Volume

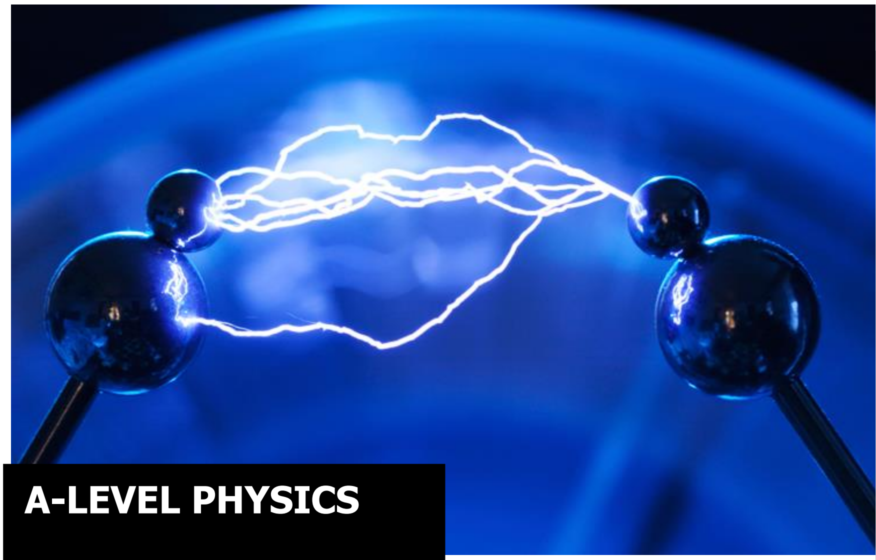
Two

ST MARY’S SCIENCE DEPARTMENT:

PHYSICS

A LEVEL PHYSICS **YEAR 1**
STUDENT CLASS BOOK
3.5.2: EMF AND POTENTIAL DIVIDERS
VOLUME **TWO**

NAME	
PHYSICS CLASS	
MODULE TEACHER	
ALPS GRADE	



A-LEVEL PHYSICS

TOPIC 5

CLASS WORKBOOK 2

THIS MUST BE BROUGHT TO ALL PHYSICS LESSONS.

Contents
3.5.1.5 Potential Divider
3.5.1.6 Electromotive Force and Internal Resistance

Overview

This section builds on and develops earlier study of these phenomena from GCSE.

It provides opportunities for the development of practical skills at an early stage in the course and lays the groundwork for later study of the many electrical applications that are important to society.

IMPORTANT NOTE

This book, along with the preparatory reading notes and independent work, must be brought to all Physics lessons with the appropriate teacher.

This book may be used as a learning resource in lessons, you are not fully equipped to learn if this is not used in lesson.

This book may also be used as a revision resource for intervention, internal assessments and external assessments.
Please keep this in your student file.

There are several activities in this book which may not be covered in lessons.

It is advised that students complete these activities outside of lessons as revision aides.

Definition List

Definitions you must learn for this module are.

Key Word	Symbol	Definition
Electromotive force, EMF (Open Circuit Voltage)	**Ɛ**	The amount of electrical energy per unit charge produced inside a source of electrical energy.
Internal Resistance	**r**	The resistance inside a source of electrical energy, the loss of pd per unit current in the source when current passes through it.
(External) Load Resistance	**R**	The resistance inside a transducer (output) of the electrical circuit, the loss of PD per unit current in the output of the circuit.
Light dependent resistor		A resistor which is designed to have a resistance that changes with light intensity.
Lost volts		The energy wasted per coulomb overcoming the internal resistance in a circuit.
Negative temperature coefficient		When the resistance of a semi-conductor decreases when its temperature is increased.
Positive temperature coefficient		When the resistance of a metal increases when its temperature is increased.
Potential divider		When two or more resistors (outputs) in series connected to a source of pd.
Terminal PD		The energy per coulomb delivered to the load outputs of the circuit.
Thermistor		A resistor which is designed to have a resistance that changes with temperature.

IMPORTANT NOTE
These definitions must be memorised by students.

You will be tested on your knowledge of these definitions.

The Language of Measurement

The following subject specific vocabulary provides definitions of key terms used in the A-level Science specifications.

Accuracy
A measurement result is considered accurate if it is judged to be close to the true value.

Calibration
Marking a scale on a measuring instrument.
This involves establishing the relationship between indications of a measuring instrument and standard or reference quantity values, which must be applied.
For example, placing a thermometer in melting ice to see whether it reads 0 °C, to check if it has been calibrated correctly.

Data
Information, either qualitative or quantitative, that has been collected.

Errors
See also uncertainties.

Measurement error
The difference between a measured value and the true value.

Anomalies
These are values in a set of results which are judged not to be part of the variation caused by random uncertainty.

Random error
These cause readings to be spread about the true value, due to results varying in an unpredictable way from one measurement to the next.
Random errors are present when any measurement is made, and cannot be corrected. The effect of random errors can be reduced by making more measurements and calculating a new mean.

Systematic error
These cause readings to differ from the true value by a consistent amount each time a measurement is made.
Sources of systematic error can include the environment, methods of observation or instruments used.
Systematic errors cannot be dealt with by simple repeats. If a systematic error is suspected, the data collection should be repeated using a different technique or a different set of equipment, and the results compared.

Zero error
Any indication that a measuring system gives a false reading when the true value of a measured quantity is zero, e.g. the needle on an ammeter failing to return to zero when no current flows.
A zero error may result in a systematic uncertainty.

Evidence
Data which has been shown to be valid.

Fair test
A fair test is one in which only the independent variable has been allowed to affect the dependent variable.

Hypothesis
A proposal intended to explain certain facts or observations.

Interval
The quantity between readings, e.g. a set of 11 readings equally spaced over a distance of 1 metre would give an interval of 10 centimetres.

Precision
Precise measurements are ones in which there is very little spread about the mean value. Precision depends only on the extent of random errors – it gives no indication of how close results are to the true value.

Prediction
A prediction is a statement suggesting what will happen in the future, based on observation, experience or a hypothesis.

Range
The maximum and minimum values of the independent or dependent variables; important in ensuring that any pattern is detected.
For example, a range of distances may be quoted as either:
'From 10 cm to 50 cm'
or
'From 50 cm to 10 cm'

Repeatable
A measurement is repeatable if the original experimenter repeats the investigation using same method and equipment and obtains the same results.

Reproducible
A measurement is reproducible if the investigation is repeated by another person, or by using different equipment or techniques, and the same results are obtained.

Resolution
This is the smallest change in the quantity being measured (input) of a measuring instrument that gives a perceptible change in the reading.

Sketch graph
A line graph, not necessarily on a grid, that shows the general shape of the relationship between two variables. It will not have any points plotted and although the axes should be labelled they may not be scaled.

True value
This is the value that would be obtained in an ideal measurement.

Uncertainty
The interval within which the true value can be expected to lie, with a given level of confidence or probability, e.g. "the temperature is 20 °C ± 2 °C, at a level of confidence of 95%.

Validity
Suitability of the investigative procedure to answer the question being asked. For example, an investigation to find out if the rate of a chemical reaction depended upon the concentration of one of the reactants would not be a valid procedure if the temperature of the reactants was not controlled.

Valid conclusion
A conclusion supported by valid data, obtained from an appropriate experimental design and based on sound reasoning.

Variables
These are physical, chemical or biological quantities or characteristics.

Categoric variables
Categoric variables have values that are labels. E.g. names of plants or types of material.

Continuous variables
Continuous variables can have values (called a quantity) that can be given a magnitude either by counting (as in the case of the number of shrimp) or by measurement (e.g. light intensity, flow rate etc.).

Control variables
A control variable is one which may, in addition to the independent variable, affect the outcome of the investigation and therefore must be kept constant or at least monitored.

Dependent variables
The dependent variable is the variable of which the value is measured for each change in the independent variable.

Independent variables
The independent variable is the variable for which values are changed or selected by the investigator.

IMPORTANT NOTE
These definitions must be memorised by students.

You will be tested on your knowledge of these definitions.

Equations

The equations below are used in this module.

Quantity/Concept	**Equation(s)**
Current/Charge	$I = \frac{\Delta Q}{\Delta t}$ $\Delta Q = I\,\Delta t$
Potential Difference/EMF	$V = PD = \varepsilon = \frac{\Delta W}{\Delta Q}$
Resistance	$R = \frac{V}{I}$
Resistance in Series	$R_T = R_1 + R_2 + R_n$
Resistance in Parallel	$1/R_T = 1/R_1 + 1/R_2 + 1/R_n$
Electrical Power	$P = VI = I^2R = \frac{V^2}{R}$
Electrical Energy Transferred	$E = Vit = \frac{V^2t}{R} = I^2Rt$
Potential Divider (Equation not given in examination book)	$Vout = \frac{R_{output}}{R_{output} + R_{wasted}} \times Vsource$
Conservation of Energy in a Circuit (Equation not given in examination book)	ε = Terminal PD + Lost Volts
EMF and Internal Resistance	$\varepsilon = I\,(R+r)$
EMF in cells in series (Equation not given in examination book)	$\mathcal{E}_{total} = \mathcal{E}_1+\mathcal{E}_2+\mathcal{E}_n$
EMF in cells in parallel (Equation not given in examination book)	$\mathcal{E}_{total} = \mathcal{E}_1=\mathcal{E}_2=\mathcal{E}_n$

IMPORTANT NOTE
These equations must be memorised by students.

You will be tested on these equations.

TOPIC: 3.5.1.5 Potential Divider

SPEC CHECK

Specification	Completed?
The potential divider used to supply constant or variable potential difference from a power supply. The use of the potentiometer as a measuring instrument is not required.	
Examples should include the use of variable resistors, thermistors, and light dependent resistors (LDR) in the potential divider.	
Students can investigate the behaviour of a potential divider circuit.	
Students should design and construct potential divider circuits to achieve various outcomes.	

NOTES

These notes are brief.

More detailed notes are found in the student preparatory reading book.

Please read the preparatory reading notes.

Kirchhoff's Laws

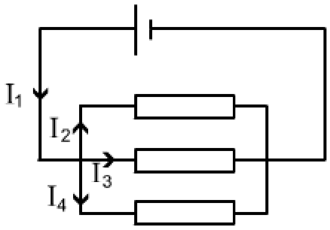
Kirchhoff came up with two laws concerning conservation in electrical circuits.

First Law

Electric charge is conserved in all circuits, all the charge that arrives at a point must leave it.

Current going in = current going out.

In the diagram, we can say that: $I_1 = I_2 + I_3 + I_4$

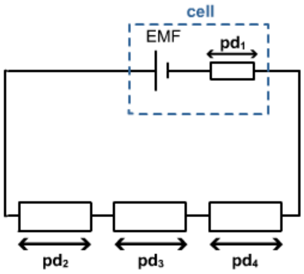


Second Law

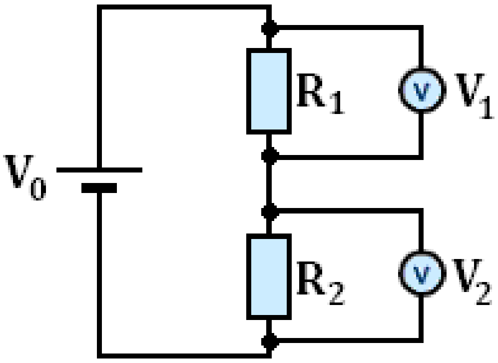
Energy is conserved in all circuits, for any complete circuit the sum of the emfs is equal to the sum of the potential differences.

Energy givers = energy takers.

In the diagram, we can say that: $\varepsilon = pd_1 + pd_2 + pd_3 + pd_4.$



Potential Dividers



A potential divider is used to produce a desired potential difference, it can be thought of as a potential selector.

A typical potential divider consists of two or more resistors that share the emf from the battery/cell.

The p.d.s across R_1 and R_2 can be calculated using the following equations:

$$\boxed{V_1 = V_0 \frac{R_1}{R_1 + R_2}}$$

$$\boxed{V_2 = V_0 \frac{R_2}{R_1 + R_2}}$$

This shows us that the size of the potential difference is equal to the input potential multiplied by what proportion of R_1 is of the total resistance.

