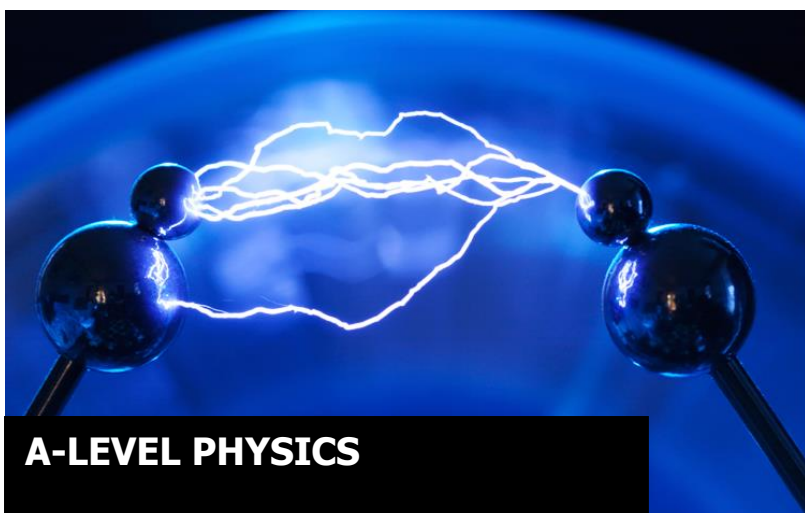


**Volume  
Two**

**ST MARY'S SCIENCE  
DEPARTMENT:  
PHYSICS**

**A LEVEL PHYSICS YEAR 1  
STUDENT INDEPENDENT WORK BOOK  
1.5.2: EMF AND POTENTIAL DIVIDERS  
VOLUME TWO**

<b>NAME</b>	
<b>PHYSICS CLASS</b>	
<b>MODULE TEACHER</b>	
<b>ALPS GRADE</b>	



**A-LEVEL PHYSICS  
TOPIC 5  
INDEPENDENT WORK BOOK 2**

**THIS MUST  
BE BROUGHT  
TO ALL  
PHYSICS  
LESSONS.**



## Contents

### 3.5.1.5 Potential Divider

### 3.5.1.6 Electromotive Force and Internal Resistance

#### Overview

This section builds on and develops earlier study of these phenomena from GCSE.

It provides opportunities for the development of practical skills at an early stage in the course and lays the groundwork for later study of the many electrical applications that are important to society.

#### **IMPORTANT NOTE**

This book contains all of the activities you can carry out independently in your study periods to enhance your understanding in A-Level Physics.

You may work through the activities in this book and mark this work yourself. Your work will then be reviewed by your teacher in KS5 file checks. This work is in addition to the class work and homework you carry out.

This book may also be used as a revision resource for intervention, internal assessments and external assessments.

**Please keep this in your student file.**

As part of this course you are expected to **read through this preparatory reading book** and **complete the independent study tasks**.

This work will not be assessed but will be monitored by your class teacher.

This must be completed by the deadline set by your class teacher.



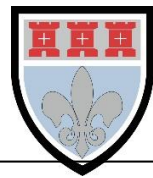
# **SECTION 1**

## **INDEPENDENT STUDY TASK**

### **Instructions**

Read through the information from the student preparatory book and then produce revision posters on the key points highlighted on the following pages.

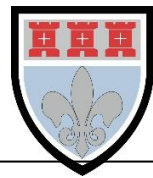
These notes should be used as a revision resource for assessments.



## INDEPENDENT STUDY TASK 1

Produce an **information sheet** on resistance in series and parallel.

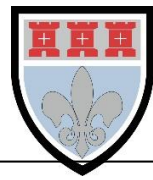
This is an independent study task to be carried out outside of lesson.



**INDEPENDENT STUDY TASK 2**

Produce an **information sheet** on potential dividers.

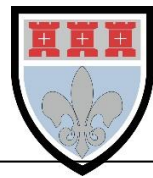
This is an independent study task to be carried out outside of lesson.



### INDEPENDENT STUDY TASK 3

Produce an **information sheet** on the equations of this module.

This is an independent study task to be carried out outside of lesson.



## INDEPENDENT STUDY TASK 4

Produce an **information sheet** on the internal resistance required practical.

This is an independent study task to be carried out outside of lesson.



## **SECTION 2**

# **KNOWLEDGE CHECKER**

### **Instructions**

Read through the information from the student preparatory book and then answer the following questions from the different parts of the topics.

These questions are designed to introduce the different parts of this module.

Use the mark schemes to review your knowledge and understanding.



# QUESTIONS

Use the preparatory reading notes to answer these questions.

**A1.** What causes internal resistance?

[1 Mark]

.....  
.....

**A2.** What is the difference between emf and terminal pd?

[1 Mark]

.....  
.....  
.....  
.....

**A3.** Write the equation used to calculate the terminal pd of a power supply.

[1 Mark]

.....  
.....

**A4.** State the formulae used to combine resistance of resistors in series.

[1 Mark]

.....  
.....

**A5.** State the formulae used to combine resistance of resistors in parallel.

[1 Mark]

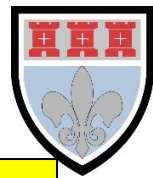
.....  
.....

**A6.** What is a potential divider?

[1 Mark]

.....  
.....

Use the preparatory reading notes to answer these questions.



**A7.** What is a potentiometer?

Use the preparatory reading notes to answer these questions.

[1 Mark]

.....  
.....

