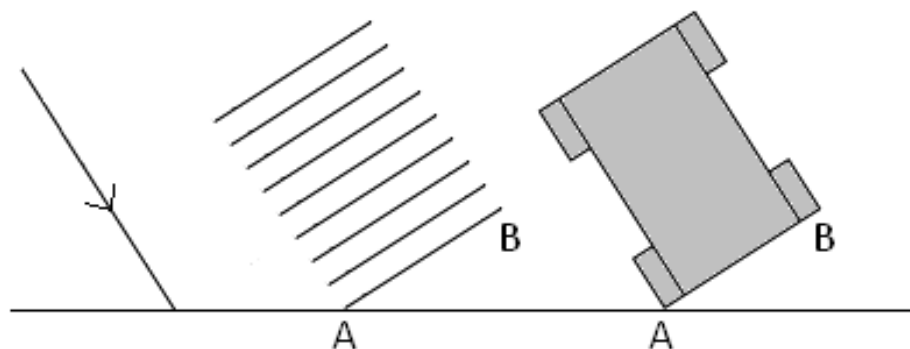
If light can travel at  $c/2$  in material  $y$  then the refractive index is:  $n = \frac{c}{c_y} \rightarrow n = \frac{c}{c/2} \rightarrow n = 2$

The higher the refractive index the slower light can travel through it

The higher the refractive index the denser the material

### Bending Light

When light passes from one material to another it is not only the speed of the light that changes, the direction can change too.

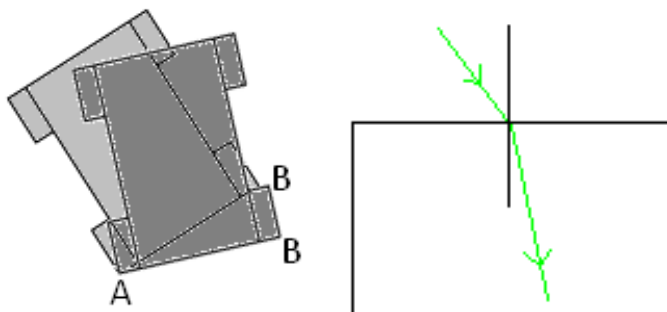


If the ray of light is incident at  $90^\circ$  to the material then there is no change in direction, only speed.



It may help to imagine the front of the ray of light as the front of a car to determine the direction the light will bend. Imagine a lower refractive index as grass and a higher refractive index as mud.

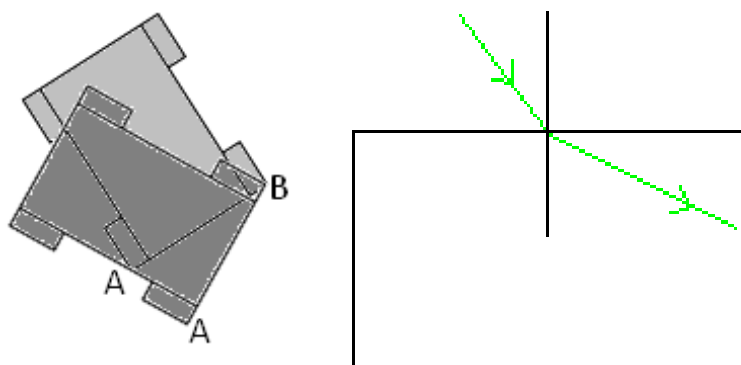
### Entering a Denser Material



The car travels on grass until tyre A reaches the mud. It is harder to move through mud so A slows down but B can keep moving at the same speed as before. The car now points in a new direction.

### Denser material – higher refractive index – bends towards the Normal

### Entering a Less Dense Material



The car travels in mud until tyre A reaches the grass. It is easier to move across grass so A can speed up but B keeps moving at the same speed as before. The car now points in a new direction.

### Less dense material – lower refractive index – bends away from the Normal

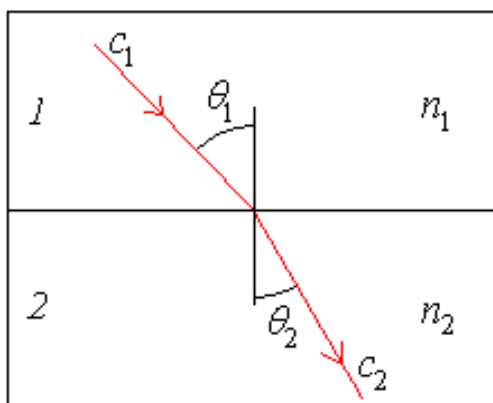


## Relative Refractive Index

Whenever two materials touch the boundary between them will have a refractive index dependent on the refractive indices of the two materials.

We call this the relative refractive index.

When light travels from material 1 to material 2 we can calculate the relative refractive index of the boundary using any of the following:



$${}_1n_2 = \frac{n_2}{n_1} = \frac{c_1}{c_2} = \frac{\sin \theta_1}{\sin \theta_2}$$

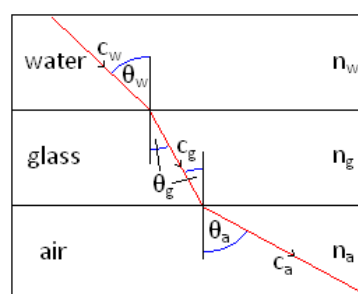
This refractive index equation can be used in any situation.

**Relative Refractive Index,  ${}_1n_2$ , has no units**

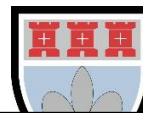
Some questions may involve light travelling through several layers of materials. Tackle one boundary at a time.

$${}_wn_g = \frac{n_g}{n_w} = \frac{c_w}{c_g} = \frac{\sin \theta_w}{\sin \theta_g} \quad \text{----->}$$

$${}_gn_a = \frac{n_a}{n_g} = \frac{c_g}{c_a} = \frac{\sin \theta_g}{\sin \theta_a} \quad \text{----->}$$



Some questions may ask to work out if the wave would reflect or refract at different mediums – do not get caught about what is taking place at each interface.



These notes are brief.

More detailed notes are found in the student preparatory reading book.

Please read the preparatory reading notes.

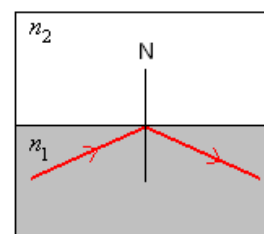
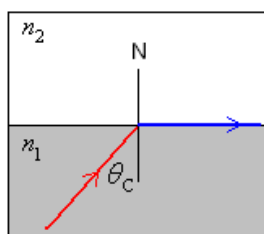
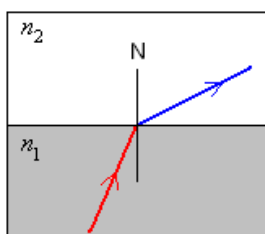
## Total Internal Reflection

We know that whenever light travels from one material to another the majority of the light refracts but a small proportion of the light also reflects off the boundary and stays in the first material.

When the incident ray strikes the boundary at an angle **less than the critical angle** the light refracts into the second material.

When the incident ray strikes the boundary at an angle **equal to the critical angle** all the light is sent along the boundary between the two materials.

When the incident ray strikes the boundary at an angle **greater than the critical angle** all the light is reflected and none refracts, we say it is total internal reflection has occurred.



At exactly the critical angle, the angle of refraction and reflection is 90 degrees.

