

**ST MARY'S SCIENCE  
DEPARTMENT:  
PHYSICS**

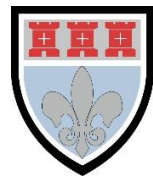
**Volume  
Two**

**A LEVEL PHYSICS YEAR 1  
PREPARATORY READING BOOK  
3.5.2: EMF AND POTENTIAL DIVIDERS  
VOLUME TWO**

<b>NAME</b>	
<b>PHYSICS CLASS</b>	
<b>MODULE TEACHER</b>	
<b>ALPS GRADE</b>	



**THIS MUST  
BE BROUGHT  
TO ALL  
PHYSICS  
LESSONS.**



## Contents

### 3.5.1.5 Potential Divider

### 3.5.1.6 Electromotive Force and Internal Resistance

## Overview

This section builds on and develops earlier study of these phenomena from GCSE.

It provides opportunities for the development of practical skills at an early stage in the course and lays the groundwork for later study of the many electrical applications that are important to society.

### **IMPORTANT NOTE**

This booklet, along with the student workbook, must be brought to all Physics lessons with the appropriate teacher.

This booklet may be used as a learning resource in lessons; you are not fully equipped to learn if this is not used in lesson.

This booklet may also be used as a revision resource for intervention, internal assessments and external assessments.

**Please keep this in your student file.**

As part of this course you are expected to **read through this preparatory reading book** and **complete the independent study tasks**.

This work will not be assessed but will be monitored by your class teacher.

This must be completed by the deadline set by your class teacher.



## Definition List

Definitions you must learn for this module are.

Key Word	Symbol	Definition
<b>Electromotive force, EMF (Open Circuit Voltage)</b>	<b><math>\mathcal{E}</math></b>	The amount of electrical energy per unit charge produced inside a source of electrical energy.
<b>Internal Resistance</b>	<b><math>r</math></b>	The resistance inside a source of electrical energy, the loss of pd per unit current in the source when current passes through it.
<b>(External) Load Resistance</b>	<b><math>R</math></b>	The resistance inside a transducer (output) of the electrical circuit, the loss of PD per unit current in the output of the circuit.
<b>Light dependent resistor</b>		A resistor which is designed to have a resistance that changes with light intensity.
<b>Lost volts</b>		The energy wasted per coulomb overcoming the internal resistance in a circuit.
<b>Negative temperature coefficient</b>		When the resistance of a semi-conductor decreases when its temperature is increased.
<b>Positive temperature coefficient</b>		When the resistance of a metal increases when its temperature is increased.
<b>Potential divider</b>		When two or more resistors (outputs) in series connected to a source of pd.
<b>Terminal PD</b>		The energy per coulomb delivered to the load outputs of the circuit.
<b>Thermistor</b>		A resistor which is designed to have a resistance that changes with temperature.

### IMPORTANT NOTE

These definitions must be memorised by students.

You will be tested on your knowledge of these definitions.



## The Language of Measurement

The following subject specific vocabulary provides definitions of key terms used in the A-level Science specifications.

### **Accuracy**

A measurement result is considered accurate if it is judged to be close to the true value.

### **Calibration**

Marking a scale on a measuring instrument.

This involves establishing the relationship between indications of a measuring instrument and standard or reference quantity values, which must be applied.

For example, placing a thermometer in melting ice to see whether it reads 0 °C, to check if it has been calibrated correctly.

### **Data**

Information, either qualitative or quantitative, that has been collected.

### **Errors**

See also uncertainties.

### **Measurement error**

The difference between a measured value and the true value.

### **Anomalies**

These are values in a set of results which are judged not to be part of the variation caused by random uncertainty.

### **Random error**

These cause readings to be spread about the true value, due to results varying in an unpredictable way from one measurement to the next.

Random errors are present when any measurement is made, and cannot be corrected. The effect of random errors can be reduced by making more measurements and calculating a new mean.

### **Systematic error**

These cause readings to differ from the true value by a consistent amount each time a measurement is made.

Sources of systematic error can include the environment, methods of observation or instruments used.

Systematic errors cannot be dealt with by simple repeats. If a systematic error is suspected, the data collection should be repeated using a different technique or a different set of equipment, and the results compared.

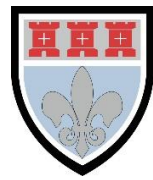
### **Zero error**

Any indication that a measuring system gives a false reading when the true value of a measured quantity is zero, e.g. the needle on an ammeter failing to return to zero when no current flows.

A zero error may result in a systematic uncertainty.

### **Evidence**

Data which has been shown to be valid.

**Fair test**

A fair test is one in which only the independent variable has been allowed to affect the dependent variable.

**Hypothesis**

A proposal intended to explain certain facts or observations.

**Interval**

The quantity between readings, e.g. a set of 11 readings equally spaced over a distance of 1 metre would give an interval of 10 centimetres.

**Precision**

Precise measurements are ones in which there is very little spread about the mean value. Precision depends only on the extent of random errors – it gives no indication of how close results are to the true value.

**Prediction**

A prediction is a statement suggesting what will happen in the future, based on observation, experience or a hypothesis.

**Range**

The maximum and minimum values of the independent or dependent variables; important in ensuring that any pattern is detected.

For example, a range of distances may be quoted as either:

'From 10 cm to 50 cm'

or

'From 50 cm to 10 cm'

**Repeatable**

A measurement is repeatable if the original experimenter repeats the investigation using same method and equipment and obtains the same results.

**Reproducible**

A measurement is reproducible if the investigation is repeated by another person, or by using different equipment or techniques, and the same results are obtained.

**Resolution**

This is the smallest change in the quantity being measured (input) of a measuring instrument that gives a perceptible change in the reading.

**Sketch graph**

A line graph, not necessarily on a grid, that shows the general shape of the relationship between two variables. It will not have any points plotted and although the axes should be labelled they may not be scaled.

**True value**

This is the value that would be obtained in an ideal measurement.

**Uncertainty**

The interval within which the true value can be expected to lie, with a given level of confidence or probability, e.g. "the temperature is  $20\text{ }^{\circ}\text{C} \pm 2\text{ }^{\circ}\text{C}$ , at a level of confidence of 95%.

**Validity**

Suitability of the investigative procedure to answer the question being asked. For example, an investigation to find out if the rate of a chemical reaction depended upon the concentration of one of the reactants would not be a valid procedure if the temperature of the reactants was not controlled.

**Valid conclusion**

A conclusion supported by valid data, obtained from an appropriate experimental design and based on sound reasoning.

**Variables**

These are physical, chemical or biological quantities or characteristics.

**Categoric variables**

Categoric variables have values that are labels. E.g. names of plants or types of material.

**Continuous variables**

Continuous variables can have values (called a quantity) that can be given a magnitude either by counting (as in the case of the number of shrimp) or by measurement (e.g. light intensity, flow rate etc.).

**Control variables**

A control variable is one which may, in addition to the independent variable, affect the outcome of the investigation and therefore must be kept constant or at least monitored.

**Dependent variables**

The dependent variable is the variable of which the value is measured for each change in the independent variable.

**Independent variables**

The independent variable is the variable for which values are changed or selected by the investigator.

**IMPORTANT NOTE**

These definitions must be memorised by students.

You will be tested on your knowledge of these definitions.



## Equations

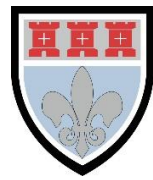
The equations below are used in this module.

Quantity/Concept	Equation(s)
<b>Current/Charge</b>	$I = \frac{\Delta Q}{\Delta t}$ $\Delta Q = I \Delta t$
<b>Potential Difference/EMF</b>	$V = PD = \varepsilon = \frac{\Delta W}{\Delta Q}$
<b>Resistance</b>	$R = \frac{V}{I}$
<b>Resistance in Series</b>	$R_T = R_1 + R_2 + R_n$
<b>Resistance in Parallel</b>	$1/R_T = 1/R_1 + 1/R_2 + 1/R_n$
<b>Electrical Power</b>	$P = VI = I^2R = \frac{V^2}{R}$
<b>Electrical Energy Transferred</b>	$E = Vit = \frac{V^2t}{R} = I^2Rt$
<b>Potential Divider (Equation not given in examination book)</b>	$V_{out} = \frac{R_{output}}{R_{output} + R_{wasted}} \times V_{source}$
<b>Conservation of Energy in a Circuit (Equation not given in examination book)</b>	$\varepsilon = \text{Terminal PD} + \text{Lost Volts}$
<b>EMF and Internal Resistance</b>	$\varepsilon = I(R+r)$
<b>EMF in cells in series (Equation not given in examination book)</b>	$\varepsilon_{total} = \varepsilon_1 + \varepsilon_2 + \varepsilon_n$
<b>EMF in cells in parallel (Equation not given in examination book)</b>	$\varepsilon_{total} = \varepsilon_1 = \varepsilon_2 = \varepsilon_n$

### IMPORTANT NOTE

These equations must be memorised by students.

You will be tested on these equations.



## VIDEO

### COURSE OVERVIEW

To watch a video looking at all of the concepts in electricity, please scan one of the following codes with your smartphone.



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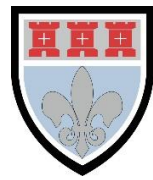
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## TOPIC: 3.5.1.5 Potential Divider

### SPEC CHECK

Specification	Completed?
The potential divider used to supply constant or variable potential difference from a power supply. The use of the potentiometer as a measuring instrument is not required.	
Examples should include the use of variable resistors, thermistors, and light dependent resistors (LDR) in the potential divider.	
Students can investigate the behaviour of a potential divider circuit.	
Students should design and construct potential divider circuits to achieve various outcomes.	

### Student Checklist

Have I.....	Yes or No?
Read through the notes of this section?	
Highlighted/underlined the key concepts of this section?	
Made my own notes based on the notes of this section?	
Brought the notes to be used in lesson?	