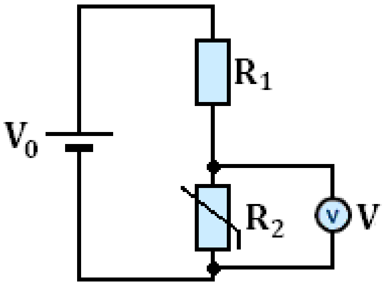
If R_1 is 10 Ω and R_2 is 90 Ω, R_1 contributes a tenth of the total resistance so R_1 has a tenth of the available potential. This can be represented using:

$$\boxed{\frac{R_1}{R_2} = \frac{V_1}{V_2}}$$

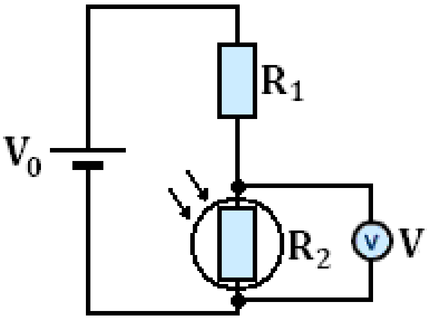
The ratio of the resistances is equal to the ratio of the output voltages.

Uses

In this potential divider, the second resistor is a thermistor. When the temperature is low the resistance (R_2) is high, this makes the output voltage high. When the temperature is high the resistance (R_2) is low, this makes the output voltage low. A use of this would be a cooling fan that works harder when it is warm.



In the second potential divider, the second resistor is a Light Dependant Resistor. When the light levels are low the resistance (R_2) is high, making the output voltage high. When the light levels increase the resistance (R_2) decreases, this makes the output voltage decrease. A use of this could be a street light sensor that lights up when the surrounding is dark.

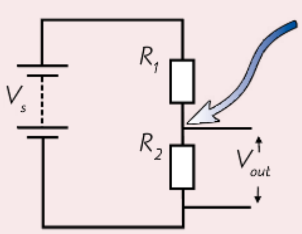


REVISION SHEET

Highlight or underline the key information on the revision sheet to consolidate your understanding.

*Use a **Potential Divider** to get a **Fraction** of a **Source Voltage***

1) At its simplest, a **potential divider** is a circuit with a **voltage source** and a couple of **resistors** in series.
2) The **potential difference** across the voltage source (e.g. a battery) is **split** in the **ratio** of the **resistances** (p.52).
3) So, if you had a **2 Ω** resistor and a **3 Ω** resistor, you'd get **2/5** of the p.d. across the **2 Ω** resistor and **3/5** across the **3**
4) You can use potential dividers to supply a potential difference, V_{out}, between **zero** and the potential difference across the voltage source. This can be useful, e.g. if you need a **varying** p.d. supply or one that is at a **lower p.d.** than the voltage source.



The voltage has **dropped** by V_1 (the voltage across R_1) by the time it gets to here. The **remaining voltage** that can be supplied, e.g. to another component, is V_{out}.

In the circuit shown, R_2 has $\frac{R_2}{R_1 + R_2}$ of the total resistance. So: $\boxed{V_{out} = \frac{R_2}{R_1 + R_2} V_s}$

E.g. if $V_s = 9$ V and you want V_{out} to be 6 V, then you need:

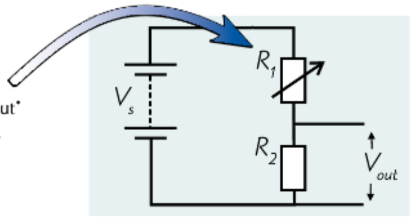
$$\boxed{\frac{R_2}{R_1 + R_2} = \frac{2}{3} \text{ which gives } R_2 = 2R_1}$$

So you could have, say, $R_1 = 100\ \Omega$, $R_2 = 200\ \Omega$

5) This circuit is mainly used for **calibrating voltmeters**, which have a **very high resistance**.
6) If you put something with a **relatively low resistance** across R_2 though, you start to run into **problems**. You've **effectively** got **two resistors** in **parallel**, which will **always** have a **total** resistance **less** than R_2. That means that V_{out} will be **less** than you've calculated, and will depend on what's connected across R_2. Hrrumph.

*Use a **Variable Resistor** to Vary the **Voltage***

If you replace R_1 with a **variable resistor**, you can change V_{out}. When $R_1 = 0$, $V_{out} = V_s$. As you increase R_1, V_{out} gets smaller.

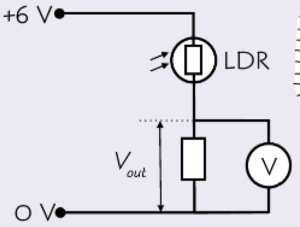


*Add an **LDR** or **Thermistor** for a **Light** or **Temperature Sensor***

1) A **light-dependent resistor** (LDR) has a very **high resistance** in the **dark**, but a **lower resistance** in the **light**.
2) An **NTC thermistor** has a **high resistance** at **low temperatures**, but a much **lower resistance** at **high temperatures** (it varies in the opposite way to a normal resistor, only much more so).
3) Either of these can be used as one of the **resistors** in a **potential divider**, giving an **output voltage** that **varies** with the **light level** or **temperature**.

See page 47 for why the resistances of LDRs and NTC thermistors change like this.

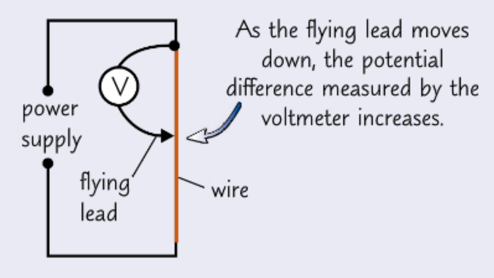
The diagram shows a **sensor** used to detect **light levels**.
When light shines on the LDR its **resistance decreases**, so V_{out} increases.
You can include LDRs and thermistors in circuits that control **switches**, e.g. to turn on a light or a heating system.



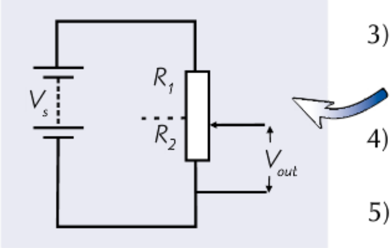
If you replace the LDR with an NTC thermistor, V_{out} will increase with temperature.

A **Potentiometer** uses a **Variable Resistor** to give a **Variable Voltage**

1) Imagine you have a long length of wire connected to a power supply. If the wire is **uniform** (i.e. same cross-sectional area and material throughout), then its **resistance** is **proportional** to its **length**.
2) This means that if you were to connect a voltmeter across different lengths of the wire, the **potential difference** you'd record would be **proportional** to the **length** you'd connected it over — you're measuring across a bigger share of the total resistance so you get a bigger potential difference.



As the flying lead moves down, the potential difference measured by the voltmeter increases.

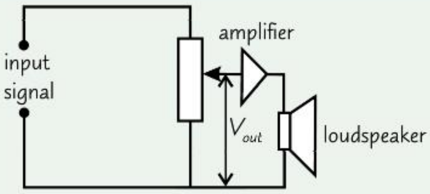


3) This is the basis of how a **potentiometer** works. A **potentiometer** is basically a potential divider, with a variable resistor replacing R_1 and R_2 (it's even sometimes **called** a potential divider just to confuse things).
4) You move a **slider** or turn a knob to **adjust** the **relative sizes** of R_1 and R_2. That way you can vary V_{out} from **0 V** up to the source voltage.
5) This is dead handy when you want to be able to **change** a **voltage continuously**, like in the **volume control** of a stereo:

I've often wished bagpipes had a volume control. Or just an off switch.

Example:

Here, V_s is replaced by the input signal (e.g. from a CD player) and V_{out} is the output to the amplifier and loudspeaker.



Reference: CGP Revision Guides

Additional Note Space

Additional Note Space

FACT RECALL

To check your understanding, answer the following questions…

A1. What is a potential divider?

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A2. Write down the equation you would use to work out the voltage output of a potential divider.

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A3. How can you make a light sensor using a potential divider?

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A4. What is a potentiometer? Give an example of when it could be used?

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ANSWERS

A1. A potential divider is a circuit containing a voltage source and a couple of resistors in series. The voltage across one of the resistor is used as an output voltage. If the resistors are not fixed, the circuit will be capable of producing a variable output voltage.

A2. $V_{out} = \frac{R_1}{R_1 + R_2} \times V_s$

A3. You can make a light sensor using a potential divider by using an LDR as one of the resistors.

A4. A potential divider containing a variable resistor instead of two resistors in series.
They are used in volume control on a stereo.

PUZZLES

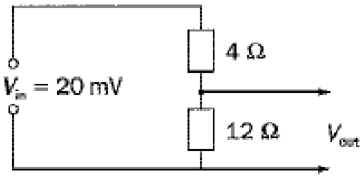
To improve your understanding, answer the following puzzles.

The answers are overleaf.

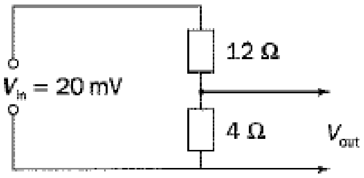
QUESTIONS

1 Find the output potential differences in the following cases.

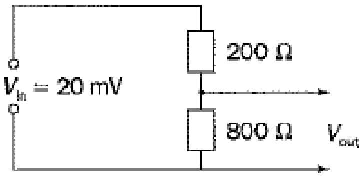
(a)



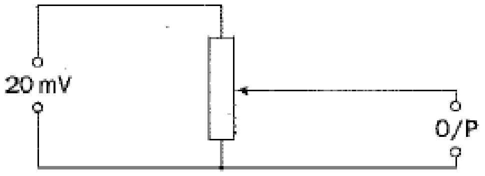
(b)



(c)



2 Plot a sketch graph showing how the output potential difference varies as the slider is moved from bottom to top.



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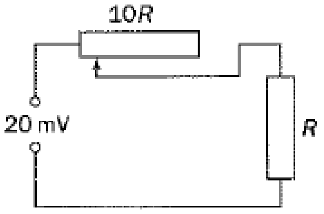
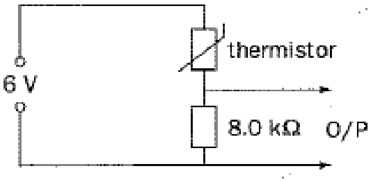
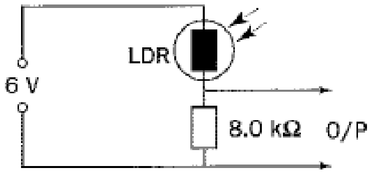


How does the potential difference vary when the slider is moved from left to right? Plot a sketch graph of the potential difference across *R* against the resistance of the variable resistance, in terms of *R*.

4 A thermistor has a resistance of 1.0 MΩ when at the ice point and 2.0 kΩ when at the steam point. It is used in the following potential divider circuit. Calculate the output potential difference when the thermistor is (a) at the ice point, (b) at the steam point.



5 A light-dependent resistor (LDR) has a resistance of 800 kΩ when in the dark and 100 Ω when in full sunlight. It is used in the following potential divider circuit.



Calculate the output potential difference when the LDR is

(a) in full sunlight

(b) in the dark.

(c) How would you alter the circuit to get a large output when the LDR is in the dark and a small output when it is in full sunlight?

Answering Space

ANSWERS

1 (a) 5 mV

(b) 15 mV

(c) 16 mV

2

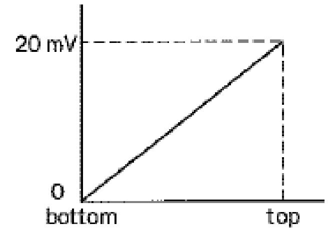


Figure 3

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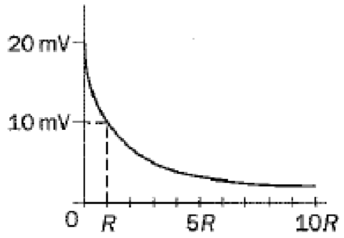


Figure 4

4 (a) 0.048 V

(b) 4.8 V

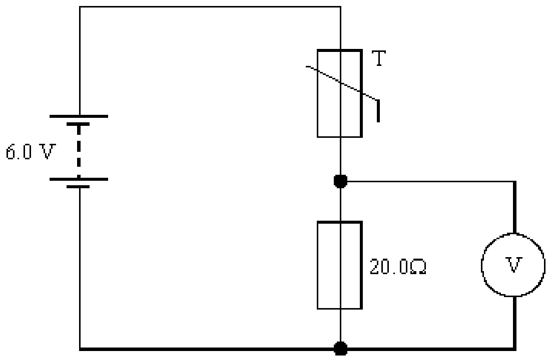
5 (a) 5.9 V

(b) 0.059 V

(c) change over the two components

SAMPLE QUESTION

S1. The circuit shown in the diagram below can be used as an electronic thermometer. The battery has negligible internal resistance.