**A8.** What is the advantage of using a potentiometer in a circuit?

[1 Mark]

.....  
.....

**A9.** How can LDR be used in a circuit to sense light intensity changes?

[1 Mark]

.....  
.....  
.....  
.....

**A10.** What three factors does the resistance of a length of wire depend on?

[1 Mark]

.....  
.....

**A11.** What device is used to measure the diameter of a piece of wire?

[1 Mark]

.....  
.....

**A12.** Define the term 'electrical insulator'?

[1 Mark]

.....  
.....  
.....

**A13.** How are diodes protected in an electrical circuit?

[1 Mark]

.....

Use the preparatory reading notes to answer these questions.



**ADVANCED SECTION**

**A14.** A 12V car battery supplies a current of 48A for 2.0 seconds to the car's starter motor. The total resistance of the connecting wires is 0.01Ω. Calculate the energy transferred from the battery.

**[1 Mark]**

.....

.....

.....

.....

Use the preparatory reading notes to answer these questions.

Questions **A25-A26** refer to the following statement.  
A battery with an internal resistance of 0.80Ω and an emf of 24V powers a dentist's drill with resistance 4.0Ω

**A15.** Calculate the current in the circuit when the drill is connected to the power supply.

**[1 Mark]**

.....

.....

.....

.....

**A16.** Calculate the potential difference wasted overcoming the internal resistance.

**[1 Mark]**

.....

.....

.....

Questions **A17-A18** refer to the following statement.  
An LDR is placed in a potential divider circuit with a fixed resistor of 100Ω. The LDR in light conditions has a resistance of 300Ω and 900Ω in dark conditions.

There is an input voltage of 6.0V

**A17.** Show that the  $V_{out}$  of a potential divider in light conditions is 1.5V.

**[1 Mark]**

.....

.....

.....

Use the preparatory reading notes to answer these questions.



Use the preparatory reading notes to answer these questions.

**A18.** Show that the  $V_{\text{out}}$  of a potential divider in dark conditions is 0.6V.

**[1 Mark]**

.....

.....

.....

.....

Use the preparatory reading notes to answer these questions.



## ANSWERS

**A1.** What causes internal resistance?

[1 Mark]

Resistance caused by the collision of charge carriers with the chemical ions of the battery and the metal ions of the wire.

**A2.** What is the difference between emf and terminal pd?

[1 Mark]

Emf is the theoretical energy supplied into the circuit per charge by the power supply. The terminal pd is the actual energy supplied into the circuit per charge by the power supply once the internal resistance has been overcome.

**A3.** Write the equation used to calculate the terminal pd of a power supply.

[1 Mark]

$$\varepsilon = I(R + r)$$

**A4.** State the formulae used to combine resistance of resistors in series.

[1 Mark]

$$R_{\text{Total}} = R_1 + R_2 + R_3 + R_n$$

**A5.** State the formulae used to combine resistance of resistors in parallel.

[1 Mark]

$$1/R_{\text{Total}} = 1/R_1 + 1/R_2 + 1/R_3 + 1/R_n$$

**A6.** What is a potential divider?

[1 Mark]

Two or more components placed in series which splits up the potential difference over the circuit.

**A7.** What is a potentiometer?

[1 Mark]

A potentiometer is a variable resistor with three connectors – it uses the principle of potential dividers to alter the output resistance of the circuit.

**A8.** What is the advantage of using a potentiometer in a circuit?

[1 Mark]

A potentiometer can be placed into a circuit and gain a maximum/minimum value for current and potential difference.

**A9.** How can LDR be used in a circuit to sense light intensity changes?

[1 Mark]

An LDR has a very high resistance in the dark, but a lower resistance in the light. When light shines on the LDR, its resistance decreases, so  $V_{\text{OUT}}$  increases.



**A10.** What three factors does the resistance of a length of wire depend on?

[1 Mark]

The material of the wire.  
The cross-sectional area of the wire.  
The temperature of the wire.

**A11.** What device is used to measure the diameter of a piece of wire?

[1 Mark]

A screw gauge micrometer or Vernier callipers.

**A12.** Define the term 'electrical insulator'?

[1 Mark]

A material which does not have enough mobile charge carriers to produce a current when a potential difference is placed over the material.

**A13.** How are diodes protected in an electrical circuit?

[1 Mark]

Diodes are protected by placing a resistor in series and always placing them in forward bias.

### ADVANCED SECTION

**A14.** A 12V car battery supplies a current of 48A for 2.0 seconds to the car's starter motor. The total resistance of the connecting wires is  $0.01\Omega$ . Calculate the energy transferred from the battery.

[1 Mark]

$$E = VIt = 12 \times 48 \times 2.0 = 1200\text{J}$$

Questions **A15-A16** refer to the following statement.

A battery with an internal resistance of  $0.80\Omega$  and an emf of 24V powers a dentist's drill with resistance  $4.0\Omega$

**A15.** Calculate the current in the circuit when the drill is connected to the power supply.

[1 Mark]

$$\varepsilon = I(R + r)$$

$$I = \varepsilon / (R + r)$$

$$I = 24 / (4.0 + 0.80)$$

$$I = 5.0\text{A}$$

**A16.** Calculate the potential difference wasted overcoming the internal resistance.

[1 Mark]

$$V = Ir = 5.0 \times 0.80 = 4.0\text{V}$$



Questions **A17-A18** refer to the following statement.

An LDR is placed in a potential divider circuit with a fixed resistor of  $100\Omega$ . The LDR in light conditions has a resistance of  $300\Omega$  and  $900\Omega$  in dark conditions.