## Kirchhoff's Laws

### Prior Knowledge Link

This is a topic found in a previous GCSE module – **Electricity**.

Kirchhoff came up with two laws concerning conservation in electrical circuits.

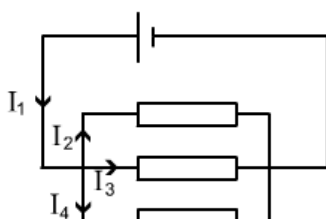
### First Law of the Obvious

Electric charge is conserved in all circuits, all the charge that arrives at a point must leave it.

**Total Current Going In = Total Current Going Out.**

In the diagram, we can say that:

$$I_1 = I_2 + I_3 + I_4$$



### Synoptic Link

This links to earlier parts of the **Electricity Module**.

### Physics Tip

This Kirchhoff's 1<sup>st</sup> law – charge cannot be created or destroyed in an electrical circuit.

### Second Law of the Obvious

Energy is conserved in all circuits, for any complete circuit **the sum of the emfs is equal to the sum of the potential differences.**

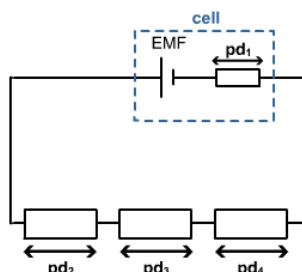
**Potential Risers = Potential Droppers**

In the diagram, we can say that:

$$\mathcal{E} = pd_1 + pd_2 + pd_3 + pd_4.$$

### Synoptic Link

This links to earlier parts of the **Electricity Module**.



### Physics Tip

This Kirchhoff's 2<sup>nd</sup> law – energy cannot be created or destroyed in an electrical circuit.

### Examination Tip

A common examination question is to discuss the limitations of a rheostat when used to collect the data for the characteristic.

Can attain neither maximum nor minimum voltage (1 mark)

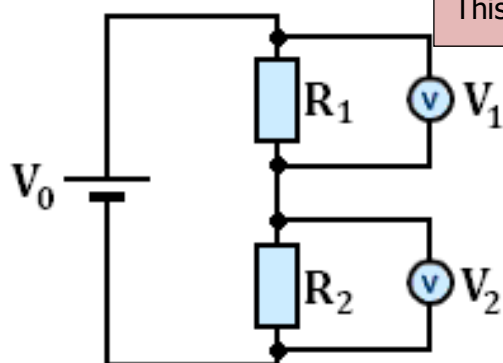
Explanation of either maximum OR minimum (1 mark)



## Potential Dividers

### Key Topic Warning

This topic is very common for questions on previous A-Level Papers.



A potential divider is used to produce a desired potential difference, it can be thought of as a potential selector.

A typical potential divider consists of two or more **resistors IN SERIES** that share the emf from the battery/cell.

### Physics Tip

This is a series circuit drawn in a different orientation.

### Physics Tip

Learn the definition of a potential divider.

The p.d.s across  $R_1$  and  $R_2$  can be calculated using the following equations:

### Physics Tip

This calculates the voltage across a component.

$$V_1 = V_0 \frac{R_1}{R_1 + R_2}$$

$$V_2 = V_0 \frac{R_2}{R_1 + R_2}$$

This shows us that the size of the potential difference is equal to the input potential multiplied by what proportion of  $R_1$  is of the total resistance.

If  $R_1$  is  $10 \Omega$  and  $R_2$  is  $90 \Omega$ ,  $R_1$  contributes a tenth of the total resistance so  $R_1$  has a tenth of the available potential. This can be represented using:

$$\frac{R_1}{R_2} = \frac{V_1}{V_2}$$

**The ratio of the resistances is equal to the ratio of the output voltages.**

### Physics Tip

Generally, the resistance across a component is directly proportional to the potential difference across it.

This can be re-arranged as the equation....

$$V_{\text{out}} = \frac{R_2}{R_1 + R_2} \times V_{\text{supply}}$$

### Physics Tip

$R_1$  is denoted as whichever resistor the current travels through first.

### Physics Tip

$V_{\text{out}}$  is equal to the voltage across  $R_2$ .

### Physics Tip

Make sure when you are using this equation that you plug in  $R_2$  as the resistor you are measuring the voltage over – otherwise you will get the wrong answer.

### Examination Tip

A common examination question is to define a potential divider.

A combination of resistors in series connected across a voltage source (to produce a required pd) (1 mark)



## Uses

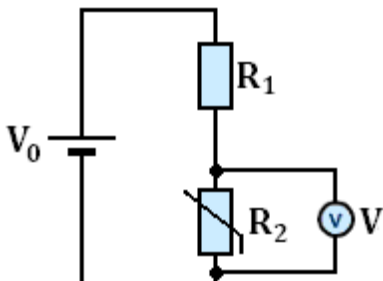
In this potential divider, the second resistor is a thermistor.

When the temperature is low the resistance ( $R_2$ ) is high, this makes the output voltage high. When the temperature is high the resistance ( $R_2$ ) is low, this makes the output voltage low.

A use of this would be a cooling fan that works harder when it is warm.

### Physics Tip

We refer to the lower component in the potential divider as the 'useful output'.



### Physics Tip

If you continually increase  $R_2$ ,  $R_1$  will eventually become negligible compared to  $R_2$ . When this happens,  $V_{out}$  will be negligible too.

Decreasing  $R_2$  until it is zero would mean that  $V_{out} = V_s$ .

### Physics Tip

This Kirchhoff's 2<sup>nd</sup> law – energy cannot be created or destroyed in an electrical circuit.

### Examination Tip

A common examination question is to state and explain what happens to the voltmeter reading when the resistance of  $R_1$  is increased while the temperature is kept constant.

When  $R$  increases, pd across  $R$  increases (1 mark)

Pd across  $R$  + pd across  $T$  = supply pd (1 mark)

### Examination Tip

A common examination question is to state and explain what happens to the ammeter reading when the temperature of the thermistor increases.

At higher temp, resistance of  $T$  is lower (1 mark)

So, circuit resistance is lower, so current/ammeter reading increases (1 mark)

So pd across  $T$ /voltmeter reading decreases (1 mark)

### Examination Tip

A common examination question is to state and explain the effect on the output pd of increasing the temperature of the thermistor in a potential divider.

Resistance of thermistor decreases (1 mark)

Output pd decreases since resistance of the parallel combination /circuit decreases (1 mark)

OR lower proportion of pd across the parallel combination (or higher proportion across the resistor) (1 mark)

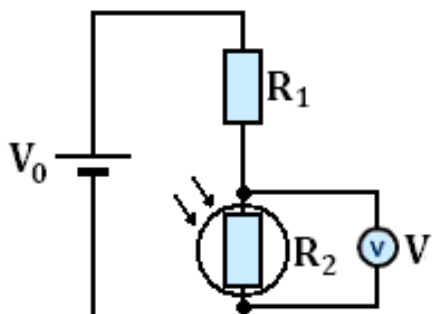
OR higher current so greater pd across the resistor. (1 mark)



In this second potential divider, the second resistor is a Light Dependent Resistor.

When the light levels are low the resistance ( $R_2$ ) is high, making the output voltage high. When the light levels increase the resistance ( $R_2$ ) decreases, this makes the output voltage decrease.

A use of this could be a street light sensor that lights up when the surrounding is dark.



### Examination Tip

Challenging questions may place components in parallel in a potential divider – this is called loading the divider.

Remember the rules of resistance in series and parallel in this case.

**A potential divider which contains a variable resistor is known as a potentiometer.**

### Physics Tip

A potentiometer can be used to measure the voltage of unknown sources.

### Physics Tip

This should explain why a potentiometer can be used to measure maximum and zero values.

### Examination Tip

A common examination question is to ask how a potential divider can be used to show how a reading on a voltmeter changes with temperature between  $0^{\circ}\text{C}$  and  $100^{\circ}\text{C}$ .

The answer should have a way to measure  $V$  and temperature.

It has a workable method of varying temperature from  $0^{\circ}\text{C}$  to  $100^{\circ}\text{C}$ . It explains why  $R$  is necessary and are able to use the thermistor to measure temperature using a graph and calibration curve.

This should include...

Measurement of  $V$  from the voltmeter

Use of a thermometer

Use of water bath

Use of ice

Importance of stirring

Explanation of the need for series resistor

Plotting of a calibration curve

Use of calibration curve to determine temperature of room



### Examination Tip

A common examination question is to ask how a potential divider of a motor and thermistor will change when a current passes through it compared to a potential divider of a motor and resistor.

Heating occurs / temperature increases when there is a current (in the thermistor) (due to  $I^2R$ ) (1 mark)

(When the temperature increases) the resistance of thermistor decreases (whereas fixed resistor remains high) (1 mark)

(Lower resistance from thermistor means) less wasted power (1 mark)

(Lower resistance from thermistor means) more pd dropped across the motor (less wasted voltage)

(1 mark)



## VIDEO

To watch a video looking at this concept, please scan one of the following codes with your smartphone.



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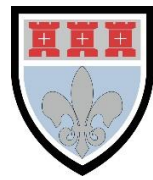
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# TOPIC: 3.5.1.6 Electromotive Force and Internal Resistance

## SPEC CHECK

Specification	Completed?
$\varepsilon = EQ$ $\varepsilon = I R + r$	
Terminal pd; emf	
Students will be expected to understand and perform calculations for circuits in which the internal resistance of the supply is not negligible.	

### Student Checklist

Have I.....	Yes or No?
Read through the notes of this section?	
Highlighted/underlined the key concepts of this section?	
Made my own notes based on the notes of this section?	
Brought the notes to be used in lesson?	



## Energy in Circuits

In circuits, there are two fundamental types of component: components which place work into the circuit – potential risers - and components which take work out of the circuit – potential droppers.

### Electromotive Force (emf), $\epsilon$

Energy givers provide an electromotive force, they force electrons around the circuit which transfer energy.