The reading on the digital voltmeter can be converted to give the temperature of the thermistor T which is used as a temperature sensor.

S1.1 Explain why the reading on the voltmeter increases as the temperature of the thermistor increases.

[2 Marks]

as the temperature of T increases its resistance decreases /more charge carriers are released

1 mark

increasing the current in the circuit /changing the ratio of resistance/reducing pd across T

1 mark

(so that so that the pd across the resister increases)

S1.2 When the thermistor is at 80.0 °C the voltmeter reading is 5.0 V. Show that the resistance of the thermistor at this temperature is 4.0 Ω.

[1 Mark]

T/ 20.0 = 1.0/5.0 OR 5.0/6.0 = 20/(20+T) OR equivalent (Therefore T = 4.0 ohms)

1 mark

S1.3 When the thermistor is at 20.0 °C its resistance is 24.5 Ω. Calculate the reading on the voltmeter.

[2 Marks]

Use of $V_{out} = R_1/(R_1 + R_2) \times V_{in}$ OR $I = 6/44.5 = 0.135$ A

1 mark

V = 2.7 V

1 mark

S1.4 The battery is replaced with another having the same emf but an internal resistance of 3.0 Ω.

S1.4.1 Calculate the new voltmeter reading when the thermistor temperature is 80.0 °C.

[2 Marks]

V/6.0 = 20.0/(20.0+4.0+3.0) OR I = 0.222 A

1 mark

V = 4.4V

1 mark

S1.4.2 State and explain the effect, if any, on the measured temperature when the thermistor is at 20.0 °C.

[1 Mark]

The measure temperature would be lower because the pd across the resistor would be less *(i.e. 2.53V)*

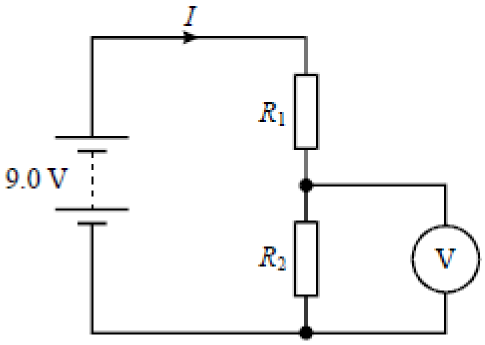
1 mark

Reference: AQA A Level Specimen Examination Materials B

SELF ASSESSMENT

To assess your understanding, answer the following questions.

Q1. In the circuit shown, the battery has negligible internal resistance.



Q1.1 If the emf of the battery = 9.0 V, R_1 = 120 Ω and R_2 = 60 Ω, calculate the current I flowing in the circuit.

[3 Marks]

……

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……

……

Q1.2 Calculate the voltage reading on the voltmeter.

[1 Mark]

……

……

Q1.3 The circuit shown in the diagram acts as a potential divider. The circuit is now modified by replacing R_1 with a temperature sensor, whose resistance decreases as the temperature increases.

Explain whether the reading on the voltmeter increases or decreases as the temperature increases from a low value.

[3 Marks]

……

……

……

……

……

……

Reference: AQA A Level Specimen Examination Materials A

Q2.1. Give the equation which relates the *electrical resistivity* of a conducting material to its *resistance*. Define the symbols in the equation.

[2 Marks]

....

....

....

Q2.2 A potential difference of 1.5 V exists across the ends of a copper wire of length 2.0 m and uniform radius 0.40 mm. Calculate the current in the wire.

resistivity of copper = 1.7×10^{-8} Ω m

[3 Marks]

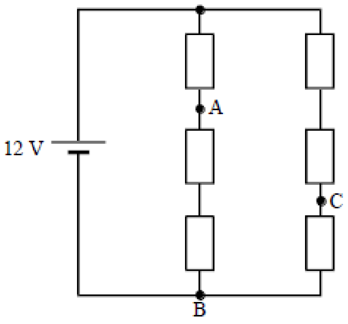
....

....

....

....

In the circuit shown, each resistor has the same resistance. The battery has an e.m.f. of 12 V and negligible internal resistance.



Q2.3 Calculate the potential difference between A and B.

[1 Mark]

....

....

Q2.4 Calculate the potential difference between B and C.

[1 Mark]

....

....

Q2.5 A high resistance voltmeter is connected between A and C. What is the reading on the voltmeter?

[3 Marks]

………

………

Reference: AQA A Level Specimen Examination Materials A

Q3.

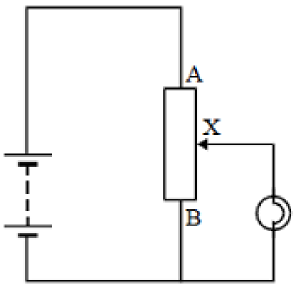


Figure 1

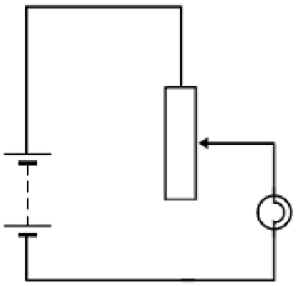


Figure 2

Q3.1 The current flowing through a torch bulb can be controlled by a variable resistor using either of the two circuit arrangements shown above. **Figure 1** is called a potential divider arrangement and **Figure 2** may be called a rheostat arrangement. For each of these two methods explain **one** advantage and **one** disadvantage.

Potential Divider

[2 Marks]

Advantage

....

....

Disadvantage

....

....

Rheostat

[2 Marks]

Advantage

....

....

Disadvantage

....

....

In **Figure 1**, the variable resistor has a total resistance of 16 Ω. When the slider of the variable resistor is set at **X**, exactly mid-way along **AB**, the bulb works according to its specification of 2.0 V, 500 mW. Calculate

Q3.2 the current through section **XB** of the variable resistance,

[1 Mark]

………

………

Q3.3 the current through section **AX** of the variable resistance.

[1 Mark]

………

………

Reference: AQA A Level Specimen Examination Materials A

ANSWERS

M1.1 (use of $V = IR$ gives) $V = I(R_1 + R_2)$ **(1)**

$$I = \frac{V}{R_1 + R_2} = \frac{9}{120 + 60} \textbf{(1)}$$

= 50 mA ✓

M1.2 V_{out} (= IR_2) = 0.05 × 60 = 3 V **(1)**
(allow C.E. for value of I from (i))

4

M1.3 (temperature increases, resistance decreases), total resistance decreases **(1)**
current increases **(1)**
voltage across R_2 increases **(1)**
[or R_2 has increased share of (total) resistance **(1)**
new current is same in both resistors **(1)** larger share of the 9 V **(1)**]

[or $V_{out} = V_{in} \dfrac{R_2}{R_1 + R_2}$ **(1)** R_1 decreases **(1)** V_{out} decreases **(1)**]

3
[7]

M2.1 resistivity defined by $\rho = \dfrac{RA}{l}$ **(1)**

Symbols defined $\dfrac{RA}{l}$

M2.2 $R = \dfrac{\rho l}{A} = \dfrac{1.7 \times 10^{-8} \times 2}{\pi(0.4 \times 10^{-3})^2}$ **(1)**

= 0.068 (Ω) **(1)** (0.0676 Ω)

$I = \dfrac{1.5}{0.068}$ = 22 A **(1)** (22.2 A)

(Allow e.c.f. from value of R)

(5)

M2.3 $pd_{AB} = \dfrac{2}{3} \times 12$ = 8 V **(1)** **(1)**

M2.4 $pd_{BC} = (\dfrac{1}{3} \times 12)$ = 4 V **(1)**

= (8 – 4) = 4 V **(1)**
(allow e.c.f. from (i) and (ii))

(5)
[10]

M3.1

Potential divider:
Advantage:
Better control from 0 to maximum **(1)**

Disadvantage:
Power wasted because lower half of resistor always carries current (or top half of resistor must be capable of carrying lower half current **and** bulb current) **(1)**

Rheostat:
advantage:

Easier to connect **(1)**

Disadvantage:

Minimum current through bulb never zero **(1)**

(4)

M3.2 $V_{XB} = V_{lamp} = 2.0\text{ V} \therefore I_{XB} = \frac{2}{16/2} = 0.25\text{ A}$ **(1)**

M3.3 $I_{AX} = I_{XB} + I_{lamp},\ I_{lamp} = I_{XB} = 0.25\text{ A}, \therefore I_{AX} = 0.5\text{ A}$ **(1)**

(2)
[6]

TOPIC: 3.5.1.6 Electromotive Force and Internal Resistance

SPEC CHECK

Specification	Completed?
ε = EQ ε= I R + r	
Terminal pd; emf	
Students will be expected to understand and perform calculations for circuits in which the internal resistance of the supply is not negligible.	

NOTES

These notes are brief.

More detailed notes are found in the student preparatory reading book.

Please read the preparatory reading notes.

Energy in Circuits

In circuits, there are two fundamental types of component: energy *givers* and energy *takers*.

Electromotive Force (emf), ε

Energy givers provide an electromotive force, they force electrons around the circuit which transfer energy.

The size of the emf can be calculate using:

$$\boxed{\varepsilon = \frac{E}{Q}}$$

This is like the equation we use to find voltage/potential difference and means the energy given to each unit of charge. We can think of this as the energy given to each electron.

The emf of a supply is the p.d. across its terminals when no current flows

EMF is measured in Joules per Coulomb, JC^{-1} or Volts, V

Energy takers have a potential difference across them, transferring energy from the circuit to the component.

emf = energy giver

p.d. = energy taker

Energy is conserved in a circuit so energy in = energy out, or:

The total of the emfs = The total of the potential differences around the whole circuit

Internal Resistance, r

The chemicals inside a cell offer a resistance to the flow of current, this is the internal resistance on the cell.

Internal Resistance is measured in Ohms, Ω

Linking emf and r

If we look at the statement in the box above and apply it to the circuit below, we can reach an equation that links emf and *r*.

Total emfs = total potential differences

ε = (p.d. across r) + (p.d. across R) {Remember that V=IR}

$\varepsilon = (I \times r) + (I \times R)$

$\varepsilon = Ir + IR$

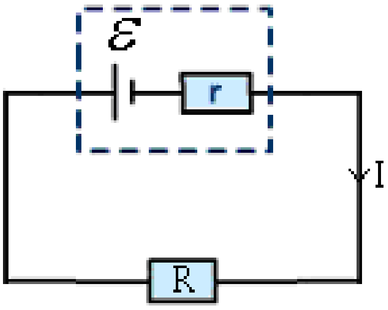
$$\boxed{\varepsilon = I(r+R)}$$

The terminal p.d. is the p.d. across the terminals of the cell when a current is flowing

ε = internal p.d + terminal p.d.

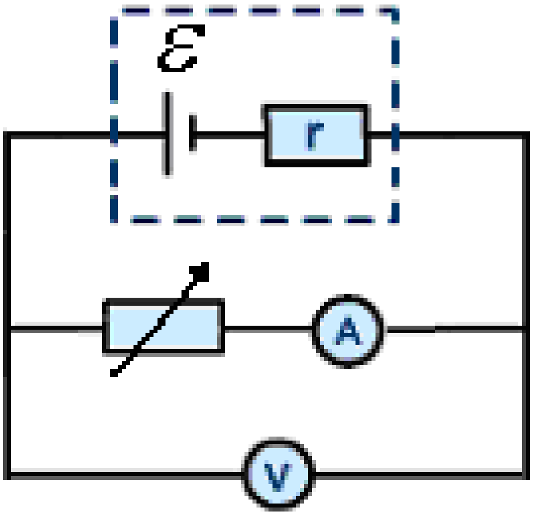
So, the above equation can be written as $\boxed{\varepsilon = Ir + V}$

where *V* is the terminal p.d.