This is another way to define critical angle.

In addition, for total internal reflection to occur, the second medium must have a lower refractive index than the first medium.

$$n_1 > n_2$$

## Critical Angle

If the **refractive index of the second medium was higher than the first medium**, then total internal reflection would not occur and **some of the wave would be refracted**.

We can derive an equation that connects the critical angle with the refractive indices of the materials.

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1}$$

But at the critical angle  $\theta_2$  is equal to  $90^\circ$  which makes  $\sin \theta_2 = 1$  this gives...

$$\frac{\sin \theta_1}{1} = \frac{n_2}{n_1}$$

$\theta_1$  is the critical angle which we represent as  $\theta_c$  making the equation:

$$\sin \theta_c = \frac{n_2}{n_1}$$



When the second material is air  $n_2 = 1$ ,

So, the equation becomes:

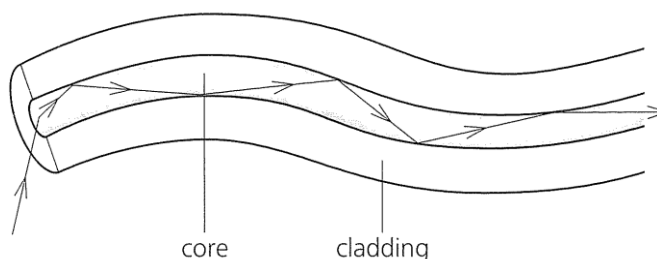
$$\sin \theta_c = \frac{1}{n_1} \quad \text{or}$$

$$n_1 = \frac{1}{\sin \theta_c}$$

## Optical Fibres/Fibre Optics

An optical fibre is a thin piece of flexible glass.

Light can travel down it due to total internal reflection. Their uses include:



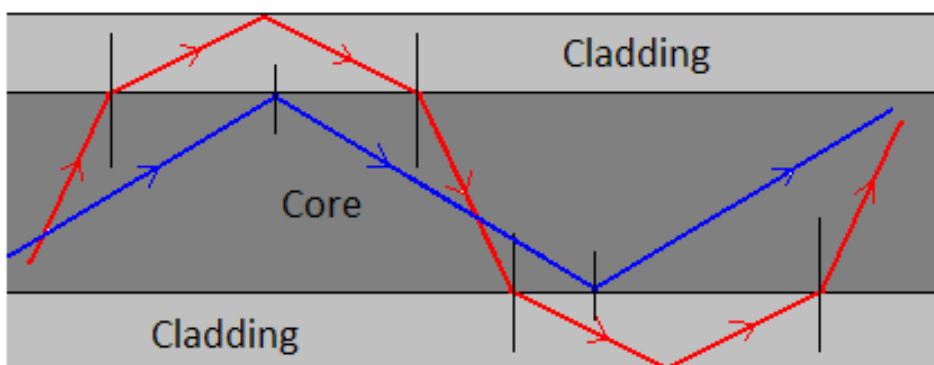
**Communication such as phone and TV signals:** they can carry more information than electricity in copper wires. The data is encrypted as a binary code of flashing of light. This requires an optical material with a high critical angle to reduce the number of possible total internal reflection paths.

**Medical endoscopes:** they allow us to see down them and are flexible so they don't cause injury to the patient.

This requires an optical material with a low critical angle to have a high number of possible total internal reflection paths.

## Cladding

Cladding is added to the outside of an optical fibre to reduce the amount of light that is lost. It does this by giving the light rays a second chance at TIR as seen in the diagram.



Cladding is also used to protect the core from damage and to stop data transferring between the cores of different fibre optics.

It does increase the critical angle but the shortest path through the optical fibre is straight through, so only letting light which stays in the core means the signal is transmitted quicker.



Consider the optical fibre with a refractive index of 1.5...

Without cladding  $n_2 = 1$        $\sin \theta_c = \frac{n_2}{n_1}$        $\sin \theta_c = \frac{1}{1.5}$        $\theta_c = 41.8^\circ$

With cladding  $n_2 = 1.4$        $\sin \theta_c = \frac{n_2}{n_1}$        $\sin \theta_c = \frac{1.4}{1.5}$        $\theta_c = 69.0^\circ$

If the cladding had a lower refractive index than the core it is easier for light to travel through so the light would bend away from the normal, **THIS GIVES TOTAL INTERNAL REFLECTION**

If the cladding had a higher refractive index than the core it is harder for light to travel through so the light would bend towards the normal, **THIS GIVES REFRACTION.**

**There can be several issues with data transmission in fibre optics.**

**Modal Dispersion** – If there are a variety of paths the incident light can totally internally reflect then the waves will spread out. Eventually, the waves will interfere with itself and produce an interference pattern. This spreads out the signal, this is called **PULSE BROADENING.**

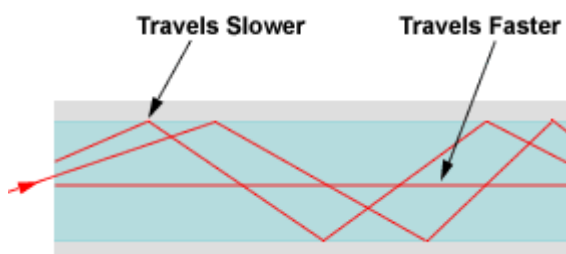
Modal dispersion is reduced by having a high critical angle in the optic core – this is achieved by placing cladding around the core.

**Material Dispersion** – Different wavelengths of a single wave (if this is white light) can travel at different speeds inside the core, making the wave spread out. Eventually, the waves will interfere with itself and produce an interference pattern. This spreads out the signal, this is called **PULSE BROADENING.**

Material dispersion is reduced by using a monochromatic light source for the wave in the optic core – this is achieved by using laser light.

**Attenuation** – Some of the wave will be absorbed by the core as it passes through it, this is called attenuation.

Attenuation can be reduced by making the optic core out of a transparent material.

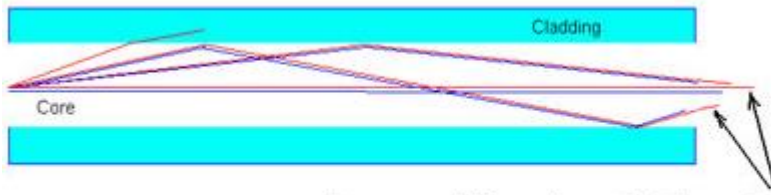


This is a diagram showing modal dispersion.

If the wave takes different paths in the core, it will spread out and pulse broaden.

This is a diagram showing material dispersion.

Different wavelengths of a wave spread out and pulse broaden.



Longer Wavelength Travels Faster



## REVISION SHEET

Highlight or underline the key information on the revision sheet to consolidate your understanding.

### Refraction Occurs When the Medium a Wave is Travelling in Changes

**Refraction** is the way a wave **changes direction** as it enters a **different medium**. The change in direction is a result of the wave **slowing down** or **speeding up**. You can tell if the wave is speeding up or slowing down by the way it **bends towards** or **away** from the normal.

- 1) If a light ray bends **towards** the normal — it is **slowing down**. The ray is going from a **less** optically dense material to a **more** optically dense material.
- 2) If the ray bends **away** from the normal — the wave is **speeding up**. It is going from an optically **denser** material to a **less** optically dense material.
- 3) The speed changes because the **wavelength** of the wave is changing and the **frequency** stays **constant** ( $v = f\lambda$ ).