The size of the emf can be calculate using:

$$\epsilon = \frac{E}{Q}$$

#### Study Tip

Learn the base units for this equation and the context in which it can be used in.

**Examination Hint:** This equation is given in the data booklet, however memorising this gives you a better chance of exam success.

#### Synoptic Link

This links to earlier parts of the **Electricity Module**.

#### Study Tip

Learn what this equation represents.

The shows the electromotive force in an electrical circuit.

This is like the equation we use to find voltage/potential difference and means the energy given to each unit of charge. We can think of this as the energy given to each electron.

**The emf of a supply is the p.d. across its terminals when no current flows**

**EMF is measured in Joules per Coulomb,  $\text{JC}^{-1}$  or Volts, V**

Energy takers have a potential difference across them, transferring energy from the circuit to the component.

emf = energy giver

p.d. = energy taker

Energy is conserved in a circuit so energy in = energy out, or:

**TOTAL EMF INTO A CIRCUIT = TOTAL POTENTIAL DIFFERENCE OUT OF A CIRCUIT**

#### Physics Tip

The terminal voltage of a battery and the voltage across the terminals of the battery mean the same thing in electrical physics.

#### Examination Tip

It is a common examination question to define the electromotive force.

emf is the work done/ energy transferred by a voltage source/battery/cell (1 mark) per unit charge (1 mark)

#### Physics Tip

This Kirchhoff's 2<sup>nd</sup> law – energy cannot be created or destroyed in an electrical circuit.



## Internal Resistance, $r$

### Key Topic Warning

This topic is very common for questions on previous A-Level Papers.

The chemicals inside a cell offer a resistance to the flow of current, this is the internal resistance on the cell.

**Internal Resistance is measured in Ohms,  $\Omega$**

### Linking emf and $r$

If we look at the statement in the box above and apply it to the circuit below, we can reach an equation that links emf and  $r$ .

$$\begin{aligned} \text{Total emfs} &= \text{total potential differences} \\ \varepsilon &= (\text{p.d. across } r) + (\text{p.d. across } R) \\ \varepsilon &= (I \times r) + (I \times R) \\ \varepsilon &= Ir + IR \end{aligned}$$

$$\boxed{\varepsilon = I(r+R)}$$

### Synoptic Link

This links to earlier parts of the **Electricity Module**.

{Remember that  $V=IR$ }

### Physics Tip

$V= IR$  is the definition of resistance from GCSE Physics.

The terminal p.d. is the p.d. across the terminals of the cell when a current is flowing

$$\varepsilon = \text{internal p.d.} + \text{terminal p.d.}$$

So, the above equation can be written as

$$\boxed{\varepsilon = Ir + V}$$

where  $V$  is the terminal p.d.

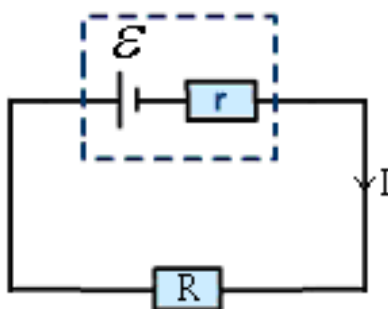
### Physics Tip

You will need to be able to re-arrange this equation to calculate different electrical quantities in a circuit.

### Study Tip

Learn what these equations represents.

The shows the internal resistance and terminal PD in an electrical circuit.



### Physics Tip

In circuit diagrams, internal resistance is shown as a tiny resistor inside the battery.

### Physics Tip

In general, you can assume the connecting wires in a circuit have no resistance – and it is no difference for internal resistance problems. But in practice, they do have a small resistance.

**Examination Tip**

A common examination question is to ask why the reading on a voltmeter does not change when the cell in the circuit is replaced with another cell of the same emf but a significant internal resistance.

correct reference to lost volts OR terminal pd OR reduced current (1 mark)

reference to resistors not changing OR resistors constant ratio (1 mark)

reference to voltmeter having high/infinite resistance (so not affecting circuit) (1 mark)

reference to pd being (very) small (due to closeness of resistance ratios in each arm) (1 mark)

voltmeter (may not be) sensitive enough (1 mark)

**Examination Tip**

A common examination question is to ask why the voltages recorded in experiments are always less than this and why the difference between the emf and the voltage becomes larger with increasing current.

reference to correct internal resistance (1 mark)

terminal pd = emf – pd across internal resistance/lost volts (1 mark)

pd/lost volts increase as current increases OR as (variable) resistance decreases greater proportion/share of emf across internal resistance (1 mark)

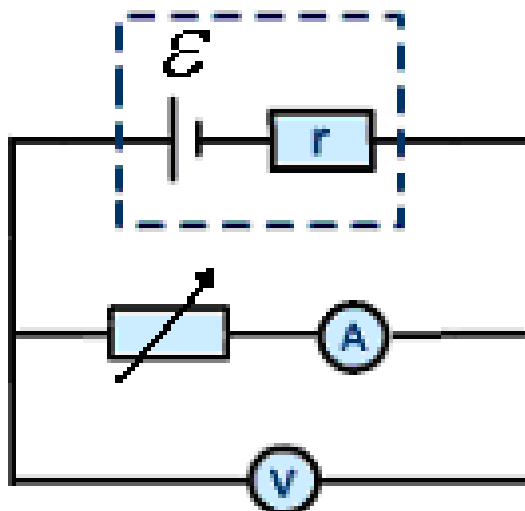


## Measuring emf and r

We can measure the emf and internal resistance of a cell by measuring the current and voltage as shown on the right, the variable resistor allows us to get a range of values. If we plot the results onto a graph of voltmeter reading against ammeter reading, we get a graph that looks like the one below.

### Working Scientifically Link

Remember when to use a potentiometer and when to use a rheostat.



### Working Scientifically Link

Remember how many values should be taken in this investigation and what range of results should be used.

### Working Scientifically Link

Remember how to remove systematic and random error from measurements.

Graphs have the general equation of  $y = mx + c$ , where  $y$  is the vertical (upwards) axis,  $x$  is the horizontal (across) axis,  $m$  is the gradient of the line and  $c$  is where the line intercepts (cuts) the  $y$  axis.

If we take  $\epsilon = Ir + V$  and arrange it into  $y = mx + c$

$$y \text{ axis} = V \text{ and } x \text{ axis} = I$$

$$\epsilon = Ir + V \rightarrow V = -Ir + \epsilon \rightarrow V = -rI + \epsilon$$

$$y = mx + c$$

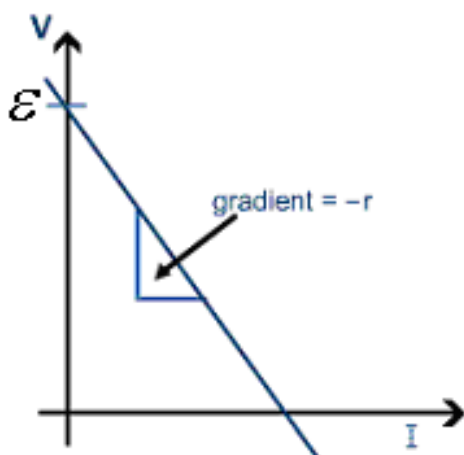
So we can see that the:

**y-intercept represents the emf**  
and

**gradient represents (-) internal resistance**

### Key Topic Warning

This topic is very common for questions on previous A-Level Papers.



### Physics Tip

This graph produced must be a straight line with a negative gradient.

### Working Scientifically Link

Remember how to determine the internal resistance and terminal PD from this graph.



**Examination Tip**

A common examination question is to ask why when a switch is closed in a circuit and there are no resistors connected – why a voltmeter reading equals the EMF of the power supply.

Power supply is on open circuit (so current is zero)

OR

Voltmeter has a (very) large resistance (so current is zero)

OR

No current (load) (so no lost volts)

OR

(Current is zero) so no lost volts

**Examination Tip**

A common examination question is to ask why when a switch is closed in a circuit and there are resistors connected – why a voltmeter reading is slightly less than the EMF of the power supply.

(Current through power supply leads to)

lost volts (across the internal resistance)

OR

(Current through power supply leads to)

voltage drop across the internal resistance

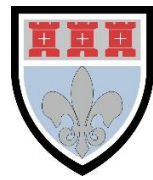
OR

(Current through power supply leads to)

Some of the emf is used in the internal resistance

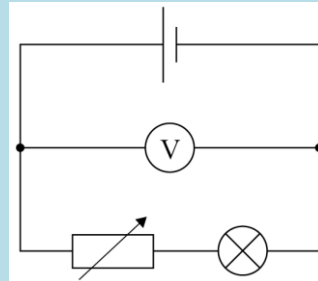
OR

Voltage is shared between the internal and external resistances



### Examination Tip

A common examination question is to ask why the reading on the voltmeter decreases as the brightness of the lamp increases in the following circuit.



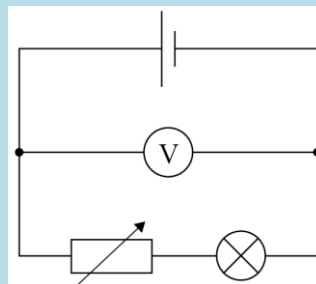
Mention of increase in lost volts/ pd across internal resistance (in cell) (1 mark) (because) current has increased (1 mark)

internal resistance is a larger proportion of total resistance (1 mark)

ratio of internal: external resistance is larger (1 mark)

### Examination Tip

A common examination question is to ask how the reading on a voltmeter would change if more cells were added in parallel to the circuit.