Measuring emf and r

We can measure the emf and internal resistance of a cell by measuring the current and voltage as shown on the right, the variable resistor allows us to get a range of values. If we plot the results onto a graph of voltmeter reading against ammeter reading, we get a graph that looks like the one below.



Graphs have the general equation of y = mx+c, where y is the vertical (upwards) axis, x is the horizontal (across) axis, m is the gradient of the line and c is where the line intercepts (cuts) the y axis.

If we take $\varepsilon = Ir + V$ and arrange it into y= mx + c

y axis = V and x axis = I

$\varepsilon = Ir + V \rightarrow V = -Ir + \varepsilon \rightarrow V = -r\,I + \varepsilon$

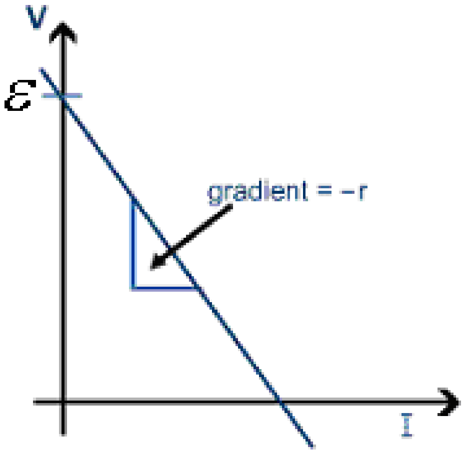
y =m x +c

So we can see that the:

y-intercept represents the emf

and

gradient represents (–) internal resistance



Highlight or underline the key information on the revision sheet to consolidate your understanding.

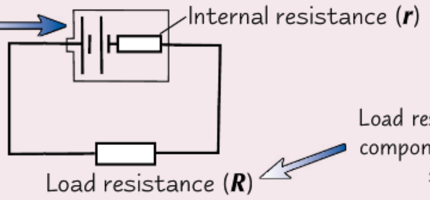
Batteries have an **Internal Resistance**

Resistance in metals comes from **electrons colliding** with **atoms** (or ions) and **losing energy** (see p.46).

In a **battery**, **chemical energy** is used to make **electrons move**. As they move, they collide with atoms inside the battery — so batteries **must** have resistance. This is called **internal resistance**.

Internal resistance is what makes **batteries** and **cells warm up** when they're used.

Because the resistance of wires is so small, on these two pages I'm assuming they have zero resistance.



Chemical reactions in the battery produce electrical energy.

In general, r is typically less than 1 ohm for a battery. This means a high current can flow.

Load resistance is the total resistance of all the components in the external circuit. You might see it called 'external resistance'.

1) The total amount of **work** the battery does on each **coulomb** of charge is called its **electromotive force** or **e.m.f.** (ε). Be careful — e.m.f. **isn't** actually a force. It's measured in **volts**.

$$\boldsymbol{W = \varepsilon Q} \quad \text{or} \quad \boldsymbol{\varepsilon = \frac{W}{Q}}$$

W is the work done on the charge (i.e. the energy transferred to the charge) in joules.

2) The **potential difference** across the **load resistance** (R) is the **work done** when **one coulomb** of charge flows through the **load resistance**. This potential difference is called the **terminal p.d.** (V).
3) If there was **no internal resistance**, the **terminal p.d**. would be the **same** as the **e.m.f.** However, in **real** power supplies, there's **always some energy lost** overcoming the internal resistance.
4) The **energy wasted per coulomb** overcoming the internal resistance is called the **lost volts** (v).

Conservation of energy tells us:

$$\textbf{energy per coulomb supplied by the source} = \textbf{energy per coulomb used in load resistance} + \textbf{energy per coulomb wasted in internal resistance}$$

There are Loads of **Calculations** with **E.m.f.** and **Internal Resistance**

Examiners can ask you to do **calculations** with **e.m.f.** and **internal resistance** in loads of **different** ways. You've got to be ready for whatever they throw at you.

$$\varepsilon = V + v \qquad \varepsilon = I(R + r) \qquad V = \varepsilon - v \qquad \varepsilon = V + Ir$$

These are all basically the **same equation**, just written differently. If you're given enough information you can calculate the e.m.f. (ε), terminal p.d. (V), lost volts (v), current (I), load resistance (R) or internal resistance (r). Which equation you should use depends on what information you've got, and what you need to calculate.

You Can Work Out the **E.m.f.** of **Multiple** Cells in **Series** or **Parallel**

For cells **in series**, you can calculate the **total e.m.f.** of the cells by **adding** their individual e.m.f.s.

$$\varepsilon_{\text{total}} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \ldots$$

This makes sense if you think about it, because each charge goes through each of the cells and so gains e.m.f. from each one.

For identical cells **in parallel**, the **total e.m.f.** of the combination of cells is the **same size** as the e.m.f. of each of the individual cells.

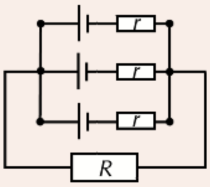
See p.52 for all the rules for parallel and series circuits.

$$\varepsilon_{\text{total}} = \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \ldots$$

This is because the current will split equally between identical cells. The charge only gains e.m.f. from the cells it travels through — so the overall e.m.f. in the circuit doesn't increase.

Time for an Example E.m.f. Calculation Question...

Example Three identical cells each with an e.m.f. of 2.0 V and an internal resistance of 0.20 Ω are connected in parallel in the circuit shown to the right. A current of 0.90 A is flowing through the circuit. Calculate the total p.d. across the cells.



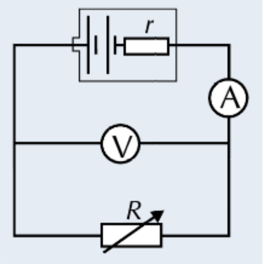
First calculate the lost volts, v, for 1 cell using $v = Ir$.

Since the current flowing through the circuit is split equally between each of the three cells, the current through one cell is $I/3$. So for 1 cell: $v = I/3 \times r = 0.90/3 \times 0.20 = 0.30 \times 0.20 = 0.06$ V

Then find the terminal p.d. across 1 cell using the equation: $V = \varepsilon - v = 2 - 0.06 = 1.94$

So the total p.d. across the cells combined = **1.9 V (to 2 s.f.)**

Investigate Internal Resistance and E.m.f. With This Circuit



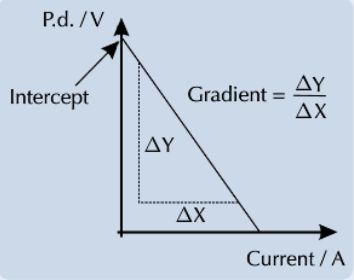
1) **Vary** the **current** in the circuit by changing the value of the **load resistance** (***R***) using the variable resistor. **Measure** the **p.d.** (***V***) for several different values of **current** (***I***).
2) Record your data for V and I in a table, and **plot the results** in a graph of V against I.

To find the **e.m.f.** and **internal resistance** of the cell, start with the equation: $\boxed{\varepsilon = V + Ir}$

1) Rearrange to give $V = -rI + \varepsilon$
2) Since ε and r are constants, that's just the equation of a **straight line**:

Equation of a straight line

$$\underbrace{y = mx}_{\text{gradient}} + \underbrace{c}_{\text{y-intercept}}$$



3) So the intercept on the vertical axis is ε.
4) And the gradient is $-r$.

You probably don't need to take repeated readings in an experiment like this one, but make sure you get plenty of data points to draw your line.

An **easier** way to **measure** the **e.m.f.** of a **power source** to just connect a **voltmeter** across its **terminals**. Voltmeters have a very **high resistance**, but a **small current** will still flow through them. This means there must be some **lost volts**, which means you measure a value **very slightly less** than the **e.m.f.** (Although in practice the difference isn't usually significant.)

Reference: CGP Revision Guides

Additional Note Space

Additional Note Space

FACT RECALL

To check your understanding, answer the following questions…

A1. Explain why batteries have an internal resistance.

………

………

………

………

A2. What is the load resistance in a circuit?

………

………

A3. What units is electromotive force measured in?

………

………

A4. Explain what lost volts are.

………

………

………

………

A5. What do the gradient and the y-intercept on a V-I graph for a power supply show?

………

………

………

………

ANSWERS

A1. In a battery, chemical energy is used to make electrons move. As they move, they collide with atoms inside the battery and lose energy – this is the internal resistance.

A2. The load resistance is the total resistance of all of the components in the external part of the circuit,
The load resistance does not include the internal resistance of the power source.

A3. Volts (V).

A4. The energy wasted per coulomb overcoming an internal resistance.

A5. The gradient is -r (where r is the internal resistance) and the y-intercept is ϵ (the electromotive force).

PUZZLES

To improve your understanding, answer the following puzzles.

The answers are overleaf.

QUESTIONS

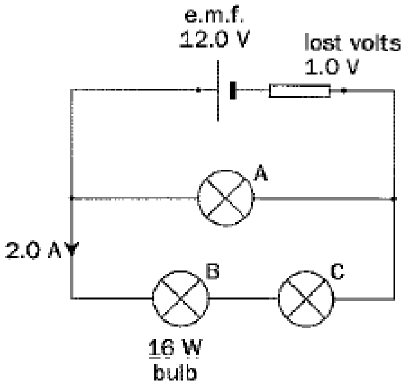
1 What is the e.m.f. of a battery that converts 6300 J of chemical energy into electrical energy while supplying 700 C of charge?

2 A battery of e.m.f. 12 V is used to supply a current of 8.0 A for 360 s. How much energy is converted from chemical energy into electrical energy during this time?

3 A kettle operating from the mains electrical supply is rated at 1500 W. It is supplied with 840 000 J during the time in which 3360 C are delivered. Assuming that the lost volts are negligible for the mains, calculate
(a) the time for which the kettle is switched on
(b) the current to the kettle
(c) the e.m.f. of the mains.

4 A torch battery transfers 5200 J of chemical energy into electrical energy while supplying a current of 0.25 A for 4400 s. The bulb transfers 4800 J of electrical energy into heat and light during this time. Calculate
(a) the charge that flows
(b) the e.m.f. of the battery
(c) the p.d. across the bulb
(d) the lost volts.

5 Find the unknown p.d.s in the circuit given. The p.d.s across components A, B and C are required.



Answering Space

ANSWERS

1 9.0 V

2 34 600 J

3 (a) 560 s
(b) 6 A
(c) 250 V, either as 1500 W per 6 A or 840 000 J per 3360 C

4 (a) 1100 C
(b) 4.72 V
(c) 4.36 V
(d) 0.36 V h

5 $V_A = 11$ V; $V_B = 8$ V; $V_C = 3$ V

SAMPLE QUESTION

Q1.1 Define the **electromotive force (emf)** of an electrical power supply.

[2 Marks]

the (total) energy transferred/work done when one unit/coulomb of charge

1 mark

is moved around a circuit/provided by the supply

1 mark

Q1.2 Explain why, when a battery is supplying a current to a circuit, the voltage measured between its terminals is less than its emf.

[2 Marks]

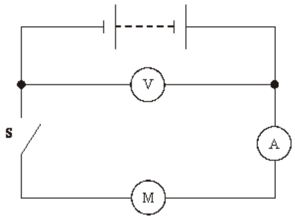
work is done inside the battery/there is resistance inside the battery

1 mark

so less energy is available for the external circuit/some voltage is lost between the terminal/mention of lost volts

1 mark

Q1.3 In the circuit shown in the figure below the voltmeter has a very high resistance and the resistance of the ammeter is negligible. The motor M is being tested using a battery with an emf of 9.00 V.



Q1.3.1 State the reading on the voltmeter when the switch S is open.

[1 Mark]

9.00 V **1 mark**

Q1.3.2 When S is closed and the motor is allowed to run freely the voltmeter reading is 8.41 V and the ammeter reads 0.82 A. Calculate the internal resistance of the battery.

[3 Marks]

Lost Volts = E - V or E = I(R + r)

1 mark

0.82r = 0.59

1 mark

internal resistance = 0.720 Ω **1 mark**

Q1.3.3 Explain why the ammeter reading is greater than 0.82 A when the motor does work by lifting a load.