The more optically dense material will have a higher refractive index (see below).

If light travels from a **less** optically dense material to a **more** optically dense material, the wave **slows down**, the **wavelength decreases** and the **frequency** stays the **same**.

### The Refractive Index of a Material Measures How Much It Slows Down Light

Light goes fastest in a **vacuum**. It **slows down** in other materials, because it **interacts** with the particles in them. The more **optically dense** a material is, the more light **slows down** when it enters it.

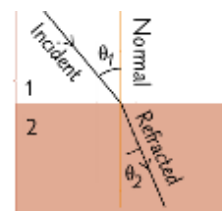
The **refractive index** of a material,  $n$ , is the **ratio** between the **speed of light** in a **vacuum**,  $c$ , and the speed of light in that **material**,  $v$ .

$$n = \frac{c}{v}$$

$$c = 3.00 \times 10^8 \text{ ms}^{-1}$$

The speed of light in air is only a tiny bit smaller than  $c$ . So you can assume the refractive index of air is 1.

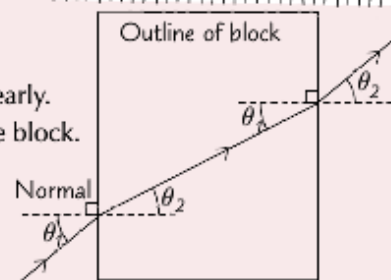
- 1) The **angle** the **incoming light** makes to the **normal** is called the **angle of incidence**,  $\theta_1$ . The **angle** the **refracted ray** makes with the **normal** is the **angle of refraction**,  $\theta_2$ .
- 2) The light is crossing a **boundary**, going from a medium with **refractive index**  $n_1$  to a medium with refractive index  $n_2$ .
- 3) When light enters an **optically denser** medium it is refracted **towards** the normal.
- 4)  $n_1$ ,  $n_2$ ,  $\theta_1$  and  $\theta_2$  are related by **Snell's law**:  $n_1 \sin \theta_1 = n_2 \sin \theta_2$



You can use a **ray box** to find the **refractive index** of a glass block:

You can do this experiment to find the refractive index of any solid, transparent material.

- 1) Place a glass block on a piece of paper and draw around it.
- 2) Use the ray box to shine a beam of light into the glass block. Turn off any other lights so you can see the path of the light beam through the block clearly.
- 3) **Trace** the path of the **incoming** and **outgoing** beams of light either side of the block.
- 4) Remove the block and join up the two paths you've drawn with a **straight line** that follows the path the light beam took through the glass block. You should be able to see from your drawing how the path of the ray **bent** when entering and leaving the block.
- 5) Measure the angles of incidence ( $\theta_1$ ) and refraction ( $\theta_2$ ) where the light enters and exits the block. Air is **less** optically dense than glass, so as the light **enters** the glass block it **bends towards** the normal ( $\theta_1 > \theta_2$ ) as it **slows down**. The beam should **bend away** from the normal as it **exits** the block ( $\theta_2 > \theta_1$ ) and **speeds up**.
- 6) Rearrange **Snell's law** to make the refractive index of the **material** the subject, and substitute in  $n = 1$  for **air** and the values you found for  $\theta_1$  and  $\theta_2$  to calculate a value for  $n$ .
- 7) The **percentage uncertainty** (see p.10) in your measurements for  $\theta_1$  and  $\theta_2$  will be **smaller** for **larger angles**, so it's better to do the experiment at large angles and then **repeat** the experiment to find an **average** of your results.
- 8) Your result should be more **precise** (see p.12) if you use a **narrower** beam of light as the **uncertainty** in the **position** of the beam will be lower.

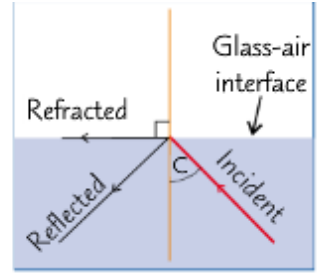


Alternatively, you could plot a graph of  $\sin \theta_1$  against  $\sin \theta_2$ . The gradient of the graph will be equal to the refractive index of the material.



When light **goes from** an optically dense material into an optically **less dense** material (e.g. glass to air), interesting things can start to happen.

Shine a ray of light at a **glass to air** boundary, then gradually **increase** the angle of incidence. As you increase the angle of incidence, the angle of **refraction** gets closer and closer to **90°**. Eventually the angle of incidence,  $\theta_1$ , reaches a **critical angle C** for which the angle of refraction,  $\theta_2 = 90^\circ$ . The light is refracted **along the boundary**.



At angles of incidence **greater than C**, refraction is **impossible**. That means **all** the light is reflected back into the material. This effect is called **total internal reflection**.

For light hitting a **material-to-air boundary** (assuming the material is more optically **dense**) at the critical angle, **Snell's law** simplifies to become:

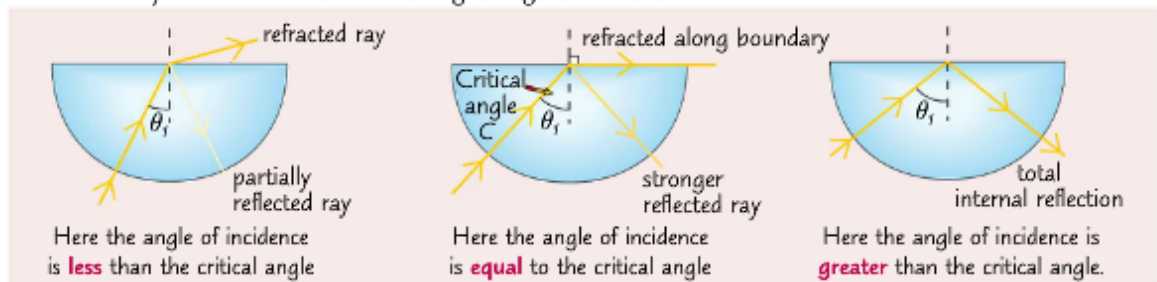
$$\sin C = \frac{1}{n}$$

This happens because  $n_{air} = 1$  and  $\sin(90^\circ) = 1$ .  $n$  is the refractive index of the material.

**You can Investigate Critical Angles and Total Internal Reflection with Glass Blocks**

- 1) Shine a light ray into the **curved face** of a semi-circular glass block so that it always enters at **right angles** to the edge — this means the ray won't **refract** as it enters the block, just when it leaves from the straight edge.
- 2) Vary the angle of **incidence**,  $\theta_1$ , until the light beam refracts so much that it exits the block along the **straight edge**. This angle of incidence is the **critical angle, C**, for glass-air boundary.
- 3) If you increase the angle of incidence so it's **greater** than C, you'll find the ray is reflected from the straight edge of the block.

You can rearrange the formula for the critical angle above and put in your value for C to find the refractive index of the block.





**Additional Note Space**



**Additional Note Space**



## PUZZLES QUESTIONS

To improve your understanding, answer the following puzzles.

The answers are overleaf.

- Using the data below, verify that the refractive index of water is 1.33.
- What is the refractive index for a ray travelling from water to air, from the data below?