**A17.** Show that the  $V_{\text{out}}$  of a potential divider in light conditions is  $1.5\text{V}$ .

[1 Mark]

$$V_{\text{out}} = V_{\text{in}} \times (R_1/R_1+R_2)$$

$$V_{\text{out}} = 6 \times (300 / 4000)$$

$$V_{\text{out}} = 6 \times 0.75 = 4.5\text{V}$$

$$V_{\text{out}} = 6 - 4.5 = 1.5\text{V}$$

**A18.** Show that the  $V_{\text{out}}$  of a potential divider in dark conditions is  $0.6\text{V}$ .

[1 Mark]

$$V_{\text{out}} = V_{\text{in}} \times (R_1/R_1+R_2)$$

$$V_{\text{out}} = 6 \times (900 / 1000)$$

$$V_{\text{out}} = 6 \times 0.9 = 5.4\text{V}$$

$$V_{\text{out}} = 6 - 5.4 = 0.6\text{V}$$



# **SECTION 3**

# **QUESTIONS**

## **Instructions**

Read through the information from the student preparatory book and then answer the following questions from the different parts of the topics.

Use the mark schemes to review your knowledge and understanding.



## TOPIC: 3.5.1.5 Potential Divider

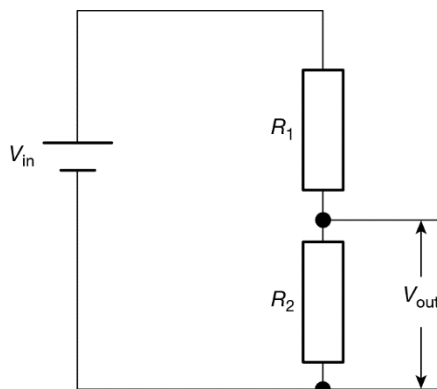
### SPEC CHECK

Specification	Completed?
The potential divider used to supply constant or variable potential difference from a power supply. The use of the potentiometer as a measuring instrument is not required.	
Examples should include the use of variable resistors, thermistors, and light dependent resistors (LDR) in the potential divider.	
Students can investigate the behaviour of a potential divider circuit.	
Students should design and construct potential divider circuits to achieve various outcomes.	



## SUPPORT

If you have a 9 V battery and you need a 6 V supply, you might use a potential divider to give the 6 V you need. The output pd  $V_{\text{out}}$  is a fraction of the input pd  $V_{\text{in}}$ . Figure 1 shows that the fraction depends on the choice of the resistors  $R_1$  and  $R_2$  – often one of these is a variable resistor so it can be adjusted.



**Figure 1**

Input circuit: the total resistance =  $R_1 + R_2$

So  $V_{\text{in}} = I(R_1 + R_2)$

Output circuit: the pd  $V_{\text{out}}$  across  $R_2 = V_{\text{out}} = IR_2$

Dividing these two equations gives:  $\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R_2}{R_1 + R_2}$  or  $V_{\text{out}} = \frac{R_2}{R_1 + R_2} V_{\text{in}}$

Also,  $V_1 = IR_1$  and  $V_2 = IR_2$  so dividing these two equations gives:

$$\frac{V_1}{V_2} = \frac{R_1}{R_2}$$

In a practical circuit, you might connect a component with resistance in parallel with  $R_2$ , so that the pd across it is  $V_{\text{out}}$ . The component must have a resistance of at least 10 times the resistance of  $R_2$ , or  $V_{\text{out}}$  is reduced.

The resistance of a light-dependent resistor (LDR) decreases with increasing light intensity.

The resistance of an NTC (negative temperature coefficient) thermistor decreases with temperature.



## Worked example

### Question

A potential divider circuit is used to switch on a light when the light intensity drops below a certain value.

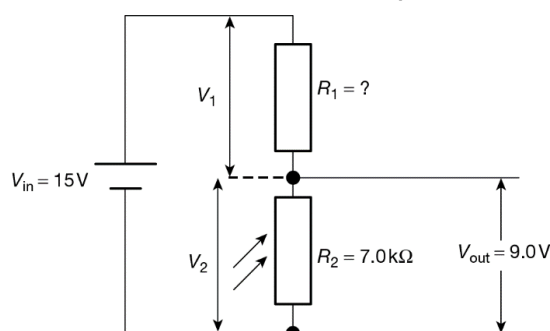
The supply pd is 15 V. The light must switch on at twilight, when the resistance of the LDR has a value of 7.0 k $\Omega$ . 9.0 V is required to switch on the light.

Calculate the fixed resistance required for the circuit.

### Answer

#### Step 1

Draw the circuit diagram and add the information from the question.



#### Step 2

Calculate  $V_1$  using that  $V_2$  must equal  $V_{out}$ .

$$V_2 = V_{out} = 9.0 \text{ V}$$

$$V_1 = V_{in} - 9.0 \text{ V}$$

$$V_1 = 15 \text{ V} - 9.0 \text{ V} = 6.0 \text{ V}$$

#### Step 3

Substitute values into equation  $\frac{V_1}{V_2} = \frac{R_1}{R_2}$  to find  $R_1$ .

$$\frac{V_1}{V_2} = \frac{R_1}{R_2}$$

$$\frac{6.0 \text{ V}}{9.0 \text{ V}} = \frac{R_1}{7.0 \text{ k}\Omega}$$

#### Step 4

Rearrange the equation to find  $R_1$ .

$$R_1 = (7.0 \text{ k}\Omega) \times 0.667$$

$$= 4.7 \text{ k}\Omega \text{ (two significant figures)}$$



## Questions

**A1.** The input pd to a potential divider is 9.0 V.

**A1.1** What is the output pd across a 1.0 kΩ resistor if the second resistor is 5 kΩ?

[1 Mark]

.....