[1 Mark]

because the battery has to provide more energy/power

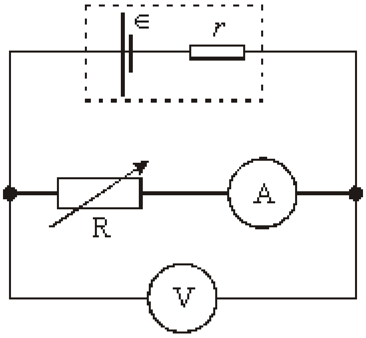
1 mark

Reference: AQA A Level Specimen Examination Materials B

SELF ASSESSMENT

To assess your understanding, answer the following questions.

Q1. In the circuit shown, a battery of emf ∈ and internal resistance *r* is connected to a variable resistor R. The current *I* and the voltage *V* are read by the ammeter and voltmeter respectively.



Q1.1 The emf is related to *V*, *I* and *r* by the equation

$\in = V + Ir$

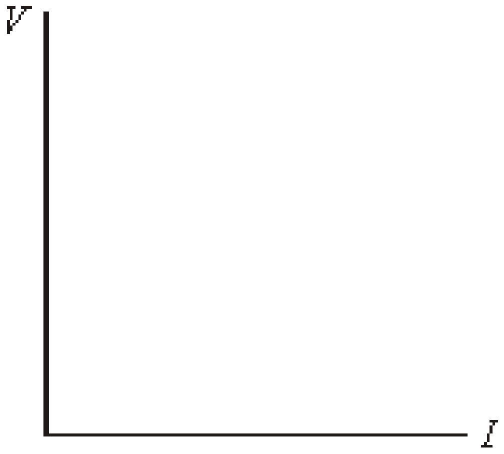
Rearrange the equation to give *V* in terms of ∈, *I* and *r*.

[1 Mark]

……

……

Q1.2 In an experiment, the value of R is altered so that a series of values of *V* and the corresponding values of *I* are obtained. Using the axes, sketch the graph you would expect to obtain as R is changed.



[2 Marks]

Q1.3 State how the values of ∈ and *r* may be obtained from the graph.

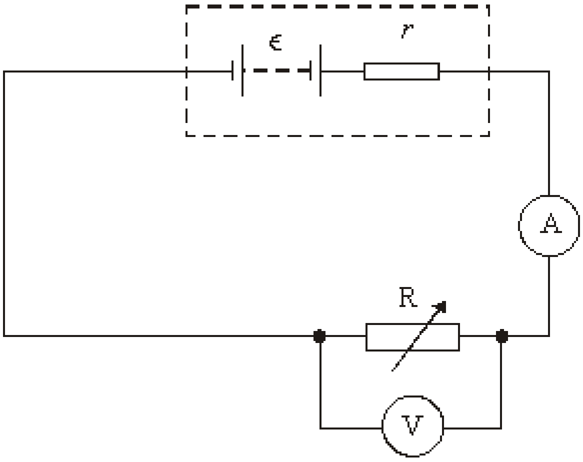
[2 Marks]

∈ ……………………………………………………………………………………………

r ……………………………………………………………………………………………

Reference: AQA A Level Specimen Examination Materials A

Q2. A battery of emf ∈ and internal resistance *r* is connected in series to a variable resistor **R** and an ammeter of negligible resistance. A voltmeter is connected across **R**, as shown in the figure below.



Q2.1 State what is meant by the emf of the battery.

[1 Mark]

……

……

Q2.2 The reading on the voltmeter is less than the emf. Explain why this is so.

[1 Mark]

……

……

A student wishes to measure ∈ and *r*. Using the circuit shown in the figure above the value of R is decreased in steps and at each step the readings *V* and *I* on the voltmeter and ammeter respectively are recorded. These are shown in the table.

reading on voltmeter/V	**reading on ammeter/A**
8.3	0.07
6.8	0.17
4.6	0.33
2.9	0.44
0.3	0.63

Q2.3 Give an expression relating V, I, ϵ and r.

[1 Mark]

……

……

Q2.4 Draw a graph of V (on the y-axis) against I (on the x-axis) on graph paper.

[5 Marks]

Q2.5 Determine the values of ϵ and r from the graph, explaining your method.

[2 Marks]

ϵ:

……

……

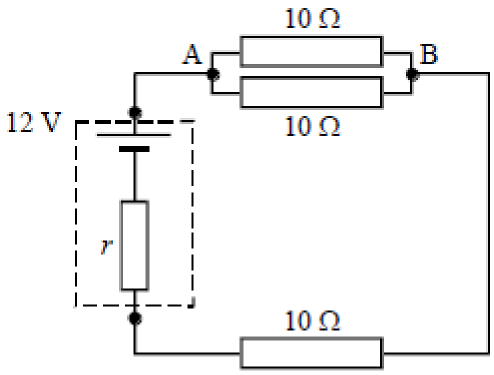
r:

……

……

Reference: AQA A Level Specimen Examination Materials A

Q3. A battery of e.m.f. 12 V and internal resistance *r* is connected in a circuit with three resistors each having a resistance of 10 Ω as shown. A current of 0.50 A flows through the battery.



Calculate

Q3.1 the potential difference between the points A and B in the circuit,

[2 Marks]

………

………

Q3.2 the internal resistance of the battery,

[2 Marks]

………

………

………

………

Q3.3 the total energy supplied by the battery in 2.0 s,

[1 Mark]

………

………

Q3.4 the fraction of the energy supplied by the battery that is dissipated within the battery.

[2 Marks]

………

………

………

Reference: AQA A Level Specimen Examination Materials A

ANSWERS

M1.1 $V = -Ir + ∈$ **(1)**

1

M1.2 straight line (within 1st quadrant) **(1)**
negative gradient **(1)**

2

M1.3 $∈$: intercept on voltage axis **(1)**
r: gradient **(1)**

2
[5]

M2.1 electrical energy produced (in the battery) per unit charge **(1)**

[Or potential/voltage across terminals when there is no current]

M2.2 there is a current (through the battery) **(1)**

voltage 'lost' across the internal resistance **(1)**

Max 2

M2.3 $∈ = V + Ir$ **(1)**

M2.4 labelled scales **(1)**
correct plotting **(1)**
best straight line **(1)**
$∈$: intercept on y axis **(1)** = 9.2 (± 0.1) V **(1)**

r : (–) gradient = $\frac{9.2}{0.65}$ = 14.2 Ω **(1)** (range 14.0 to 14.3)

8
[10]

M3.1 R_{AB} = 5.0 (Ω) **(1)**
V (= 5.0 × 0.50) = 2.5 V **(1)**

M3.2 V_r = 12 – 2.5 + 5.0 **(1)** = 4.5 (V) **(1)**

$r = \left(\frac{V_r}{I} = \frac{4.5}{0.5}\right)$ = 9.0 Ω **(1)**

M3.3 W (= EIt) = 12 J **(1)**

M3.4 W_r (= V_rIt) = 4.5 (J) **(1)**

$\frac{W_r}{W}\left(= \frac{4.5}{12}\right)$ = 0.375 **(1)**

REVISION CHECKLIST

Specification reference	**Checklist questions**	
3.5.1.4	Can you carry out calculations for resistors in series and in parallel?	☐
3.5.1.4	Can you explain and use the energy and power equations: $E = Ivt$ and $P = IV = I^2R = \frac{V^2}{R}$?	☐
3.5.1.4	Can you explain the relationships between currents, voltages and resistances in series and parallel circuits, including cells in series and identical cells in parallel?	☐
3.5.1.4	Can you explain conservation of charge and conservation of energy in dc circuits?	☐
3.5.1.5	Can you describe how the potential divider is used to supply constant or variable potential difference from a power supply?	☐
3.5.1.5	Can you explain the use of variable resistors, thermistors, and light dependent resistors in the potential divider?	☐
3.5.1.6	Can you use the formulae $\varepsilon = \frac{E}{Q}$ and $\varepsilon = I(R + r)$?	☐
3.5.1.6	Can you explain terminal pd and emf?	☐
3.5.1.6	Can you understand and perform calculations for circuits in which the internal resistance of the supply is not negligible?	☐
3.5.1.6	Have you carried out an investigation into the emf and internal resistance of electric cells and batteries by measuring the variation of the terminal pd across the cell with the current in it?	☐

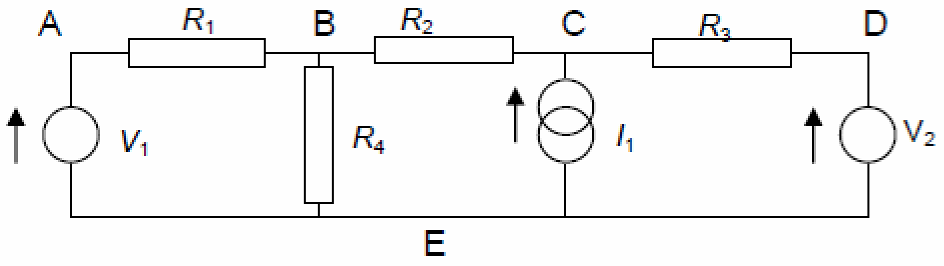
The following section include information beyond the A-Level Physics.
This information will further your understanding and provide a bridge to University Level Physics.

6.3 Circuits – putting it together

In this section, we look at combining resistors, capacitors and inductors in electrical circuits. There are two reasons for doing this. Firstly, once you have left school, you will be faced with complicated electronic networks, and you need to be able to analyse these just as well as the simple series and parallel arrangements you dealt with in the classroom. Secondly, engineers frequently use electric circuits as models or analogies for other systems (say, an oscillating bridge or the control of the nervous system over the muscles in a leg) – the better you understand electric circuits, the better you will understand any linked system.

6.3.1 Circuit Analysis

Our aim here is to be able to solve a circuit like the one below. The circles represent constant-voltage sources (a bit like cells or batteries) and the linked circles represent constant-current sources. Our aim is to find voltage difference across each component, and also to work out the current in each resistor.



In order to solve the circuit, we use two rules – the Kirchoff Laws. Kirchoff's 1st says that the total current going into a junction is equal to the total current leaving it. Therefore, at B in the circuit below, we would say that $I_{BE} = I_{AB} + I_{CB}$, where I_{BE} means the current flowing from B to E (through R_4).

Kirchoff's 2nd Law is that voltages always add up correctly. In other words, no matter which route we took from E to B, say, we would agree on the voltage difference between E and B. In symbols, if V_{BE} means the difference in potential (as measured by a voltmeter) between B and E, then we have $V_{BE} = V_{AE} + V_{BA}$. This is basically the same thing as the law of conservation of energy. The voltage (or, more strictly, the *potential*) at B, V_B, is the energy content of one coulomb of charge at B. In travelling to E, it will lose V_B-V_E joules, irrespective of the route taken.[23] In fact, we assume the truth of Kirchoff's 2nd Law whenever we say, "let's call the voltage at A 'V_A'," for we are assuming that the voltage of A does not depend on the route used to measure it.

Using these two rules, and the equation for the current through a resistor (for example, $V_{BA} = I_{BA} R_1$), we may write down a set of equations for the circuit. Notice that because currents are said to go from + to –, this means that if V_{AB} (the voltage of A, measured relative to B) is positive, then V_A is bigger than V_B, and hence I_{AB} will be positive too. To make the notation easier we will take the potential at E to be zero. In symbolic form, this means that we shall call V_{BE} (that is, $V_B – V_E$) V_B for short.

Kirchoff's First Law:

$$I_{EA} = I_{AB}; \quad I_{BE} = I_{AB}+I_{CB}; \quad I_1 = I_{CB}+I_{CD}; \quad I_{CD} = I_{DE}$$

Kirchoff's Second Law:

$$\begin{aligned} V_B &= I_{BE} R_4 \\ &= V_A + V_{BA} = V_1 - I_{AB} R_1 \\ &= V_D + V_{CD} + V_{BC} = V_2 + I_{CD} R_3 - I_{CB} R_2 \end{aligned}$$

After elimination, the equations reduce to two:

$$V_1 - I_{AB} R_1 = V_2 + I_1 R_3 - (R_3 + R_2) I_{CB} = (I_{AB} + I_{CB}) R_4,$$

and from these the currents I_{AB} and I_{CB} can be found (after a bit of messy algebra). After this, the remaining currents and voltages are straightforward to determine.