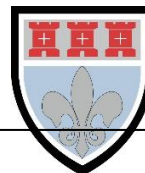
Lost volts reduced (current remains the same,  $V_2 > V_1$ )

Effective internal resistance is a smaller proportion of total resistance / ratio of internal: external resistance is smaller (1 mark) (because)

two cells in parallel behave as a single cell (with the same emf) but with half the internal resistance / reduced internal resistance (1 mark)

Current through each cell is less than cell on its own (1 mark)

Decreased current through cell decreases lost volts / pd dropped across internal resistance (1 mark)

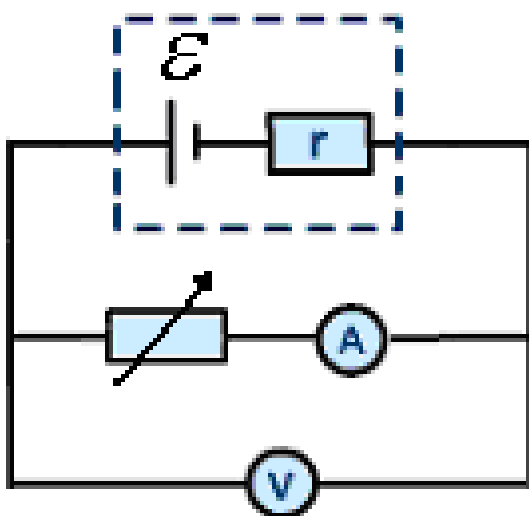


## Measuring Internal Resistance Required Practical

You can measure the internal resistance and EMF of a cell or battery using the following circuit.

### Study Tip

Learn the key skills associated with this practical.



### Physics Tip

Remember that ammeters are always connected in series and voltmeters are always connected in parallel.

### Study Tip

Learn the required practical process, how to gain accurate results and how the observations can be analysed.

1. Set the variable resistor (the load resistance) to its highest resistance.
2. Close the switch and record the current going through and potential difference across the circuit. Open the switch and close it again to get two more sets of current and potential difference readings for this load resistance.
3. Calculate the mean current and potential difference for this resistance from your results.
4. Decrease the resistance of the variable resistor by a small amount and repeat the previous step for this resistance.
5. Repeat the previous steps until you have a set of mean currents and potential differences for 10 different load resistances (over the widest possible range). This allows you to draw a line of best fit with confidence.
6. Plot a V-I graph for your mean data and draw a line of best fit. This should be a straight line with a negative gradient.
7. Make sure all of the other variables are kept constant when carrying out the experiment, including external factors (like temperature) and physical conditions – which would affect the resistivity of the material.

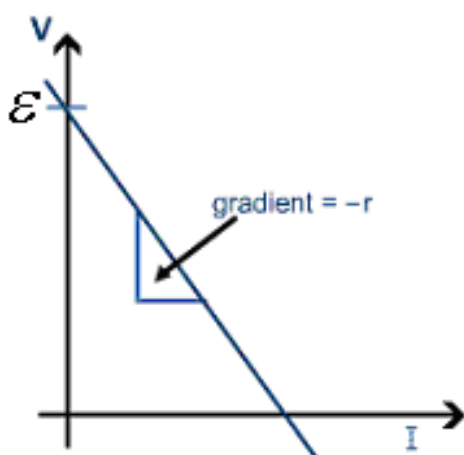
You can re-arrange the equation  $V = \epsilon - Ir$  to get  $V = -rI + \epsilon$ .

The equation of the straight line is  $y = mx + c$ .

Since  $\epsilon$  and  $r$  are constants,  $V = -rI + \epsilon$  is just the equation of the straight line.

You can just read  $\epsilon$  and  $r$  from the graph -  $\epsilon$  is the y-intercept and the gradient is  $-r$ .

You can find the gradient of the line by calculating change in the pd by the change in the current.



### Physics Tip

Remember not to include any anomalous results when calculating mean values.

### Physics Tip

The EMF of a power source could be also measured by connecting a high resistance voltmeter across its terminals. There is a small current that flows through the voltmeter so there must be some lost volts.



## VIDEO

To watch a video looking at this concept, please scan one of the following codes with your smartphone.



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## REQUIRED PRACTICAL



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## UPGRADE YOUR PHYSICS

The following section include information beyond the **A-Level Physics**.

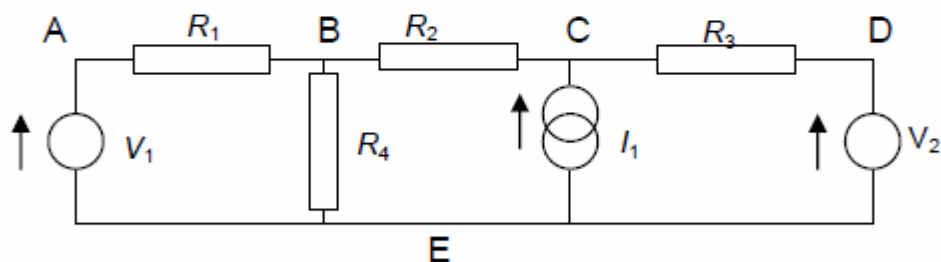
This information will further your understanding and provide a bridge to **University Level Physics**.

### 6.3 Circuits – putting it together

In this section, we look at combining resistors, capacitors and inductors in electrical circuits. There are two reasons for doing this. Firstly, once you have left school, you will be faced with complicated electronic networks, and you need to be able to analyse these just as well as the simple series and parallel arrangements you dealt with in the classroom. Secondly, engineers frequently use electric circuits as models or analogies for other systems (say, an oscillating bridge or the control of the nervous system over the muscles in a leg) – the better you understand electric circuits, the better you will understand any linked system.

#### 6.3.1 Circuit Analysis

Our aim here is to be able to solve a circuit like the one below. The circles represent constant-voltage sources (a bit like cells or batteries) and the linked circles represent constant-current sources. Our aim is to find voltage difference across each component, and also to work out the current in each resistor.



In order to solve the circuit, we use two rules – the Kirchoff Laws. Kirchoff's 1<sup>st</sup> says that the total current going into a junction is equal to the total current leaving it. Therefore, at B in the circuit below, we would say that  $I_{BE} = I_{AB} + I_{CB}$ , where  $I_{BE}$  means the current flowing from B to E (through  $R_4$ ).

Kirchoff's 2<sup>nd</sup> Law is that voltages always add up correctly. In other words, no matter which route we took from E to B, say, we would agree on the voltage difference between E and B. In symbols, if  $V_{BE}$  means



the difference in potential (as measured by a voltmeter) between B and E, then we have  $V_{BE} = V_{AE} + V_{BA}$ . This is basically the same thing as the law of conservation of energy. The voltage (or, more strictly, the *potential*) at B,  $V_B$ , is the energy content of one coulomb of charge at B. In travelling to E, it will lose  $V_B - V_E$  joules, irrespective of the route taken.<sup>23</sup> In fact, we assume the truth of Kirchoff's 2<sup>nd</sup> Law whenever we say, "let's call the voltage at A ' $V_A$ ,'" for we are assuming that the voltage of A does not depend on the route used to measure it.

Using these two rules, and the equation for the current through a resistor (for example,  $V_{BA} = I_{BA} R_1$ ), we may write down a set of equations for the circuit. Notice that because currents are said to go from + to -, this means that if  $V_{AB}$  (the voltage of A, measured relative to B) is positive, then  $V_A$  is bigger than  $V_B$ , and hence  $I_{AB}$  will be positive too. To make the notation easier we will take the potential at E to be zero. In symbolic form, this means that we shall call  $V_{BE}$  (that is,  $V_B - V_E$ )  $V_B$  for short.

Kirchoff's First Law:

$$I_{EA} = I_{AB}; \quad I_{BE} = I_{AB} + I_{CB}; \quad I_1 = I_{CB} + I_{CD}; \quad I_{CD} = I_{DE}$$

Kirchoff's Second Law:

$$\begin{aligned} V_B &= I_{BE} R_4 \\ &= V_A + V_{BA} = V_1 - I_{AB} R_1 \\ &= V_D + V_{CD} + V_{BC} = V_2 + I_{CD} R_3 - I_{CB} R_2 \end{aligned}$$

After elimination, the equations reduce to two:

$$V_1 - I_{AB} R_1 = V_2 + I_1 R_3 - (R_3 + R_2) I_{CB} = (I_{AB} + I_{CB}) R_4,$$

and from these the currents  $I_{AB}$  and  $I_{CB}$  can be found (after a bit of messy algebra). After this, the remaining currents and voltages are straightforward to determine.

These principles can be used to solve any circuit. However, as networks get bigger, it is useful to find more prescriptive methods of solution, which could be used by a computer. We shall cover two methods here – for certain problems, they may be more efficient than the direct application of Kirchoff's Laws.

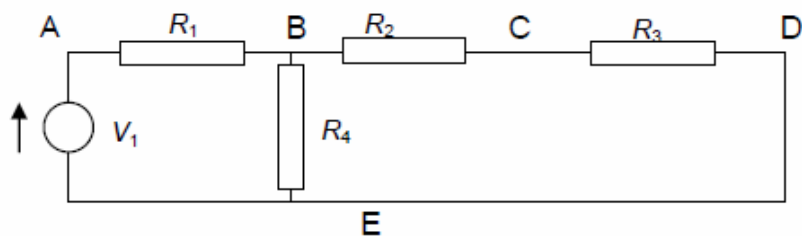


### 6.3.1.1 Method of Superposition

The method of superposition relies on the fact that for a simple resistor, the current is proportional to the voltage. It follows that if current  $I_1$  causes a voltage difference of 3V, and current  $I_2$  causes a voltage difference of 5V, then current  $I_1+I_2$  will cause an 8V p.d. across the component. Here is the procedure:

- Choose one of the supply components.
- Remove the other supply components from the circuit. Replace voltage sources with direct connections (short circuits), and leave breaks in the circuit where the current sources were (open circuits).
- Calculate the current in each wire, and the voltage across each component.
- Repeat the procedure for each supply component in turn.
- The current in each wire for the original (whole) circuit is equal to the sum of the currents in that wire due to each supply unit.
- The voltage across each component in the original (whole) circuit is equal to the sum of the voltages across that component due to each supply unit.

Let's use this method to analyse the circuit above. We start by considering only source  $V_1$ . Removing the other supply components gives us a circuit like this.