Angle of incidence in air	Angle of refraction in water
10.0°	7.5°
20.0°	14.9°
30.0°	22.0°
40.0°	29.0°
50.0°	35.0°
60.0°	40.5°
70.0°	45.0°
80.0°	48.0°

- A ray of light strikes a glass block at an angle of 30°. If the refractive index of glass is 1.5, what is the angle of refraction?
  - Sketch the graph you would obtain if you plotted  $\sin i$  against  $\sin r$ .
  - Look up the refractive index of glass in your data book. Why do you think that there are several values given?
  - If the refractive index of glass is 1.5 and the speed of light in air is  $300\,000\,000\text{ m s}^{-1}$ . What is the speed of light in glass?
  - Find the deviation produced by a dense flint equilateral glass prism of refractive index 1.66 when a ray of light strikes it at an angle of 32°. Draw a sketch of the path of the ray through the prism.
  - Calculate the speed of light in a diamond, given that the speed of light in a vacuum is  $2.998 \times 10^8\text{ m s}^{-1}$  and that the refractive index of diamond is 2.42.
  - You look at the bottom of a 2 m swimming pool. How deep does it appear to your eye? Take the refractive index of water to be 1.33.
- TIP:* Do not forget to include any reflected rays on your diagrams for refraction. Remember reflection and refraction can both occur.



## ANSWERS

- 1** This provides practice with sine for non-mathematicians.
- 2** 0.75
- 3**  $19.47^\circ$
- 4** This question provides an opportunity to go over rearranging equations and relationships.
- 5** Different types of glass are available and different colours of light are passed through it.
- 6**  $200\,000\,000\text{ m s}^{-1}$
- 7**  $52.28^\circ$ . This can be done in stages and some students will need help with the geometry.
- 8**  $1.239 \times 10^8\text{ m s}^{-1}$
- 9** 1.50 m

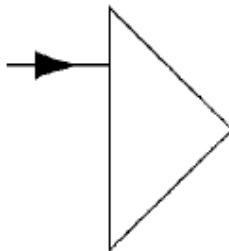


## PUZZLES QUESTIONS

To improve your understanding, answer the following puzzles.

The answers are overleaf.

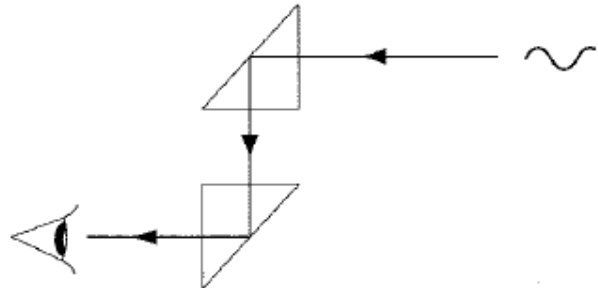
- The refractive index of glass is 1.5. Calculate the critical angle for a ray travelling from glass to air. (Take care with this – think about the value of refractive index.)
- Diamonds have a refractive index of 2.42. What is the critical angle for a diamond engagement ring? Why does the diamond ring sparkle?
- The critical angle for light travelling from glass to air is  $41^\circ$ . What is the refractive index of the glass block?
- Right-angled isosceles prisms, rather than plane mirrors, are used in periscopes. Sketch a diagram to show the path of the ray through the prism so that it is turned through  $90^\circ$ . Why are plane mirrors not used in this instrument?
- Complete the diagram below to show the path of the light through the prism. This is of use in bicycle reflectors and Cat's eyes. Say how these items work.
- The prism (right) can be used in a pair of binoculars. Why do you use two prisms in each eyepiece?
- Explain the terms
  - refractive index
  - critical angle.
  - You are driving along on a hot summer's day and the road ahead looks wet, as though there is a pool of water in the middle of it. Explain your answer in terms of (a) and (b).





## ANSWERS

- 1 41.81°
- 2 24.41°. This answer indicates that any light incident on a face of the stone at an angle greater than 24.41° will be totally internally reflected.
- 3 1.52
- 4 Prisms are better in a periscope rather than mirrors because you can get multiple reflections (one from the front and one from the back surface of the mirror) and mirrors reflect less light than the prism does and are damaged more easily.



- 5
 
 Cat's eyes and bike reflectors are made like this to reflect the car headlight rays back.

- 6 A second prism inverts the rays of light so that the image is the right way up and the right way round. The arrangement ensures that the binoculars are compact.
- 7 (c) The road surface has absorbed heat energy and is hotter than the surrounding air. This means that there are hot layers of air near the tarmac and cooler layers higher up. The density of the layer of air depends on its temperature and light from the sky is refracted more and more as it travels through the atmosphere. The hotter layers refract it the most, and total internal reflection takes place when the angle of incidence is greater than the critical angle. You think that the reflection of the sky is a pool of water.

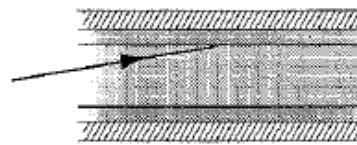
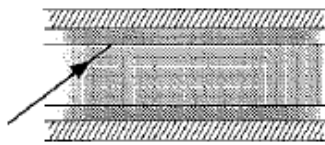


## PUZZLES QUESTIONS

To improve your understanding, answer the following puzzles.

The answers are overleaf.

- 1 What would be the advantage of using a low-powered laser to transmit messages along a fibre-optic tube?
- 2 If the refractive index of the core of an optical fibre is 1.520 and the speed of light in air is  $300\,000\,000\text{ m s}^{-1}$ , how long does it take a ray of light to travel down the axis of a fibre 50 km long?
- 3 It is essential that the ray travel down the tube as quickly as possible. Complete the paths of the two rays in the diagrams below, paying attention to the laws of physics, to demonstrate that the critical angle must be as high as possible.



- 4 The critical angle of a particular fibre optic system is  $78^\circ$ . The refractive index of the core is 1.53. What is the refractive index of the cladding?
- 5 If the refractive index of the core of a fibre is 1.494 and the cladding has a refractive index of 1.476, what is the critical angle for the boundary of the two glasses?
- 6 A piece of optical fibre is 150 m long. Two rays of light travel along the fibre. One goes along the axis and the other is reflected off the boundary at the critical angle of  $85^\circ$ . What is the difference in the time taken by the two rays? Speed of light can be taken to be  $300\,000\,000\text{ m s}^{-1}$  in air and the refractive index of the core is 1.492.
- 7 Communications was one use of fibre optics. Find out other uses of these fibres.



## ANSWERS

- 1** A low-powered laser would consist of an intense beam of monochromatic light. There would be no dispersion of the beam so the signals would arrive at the receiver without distortion.
- 2** Find the speed of light in the fibre-optic tube ( $1.974 \times 10^8 \text{ m s}^{-1}$ ) and then find the time taken to travel 50 km, i.e.  $2.5 \times 10^{-4} \text{ s}$ .
- 3** It takes less time for the ray to travel down the tube if the critical angle is large.
- 4** 1.497
- 5**  $81.1^\circ$
- 6** Speed of the light in core =  $2.01 \times 10^8 \text{ m s}^{-1}$ . The ray at  $85^\circ$  travels an extra 0.57 m; the time for this is  $2.85 \times 10^{-9} \text{ s}$ .
- 7** Medical uses can be investigated here, as well as industrial uses.



## SAMPLE QUESTION

**S1.** The diagram shows a ray of monochromatic light, in the plane of the paper, incident on the end face of an optical fibre.