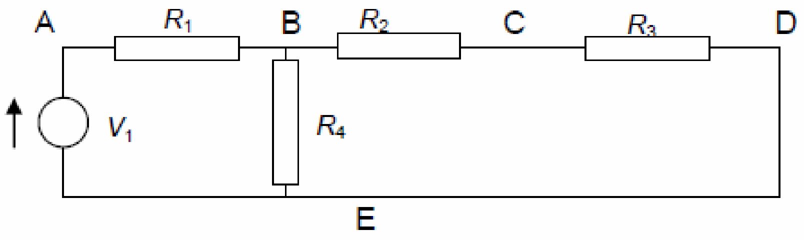
These principles can be used to solve any circuit. However, as networks get bigger, it is useful to find more prescriptive methods of solution, which could be used by a computer. We shall cover two methods here – for certain problems, they may be more efficient than the direct application of Kirchoff's Laws.

6.3.1.1 Method of Superposition

The method of superposition relies on the fact that for a simple resistor, the current is proportional to the voltage. It follows that if current I_1 causes a voltage difference of 3V, and current I_2 causes a voltage difference of 5V, then current I_1+I_2 will cause an 8V p.d. across the component. Here is the procedure:

- Choose one of the supply components.
- Remove the other supply components from the circuit. Replace voltage sources with direct connections (short circuits), and leave breaks in the circuit where the current sources were (open circuits).
- Calculate the current in each wire, and the voltage across each component.
- Repeat the procedure for each supply component in turn.
- The current in each wire for the original (whole) circuit is equal to the sum of the currents in that wire due to each supply unit.
- The voltage across each component in the original (whole) circuit is equal to the sum of the voltages across that component due to each supply unit.

Let's use this method to analyse the circuit above. We start by considering only source V_1. Removing the other supply components gives us a circuit like this.



This circuit is easier to analyse as it only has one supply. Supply V_1 feeds a circuit with resistance

$$R_1 + \{R_4 \,//\, (R_2+R_3)\}$$

$$= R_1 + \frac{R_4\left(R_2 + R_3\right)}{R_4 + R_2 + R_3}$$

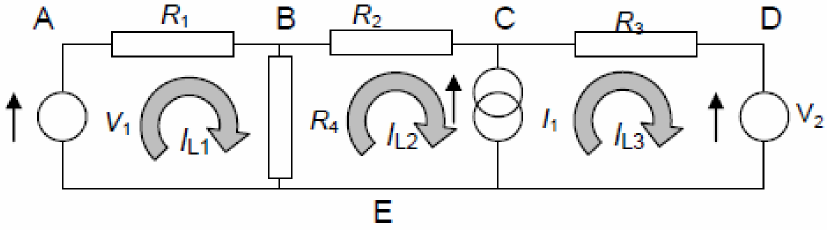
where // means 'in parallel with.' Accordingly, the current supplied by V_1 (and the current through R_1 which is in series with it) is equal to V_1 divided by this resistance. The voltages of points B, C and D can be calculated, as can the current in each wire. We make a note of the values, and add to them the results of analyses of circuits only containing I_1 and only containing V_2.

You may find this method good in the sense that you only have to deal with one supply component at a time – and therefore all you need to know is how to combine resistors (something you've done before). Having said that, we end up analysing three circuits rather than one, so it is more time consuming.

Before leaving the method, you may be curious why voltage sources were replaced with short circuits, and current sources with open circuits. Here's the reason. A voltage source does not change the voltage across its terminals, no matter what the current is (d Voltage / d Current = $R_{equivalent}$ = 0). The only type of resistor which behaves likewise is a perfect conductor (0Ω). Similarly, a current source does not change its current, no matter what the voltage (d Current / d Voltage = 1 / $R_{equivalent}$ = 0). The equivalent resistor in this case is a perfect insulator (∞Ω) which lets no current through ever.

6.3.1.2 *Method of Loop Currents*

Here we break the circuit down into the smallest loops it contains. Here there are three loops:



- E to A to B and back to E (loop 1),
- E to B to C to E (loop 2), and
- E to C to D to E (loop 3).

We call the current in loop 1 "loop current" number one (I_{L1}), with I_{L2} and I_{L3} representing the currents in the other two loops. We then express all other currents in terms of the loop currents. Clearly, $I_{AB} = I_{L1}$, since R_1 is in the first loop alone. Similarly, $I_{BC} = I_{L2}$, and $I_{CD} = I_{L3}$.

The current through R_4 is more complex, since this resistor is part of two of the loops. We write $I_{BE} = I_{L1} – I_{L2}$. Here I_{L1} is positive, since I_{BE} is in the same direction as I_{L1}, whereas I_{L2} (which goes from E to B then on to C) is in the opposite direction. These designations automatically take care of Kirchoff's First Law. Notice that by this method, $I_1 = I_{L3} – I_{L2}$.

Each loop now contributes one equation – Kirchoff's 2nd law around that loop. Clearly, if you go all the way round the loop, you must return to the voltage you started with. Taking the first loop as an example, we have:

$$0 \quad = V_{AE} + V_{BA} + V_{EB}$$

$= V_1 - I_{AB} R_1 - I_{BE} R_4$
$= V_1 - I_{L1} R_1 + (I_{L1} - I_{L2}) R_4.$

In a similar way, we write equations for each of the other two loops[24]. We then have three equations in three unknowns (the three loop currents), which can be solved. The end result is the same as for a direct 'sledgehammer' approach with Kirchoff's Laws – but the method is more organized.

6.3.2 Alternating Current

Having looked at circuits with resistors in them, we next turn our attention to circuits with inductors and capacitors as well. For a direct current, the situation is easy. After a brief period of settling down, there is no voltage drop across an inductor (because the current isn't changing), and a capacitor doesn't conduct at all.

For alternating currents the situation is more complicated. Let us suppose that the supply voltage is given by $V=V_0 \cos \omega t$. It turns out that the circuit will settle down to a steady behaviour (called the steady state). Once this has happened, the voltage across each component (and the current through each component) will also be a cosine wave with frequency ω, however it may not be in phase with the original V.

6.3.2.1 Resistor, capacitor and inductor

We start with the three simplest circuits – the lone resistor, the lone capacitor and the lone inductor, each supplied with a voltage $V=V_0 \cos \omega t$.

For the resistor, I=V/R, so the result is straightforward.

For the capacitor, Q=VC, and if we take I as positive in the direction which charges the capacitor, then

$$\begin{aligned} I &= \frac{dQ}{dt} = \frac{d}{dt} CV_0 \cos \omega t \\ &= -\omega CV_0 \sin \omega t \\ &= \omega CV_0 \cos\left(\omega t + \tfrac{1}{2}\pi\right) \\ &= -\omega CV_0 \cos\left(\omega t - \tfrac{1}{2}\pi\right) \end{aligned} \tag{31}$$

For the inductor, $V = L\, dI/dt$, so

$$\frac{dI}{dt} = \frac{V_0}{L}\cos\omega t$$

$$I = \frac{V_0}{L\omega}\sin\omega t \qquad (32)$$

$$= \frac{V_0}{L\omega}\cos\left(\omega t - \tfrac{1}{2}\pi\right)$$

where we have taken the constant of integration to be zero. Failure to do so would lead to a non-zero mean current, which is clearly impossible as the mean supply voltage is zero.

6.3.2.2 Reactance and Impedance

For resistors, the current and voltage are proportional, and consequently are in phase – one peaks at the same time as the other. For the other two components, this is not the case. The voltage is π/2 radians (or 90°) out of phase with respect to the current. Inductor currents peak 90° later than the voltage (the current *lags* the voltage), whereas capacitor currents peak 90° before the voltage (the current *leads* the voltage). Nevertheless, the amplitude of the voltage is still proportional to the amplitude of the current, and we call the ratio of the amplitudes the reactance (X).

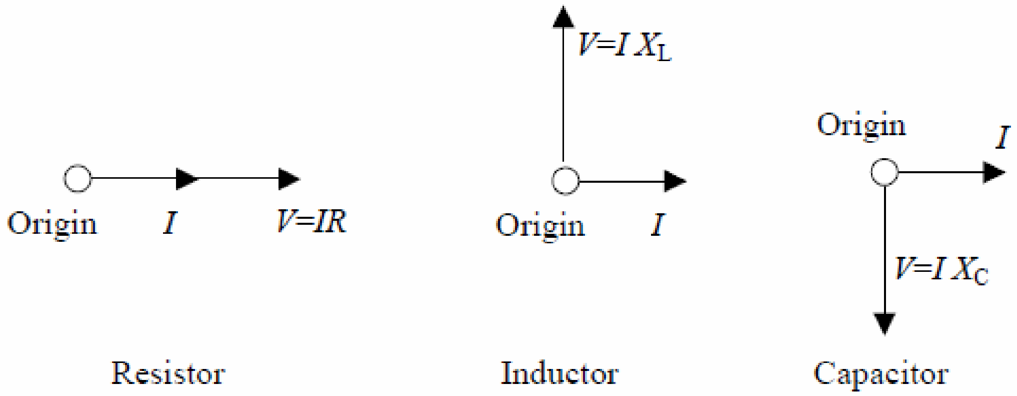
$$X_L = \omega L$$

$$X_C = -\frac{1}{\omega C} \qquad (33)$$

By convention, we take reactance to be positive if the current lags the voltage by 90°, and negative if it leads by 90°. For capacitors and inductors in series, the total reactance is equal to the sum of the individual components' reactances – just as resistances add in series. Similarly, the formula for combining reactances in parallel is the same for that used for the resistance of resistors wired in parallel.

When a circuit is constructed with resistors, capacitors and inductors, then we need a way of analysing a circuit with both resistances and reactances. We visualise the situation using a 2D (phasor) diagram.

For any component or circuit, both voltage and current are represented by vectors. The length of the lines gives the amplitude, and the angle between the vectors gives the phase difference. By convention, we imagine the vectors to rotate about the origin in an anticlockwise direction (once per time period of the alternating current). The vectors for a resistor, capacitor and inductor are shown below.



As the arrows rotate anticlockwise, for the inductor, *V* comes before *I*. With the capacitor, *I* comes before *V*. This accurately represents the phase relationships between voltage and current for these components.

For a set of components in series, the current *I* will be the same for all of them. We usually draw the current pointing to the right. Voltages across inductors will then point up, those across resistors point right, and those across capacitors point down. By adding these voltages vectorially, we arrive at the voltage across the set of components – and can calculate its amplitude and phase relationship with respect to the current.

Similarly, for components in parallel, the voltage will be the same for each. We thus put voltage pointing to the right. Currents in capacitors now point up, currents in resistors point right, and currents in inductors point down. The total current is given by the vector sum of the individual currents.

In all cases, we call the ratio of the voltage amplitude to the current amplitude the impedance (*Z*) irrespective of the phase difference between the current and voltage.[25] In general the impedance of a component is related to resistance and reactance by $Z^2 = R^2 + X^2$.

6.3.2.3 *Complex Numbers and Impedance*

If you are familiar with complex numbers, there is an easier way of describing all of this, using the Argand diagram in place of 2-dimensional vectors. The impedance *Z* is a now a complex number $Z = R + iX$, with *R* as its real part and *X* as its imaginary part.

The complex impedances of a resistor, capacitor and inductor are accordingly written as R, $-i/\omega C$ and $i\omega L$ respectively. The impedance of a set of components in series is given by the sum of the individual impedances. For a parallel network, Z^{-1} of the network is given by the sum of Z^{-1} for each component, where 'inverse' (or 'reciprocal') is calculated in the usual way for complex numbers.