.....

**A1.2** What is the output pd across a 330 Ω if the second resistor is 990 Ω?

[1 Mark]

.....

.....

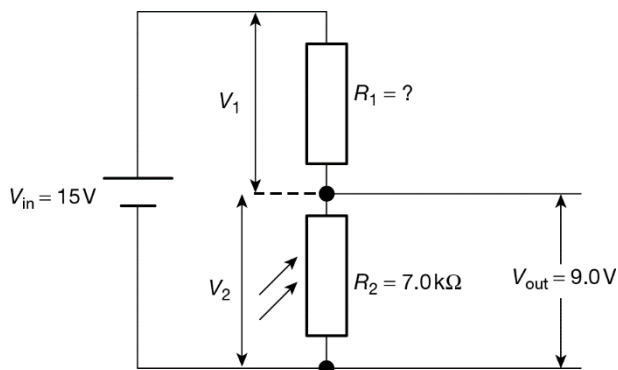
**A1.3** If the output pd across a 680 Ω resistor is 7.0 V, what is the resistance of the other resistor

[1 Mark]

.....

.....

**A2.** A potential divider circuit, like that shown in **Figure 2**, is used to switch on a light when the light intensity drops below a certain value.



**Figure 2**

The supply pd is 12 V. The light must switch on at twilight, when the resistance of the LDR has a value of 5.0 kΩ. A pd of 4.0 V is required to switch on the light.

Calculate the fixed resistance required for the circuit.

[2 Marks]

.....

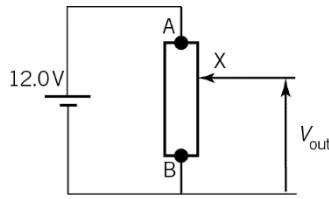
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**A3. Figure 3** shows a length of uniform resistance wire AB connected in a circuit with a 12.0 V power supply to make a potentiometer.



**Figure 3**

What will be the value of the output pd when:

**A3.1** the length AX = half the length of AB

**[2 Marks]**

.....

.....

.....

**A3.2** the length AX =  $\frac{1}{3}$  the length of AB

**[2 Marks]**

.....

.....

.....

**A3.3** the length of AX =  $\frac{3}{4}$  the length of AB.

**[2 Marks]**

.....

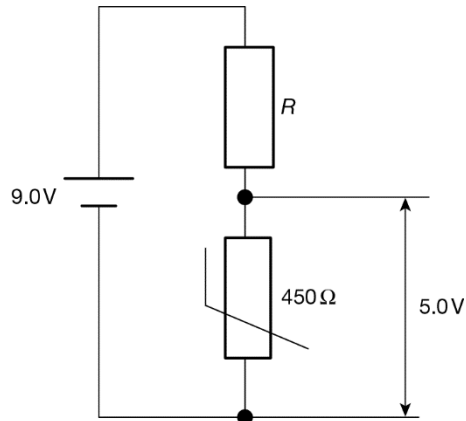
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**A3.** A heater switches on when the pd is 5.0 V. You have a supply pd of 9.0 V and a thermistor with a resistance that decreases with temperature, which has a resistance of  $450\ \Omega$  at a temperature of  $20^\circ\text{C}$ .

Calculate the value of the series resistor that should be used in the circuit, shown in **Figure 4**, to switch on the heater when the temperature falls to  $20^\circ\text{C}$ .



**Figure 4**

**[3 Marks]**

.....

.....

.....

.....



## ANSWERS

$$\mathbf{A1.1} \quad V_{\text{out}} = \frac{1.0 \text{ k}\Omega}{5.0 \text{ k}\Omega + 1.0 \Omega} \times 9.0 \text{ V} = 1.5 \text{ V} \quad (1 \text{ mark})$$

$$\mathbf{A1.2} \quad V_{\text{out}} = \frac{330 \Omega}{330 \Omega + 990 \Omega} \times 9.0 \text{ V} = 2.3 \text{ V (two significant figures)} \quad (1 \text{ mark})$$

$$\mathbf{A1.3} \quad \frac{9.0 - 7 \text{ V}}{7.0 \text{ V}} = \frac{R_1}{680 \Omega}$$

$$R_1 = 680 \Omega \times \frac{2.0}{7.0} = 190 \Omega \text{ (two significant figures)} \quad (1 \text{ mark})$$

$$\mathbf{A2.} \quad V_1 = 12 \text{ V} - 4.0 \text{ V} = 8.0 \text{ V} \quad (1 \text{ mark})$$

$$\frac{8.0 \text{ V}}{4.0 \text{ V}} = \frac{R_1}{5.0 \text{ k}\Omega}$$

$$R_1 = 10 \text{ k}\Omega \quad (1 \text{ mark})$$

$$\mathbf{A3.1} \quad AX = 0.5 \times AB, \text{ so } XB = 0.5 AB$$

$$\text{So } R_2 = 0.5 (R_1 + R_2) \quad (1 \text{ mark})$$

$$V_2 = 0.5 (V_1 + V_2) \text{ (or } V_{\text{out}} = 0.5 V_{\text{in}})$$

$$V_{\text{out}} = 0.5 \times 12.0 \text{ V}$$

$$V_{\text{out}} = 6.0 \text{ V} \quad (1 \text{ mark})$$

$$\mathbf{A3.2} \quad AX = \frac{1}{3} \times AB \text{ so } XB = \frac{2}{3} AB$$