**1.1** Draw on the diagram the complete path followed by the incident ray, showing it entering into the fibre and emerging from the fibre at the far end.

[3 Marks]

**diagram to show: refraction towards normal on entry (1 mark)**

**total internal reflection shown along fibre (1 mark)**

**refraction away from normal on leaving glass (1 mark)**

**1.2** State any changes that occur in the speed of the ray as it follows this path from the source. Calculations are not required.

[1 Mark]

**Speed of light decreases on entry into glass and increases on leaving**

**1.3** Calculate the critical angle for the optical fibre at the air boundary.

Refractive index of the optical fibre glass = 1.57

use of  $\sin \theta_c = \frac{1}{n}$  gives  $\sin \theta_c = \frac{1}{1.57}$  (1 mark)

$\theta_c = 39.6^\circ$  (1 mark)

**1.4** The optical fibre is now surrounded by cladding of refractive index 1.47. Calculate the critical angle at the core-cladding boundary.

[3 Marks]

$\left(\frac{n_2}{n_1}\right) = \frac{1.57}{1.47}$  (1 mark) (=1.07)

$\sin \theta_c = \frac{1}{1.07}$  (1 mark)

$\theta_c = 69.4^\circ$  (1 mark)

**1.5** State **one** advantage of cladding an optical fibre.

[1 Mark]

**to protect the core surface  
[or to prevent cross-over]**

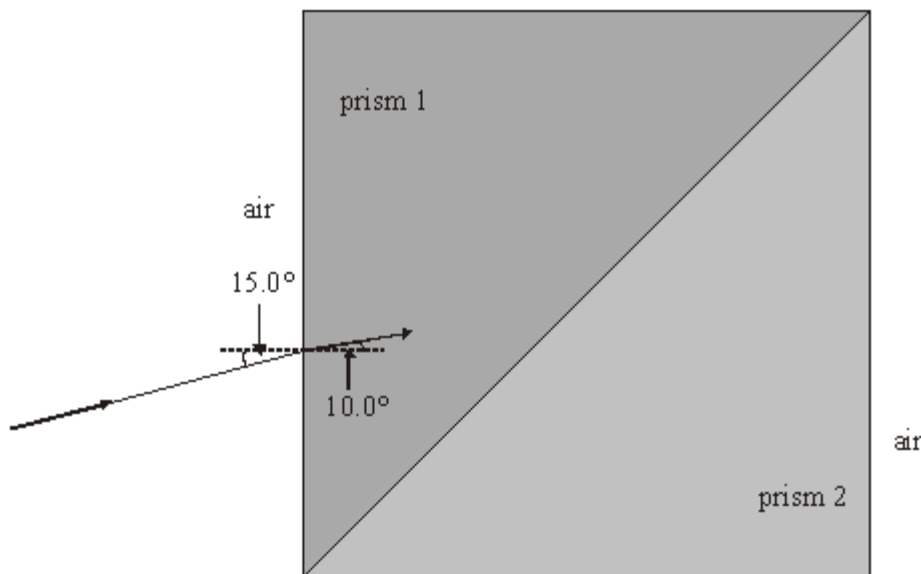
**(1 mark)**



# SELF ASSESSMENT

**A1.** A ray of light passes from air into a glass prism as shown in **Figure 1**.

**Figure 1**



**A1.1** Confirm, by calculation, that the refractive index of the glass from which the prism was made is 1.49.

**[1 Mark]**

.....

.....

.....

.....

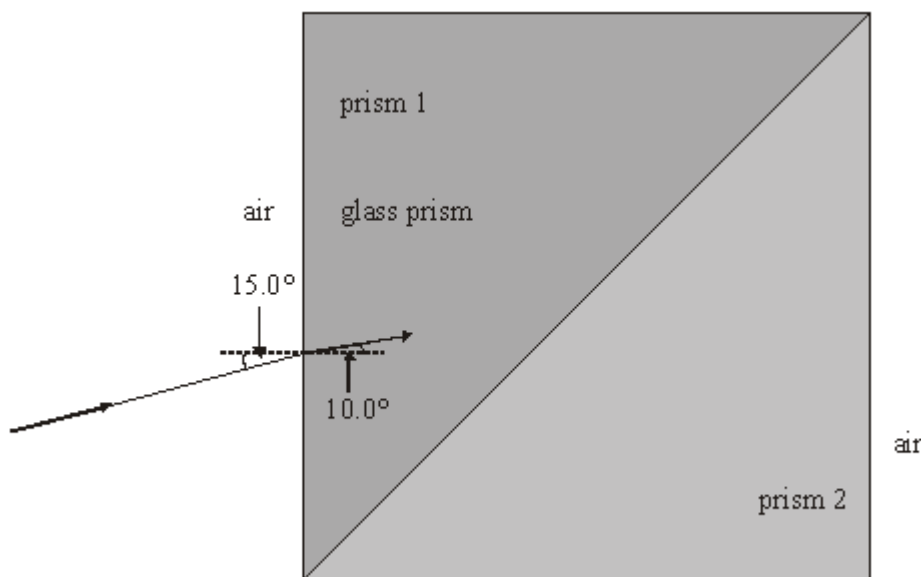
**A1.2** On **Figure 1**, draw the continuation of the path of the ray of light until it emerges back into the air. Write on **Figure 1** the values of the angles between the ray and any normals you have drawn. The critical angle from glass to air is less than  $45^\circ$

**[2 Marks]**



A second prism, **prism 2**, made from transparent material of refractive index 1.37 is placed firmly against the original prism, **prism 1**, to form a cube as shown in **Figure 2**.

**Figure 2**



**A1.3** The ray strikes the boundary between the prisms.

**[7 Marks]**

Calculate the angle of refraction of the ray in **prism 2**.

.....

.....

.....

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**A1.4** Calculate the speed of light in **prism 2**.

.....

.....

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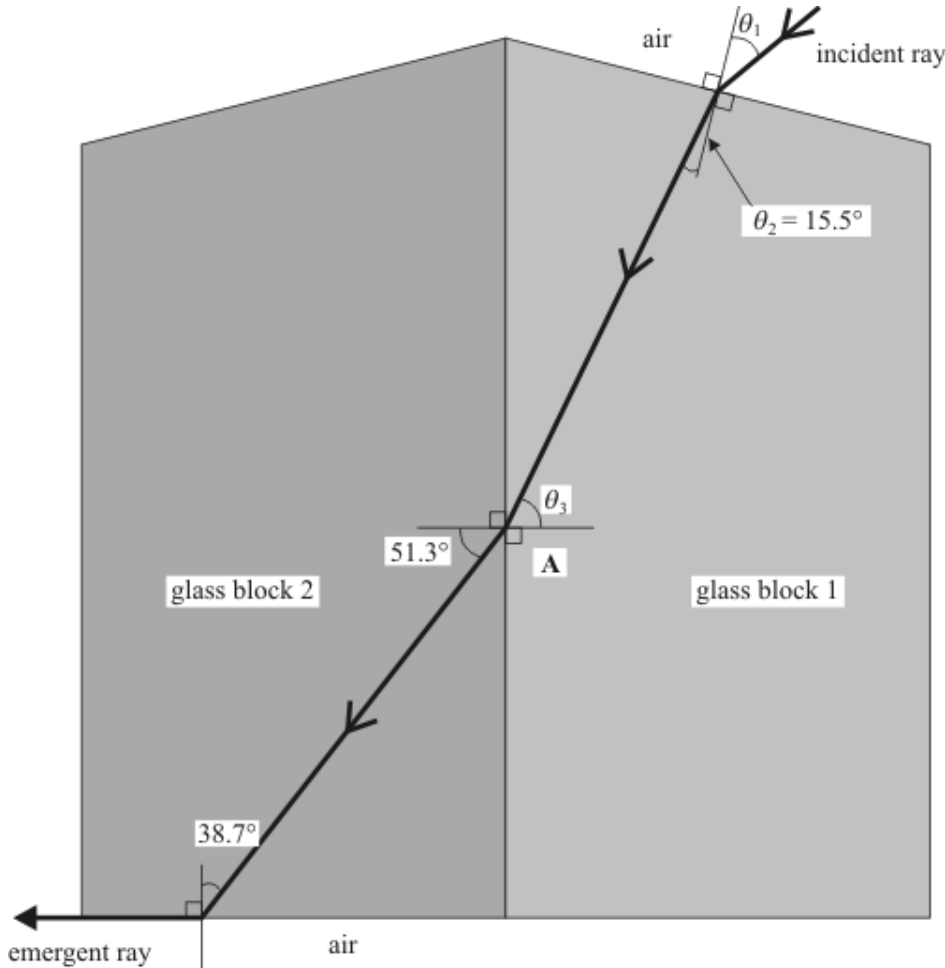
**A1.5** Draw a path the ray could follow to emerge from **prism 2** into the air.