This circuit is easier to analyse as it only has one supply. Supply  $V_1$  feeds a circuit with resistance

$$R_1 + \{R_4 // (R_2 + R_3)\}$$

$$= R_1 + \frac{R_4 (R_2 + R_3)}{R_4 + R_2 + R_3}$$

where // means 'in parallel with.' Accordingly, the current supplied by  $V_1$  (and the current through  $R_1$  which is in series with it) is equal to  $V_1$  divided by this resistance. The voltages of points B, C and D can be calculated, as can the current in each wire. We make a note of the values, and add to them the results of analyses of circuits only containing  $I_1$  and only containing  $V_2$ .

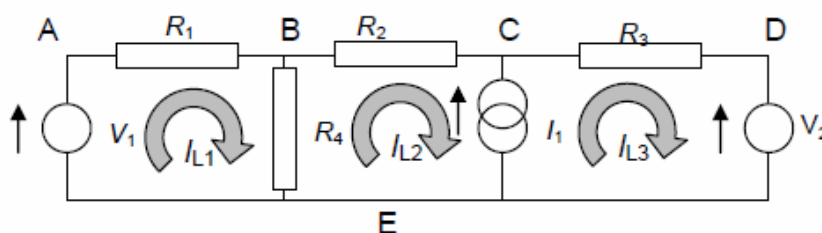


You may find this method good in the sense that you only have to deal with one supply component at a time – and therefore all you need to know is how to combine resistors (something you've done before). Having said that, we end up analysing three circuits rather than one, so it is more time consuming.

Before leaving the method, you may be curious why voltage sources were replaced with short circuits, and current sources with open circuits. Here's the reason. A voltage source does not change the voltage across its terminals, no matter what the current is ( $d \text{ Voltage} / d \text{ Current} = R_{\text{equivalent}} = 0$ ). The only type of resistor which behaves likewise is a perfect conductor ( $0\Omega$ ). Similarly, a current source does not change its current, no matter what the voltage ( $d \text{ Current} / d \text{ Voltage} = 1 / R_{\text{equivalent}} = 0$ ). The equivalent resistor in this case is a perfect insulator ( $\infty\Omega$ ) which lets no current through ever.

### 6.3.1.2 Method of Loop Currents

Here we break the circuit down into the smallest loops it contains. Here there are three loops:



- E to A to B and back to E (loop 1),
- E to B to C to E (loop 2), and
- E to C to D to E (loop 3).

We call the current in loop 1 “loop current” number one ( $I_{L1}$ ), with  $I_{L2}$  and  $I_{L3}$  representing the currents in the other two loops. We then express all other currents in terms of the loop currents. Clearly,  $I_{AB} = I_{L1}$ , since  $R_1$  is in the first loop alone. Similarly,  $I_{BC} = I_{L2}$ , and  $I_{CD} = I_{L3}$ .

The current through  $R_4$  is more complex, since this resistor is part of two of the loops. We write  $I_{BE} = I_{L1} - I_{L2}$ . Here  $I_{L1}$  is positive, since  $I_{BE}$  is in the same direction as  $I_{L1}$ , whereas  $I_{L2}$  (which goes from E to B then on to C) is in the opposite direction. These designations automatically take care of Kirchoff's First Law. Notice that by this method,  $I_1 = I_{L3} - I_{L2}$ .

Each loop now contributes one equation – Kirchoff's 2<sup>nd</sup> law around that loop. Clearly, if you go all the way round the loop, you must return to the voltage you started with. Taking the first loop as an example, we have:

$$0 = V_{AE} + V_{BA} + V_{EB}$$



$$\begin{aligned}
 &= V_1 - I_{AB} R_1 - I_{BE} R_4 \\
 &= V_1 - I_{L1} R_1 + (I_{L1} - I_{L2}) R_4.
 \end{aligned}$$

In a similar way, we write equations for each of the other two loops<sup>24</sup>. We then have three equations in three unknowns (the three loop currents), which can be solved. The end result is the same as for a direct 'sledgehammer' approach with Kirchoff's Laws – but the method is more organized.

### 6.3.2 Alternating Current

Having looked at circuits with resistors in them, we next turn our attention to circuits with inductors and capacitors as well. For a direct current, the situation is easy. After a brief period of settling down, there is no voltage drop across an inductor (because the current isn't changing), and a capacitor doesn't conduct at all.

For alternating currents the situation is more complicated. Let us suppose that the supply voltage is given by  $V = V_0 \cos \omega t$ . It turns out that the circuit will settle down to a steady behaviour (called the steady state). Once this has happened, the voltage across each component (and the current through each component) will also be a cosine wave with frequency  $\omega$ , however it may not be in phase with the original  $V$ .

#### 6.3.2.1 Resistor, capacitor and inductor

We start with the three simplest circuits – the lone resistor, the lone capacitor and the lone inductor, each supplied with a voltage  $V = V_0 \cos \omega t$ .

For the resistor,  $I = V/R$ , so the result is straightforward.

For the capacitor,  $Q = VC$ , and if we take  $I$  as positive in the direction which charges the capacitor, then

$$\begin{aligned}
 I &= \frac{dQ}{dt} = \frac{d}{dt} CV_0 \cos \omega t \\
 &= -\omega CV_0 \sin \omega t \\
 &= \omega CV_0 \cos\left(\omega t + \frac{1}{2}\pi\right) \\
 &= -\omega CV_0 \cos\left(\omega t - \frac{1}{2}\pi\right)
 \end{aligned} \tag{31}$$

For the inductor,  $V = L dI/dt$ , so



$$\begin{aligned}\frac{dI}{dt} &= \frac{V_0}{L} \cos \omega t \\ I &= \frac{V_0}{L\omega} \sin \omega t \\ &= \frac{V_0}{L\omega} \cos\left(\omega t - \frac{1}{2}\pi\right)\end{aligned}\quad (32)$$

where we have taken the constant of integration to be zero. Failure to do so would lead to a non-zero mean current, which is clearly impossible as the mean supply voltage is zero.

### 6.3.2.2 Reactance and Impedance

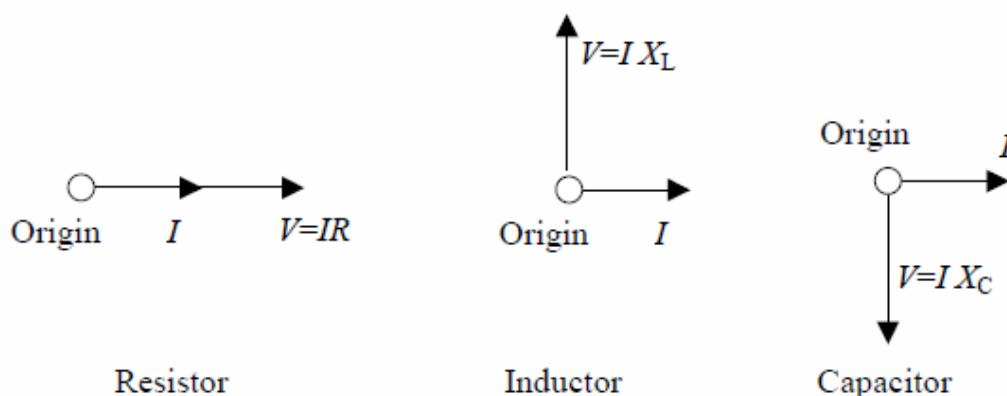
For resistors, the current and voltage are proportional, and consequently are in phase – one peaks at the same time as the other. For the other two components, this is not the case. The voltage is  $\pi/2$  radians (or  $90^\circ$ ) out of phase with respect to the current. Inductor currents peak  $90^\circ$  later than the voltage (the current *lags* the voltage), whereas capacitor currents peak  $90^\circ$  before the voltage (the current *leads* the voltage). Nevertheless, the amplitude of the voltage is still proportional to the amplitude of the current, and we call the ratio of the amplitudes the reactance ( $X$ ).

$$\begin{aligned}X_L &= \omega L \\ X_C &= -\frac{1}{\omega C}\end{aligned}\quad (33)$$

By convention, we take reactance to be positive if the current lags the voltage by  $90^\circ$ , and negative if it leads by  $90^\circ$ . For capacitors and inductors in series, the total reactance is equal to the sum of the individual components' reactances – just as resistances add in series. Similarly, the formula for combining reactances in parallel is the same for that used for the resistance of resistors wired in parallel.

When a circuit is constructed with resistors, capacitors and inductors, then we need a way of analysing a circuit with both resistances and reactances. We visualise the situation using a 2D (phasor) diagram.

For any component or circuit, both voltage and current are represented by vectors. The length of the lines gives the amplitude, and the angle between the vectors gives the phase difference. By convention, we imagine the vectors to rotate about the origin in an anticlockwise direction (once per time period of the alternating current). The vectors for a resistor, capacitor and inductor are shown below.



As the arrows rotate anticlockwise, for the inductor,  $V$  comes before  $I$ . With the capacitor,  $I$  comes before  $V$ . This accurately represents the phase relationships between voltage and current for these components.

For a set of components in series, the current  $I$  will be the same for all of them. We usually draw the current pointing to the right. Voltages across inductors will then point up, those across resistors point right, and those across capacitors point down. By adding these voltages vectorially, we arrive at the voltage across the set of components – and can calculate its amplitude and phase relationship with respect to the current.

Similarly, for components in parallel, the voltage will be the same for each. We thus put voltage pointing to the right. Currents in capacitors now point up, currents in resistors point right, and currents in inductors point down. The total current is given by the vector sum of the individual currents.

In all cases, we call the ratio of the voltage amplitude to the current amplitude the impedance ( $Z$ ) irrespective of the phase difference between the current and voltage.<sup>25</sup> In general the impedance of a component is related to resistance and reactance by  $Z^2 = R^2 + X^2$ .