$$\text{So } R_2 = \frac{2}{3} (R_1 + R_2) \quad (1 \text{ mark})$$

$$V_2 = \frac{2}{3} (V_1 + V_2) \text{ (or } V_{\text{out}} = \frac{2}{3} V_{\text{in}})$$

$$V_{\text{out}} = \frac{2}{3} \times 12.0 \text{ V}$$

$$V_{\text{out}} = 8.0 \text{ V} \quad (1 \text{ mark})$$

$$\mathbf{A3.3} \quad AX = \frac{3}{4} \times AB \text{ so } XB = \frac{1}{4} AB$$

$$\text{So } R_2 = \frac{1}{4} (R_1 + R_2) \quad (1 \text{ mark})$$

$$V_2 = \frac{1}{4} (V_1 + V_2) \text{ (or } V_{\text{out}} = \frac{1}{4} V_{\text{in}})$$

$$V_{\text{out}} = \frac{1}{4} \times 12.0 \text{ V}$$

$$V_{\text{out}} = 3.0 \text{ V} \quad (1 \text{ mark})$$

$$\mathbf{A4.} \quad V_{\text{out}} = 9.0 \text{ V} - 5.0 \text{ V} \quad (1 \text{ mark})$$

$$\frac{9.0 - 5.0 \text{ V}}{5.0 \text{ V}} = \frac{R_1}{450 \Omega} \quad (1 \text{ mark})$$

$$R_1 = 450 \Omega \times \frac{4.0}{5.0} = 360 \Omega \quad (1 \text{ mark})$$



# PRACTICAL SKILLS

Complete the following practical skills-based task to improve your experimental understanding of this part of the course in preparation for **Physics Paper 3**.

The circuit in **Figure 1** operates as an automatic light switch. The light-emitting diode (LED) is switched on when the surroundings are dark enough.

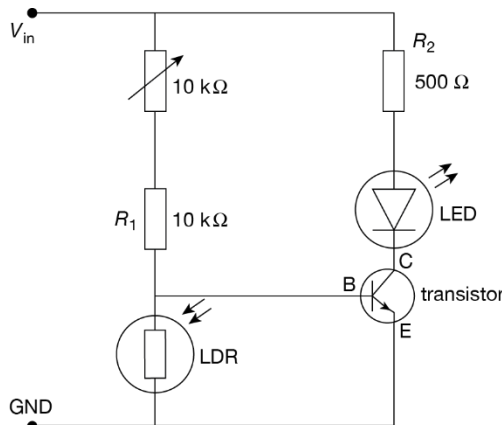
The circuit contains a *transistor*. The transistor operates as an electronic switch. When the potential difference at its base (labelled **B**) is below a certain threshold (the switching threshold) it has a very high resistance between the collector (**C**) and emitter (**E**) pins. When the potential difference rises above the switching threshold, the resistance drops dramatically and a current can flow between the collector and emitter. Different types of transistor have different switching thresholds, the transistor in **Figure 1** is a 2N2222 NPN bipolar junction transistor which is one of the most common.

The  $V_{in}$  should be 10 V.

The resistor  $R_1$  limits the current through the light-dependent resistor (LDR) to prevent damage.

The resistor  $R_2$  limits the current through the LED and transistor to prevent damage.

The variable resistor is used to adjust the threshold at which the circuit turns on.



**Figure 1**

Construct the circuit as shown in **Figure 1**. It is important to connect the circuit as shown because the diode and the transistor will only operate with the current in one direction. Adjust the variable resistor so that the LED is 'just off'.

**P1.** Measure the current into the collector of the transistor. Explain why a normal lamp would not operate in this circuit.

**[1 Mark]**

.....

.....

**P2.** Cover the LDR and describe the action of the circuit. Explain in detail why the LED is not illuminated.

**[4 Marks]**

.....

.....

.....



.....

.....

.....

**P3.** Uncover the LDR and describe the action of the circuit.  
Explain in detail why the LED is illuminated.

**[4 Marks]**

.....

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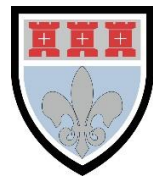
**P4.** Replace the LED with a relay which controls a lamp in a second circuit so that the lamp will come on when it is dark. Describe how this combination of circuits operates.

**[2 Marks]**

.....

.....

.....



## ANSWERS

**P1.** The current is too small to illuminate a lamp.

*(1 mark)*

**P2.** Key points which should be mentioned:

The resistance of the LDR is very high.

*(1 mark)*

The pd at the base of the transistor is below the switching threshold.

*(1 mark)*

The resistance between the collector and the base of the transistor is very high.

*(1 mark)*

There is either no or a very low current in the LED so it does not light up.

*(1 mark)*

**P3.** Key points which should be mentioned:

The resistance of the LDR is lower.

*(1 mark)*

The pd at the base of the transistor is above the switching threshold.

*(1 mark)*

The resistance between the collector and base of the transistor is low.

*(1 mark)*

There is current in the LED so it lights up.

*(1 mark)*

**P4.** Key points which should be mentioned:

The relay switches on the second circuit when the light level is low (see question 2).

*(1 mark)*

The current in the second circuit can be much larger and so can illuminate a lamp.