**Reference:** AQA A Level Physics Legacy A Examinations



**A2.** The figure below shows a ray of light passing from air into glass at the top face of **glass block 1** and emerging along the bottom face of **glass block 2**.

Refractive index of the glass in block 1 = 1.45



Calculate

[7 Marks]

**A2.1** the incident angle  $\theta_1$ ,

.....  
 .....

**A2.2** the refractive index of the glass in block 2,

.....  
 .....

**A2.3** the angle  $\theta_3$  by considering the refraction at point A.

.....  
 .....



**A2.4** In which of the two blocks of glass will the speed of light be greater?

[2 Marks]

.....

.....

Explain your reasoning.

.....

.....

.....

.....

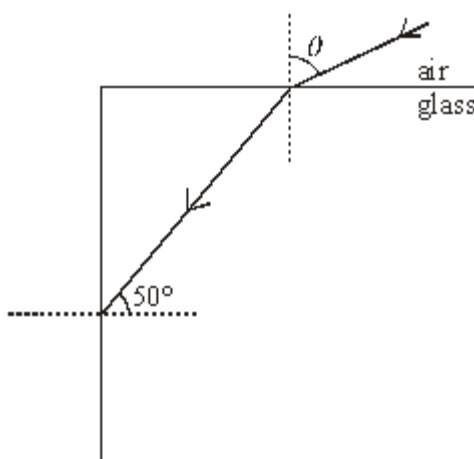
**A2.5** Using a ruler, draw the path of a ray partially reflected at **A** on the figure above. Continue the ray to show it emerging into the air. No calculations are expected.

[2 Marks]

**Reference:** AQA A Level Physics Legacy A Examinations

**A3.** The diagram shows a cube of glass. A ray of light, incident at the centre of a face of the cube, at an angle of incidence  $\theta$ , goes on to meet another face at an angle of incidence of  $50^\circ$ , as shown in the figure below

critical angle at the glass-air boundary =  $45^\circ$

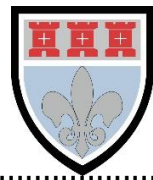


**A3.1** Draw on the diagram the continuation of the path of the ray, showing it passing through the glass and out into the air.

[3 Marks]

**A3.2** Show that the refractive index of the glass is 1.41

[2 Marks]



.....  
.....  
**A3.3** Calculate the angle of incidence,  $\theta$ .

**[3 Marks]**

.....  
.....  
.....  
.....

**Reference:** AQA A Level Physics Legacy A Examinations



## REVISION CHECKLIST

Specification reference	Checklist questions	
3.3.2.1	Can you define path difference and coherence?	<input type="checkbox"/>
3.3.2.1	Can you explain interference and diffraction using a laser as a source of monochromatic light?	<input type="checkbox"/>
3.3.2.1	Can you describe Young's double-slit experiment?	<input type="checkbox"/>
3.3.2.1	Can you describe the use of two coherent sources or the use of a single source with double slits to produce an interference pattern?	<input type="checkbox"/>
3.3.2.1	Can you explain fringe spacing using the equation $w = \frac{\lambda D}{s}$ ?	<input type="checkbox"/>
3.3.2.1	Can you describe the production of an interference pattern using white light?	<input type="checkbox"/>
3.3.2.1	Can you describe safety issues associated with using lasers?	<input type="checkbox"/>
3.3.2.1	Can you describe and explain interference produced with sound and electromagnetic waves?	<input type="checkbox"/>
3.3.2.1	Can you explain how our knowledge and understanding of the nature of electromagnetic radiation has changed over time?	<input type="checkbox"/>
3.3.2.1	Have you carried out an investigation of interference effects using the Young double-slit experiment and the diffraction grating?	<input type="checkbox"/>
3.3.2.2	Can you describe the appearance of the diffraction pattern from a single slit using monochromatic and white light?	<input type="checkbox"/>
3.3.2.2	Can you describe how the width of the central diffraction maximum varies with wavelength and slit width?	<input type="checkbox"/>
3.3.2.2	Can you describe the diffraction pattern when light is shone on a plane transmission diffraction grating at normal incidence?	<input type="checkbox"/>



Specification reference	Checklist questions	
3.3.2.2	Can you derive $d\sin\theta = n\lambda$ ?	<input type="checkbox"/>
3.3.2.2	Can you suggest some applications of diffraction gratings?	<input type="checkbox"/>
3.2.2.3	Can you calculate the refractive index of a substance using $n = \frac{c}{c_s}$ ?	<input type="checkbox"/>
3.2.2.3	Can you recall that the refractive index of air is approximately 1?	<input type="checkbox"/>
3.2.2.3	Can you recall and use Snell's law of refraction ( $n_1\sin\theta_1 = n_2\sin\theta_2$ ) for a boundary?	<input type="checkbox"/>
3.2.2.3	Can you explain total internal reflection using $\sin\theta_c = \frac{n_2}{n_1}$ ?	<input type="checkbox"/>
3.2.2.3	Can you explain fibre optics, including the function of the cladding?	<input type="checkbox"/>
3.2.2.3	Can you explain material and modal dispersion?	<input type="checkbox"/>
3.2.2.3	Can you explain the principles and consequences of pulse broadening and absorption?	<input type="checkbox"/>