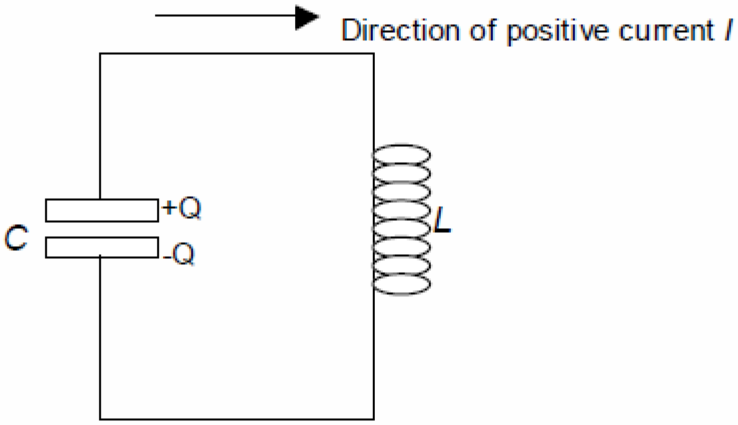
6.3.2.4 *Root Mean Square values*

You will also need to remember the definition of RMS voltage and current in an a.c. circuit. For a resistor, remember that the RMS supply voltage is the d.c. voltage which would supply the same mean **power** to the device.

$$\overline{P} \equiv \frac{V_{rms}^2}{R} = \frac{\overline{V_0^2 \cos^2 \omega t}}{R} = \frac{V_0^2}{2R}$$
$$\Rightarrow \quad V_{rms} = \frac{1}{\sqrt{2}} V_0 \tag{34}$$

6.3.3 Resonance

One further circuit needs a mention, and that is the simple circuit of an inductor and a capacitor connected together, as shown in the diagram below. Both the voltage and current for the two components must be the same, and so with the sign conventions chosen in the diagram:



$$\frac{Q}{C} = L\frac{dI}{dt} = -L\frac{d^2Q}{dt^2}$$
$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q \tag{35}$$

This is an equation of 'simple harmonic motion' with angular frequency ω, where $\omega^2 = 1/LC$. This circuit can therefore oscillate at this frequency, and this makes it useful in radio receivers for selecting the frequency (and hence radio station) which the listener wants to detect.

8 Practical Physics

8.1 Errors, and how to make them[29]

Every dog has its day, every silver lining has its cloud, and every measurement has its error.

If you doubt this, take (sorry – borrow with permission) a school metre stick, and try and measure the length of a corridor in your school. Try and measure it to the nearest centimetre. Then measure it again. Unless you cheated by choosing a short corridor, you should find that the measurements are different. What's gone wrong?

Nothing has gone wrong. No measurement is exact, and if you take a series of readings, you will find that they cluster around the 'true value'. This spread of readings is called random error – and will be determined by the instrument you use and the observation technique. To be more precise and polite, this kind of 'error' is usually called uncertainty, as this word doesn't imply any mistake or incompetence on the part of the scientist.

So, whenever you write down a measurement, you should also write down its uncertainty. This can be expressed in two ways – absolute and relative.

The absolute uncertainty gives the size of the spread of readings. You might conclude that your corridor was (12.3±0.2)m long. In other words, your measurements are usually within 20cm of 12.3m. In this case the absolute uncertainty is 20cm.

The absolute uncertainty only gives part of the story. A 10cm error in the length of a curtain track implies sloppy work. A 10cm error in the total length of the M1 motorway is an impressive measurement. To make this clearer, we often state errors (or uncertainties) in percentage form – and this is called relative uncertainty. The relative uncertainty in the length of the corridor is

$$\text{Relative Error} = \frac{\text{Abs. Uncertainty}}{\text{Measurement}} = \frac{0.2\text{ m}}{12.3\text{ m}} = 1.6\% \approx 2\%. \qquad (1)$$

Notice the rounding off at the end. It is usually pointless to give uncertainties to more than one significant figure.

Every measurement has its uncertainty, and the only way of determining this is to take more than one measurement, and work out the standard deviation – to measure the spread. In practice the spread can be 'eyeballed' rather than calculated. If the measurements were 54.5cm, 54.7cm and 54.3cm, then there is no need to use a calculator and the technical definition of deviation. The observation that the spread is about ±0.2cm is perfectly good enough.

Notice that the more readings you take, the better idea you get of the spread of the measurements – and hence the better estimate you can make for the middle, which is indicative of 'true' value. Therefore we find, from statistics, that if you take *n* measurements, and the absolute uncertainty is *x*, then the uncertainty of the mean of those measurements is approximately:[30]

$$\text{Uncertainty of mean} = \frac{x}{\sqrt{n}}. \qquad (2)$$

Therefore, the more measurements you take, the more accurate the work. Notice that if you wish to halve the uncertainty, you need to take *four* times as many readings. This is subject to one proviso:

Measurements also have a resolution. This is the smallest distinguishable difference that the measuring device (including the technique) can detect. For a simple length measurement with a metre ruler, the resolution is probably 1mm. However if, by years of practice with a magnifying lens, you could divide millimetres into tenths by eye, you would have a resolution of 0.1mm using the same metre stick. That is why we say that the resolution depends on the technique as well as on the apparatus.

The uncertainty of a measurement can never be less than the resolution. This is the proviso we mentioned below equation (2). Why should this be the case? Let us have a parable.

Many years ago, the great nation of China had an emperor. The masses of the population were not permitted to see him. One day, a citizen had the sudden desire to know the length of the emperor's nose. He could not do this directly, since he was not permitted to visit the emperor. So, using the apparatus of the imperial administration, he asked all the regional mandarins to ask the entire population to make a guess. Each person would make some guess at the imperial nasal length – and the error of each guess would probably be no more than ±2cm – since nose lengths tend not to vary by more than about 4cm.

However, the mean would be a different matter. Averaged over the 1000 million measurements, the error in the mean would be 0.7μm. So the emperor's nose had been measured incredibly accurately – without a single observation having been made!

The moral of the story: uncertainties *are* reduced by repeated measurement, but the error can never be reduced below the resolution of the technique – here 2cm – since ignorance can not be circumvented by pooling it with more ignorance.

8.2 Errors, and how to make them worse

Errors are one thing. The trouble is that usually we want to put our measurements into a formula to calculate something else. For example, we might want to measure the strength of a magnetic field by measuring the force on a current-carrying wire $B = F/IL$.

If there is a 7% uncertainty in the current, 2% in the force and 1% in the length – what is the uncertainty in the magnetic field?

There are two rules you need:

8.2.1 Rule 1 – Adding or subtracting measurements

If two measurements are added or subtracted, the **absolute** uncertainty in the result equals the sum (never the difference) of the absolute uncertainties of the individual measurements.

Therefore if a car is (3.2±0.1)m long, and a caravan is (5.2±0.2)m long, the total length is (8.4±0.3)m long. Similarly if the height of a two-storey house is (8.3±0.2)m and the height of the ground floor is (3.1±0.1)m, the height of the upper floor is (5.2±0.3)m.

Even in the second case, we do not subtract the uncertainties, since there is nothing stopping one measurements being high, while the other is low.[31]

8.2.2 Rule 2 – Multiplying or dividing measurements

If two measurements are multiplied or divided, the **relative** uncertainty in the result equals the sum (never the difference) of the relative uncertainties of the individual measurements.

Therefore if the speed of a car is 30mph ± 10%, and the time for a journey is 6 hours ± 2%, the uncertainty in the distance travelled is 12%.

Notice that one consequence of this is that if a measurement, with relative uncertainty $p\%$ is squared (multiplied by itself), the relative error in the square is $2p\%$ - i.e. doubled. Similarly if the error in measurement L is $p\%$, the error in L^n is $p\times n\%$. Notice that while a square root will halve the relative error, an inverse square ($n=-2$) doubles it. All the minus sign does is to turn overestimates into underestimates. It does not reduce the magnitude of the relative error.[32]

Now we can answer our question about the magnetic field measurement at the beginning of the section. All three relative errors (in length, force and current) must be added to give the relative error in the magnetic field, which is therefore 10%.

8.3 Systematic Errors

All the 'errors' mentioned so far are called 'random', since we assume that the measurements will be clustered around the true value. However often an oversight in our technique will cause a measurement to be overestimated more often than underestimated or vice-versa. This kind of error is called 'systematic error', and can't be reduced by averaging readings. The only way of spotting this kind of error (which is a true error in that there is something wrong with the measurement) is to repeat the measurement using a completely different technique, and compare the results. Just thinking hard about the method can help you spot some systematic errors, but it is still a good idea to perform the experiment a different way if time allows.

8.4 Which Graph?

You will often have to use graphs to check the functional form of relationships. You may also have to make measurements using the graph. In order to do either of these, you usually need to manipulate the data until you can plot a straight line. A straight line is conclusive proof that you have got the form of the formula right!

The gradient and y-intercept can then be read, and these enable other measurements to be made. For example, your aim may be to measure the acceleration due to gravity. You may plot velocity of falling against time, in which case you will need to find the gradient of the line.

At its most general, you will have a suspected functional form *y=f(x)*, and you will need to work out what is going on in the function *f*. Notice that our experiment will give us pairs of *(x,y)* values – what is not known are the parameters in the function *f*. We find them by manipulating the equation:

$$\begin{aligned} y &= f(x) \\ &\vdots \\ g(x,y) &= A\,h(x,y) + B \end{aligned}.$$

We can then plot *g(x,y)* against *h(x,y)*, and obtain the parameters *A* and *B* from the gradient and intercept of the line. Furthermore, the presence of the straight line on the graph assures us that our function *f* was a good guess. We shall now look at the most common examples.

8.4.1 Exponential growth or decay

Here we have the functional form $y = Ae^{Bx}$, where *A* and *B* need to be determined. We manipulate the equation:

$$\begin{aligned} y &= Ae^{Bx} \\ \ln y &= \ln A + Bx \end{aligned}.$$

So we plot (ln *y*) on the vertical axis, and (*x*) on the horizontal. The y-intercept gives ln *A*, and the gradient gives *B*.

8.4.2 Logarithmic growth or decay

Here we have the functional form $y = A + B\ln x$, and again, we need to work out the values of *A* and *B*. This equation is already in linear form – we plot *y* on the vertical, and (ln *x*) on the horizontal. The y-intercept gives *A*, and the gradient gives *B*.

8.4.3 Power laws

This covers all equations with unknown powers: $y = Ax^B$. The manipulation involves logarithms:

$$y = Ax^B$$
$$\ln y = \ln A + \ln x^B \, .$$
$$\ln y = \ln A + B \ln x$$

Here we plot (ln *y*) against (ln *x*), and find the power (*B*) as the gradient of the line. The *A* value can be inferred from the y-intercept, which is equal to ln *A*.

8.4.4 Other forms

Even hideous looking equations can be reduced to straight lines if you crack the whip hard enough. How about $y = A\sqrt{x} + Bx^3$? Is it tasty enough for your breakfast? Actually it's fine if digested slowly:

$$y = A\sqrt{x} + Bx^3$$
$$\frac{y}{\sqrt{x}} = A + Bx^{5/2} \, .$$

This looks even worse, doesn't it? But remember that it is *x* and *y* that are *known*. If we plot ($y/\sqrt{x}$) on the vertical, and ($x^{5/2}$) on the horizontal, a straight line appears, and we can read *A* and *B* from the y-intercept and gradient respectively.