*(1 mark)*



## CHALLENGE QUESTION

To assess your understanding, answer the following higher-level question on this topic.

A battery of electromotive force 12V and negligible internal resistance is connected to two resistors and a light-dependent resistor (LDR), as shown in Fig. 4.1.

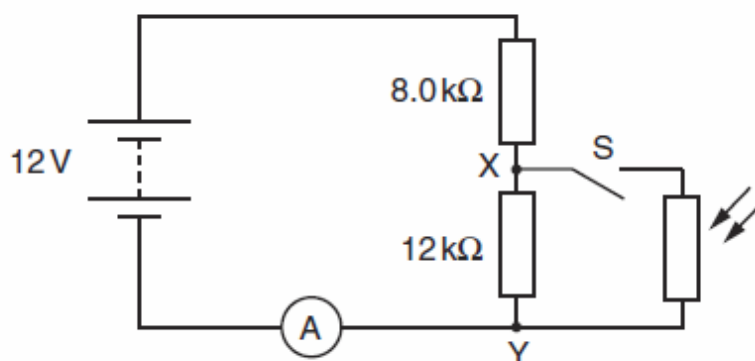


Fig. 4.1

An ammeter is connected in series with the battery. The LDR and switch S are connected across the points XY.

- (a) The switch S is open. Calculate the potential difference (p.d.) across XY.

p. d. = ..... V [3]

- (b) The switch S is closed. The resistance of the LDR is  $4.0 \text{ k}\Omega$ . Calculate the current in the ammeter.

current = ..... A [3]



(c) The switch S remains closed. The intensity of the light on the LDR is increased. State and explain the change to

(i) the ammeter reading,

.....  
.....  
..... [2]

(ii) the p.d. across XY.

.....  
.....  
..... [2]

**Reference:** Cambridge International A Level Physics AS Unit 2 June 2012



## MARK SCHEME

- (a) total resistance =  $20 \text{ (k}\Omega\text{)}$   
current =  $12 / 20 \text{ (mA)}$  or potential divider formula  
p.d. =  $[12 / 20] \times 12 = 7.2 \text{ V}$
- (b) parallel resistance =  $3 \text{ (k}\Omega\text{)}$   
total resistance  $8 + 3 = 11 \text{ (k}\Omega\text{)}$   
current =  $12 / 11 \times 10^3 = 1.09 \times 10^{-3}$  or  $1.1 \times 10^{-3} \text{ A}$
- (c) (i) LDR resistance decreases  
total resistance (of circuit) is less hence current increases
- (ii) resistance across XY is less  
less proportion of  $12 \text{ V}$  across XY hence p.d. is less

C1  
C1  
A1 [3]

C1  
C1  
A1 [3]

M1  
A1 [2]

M1  
A1 [2]



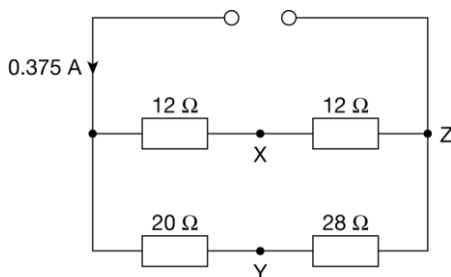
# PRACTICAL SKILLS

Complete the following practical skills-based task to improve your experimental understanding of this part of the course in preparation for the **Physics Paper 3 questions**.

This first set of questions checks your ability to apply Kirchhoff's laws in a range of situations.

The questions become more difficult as you go down the list.

**Q1.** The circuit diagram in **Figure 1** represents a simple circuit containing four resistors and a low voltage power supply.



**Figure 1** Conservation of energy in circuits

**Q1.1** State Kirchhoff's second law.

[1 Mark]

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**Q1.2** Calculate the current at point **Y** in the circuit.

[1 Mark]

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**Q1.3** Calculate the potential difference across the 28 Ω resistor.

[1 Mark]

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**Q1.4** Find the electromotive force (emf) of the power supply.

[1 Mark]

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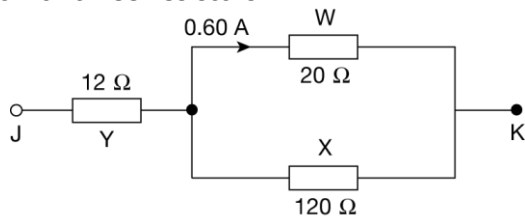
**Q1.5** State the potential difference between points **X** and **Z** in the circuit.

[1 Mark]

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.....

**Q2. Figure 2** shows a combination of three resistors.



**Figure 2**

**Q2.1** Calculate the current in resistor **X**.

[3 Marks]

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**Q2.2** State Kirchhoff's first law.

[1 Mark]

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.....

**Q2.3** State the current in resistor **Y**.

[1 Mark]

.....

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**Q2.4** Calculate the potential difference across resistor **Y**.

[2 Marks]

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**Q2.5** Calculate the potential difference between points **J** and **K**.

[1 Mark]

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.....

**Q2.6** Calculate the total resistance of the combination of resistors.

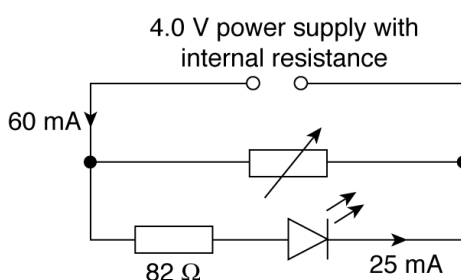
[2 Marks]

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**Q3.** A student is using the circuit shown in **Figure 3** to investigate the characteristics of a light-emitting diode (LED). The diode is operating with a potential difference of 2.0 V across it.