## DATASHEET

## DATA - FUNDAMENTAL CONSTANTS AND VALUES

Quantity	Symbol	Value	Units
speed of light in vacuo	$c$	$3.00 \times 10^8$	$\text{m s}^{-1}$
permeability of free space	$\mu_0$	$4\pi \times 10^{-7}$	$\text{H m}^{-1}$
permittivity of free space	$\epsilon_0$	$8.85 \times 10^{-12}$	$\text{F m}^{-1}$
magnitude of the charge of electron	$e$	$1.60 \times 10^{-19}$	C
the Planck constant	$h$	$6.63 \times 10^{-34}$	J s
gravitational constant	$G$	$6.67 \times 10^{-11}$	$\text{N m}^2 \text{kg}^{-2}$
the Avogadro constant	$N_A$	$6.02 \times 10^{23}$	$\text{mol}^{-1}$
molar gas constant	$R$	8.31	$\text{J K}^{-1} \text{mol}^{-1}$
the Boltzmann constant	$k$	$1.38 \times 10^{-23}$	$\text{J K}^{-1}$
the Stefan constant	$\sigma$	$5.67 \times 10^{-8}$	$\text{W m}^{-2} \text{K}^{-4}$
the Wien constant	$\alpha$	$2.90 \times 10^{-3}$	m K
electron rest mass (equivalent to $5.5 \times 10^{-4}$ u)	$m_e$	$9.11 \times 10^{-31}$	kg
electron charge/mass ratio	$\frac{e}{m_e}$	$1.76 \times 10^{11}$	$\text{C kg}^{-1}$
proton rest mass (equivalent to 1.00728 u)	$m_p$	$1.67(3) \times 10^{-27}$	kg
proton charge/mass ratio	$\frac{e}{m_p}$	$9.58 \times 10^7$	$\text{C kg}^{-1}$
neutron rest mass (equivalent to 1.00867 u)	$m_n$	$1.67(5) \times 10^{-27}$	kg
gravitational field strength	$g$	9.81	$\text{N kg}^{-1}$
acceleration due to gravity	$g$	9.81	$\text{m s}^{-2}$
atomic mass unit (1u is equivalent to 931.5 MeV)	u	$1.661 \times 10^{-27}$	kg

## ALGEBRAIC EQUATION

quadratic equation  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

## ASTRONOMICAL DATA

Body	Mass/kg	Mean radius/m
Sun	$1.99 \times 10^{30}$	$6.96 \times 10^8$
Earth	$5.97 \times 10^{24}$	$6.37 \times 10^6$

## GEOMETRICAL EQUATIONS

arc length =  $r\theta$

circumference of circle =  $2\pi r$

area of circle =  $\pi r^2$

curved surface area of cylinder =  $2\pi r h$

area of sphere =  $4\pi r^2$

volume of sphere =  $\frac{4}{3}\pi r^3$



### Particle Physics

Class	Name	Symbol	Rest energy/MeV
photon	photon	$\gamma$	0
lepton	neutrino	$\nu_e$	0
		$\nu_\mu$	0
	electron	$e^\pm$	0.510999
	muon	$\mu^\pm$	105.659
mesons	$\pi$ meson	$\pi^\pm$	139.576
		$\pi^0$	134.972
	K meson	$K^\pm$	493.821
		$K^0$	497.762
baryons	proton	p	938.257
	neutron	n	939.551

### Properties of quarks

antiquarks have opposite signs

Type	Charge	Baryon number	Strangeness
<b>u</b>	$+\frac{2}{3}e$	$+\frac{1}{3}$	0
<b>d</b>	$-\frac{1}{3}e$	$+\frac{1}{3}$	0
<b>s</b>	$-\frac{1}{3}e$	$+\frac{1}{3}$	-1

### Properties of Leptons

		Lepton number
Particles:	$e^-, \nu_e; \mu^-, \nu_\mu$	+1
Antiparticles:	$e^+, \bar{\nu}_e, \mu^+, \bar{\nu}_\mu$	-1

### Photons and energy levels

photon energy  $E = hf = hc / \lambda$

photoelectricity  $hf = \phi + E_{k(\max)}$

energy levels  $hf = E_1 - E_2$

de Broglie wavelength  $\lambda = \frac{h}{p} = \frac{h}{mv}$

### Waves

wave speed  $c = f\lambda$  period  $f = \frac{1}{T}$

first harmonic  $f = \frac{1}{2l} \sqrt{\frac{T}{\mu}}$

fringe spacing  $w = \frac{\lambda D}{s}$  diffraction grating  $d \sin \theta = n\lambda$

refractive index of a substance s,  $n = \frac{c}{c_s}$

for two different substances of refractive indices  $n_1$  and  $n_2$ ,

law of refraction  $n_1 \sin \theta_1 = n_2 \sin \theta_2$

critical angle  $\sin \theta_c = \frac{n_2}{n_1}$  for  $n_1 > n_2$

### Mechanics

moments moment =  $Fd$

velocity and acceleration  $v = \frac{\Delta s}{\Delta t}$   $a = \frac{\Delta v}{\Delta t}$

equations of motion  $v = u + at$   $s = \left(\frac{u+v}{2}\right) t$

$v^2 = u^2 + 2as$   $s = ut + \frac{at^2}{2}$

force  $F = ma$

force  $F = \frac{\Delta(mv)}{\Delta t}$

impulse  $F \Delta t = \Delta(mv)$

work, energy and power  $W = F s \cos \theta$

$E_k = \frac{1}{2} m v^2$   $\Delta E_p = mg\Delta h$

$P = \frac{\Delta W}{\Delta t}$ ,  $P = Fv$

efficiency =  $\frac{\text{useful output power}}{\text{input power}}$

### Materials

density  $\rho = \frac{m}{v}$  Hooke's law  $F = k \Delta L$

Young modulus =  $\frac{\text{tensile stress}}{\text{tensile strain}}$  tensile stress =  $\frac{F}{A}$

tensile strain =  $\frac{\Delta L}{L}$

energy stored  $E = \frac{1}{2} F \Delta L$



## Electricity

current and pd  $I = \frac{\Delta Q}{\Delta t}$   $V = \frac{W}{Q}$   $R = \frac{V}{I}$

resistivity  $\rho = \frac{RA}{L}$

resistors in series  $R_T = R_1 + R_2 + R_3 + \dots$

resistors in parallel  $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$

power  $P = VI = I^2R = \frac{V^2}{R}$

emf  $\varepsilon = \frac{E}{Q}$   $\varepsilon = I(R + r)$

## Circular motion

magnitude of angular speed  $\omega = \frac{v}{r}$

$$\omega = 2\pi f$$

centripetal acceleration  $a = \frac{v^2}{r} = \omega^2 r$

centripetal force  $F = \frac{mv^2}{r} = m\omega^2 r$

## Simple harmonic motion

acceleration  $a = -\omega^2 x$

displacement  $x = A \cos(\omega t)$

speed  $v = \pm \omega \sqrt{(A^2 - x^2)}$

maximum speed  $v_{\max} = \omega A$

maximum acceleration  $a_{\max} = \omega^2 A$

for a mass-spring system  $T = 2\pi \sqrt{\frac{m}{k}}$

for a simple pendulum  $T = 2\pi \sqrt{\frac{l}{g}}$

## Thermal physics

energy to change temperature  $Q = mc\Delta\theta$

energy to change state  $Q = ml$

gas law  $pV = nRT$   
 $pV = NkT$

kinetic theory model  $pV = \frac{1}{3} N m (c_{\text{rms}})^2$

kinetic energy of gas molecule  $\frac{1}{2} m (c_{\text{rms}})^2 = \frac{3}{2} kT = \frac{3RT}{2N_A}$

## Gravitational fields

force between two masses  $F = \frac{Gm_1m_2}{r^2}$

gravitational field strength  $g = \frac{F}{m}$

magnitude of gravitational field strength in a radial field  $g = \frac{GM}{r^2}$

work done  $\Delta W = m\Delta V$

gravitational potential  $V = -\frac{GM}{r}$   
 $g = -\frac{\Delta V}{\Delta r}$