DATASHEET

DATA – FUNDAMENTAL CONSTANTS AND VALUES

Quantity	Symbol	Value	Units
speed of light in vacuo	c	3.00×10^{8}	$\mathrm{m\,s^{-1}}$
permeability of free space	μ_0	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
permittivity of free space	ε_0	8.85×10^{-12}	$\mathrm{F\,m^{-1}}$
magnitude of the charge of electron	e	1.60×10^{-19}	C
the Planck constant	h	6.63×10^{-34}	$\mathrm{J\,s}$
gravitational constant	G	6.67×10^{-11}	$\mathrm{N\,m^{2}\,kg^{-2}}$
the Avogadro constant	N_A	6.02×10^{23}	$\mathrm{mol^{-1}}$
molar gas constant	R	8.31	$\mathrm{J\,K^{-1}\,mol^{-1}}$
the Boltzmann constant	k	1.38×10^{-23}	$\mathrm{J\,K^{-1}}$
the Stefan constant	σ	5.67×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
the Wien constant	α	2.90×10^{-3}	$\mathrm{m\,K}$
electron rest mass (equivalent to 5.5×10^{-4} u)	m_e	9.11×10^{-31}	kg
electron charge/mass ratio	$\frac{e}{m_e}$	1.76×10^{11}	$\mathrm{C\,kg^{-1}}$
proton rest mass (equivalent to 1.00728 u)	m_p	$1.67(3) \times 10^{-27}$	kg
proton charge/mass ratio	$\frac{e}{m_p}$	9.58×10^{7}	$\mathrm{C\,kg^{-1}}$
neutron rest mass (equivalent to 1.00867 u)	m_n	$1.67(5) \times 10^{-27}$	kg
gravitational field strength	g	9.81	$\mathrm{N\,kg^{-1}}$
acceleration due to gravity	g	9.81	$\mathrm{m\,s^{-2}}$
atomic mass unit (1u is equivalent to 931.5 MeV)	u	1.661×10^{-27}	kg

ALGEBRAIC EQUATION

quadratic equation $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

ASTRONOMICAL DATA

Body	Mass/kg	Mean radius/m
Sun	1.99×10^{30}	6.96×10^{8}
Earth	5.97×10^{24}	6.37×10^{6}

GEOMETRICAL EQUATIONS

arc length $= r\theta$

circumference of circle $= 2\pi r$

area of circle $= \pi r^2$

curved surface area of cylinder $= 2\pi rh$

area of sphere $= 4\pi r^2$

volume of sphere $= \frac{4}{3}\pi r^3$

Particle Physics

Class	Name	Symbol	Rest energy/MeV
photon	photon	γ	0
lepton	neutrino	ν_e	0
		ν_μ	0
	electron	$e^{\pm}$	0.510999
	muon	$\mu^{\pm}$	105.659
mesons	π meson	$\pi^{\pm}$	139.576
		π^0	134.972
	K meson	$K^{\pm}$	493.821
		K^0	497.762
baryons	proton	p	938.257
	neutron	n	939.551

Properties of quarks

antiquarks have opposite signs

Type	Charge	Baryon number	Strangeness
u	$+\frac{2}{3}e$	$+\frac{1}{3}$	0
d	$-\frac{1}{3}e$	$+\frac{1}{3}$	0
s	$-\frac{1}{3}e$	$+\frac{1}{3}$	-1

Properties of Leptons

		Lepton number
Particles:	e^-, ν_e ; μ^-, ν_μ	$+1$
Antiparticles:	e^+, $\overline{\nu_e}$, μ^+, $\overline{\nu_\mu}$	-1

Photons and energy levels

photon energy $E = hf = hc/\lambda$

photoelectricity $hf = \phi + E_{k\,(max)}$

energy levels $hf = E_1 - E_2$

de Broglie wavelength $\lambda = \frac{h}{p} = \frac{h}{mv}$

Waves

wave speed $c = f\lambda$ *period* $f = \frac{1}{T}$

first harmonic $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

fringe spacing $w = \frac{\lambda D}{s}$ *diffraction grating* $d \sin\theta = n\lambda$

refractive index of a substance s, $n = \frac{c}{c_s}$

for two different substances of refractive indices n_1 *and* n_2,

law of refraction $n_1 \sin\theta_1 = n_2 \sin\theta_2$

critical angle $\sin\theta_c = \frac{n_2}{n_1}$ for $n_1 > n_2$

Mechanics

moments moment $= Fd$

velocity and acceleration $v = \frac{\Delta s}{\Delta t}$ $a = \frac{\Delta v}{\Delta t}$

equations of motion $v = u + at$ $s = \left(\frac{u+v}{2}\right)t$

$v^2 = u^2 + 2as$ $s = ut + \frac{at^2}{2}$

force $F = ma$

force $F = \frac{\Delta(mv)}{\Delta t}$

impulse $F\,\Delta t = \Delta(mv)$

work, energy and power $W = F\,s\cos\theta$

$E_k = \frac{1}{2}m\,v^2$ $\Delta E_p = mg\Delta h$

$P = \frac{\Delta W}{\Delta t}$, $P = Fv$

$efficiency = \frac{useful\ output\ power}{input\ power}$

Materials

density $\rho = \frac{m}{V}$ *Hooke's law* $F = k\,\Delta L$

Young modulus $= \frac{tensile\ stress}{tensile\ strain}$ *tensile stress* $= \frac{F}{A}$

tensile strain $= \frac{\Delta L}{L}$

energy stored $E = \frac{1}{2}F\Delta L$

Electricity

current and pd	$I = \frac{\Delta Q}{\Delta t}$ $\quad V = \frac{W}{Q}$ $\quad R = \frac{V}{I}$
resistivity	$\rho = \frac{RA}{L}$
resistors in series	$R_T = R_1 + R_2 + R_3 + \ldots$
resistors in parallel	$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$
power	$P = VI = I^2R = \frac{V^2}{R}$
emf	$\varepsilon = \frac{E}{Q}$ $\quad \varepsilon = I(R + r)$

Circular motion

magnitude of angular speed	$\omega = \frac{v}{r}$
	$\omega = 2\pi f$
centripetal acceleration	$a = \frac{v^2}{r} = \omega^2 r$
centripetal force	$F = \frac{mv^2}{r} = m\omega^2 r$

Simple harmonic motion

acceleration	$a = -\omega^2 x$
displacement	$x = A \cos(\omega t)$
speed	$v = \pm\omega\sqrt{(A^2 - x^2)}$
maximum speed	$v_{max} = \omega A$
maximum acceleration	$a_{max} = \omega^2 A$
for a mass-spring system	$T = 2\pi\sqrt{\frac{m}{k}}$
for a simple pendulum	$T = 2\pi\sqrt{\frac{l}{g}}$

Thermal physics

energy to change temperature	$Q = mc\Delta\theta$
energy to change state	$Q = m\,l$
gas law	$pV = nRT$
	$pV = NkT$
kinetic theory model	$pV = \frac{1}{3}N\,m\,(c_{\mathrm{rms}})^2$
kinetic energy of gas molecule	$\frac{1}{2}m\,(c_{\mathrm{rms}})^2 = \frac{3}{2}kT = \frac{3RT}{2N_A}$

Gravitational fields

force between two masses	$F = \frac{Gm_1m_2}{r^2}$
gravitational field strength	$g = \frac{F}{m}$
magnitude of gravitational field strength in a radial field	$g = \frac{GM}{r^2}$
work done	$\Delta W = m\Delta V$
gravitational potential	$V = -\frac{GM}{r}$
	$g = -\frac{\Delta V}{\Delta r}$

Electric fields and capacitors

force between two point charges	$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$
force on a charge	$F = EQ$
field strength for a uniform field	$E = \frac{V}{d}$
work done	$\Delta W = Q\Delta V$
field strength for a radial field	$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$
electric potential	$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$
	$E = \frac{\Delta V}{\Delta r}$
capacitance	$C = \frac{Q}{V}$
	$C = \frac{A\varepsilon_0\varepsilon_r}{d}$
capacitor energy stored	$E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$
capacitor charging	$Q = Q_0(1 - \mathrm{e}^{-t/RC})$
decay of charge	$Q = Q_0\mathrm{e}^{-t/RC}$
time constant	RC

Magnetic fields

force on a current	$F = BIl$
force on a moving charge	$F = BQv$
magnetic flux	$\Phi = BA$
magnetic flux linkage	$N\Phi = BAN\cos\theta$
magnitude of induced emf	$\varepsilon = N\dfrac{\Delta\Phi}{\Delta t}$
	$N\Phi = BAN\cos\theta$
emf induced in a rotating coil	$\varepsilon = BAN\omega\sin\omega t$
alternating current	$I_{\mathrm{rms}} = \dfrac{I_0}{\sqrt{2}} \quad V_{\mathrm{rms}} = \dfrac{V_0}{\sqrt{2}}$
transformer equations	$\dfrac{N_s}{N_p} = \dfrac{V_s}{V_p}$
	$efficiency = \dfrac{I_s V_s}{I_p V_p}$

Nuclear physics

the inverse square law for γ radiation	$I = \dfrac{k}{x^2}$
radioactive decay	$\dfrac{\Delta N}{\Delta t} = -\lambda N,\ N = N_0 e^{-\lambda t}$
activity	$A = \lambda N$
half-life	$T_{½} = \dfrac{\ln 2}{\lambda}$
nuclear radius	$R = R_0 A^{1/3}$
energy-mass equation	$E = mc^2$

OPTIONS

Astrophysics

1 astronomical unit = 1.50×10^{11} m

1 light year = 9.46×10^{15} m

1 parsec = 206265 AU = 3.08×10^{16} m
= 3.26 light year

Hubble constant, H = 65 km s^{-1} Mpc^{-1}

$$M = \frac{\textit{angle subtended by image at eye}}{\textit{angle subtended by object at unaided eye}}$$

in normal adjustment	$M = \dfrac{f_o}{f_e}$
Rayleigh criterion	$\theta \approx \dfrac{\lambda}{D}$
magnitude equation	$m - M = 5\log\dfrac{d}{10}$
Wien's law	$\lambda_{\max} T = 2.9 \times 10^{-3}$ m K
Stefan's law	$P = \sigma A T^4$
Schwarzschild radius	$R_s \approx \dfrac{2GM}{c^2}$
Doppler shift for v << c	$\dfrac{\Delta f}{f} = -\dfrac{\Delta\lambda}{\lambda} = \dfrac{v}{c}$
red shift	$z = -\dfrac{v}{c}$
Hubble's law	$v = Hd$

Medical physics

lens equations	$P = \dfrac{1}{f}$
	$m = \dfrac{v}{u}$
	$\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$
threshold of hearing	$I_0 = 1.0 \times 10^{-12}$ W m^{-2}
intensity level	$intensity\ level = 10\log\dfrac{I}{I}$
absorption	$I = I_0 e^{-\mu x}$
	$\mu_m = \dfrac{\mu}{\rho}$
ultrasound imaging	$Z = pc$
	$\dfrac{I_r}{I_i} = \left(\dfrac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$
half-lives	$\dfrac{1}{T_E} = \dfrac{1}{T_B} + \dfrac{1}{T_P}$

Engineering physics

moment of inertia $I = \Sigma mr^2$

angular kinetic energy $E_k = \frac{1}{2} I \omega^2$

equations of angular motion

$$\omega_2 = \omega_1 + \alpha t$$

$$\omega_2{}^2 = \omega_1{}^2 + 2\alpha\theta$$

$$\theta = \omega_1 t + \frac{\alpha t^2}{2}$$

$$\theta = \frac{(\omega_1 + \omega_2)\, t}{2}$$

torque $T = I\,\alpha$

$T = F\,r$

angular momentum $\text{angular momentum} = I\omega$

angular impulse $T\Delta t = \Delta(I\omega)$

work done $W = T\theta$

power $P = T\omega$

thermodynamics $Q = \Delta U + W$

$W = p\Delta V$

adiabatic change $pV^{\gamma} = \text{constant}$

isothermal change $pV = \text{constant}$

heat engines

$$efficiency = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H}$$

$$\text{maximum theoretical efficiency} = \frac{T_H - T_C}{T_H}$$

work done per cycle = area of loop

input power = calorific value × fuel flow rate

$$\text{indicated power} = (\text{area of } p - V \text{ loop}) \times (\text{number of cycles per second}) \times (\text{number of cylinders})$$

output or brake power $P = T\omega$

friction power = indicated power – brake power

heat pumps and refrigerators

refrigerator: $COP_{\text{ref}} = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C}$

heat pump: $COP_{\text{hp}} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_C}$

Turning points in physics

electrons in fields $F = \frac{eV}{d}$

$F = Bev$

$r = \frac{mv}{Be}$

$½\, mv^2 = eV$

Millikan's experiment $\frac{QV}{d} = mg$

$F = 6\pi\eta r v$

Maxwell's formula $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$

special relativity $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$E = m c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Electronics

resonant frequency $f_0 = \frac{1}{2\pi \sqrt{LC}}$

Q-factor $Q = \frac{f_0}{f_B}$

operational amplifiers: open loop $V_{\text{out}} = A_{\text{OL}}(V_+ - V_-)$

inverting amplifier $\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_f}{R_{\text{in}}}$

non-inverting amplifier $\frac{V_{\text{out}}}{V_{\text{in}}} = 1 + \frac{R_f}{R_1}$

summing amplifier $V_{\text{out}} = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \cdots\right)$

difference amplifier $V_{\text{out}} = (V_+ - V_-)\frac{R_f}{R_1}$

Bandwidth requirement:

for AM $\text{bandwidth} = 2f_M$

for FM $\text{bandwidth} = 2(\Delta f + f_M)$

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This document has been produced for the AQA A Level Physics Specification.

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Only constructive and reasoned feedback will be considered.