### 6.3.2.3 Complex Numbers and Impedance

If you are familiar with complex numbers, there is an easier way of describing all of this, using the Argand diagram in place of 2-dimensional vectors. The impedance  $Z$  is now a complex number  $Z = R + iX$ , with  $R$  as its real part and  $X$  as its imaginary part.

The complex impedances of a resistor, capacitor and inductor are accordingly written as  $R$ ,  $-i/\omega C$  and  $i\omega L$  respectively. The impedance of a set of components in series is given by the sum of the individual impedances. For a parallel network,  $Z^{-1}$  of the network is given by the



sum of  $Z^{-1}$  for each component, where 'inverse' (or 'reciprocal') is calculated in the usual way for complex numbers.

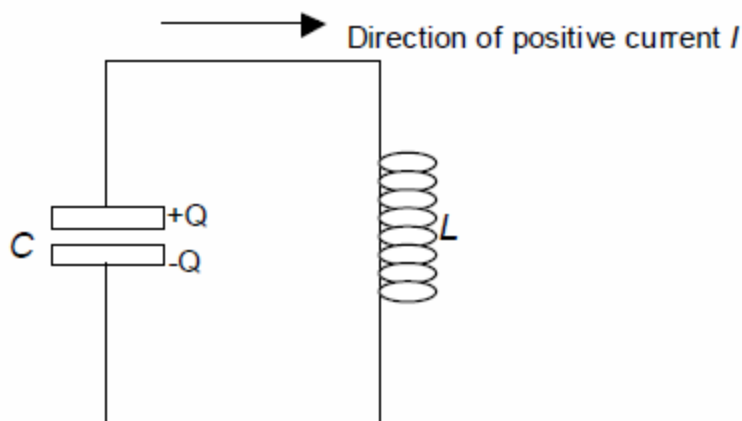
#### 6.3.2.4 Root Mean Square values

You will also need to remember the definition of RMS voltage and current in an a.c. circuit. For a resistor, remember that the RMS supply voltage is the d.c. voltage which would supply the same mean power to the device.

$$\begin{aligned}\bar{P} &\equiv \frac{V_{rms}^2}{R} = \frac{\overline{V_0^2 \cos^2 \omega t}}{R} = \frac{V_0^2}{2R} \\ \Rightarrow V_{rms} &= \frac{1}{\sqrt{2}} V_0\end{aligned}\quad (34)$$

### 6.3.3 Resonance

One further circuit needs a mention, and that is the simple circuit of an inductor and a capacitor connected together, as shown in the diagram below. Both the voltage and current for the two components must be the same, and so with the sign conventions chosen in the diagram:



$$\begin{aligned}\frac{Q}{C} &= L \frac{dI}{dt} = -L \frac{d^2 Q}{dt^2} \\ \frac{d^2 Q}{dt^2} &= -\frac{1}{LC} Q\end{aligned}\quad (35)$$

This is an equation of 'simple harmonic motion' with angular frequency  $\omega$ , where  $\omega^2 = 1/LC$ . This circuit can therefore oscillate at this frequency, and this makes it useful in radio receivers for selecting the frequency (and hence radio station) which the listener wants to detect.



## 8 Practical Physics

### 8.1 *Errors, and how to make them*<sup>29</sup>

Every dog has its day, every silver lining has its cloud, and every measurement has its error.

If you doubt this, take (sorry – borrow with permission) a school metre stick, and try and measure the length of a corridor in your school. Try and measure it to the nearest centimetre. Then measure it again. Unless you cheated by choosing a short corridor, you should find that the measurements are different. What's gone wrong?

Nothing has gone wrong. No measurement is exact, and if you take a series of readings, you will find that they cluster around the 'true value'. This spread of readings is called random error – and will be determined by the instrument you use and the observation technique. To be more precise and polite, this kind of 'error' is usually called uncertainty, as this word doesn't imply any mistake or incompetence on the part of the scientist.

So, whenever you write down a measurement, you should also write down its uncertainty. This can be expressed in two ways – absolute and relative.

The absolute uncertainty gives the size of the spread of readings. You might conclude that your corridor was  $(12.3 \pm 0.2)\text{m}$  long. In other words, your measurements are usually within 20cm of 12.3m. In this case the absolute uncertainty is 20cm.

The absolute uncertainty only gives part of the story. A 10cm error in the length of a curtain track implies sloppy work. A 10cm error in the total length of the M1 motorway is an impressive measurement. To make this clearer, we often state errors (or uncertainties) in percentage form – and this is called relative uncertainty. The relative uncertainty in the length of the corridor is



$$\text{Relative Error} = \frac{\text{Abs. Uncertainty}}{\text{Measurement}} = \frac{0.2 \text{ m}}{12.3 \text{ m}} = 1.6\% \approx 2\%. \quad (1)$$

Notice the rounding off at the end. It is usually pointless to give uncertainties to more than one significant figure.

Every measurement has its uncertainty, and the only way of determining this is to take more than one measurement, and work out the standard deviation – to measure the spread. In practice the spread can be 'eyeballed' rather than calculated. If the measurements were 54.5cm, 54.7cm and 54.3cm, then there is no need to use a calculator and the technical definition of deviation. The observation that the spread is about  $\pm 0.2\text{cm}$  is perfectly good enough.

Notice that the more readings you take, the better idea you get of the spread of the measurements – and hence the better estimate you can make for the middle, which is indicative of 'true' value. Therefore we find, from statistics, that if you take  $n$  measurements, and the absolute uncertainty is  $x$ , then the uncertainty of the mean of those measurements is approximately:<sup>30</sup>

$$\text{Uncertainty of mean} = \frac{x}{\sqrt{n}}. \quad (2)$$

Therefore, the more measurements you take, the more accurate the work. Notice that if you wish to halve the uncertainty, you need to take *four* times as many readings. This is subject to one proviso:

Measurements also have a resolution. This is the smallest distinguishable difference that the measuring device (including the technique) can detect. For a simple length measurement with a metre ruler, the resolution is probably 1mm. However if, by years of practice with a magnifying lens, you could divide millimetres into tenths by eye, you would have a resolution of 0.1mm using the same metre stick. That is why we say that the resolution depends on the technique as well as on the apparatus.

The uncertainty of a measurement can never be less than the resolution. This is the proviso we mentioned below equation (2). Why should this be the case? Let us have a parable.

Many years ago, the great nation of China had an emperor. The masses of the population were not permitted to see him. One day, a citizen had



the sudden desire to know the length of the emperor's nose. He could not do this directly, since he was not permitted to visit the emperor. So, using the apparatus of the imperial administration, he asked all the regional mandarins to ask the entire population to make a guess. Each person would make some guess at the imperial nasal length – and the error of each guess would probably be no more than  $\pm 2\text{cm}$  – since nose lengths tend not to vary by more than about 4cm.

However, the mean would be a different matter. Averaged over the 1000 million measurements, the error in the mean would be  $0.7\mu\text{m}$ . So the emperor's nose had been measured incredibly accurately – without a single observation having been made!

The moral of the story: uncertainties *are* reduced by repeated measurement, but the error can never be reduced below the resolution of the technique – here 2cm – since ignorance can not be circumvented by pooling it with more ignorance.

## 8.2 *Errors, and how to make them worse*

Errors are one thing. The trouble is that usually we want to put our measurements into a formula to calculate something else. For example, we might want to measure the strength of a magnetic field by measuring the force on a current-carrying wire  $B = F/IL$ .

If there is a 7% uncertainty in the current, 2% in the force and 1% in the length – what is the uncertainty in the magnetic field?

There are two rules you need:

### 8.2.1 Rule 1 – Adding or subtracting measurements

If two measurements are added or subtracted, the **absolute** uncertainty in the result equals the sum (never the difference) of the absolute uncertainties of the individual measurements.

Therefore if a car is  $(3.2 \pm 0.1)\text{m}$  long, and a caravan is  $(5.2 \pm 0.2)\text{m}$  long, the total length is  $(8.4 \pm 0.3)\text{m}$  long. Similarly if the height of a two-storey house is  $(8.3 \pm 0.2)\text{m}$  and the height of the ground floor is  $(3.1 \pm 0.1)\text{m}$ , the height of the upper floor is  $(5.2 \pm 0.3)\text{m}$ .

Even in the second case, we do not subtract the uncertainties, since there is nothing stopping one measurements being high, while the other is low.<sup>31</sup>



### 8.2.2 Rule 2 – Multiplying or dividing measurements

If two measurements are multiplied or divided, the relative uncertainty in the result equals the sum (never the difference) of the relative uncertainties of the individual measurements.

Therefore if the speed of a car is  $30\text{mph} \pm 10\%$ , and the time for a journey is  $6\text{ hours} \pm 2\%$ , the uncertainty in the distance travelled is  $12\%$ .

Notice that one consequence of this is that if a measurement, with relative uncertainty  $p\%$  is squared (multiplied by itself), the relative error in the square is  $2p\%$  - i.e. doubled. Similarly if the error in measurement  $L$  is  $p\%$ , the error in  $L^n$  is  $p \times n\%$ . Notice that while a square root will halve the relative error, an inverse square ( $n=-2$ ) doubles it. All the minus sign does is to turn overestimates into underestimates. It does not reduce the magnitude of the relative error.<sup>32</sup>

Now we can answer our question about the magnetic field measurement at the beginning of the section. All three relative errors (in length, force and current) must be added to give the relative error in the magnetic field, which is therefore  $10\%$ .

## 8.3 Systematic Errors

All the 'errors' mentioned so far are called 'random', since we assume that the measurements will be clustered around the true value. However often an oversight in our technique will cause a measurement to be overestimated more often than underestimated or vice-versa. This kind of error is called 'systematic error', and can't be reduced by averaging readings. The only way of spotting this kind of error (which is a true error in that there is something wrong with the measurement) is to repeat the measurement using a completely different technique, and compare the results. Just thinking hard about the method can help you spot some



systematic errors, but it is still a good idea to perform the experiment a different way if time allows.