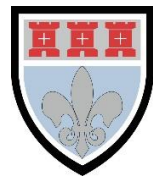
**Figure 3**

**Q3.1** State the potential difference across the 82 Ω resistor.

[1 Mark]

.....

.....



**Q3.2** Calculate the resistance of the LED.

**[2 Marks]**

.....

.....

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**Q3.3** Calculate the resistance setting of the variable resistor when the diode is operating.

**[2 Marks]**

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**Q3.4** Assuming that each electron passing through the LED produces a single photon, calculate the rate of photon emission from the LED.

**[3 Marks]**

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.....



## ANSWERS

**1.1** Accept any suitable variant of 'Around any closed loop in a circuit, the sum of the emfs is equal to the sum of the products of current and resistance in that loop'. (1 mark)

**1.2** 0.125 A (1 mark)

**1.3** 3.5 V (1 mark)

**1.4** 6.0 V (1 mark)

**1.5** 3.0 V (1 mark)

**2.1** pd across **X** = pd across **W** (1 mark)

$$= 0.60 \times 20 = 12 \text{ V}$$

(1 mark)

$$I_X = \frac{12}{120} = 0.10 \text{ A} \quad (1 \text{ mark})$$

**2.1** sum of currents into junction = sum of currents out of junction (or equivalent). (1 mark)

**2.2** 0.70 A (1 mark)

**2.3**  $R = 12 \Omega$

$$V = IR = 0.70 \times 12 \quad (1 \text{ mark})$$

$$= 8.4 \text{ V} \quad (1 \text{ mark})$$

**2.4**  $8.4 + 12 = 20.4 \text{ V}$  (1 mark)

**2.5** resistance of parallel section =  $17.1 \Omega$  (1 mark)

total resistance =  $29.1 \Omega$  (1 mark)

**3.1** 2.0 V (1 mark)

$$\mathbf{3.2} \quad R = \frac{V}{I} = \frac{2.0}{25} \times 10^{-3} \quad (1 \text{ mark})$$

$$= 80 \Omega \quad (1 \text{ mark})$$

**3.3** current in upper branch = 35 mA (1 mark)

$$R = \frac{V}{I} = \frac{4.0}{35} \times 10^{-3} = 114 \Omega \quad (1 \text{ mark})$$

**3.4** rate of electrons entering light-emitting diode =  $\frac{\text{current}}{\text{charge on one electron}}$  (1 mark)

$$= 25 \times \frac{10^{-3}}{1.60} \times 10^{-19} \quad (1 \text{ mark})$$

$$= 1.56 \times 10^{17} \text{ photons per second} \quad (1 \text{ mark})$$



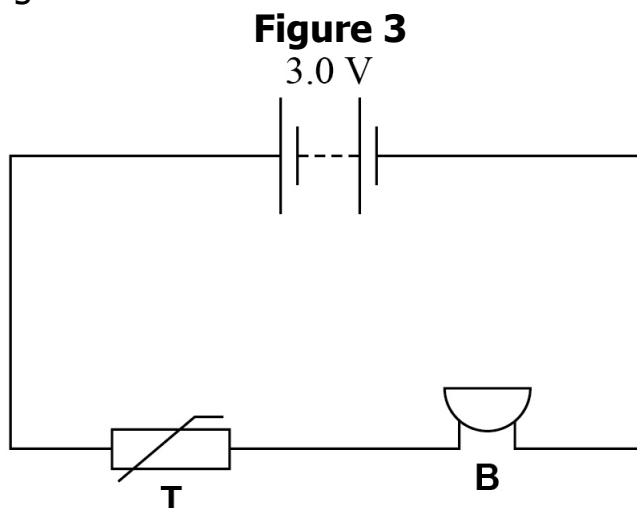
## CHALLENGE QUESTION

To assess your understanding, answer the following higher-level question on this topic.

**1. Figure 3** shows a thermistor **T** used in an alarm circuit for a refrigerator. The alarm is designed to sound a buzzer **B** when the temperature exceeds a threshold value.

**B** has a constant resistance of  $123 \Omega$ .

The battery has a negligible internal resistance.



The buzzer sounds when the potential difference across it is greater than 1.8 V.

**1.1** Explain why the potential difference across the buzzer increases when the temperature increases.

**[3 marks]**

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**1.2** Show that, when the potential difference across the buzzer is 1.8 V, the resistance of the thermistor is about  $80 \Omega$ .