## Electric fields and capacitors

force between two point charges  $F = \frac{1}{4\pi\epsilon_0} \frac{Q_1Q_2}{r^2}$

force on a charge  $F = EQ$

field strength for a uniform field  $E = \frac{V}{d}$

work done  $\Delta W = Q\Delta V$

field strength for a radial field  $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$

electric potential  $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$

$$E = \frac{\Delta V}{\Delta r}$$

capacitance  $C = \frac{Q}{V}$

$$C = \frac{A\epsilon_0\epsilon_r}{d}$$

capacitor energy stored  $E = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C}$

capacitor charging  $Q = Q_0(1 - e^{-t/RC})$

decay of charge  $Q = Q_0 e^{-t/RC}$

time constant  $RC$



## Magnetic fields

<i>force on a current</i>	$F = BIl$
<i>force on a moving charge</i>	$F = BQv$
<i>magnetic flux</i>	$\Phi = BA$
<i>magnetic flux linkage</i>	$N\Phi = BAN \cos \theta$
<i>magnitude of induced emf</i>	$\varepsilon = N \frac{\Delta \Phi}{\Delta t}$
	$N\Phi = BAN \cos \theta$
<i>emf induced in a rotating coil</i>	$\varepsilon = BAN\omega \sin \omega t$
<i>alternating current</i>	$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \quad V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$
<i>transformer equations</i>	$\frac{N_s}{N_p} = \frac{V_s}{V_p}$
	$\text{efficiency} = \frac{I_s V_s}{I_p V_p}$

## Nuclear physics

<i>the inverse square law for <math>\gamma</math> radiation</i>	$I = \frac{k}{x^2}$
<i>radioactive decay</i>	$\frac{\Delta N}{\Delta t} = -\lambda N, N = N_0 e^{-\lambda t}$
<i>activity</i>	$A = \lambda N$
<i>half-life</i>	$T_{1/2} = \frac{\ln 2}{\lambda}$
<i>nuclear radius</i>	$R = R_0 A^{1/3}$
<i>energy-mass equation</i>	$E = mc^2$

## OPTIONS

### Astrophysics

1 astronomical unit	$= 1.50 \times 10^{11} \text{ m}$
1 light year	$= 9.46 \times 10^{15} \text{ m}$
1 parsec	$= 206265 \text{ AU} = 3.08 \times 10^{16} \text{ m}$
	$= 3.26 \text{ light year}$

$$\text{Hubble constant, } H = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

$$M = \frac{\text{angle subtended by image at eye}}{\text{angle subtended by object at unaided eye}}$$

$$\text{in normal adjustment} \quad M = \frac{f_o}{f_e}$$

$$\text{Rayleigh criterion} \quad \theta \approx \frac{\lambda}{D}$$

$$\text{magnitude equation} \quad m - M = 5 \log \frac{d}{10}$$

$$\text{Wien's law} \quad \lambda_{\text{max}} T = 2.9 \times 10^{-3} \text{ m K}$$

$$\text{Stefan's law} \quad P = \sigma AT^4$$

$$\text{Schwarzschild radius} \quad R_s \approx \frac{2GM}{c^2}$$

$$\text{Doppler shift for } v \ll c \quad \frac{\Delta f}{f} = -\frac{\Delta \lambda}{\lambda} = \frac{v}{c}$$

$$\text{red shift} \quad z = -\frac{v}{c}$$

$$\text{Hubble's law} \quad v = Hd$$

### Medical physics

$$\text{lens equations} \quad P = \frac{1}{f}$$

$$m = \frac{v}{u}$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\text{threshold of hearing} \quad I_0 = 1.0 \times 10^{-12} \text{ W m}^{-2}$$

$$\text{intensity level} \quad \text{intensity level} = 10 \log \frac{I}{I_0}$$

$$\text{absorption} \quad I = I_0 e^{-\mu x}$$

$$\mu_m = \frac{\mu}{\rho}$$

$$\text{ultrasound imaging} \quad Z = \rho c$$

$$\frac{I_r}{I_i} = \left( \frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2$$

$$\text{half-lives} \quad \frac{1}{T_E} = \frac{1}{T_B} + \frac{1}{T_P}$$



## Engineering physics

*moment of inertia*  $I = \Sigma mr^2$

*angular kinetic energy*  $E_k = \frac{1}{2} I \omega^2$

*equations of angular motion*

$$\omega_2 = \omega_1 + \alpha t$$

$$\omega_2^2 = \omega_1^2 + 2\alpha\theta$$

$$\theta = \omega_1 t + \frac{\alpha t^2}{2}$$

$$\theta = \frac{(\omega_1 + \omega_2) t}{2}$$

*torque*  $T = I \alpha$

$$T = F r$$

*angular momentum*  $\text{angular momentum} = I \omega$

*angular impulse*  $T \Delta t = \Delta(I \omega)$

*work done*  $W = T \theta$

*power*  $P = T \omega$

*thermodynamics*  $Q = \Delta U + W$

$$W = p \Delta V$$

*adiabatic change*  $pV^\gamma = \text{constant}$

*isothermal change*  $pV = \text{constant}$

*heat engines*

$$\text{efficiency} = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H}$$

$$\text{maximum theoretical efficiency} = \frac{T_H - T_C}{T_H}$$

*work done per cycle = area of loop*

*input power = calorific value  $\times$  fuel flow rate*

$$\text{indicated power} = \frac{\text{area of } p - V \text{ loop}}{\text{number of cycles per second}} \times \text{number of cylinders}$$

*output or brake power*  $P = T \omega$

*friction power = indicated power - brake power*

*heat pumps and refrigerators*

*refrigerator:*  $COP_{\text{ref}} = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C}$

*heat pump:*  $COP_{\text{hp}} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_C}$

## Turning points in physics

*electrons in fields*  $F = \frac{eV}{d}$

$$F = Bev$$

$$r = \frac{mv}{Be}$$

$$\frac{1}{2} mv^2 = eV$$

*Millikan's experiment*  $\frac{QV}{d} = mg$

$$F = 6\pi\eta r v$$

*Maxwell's formula*  $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$$

*special relativity*

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

## Electronics

*resonant frequency*  $f_0 = \frac{1}{2\pi \sqrt{LC}}$

*Q-factor*  $Q = \frac{f_0}{f_B}$

*operational amplifiers: open loop*  $V_{\text{out}} = A_{\text{OL}}(V_+ - V_-)$

*inverting amplifier*  $\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_f}{R_{\text{in}}}$

*non-inverting amplifier*  $\frac{V_{\text{out}}}{V_{\text{in}}} = 1 + \frac{R_f}{R_1}$

*summing amplifier*  $V_{\text{out}} = -R_f \left( \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \dots \right)$

*difference amplifier*  $V_{\text{out}} = (V_+ - V_-) \frac{R_f}{R_1}$

*Bandwidth requirement:*

*for AM*  $\text{bandwidth} = 2f_M$

*for FM*  $\text{bandwidth} = 2(\Delta f + f_M)$



### **Acknowledgements**

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This document has been produced for educational purposes only.

This document has been produced for the AQA A Level Physics Specification.

### **Student Voice**

If you when using this document, you believe there is an improvement to made, please state this in the space below....

Only constructive and reasoned feedback will be considered.