## 8.4 Which Graph?

You will often have to use graphs to check the functional form of relationships. You may also have to make measurements using the graph. In order to do either of these, you usually need to manipulate the data until you can plot a straight line. A straight line is conclusive proof that you have got the form of the formula right!

The gradient and y-intercept can then be read, and these enable other measurements to be made. For example, your aim may be to measure the acceleration due to gravity. You may plot velocity of falling against time, in which case you will need to find the gradient of the line.

At its most general, you will have a suspected functional form  $y=f(x)$ , and you will need to work out what is going on in the function  $f$ . Notice that our experiment will give us pairs of  $(x,y)$  values – what is not known are the parameters in the function  $f$ . We find them by manipulating the equation:

$$y = f(x)$$

$$\vdots$$

$$g(x, y) = Ah(x, y) + B$$

We can then plot  $g(x,y)$  against  $h(x,y)$ , and obtain the parameters  $A$  and  $B$  from the gradient and intercept of the line. Furthermore, the presence of the straight line on the graph assures us that our function  $f$  was a good guess. We shall now look at the most common examples.

### 8.4.1 Exponential growth or decay

Here we have the functional form  $y = Ae^{Bx}$ , where  $A$  and  $B$  need to be determined. We manipulate the equation:

$$y = Ae^{Bx}$$

$$\ln y = \ln A + Bx$$

So we plot  $(\ln y)$  on the vertical axis, and  $(x)$  on the horizontal. The y-intercept gives  $\ln A$ , and the gradient gives  $B$ .

### 8.4.2 Logarithmic growth or decay

Here we have the functional form  $y = A + B \ln x$ , and again, we need to work out the values of  $A$  and  $B$ . This equation is already in linear form – we plot  $y$  on the vertical, and  $(\ln x)$  on the horizontal. The y-intercept gives  $A$ , and the gradient gives  $B$ .



### 8.4.3 Power laws

This covers all equations with unknown powers:  $y = Ax^B$ . The manipulation involves logarithms:

$$y = Ax^B$$

$$\ln y = \ln A + \ln x^B$$

$$\ln y = \ln A + B \ln x$$

Here we plot  $(\ln y)$  against  $(\ln x)$ , and find the power ( $B$ ) as the gradient of the line. The  $A$  value can be inferred from the y-intercept, which is equal to  $\ln A$ .

### 8.4.4 Other forms

Even hideous looking equations can be reduced to straight lines if you crack the whip hard enough. How about  $y = A\sqrt{x} + Bx^3$ ? Is it tasty enough for your breakfast? Actually it's fine if digested slowly:

$$y = A\sqrt{x} + Bx^3$$

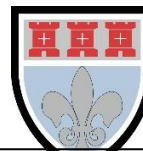
$$\frac{y}{\sqrt{x}} = A + Bx^{5/2}$$

This looks even worse, doesn't it? But remember that it is  $x$  and  $y$  that are *known*. If we plot  $(y/\sqrt{x})$  on the vertical, and  $(x^{5/2})$  on the horizontal, a straight line appears, and we can read  $A$  and  $B$  from the y-intercept and gradient respectively.



## REVISION CHECKLIST

Specification reference	Checklist questions	
3.5.1.4	Can you carry out calculations for resistors in series and in parallel?	<input type="checkbox"/>
3.5.1.4	Can you explain and use the energy and power equations: $E = Ivt$ and $P = IV = I^2R = \frac{V^2}{R}$ ?	<input type="checkbox"/>
3.5.1.4	Can you explain the relationships between currents, voltages and resistances in series and parallel circuits, including cells in series and identical cells in parallel?	<input type="checkbox"/>
3.5.1.4	Can you explain conservation of charge and conservation of energy in dc circuits?	<input type="checkbox"/>
3.5.1.5	Can you describe how the potential divider is used to supply constant or variable potential difference from a power supply?	<input type="checkbox"/>
3.5.1.5	Can you explain the use of variable resistors, thermistors, and light dependent resistors in the potential divider?	<input type="checkbox"/>
3.5.1.6	Can you use the formulae $\varepsilon = \frac{E}{Q}$ and $\varepsilon = I(R + r)$ ?	<input type="checkbox"/>
3.5.1.6	Can you explain terminal pd and emf?	<input type="checkbox"/>
3.5.1.6	Can you understand and perform calculations for circuits in which the internal resistance of the supply is not negligible?	<input type="checkbox"/>
3.5.1.6	Have you carried out an investigation into the emf and internal resistance of electric cells and batteries by measuring the variation of the terminal pd across the cell with the current in it?	<input type="checkbox"/>



## DATASHEET

## DATA - FUNDAMENTAL CONSTANTS AND VALUES

Quantity	Symbol	Value	Units
speed of light in vacuo	$c$	$3.00 \times 10^8$	$\text{m s}^{-1}$
permeability of free space	$\mu_0$	$4\pi \times 10^{-7}$	$\text{H m}^{-1}$
permittivity of free space	$\epsilon_0$	$8.85 \times 10^{-12}$	$\text{F m}^{-1}$
magnitude of the charge of electron	$e$	$1.60 \times 10^{-19}$	C
the Planck constant	$h$	$6.63 \times 10^{-34}$	J s
gravitational constant	$G$	$6.67 \times 10^{-11}$	$\text{N m}^2 \text{kg}^{-2}$
the Avogadro constant	$N_A$	$6.02 \times 10^{23}$	$\text{mol}^{-1}$
molar gas constant	$R$	8.31	$\text{J K}^{-1} \text{mol}^{-1}$
the Boltzmann constant	$k$	$1.38 \times 10^{-23}$	$\text{J K}^{-1}$
the Stefan constant	$\sigma$	$5.67 \times 10^{-8}$	$\text{W m}^{-2} \text{K}^{-4}$
the Wien constant	$\alpha$	$2.90 \times 10^{-3}$	m K
electron rest mass (equivalent to $5.5 \times 10^{-4}$ u)	$m_e$	$9.11 \times 10^{-31}$	kg
electron charge/mass ratio	$\frac{e}{m_e}$	$1.76 \times 10^{11}$	$\text{C kg}^{-1}$
proton rest mass (equivalent to 1.00728 u)	$m_p$	$1.67(3) \times 10^{-27}$	kg
proton charge/mass ratio	$\frac{e}{m_p}$	$9.58 \times 10^7$	$\text{C kg}^{-1}$
neutron rest mass (equivalent to 1.00867 u)	$m_n$	$1.67(5) \times 10^{-27}$	kg
gravitational field strength	$g$	9.81	$\text{N kg}^{-1}$
acceleration due to gravity	$g$	9.81	$\text{m s}^{-2}$
atomic mass unit (1u is equivalent to 931.5 MeV)	u	$1.661 \times 10^{-27}$	kg

## ALGEBRAIC EQUATION

quadratic equation  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

## ASTRONOMICAL DATA

Body	Mass/kg	Mean radius/m
Sun	$1.99 \times 10^{30}$	$6.96 \times 10^8$
Earth	$5.97 \times 10^{24}$	$6.37 \times 10^6$

## GEOMETRICAL EQUATIONS

arc length =  $r\theta$

circumference of circle =  $2\pi r$

area of circle =  $\pi r^2$

curved surface area of cylinder =  $2\pi r h$

area of sphere =  $4\pi r^2$

volume of sphere =  $\frac{4}{3}\pi r^3$



### Particle Physics

Class	Name	Symbol	Rest energy/MeV
photon	photon	$\gamma$	0
lepton	neutrino	$\nu_e$	0
		$\nu_\mu$	0
	electron	$e^\pm$	0.510999
	muon	$\mu^\pm$	105.659
mesons	$\pi$ meson	$\pi^\pm$	139.576
		$\pi^0$	134.972
	K meson	$K^\pm$	493.821
		$K^0$	497.762
baryons	proton	p	938.257
	neutron	n	939.551

### Properties of quarks

antiquarks have opposite signs

Type	Charge	Baryon number	Strangeness
<b>u</b>	$+\frac{2}{3}e$	$+\frac{1}{3}$	0
<b>d</b>	$-\frac{1}{3}e$	$+\frac{1}{3}$	0
<b>s</b>	$-\frac{1}{3}e$	$+\frac{1}{3}$	-1

### Properties of Leptons

	Lepton number
Particles: $e^-, \nu_e; \mu^-, \nu_\mu$	+1
Antiparticles: $e^+, \bar{\nu}_e, \mu^+, \bar{\nu}_\mu$	-1

### Photons and energy levels

photon energy	$E = hf = hc / \lambda$
photoelectricity	$hf = \phi + E_{k(\max)}$
energy levels	$hf = E_1 - E_2$
de Broglie wavelength	$\lambda = \frac{h}{p} = \frac{h}{mv}$

### Waves

wave speed  $c = f\lambda$  period  $f = \frac{1}{T}$

first harmonic  $f = \frac{1}{2l} \sqrt{\frac{T}{\mu}}$

fringe spacing  $w = \frac{\lambda D}{s}$  diffraction grating  $d \sin \theta = n\lambda$

refractive index of a substance s,  $n = \frac{c}{c_s}$

for two different substances of refractive indices  $n_1$  and  $n_2$ ,

law of refraction  $n_1 \sin \theta_1 = n_2 \sin \theta_2$

critical angle  $\sin \theta_c = \frac{n_2}{n_1}$  for  $n_1 > n_2$

### Mechanics

moments moment =  $Fd$

velocity and acceleration  $v = \frac{\Delta s}{\Delta t}$   $a = \frac{\Delta v}{\Delta t}$

equations of motion  $v = u + at$   $s = \left(\frac{u+v}{2}\right) t$

$v^2 = u^2 + 2as$   $s = ut + \frac{at^2}{2}$

force  $F = ma$

force  $F = \frac{\Delta(mv)}{\Delta t}$

impulse  $F \Delta t = \Delta(mv)$

work, energy and power  $W = F s \cos \theta$

$E_k = \frac{1}{2} m v^2$   $\Delta E_p = mg\Delta h$

$P = \frac{\Delta W}{\Delta t}$ ,  $P = Fv$

efficiency =  $\frac{\text{useful output power}}{\text{input power}}$

### Materials

density  $\rho = \frac{m}{v}$  Hooke's law  $F = k \Delta L$

Young modulus =  $\frac{\text{tensile stress}}{\text{tensile strain}}$  tensile stress =  $\frac{F}{A}$

tensile strain =  $\frac{\Delta L}{L}$

energy stored  $E = \frac{1}{2} F \Delta L$



## Electricity

current and pd  $I = \frac{\Delta Q}{\Delta t}$   $V = \frac{W}{Q}$   $R = \frac{V}{I}$

resistivity  $\rho = \frac{RA}{L}$

resistors in series  $R_T = R_1 + R_2 + R_3 + \dots$

resistors in parallel  $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$

power  $P = VI = I^2R = \frac{V^2}{R}$

emf  $\varepsilon = \frac{E}{Q}$   $\varepsilon = I(R + r)$

## Circular motion

magnitude of angular speed  $\omega = \frac{v}{r}$

$$\omega = 2\pi f$$

centripetal acceleration  $a = \frac{v^2}{r} = \omega^2 r$

centripetal force  $F = \frac{mv^2}{r} = m\omega^2 r$

## Simple harmonic motion

acceleration  $a = -\omega^2 x$

displacement  $x = A \cos(\omega t)$

speed  $v = \pm \omega \sqrt{(A^2 - x^2)}$

maximum speed  $v_{\max} = \omega A$

maximum acceleration  $a_{\max} = \omega^2 A$

for a mass-spring system  $T = 2\pi \sqrt{\frac{m}{k}}$

for a simple pendulum  $T = 2\pi \sqrt{\frac{l}{g}}$

## Thermal physics

energy to change temperature  $Q = mc\Delta\theta$

energy to change state  $Q = ml$

gas law  $pV = nRT$   
 $pV = NkT$

kinetic theory model  $pV = \frac{1}{3} N m (c_{\text{rms}})^2$

kinetic energy of gas molecule  $\frac{1}{2} m (c_{\text{rms}})^2 = \frac{3}{2} kT = \frac{3RT}{2N_A}$

## Gravitational fields

force between two masses  $F = \frac{Gm_1m_2}{r^2}$

gravitational field strength  $g = \frac{F}{m}$

magnitude of gravitational field strength in a radial field  $g = \frac{GM}{r^2}$

work done  $\Delta W = m\Delta V$

gravitational potential  $V = -\frac{GM}{r}$

$$g = -\frac{\Delta V}{\Delta r}$$

## Electric fields and capacitors

force between two point charges  $F = \frac{1}{4\pi\epsilon_0} \frac{Q_1Q_2}{r^2}$

force on a charge  $F = EQ$

field strength for a uniform field  $E = \frac{V}{d}$

work done  $\Delta W = Q\Delta V$

field strength for a radial field  $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$

electric potential  $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$

$$E = \frac{\Delta V}{\Delta r}$$

capacitance  $C = \frac{Q}{V}$

$$C = \frac{A\epsilon_0\epsilon_r}{d}$$

capacitor energy stored  $E = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C}$

capacitor charging  $Q = Q_0(1 - e^{-t/RC})$

decay of charge  $Q = Q_0 e^{-t/RC}$

time constant  $RC$



## Magnetic fields

<i>force on a current</i>	$F = BIl$
<i>force on a moving charge</i>	$F = BQv$
<i>magnetic flux</i>	$\Phi = BA$
<i>magnetic flux linkage</i>	$N\Phi = BAN \cos \theta$
<i>magnitude of induced emf</i>	$\varepsilon = N \frac{\Delta\Phi}{\Delta t}$
	$N\Phi = BAN \cos \theta$
<i>emf induced in a rotating coil</i>	$\varepsilon = BAN\omega \sin \omega t$
<i>alternating current</i>	$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \quad V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$
<i>transformer equations</i>	$\frac{N_s}{N_p} = \frac{V_s}{V_p}$
	$\text{efficiency} = \frac{I_s V_s}{I_p V_p}$

## Nuclear physics

<i>the inverse square law for <math>\gamma</math> radiation</i>	$I = \frac{k}{x^2}$
<i>radioactive decay</i>	$\frac{\Delta N}{\Delta t} = -\lambda N, N = N_0 e^{-\lambda t}$
<i>activity</i>	$A = \lambda N$
<i>half-life</i>	$T_{1/2} = \frac{\ln 2}{\lambda}$
<i>nuclear radius</i>	$R = R_0 A^{1/3}$
<i>energy-mass equation</i>	$E = mc^2$

## OPTIONS

### Astrophysics

1 astronomical unit	$= 1.50 \times 10^{11} \text{ m}$
1 light year	$= 9.46 \times 10^{15} \text{ m}$
1 parsec	$= 206265 \text{ AU} = 3.08 \times 10^{16} \text{ m}$
	$= 3.26 \text{ light year}$

$$\text{Hubble constant, } H = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

$$M = \frac{\text{angle subtended by image at eye}}{\text{angle subtended by object at unaided eye}}$$

<i>in normal adjustment</i>	$M = \frac{f_o}{f_e}$
<i>Rayleigh criterion</i>	$\theta \approx \frac{\lambda}{D}$
<i>magnitude equation</i>	$m - M = 5 \log \frac{d}{10}$
<i>Wien's law</i>	$\lambda_{\text{max}} T = 2.9 \times 10^{-3} \text{ m K}$
<i>Stefan's law</i>	$P = \sigma AT^4$
<i>Schwarzschild radius</i>	$R_s \approx \frac{2GM}{c^2}$
<i>Doppler shift for <math>v \ll c</math></i>	$\frac{\Delta f}{f} = -\frac{\Delta \lambda}{\lambda} = \frac{v}{c}$
<i>red shift</i>	$z = -\frac{v}{c}$
<i>Hubble's law</i>	$v = Hd$

### Medical physics

<i>lens equations</i>	$P = \frac{1}{f}$
	$m = \frac{v}{u}$
	$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$
<i>threshold of hearing</i>	$I_0 = 1.0 \times 10^{-12} \text{ W m}^{-2}$
<i>intensity level</i>	$\text{intensity level} = 10 \log \frac{I}{I_0}$
<i>absorption</i>	$I = I_0 e^{-\mu x}$
	$\mu_m = \frac{\mu}{\rho}$
<i>ultrasound imaging</i>	$Z = \rho c$
	$\frac{I_r}{I_i} = \left( \frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2$
<i>half-lives</i>	$\frac{1}{T_B} = \frac{1}{T_E} + \frac{1}{T_P}$



## Engineering physics

<i>moment of inertia</i>	$I = \Sigma mr^2$
<i>angular kinetic energy</i>	$E_k = \frac{1}{2} I \omega^2$
<i>equations of angular motion</i>	$\omega_2 = \omega_1 + \alpha t$ $\omega_2^2 = \omega_1^2 + 2\alpha\theta$ $\theta = \omega_1 t + \frac{\alpha t^2}{2}$ $\theta = \frac{(\omega_1 + \omega_2) t}{2}$
<i>torque</i>	$T = I \alpha$ $T = F r$
<i>angular momentum</i>	<i>angular momentum</i> = $I \omega$
<i>angular impulse</i>	$T \Delta t = \Delta(I \omega)$
<i>work done</i>	$W = T \theta$
<i>power</i>	$P = T \omega$
<i>thermodynamics</i>	$Q = \Delta U + W$ $W = p \Delta V$
<i>adiabatic change</i>	$pV^\gamma = \text{constant}$
<i>isothermal change</i>	$pV = \text{constant}$
<i>heat engines</i>	
	$\text{efficiency} = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H}$
	$\text{maximum theoretical efficiency} = \frac{T_H - T_C}{T_H}$
<i>work done per cycle</i>	= area of loop
<i>input power</i>	= calorific value $\times$ fuel flow rate
	$\text{indicated power} = \frac{\text{area of } p - V \text{ loop}}{\text{number of cycles per second}} \times \text{number of cylinders}$
<i>output or brake power</i>	$P = T \omega$
<i>friction power</i>	= indicated power - brake power
<i>heat pumps and refrigerators</i>	
<i>refrigerator:</i>	$COP_{\text{ref}} = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C}$
<i>heat pump:</i>	$COP_{\text{hp}} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_C}$

## Turning points in physics

<i>electrons in fields</i>	$F = \frac{eV}{d}$ $F = Bev$ $r = \frac{mv}{Be}$ $\frac{1}{2} mv^2 = eV$
<i>Millikan's experiment</i>	$\frac{QV}{d} = mg$ $F = 6\pi\eta r v$
<i>Maxwell's formula</i>	$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$
<i>special relativity</i>	$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ $l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$ $E = m c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$

## Electronics

<i>resonant frequency</i>	$f_0 = \frac{1}{2\pi \sqrt{LC}}$
<i>Q-factor</i>	$Q = \frac{f_0}{f_B}$
<i>operational amplifiers: open loop</i>	$V_{\text{out}} = A_{\text{OL}}(V_+ - V_-)$
<i>inverting amplifier</i>	$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_f}{R_{\text{in}}}$
<i>non-inverting amplifier</i>	$\frac{V_{\text{out}}}{V_{\text{in}}} = 1 + \frac{R_f}{R_1}$
<i>summing amplifier</i>	$V_{\text{out}} = -R_f \left( \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \dots \right)$
<i>difference amplifier</i>	$V_{\text{out}} = (V_+ - V_-) \frac{R_f}{R_1}$
<i>Bandwidth requirement:</i>	
<i>for AM</i>	$\text{bandwidth} = 2f_M$
<i>for FM</i>	$\text{bandwidth} = 2(\Delta f + f_M)$



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Only constructive and reasoned feedback will be considered.