**[2 marks]**

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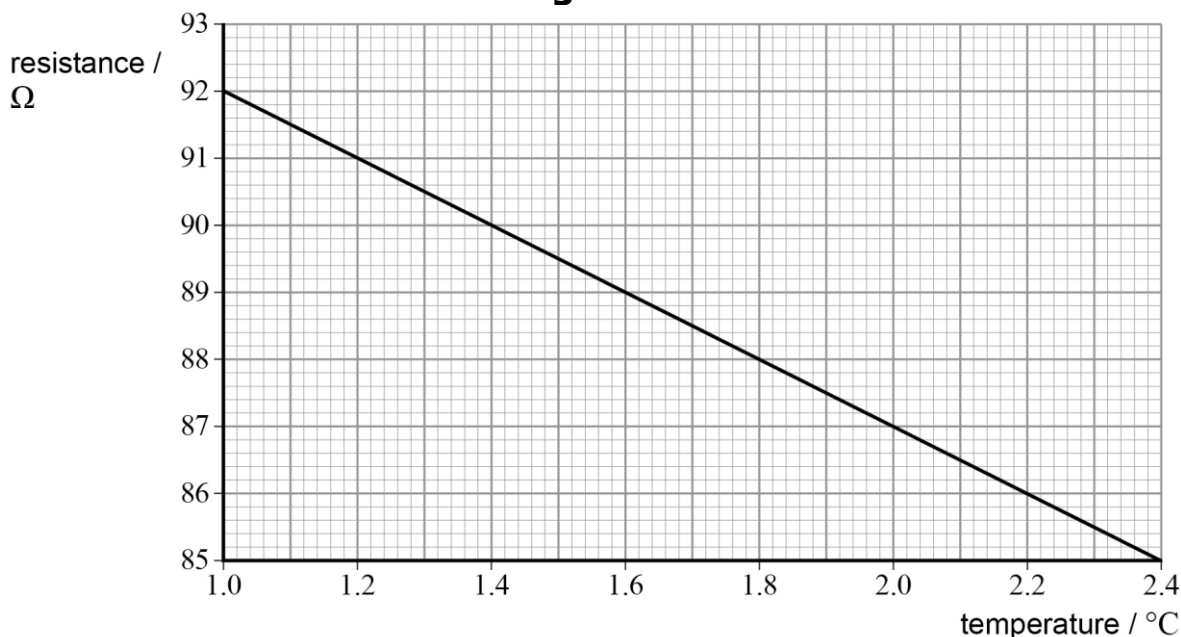
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**Figure 4** shows how the resistance of the thermistor varies with temperature over a small range of temperatures.

**Figure 4**



**1.3** Calculate the gradient of the graph.

**[2 marks]**

.....

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.....

.....

Gradient = ..... Ω K<sup>-1</sup>

**1.4** Determine the temperature at which the buzzer will start to sound. Assume that the gradient of the graph remains constant when the temperature rises above 2.4 °C.

**[1 mark]**

.....

.....

Temperature = ..... °C



# MARK SCHEME

Question	Marking guidance	Mark	Comments
<b>1.1</b>	<p>As temperature increases, the resistance of the thermistor decreases ✓</p> <p>As more charge carriers become available for conduction/ are raised to the conduction band ✓</p> <p>A larger share/proportion of the battery pd across the buzzer (or reference to a potential divider) ✓</p>	3	
<b>1.2</b>	<p>Use of <math>\frac{1.2}{1.8} = \frac{R}{123}</math> or <math>I = \frac{1.8}{123}</math> or <math>I = 0.0146</math> (A) ✓</p> <p><math>R = 82</math> (<math>\Omega</math>) ✓</p>	2	Must see 82 not just 80. Answers based on checking can only score both marks if unrounded values are seen.
<b>1.3</b>	<p>Attempt to find gradient as <math>\Delta y/\Delta x</math> with correct values read from graph</p> <p>Gradient = <math>-5</math> (<math>\Omega</math> K<math>^{-1}</math>)</p>	2	Must have negative sign for second mark.
<b>1.4</b>	T = 3.0 ( $^{\circ}$ C)	1	If clearly use 80 $\Omega$ from 7.2 can gain full marks. In this case T=3.4 $^{\circ}$ C)



## TOPIC: 3.5.1.6 Electromotive Force and Internal Resistance

### SPEC CHECK

Specification	Completed?
$\varepsilon = EQ$ $\varepsilon = IR + r$	
Terminal pd; emf	
Students will be expected to understand and perform calculations for circuits in which the internal resistance of the supply is not negligible.	



## SUPPORT

The emf of a battery (or cell),  $\varepsilon$ , is the **electrical energy** per unit **charge** *produced* by the battery, its unit is volts (V)

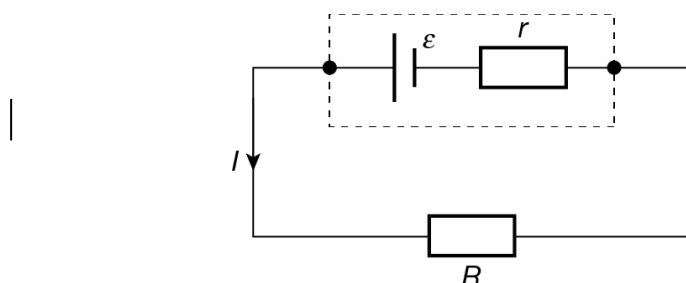
If electrical energy,  $E$ , is given to a charge,  $Q$ , in the battery, then  $\text{emf} = \frac{\text{electrical energy}}{\text{charge}}$ ,  $\varepsilon = \frac{E}{Q}$

The pd across the terminals of a battery (or cell), also known as the terminal pd, is the electrical energy per unit charge *delivered* by the battery when it is in a circuit.

The terminal pd is less than the emf whenever current passes through the battery, due to the internal resistance of the battery.

The internal resistance,  $r$ , of a battery or a cell is defined as the loss of pd per unit current in the battery when current passes through the battery. The internal resistance is due to opposition to the flow of charge through the battery – this causes electrical energy produced by the battery to be dissipated inside the source when charge flows through it.

When a cell of emf  $\varepsilon$  and internal resistance  $r$  is connected to an external resistor of resistance  $R$ , as shown in Figure 1, then  $\varepsilon = I(R + r)$  or  $\varepsilon = V + Ir$ .



**Figure 1** The internal resistance of a source is often shown as a resistor (labelled 'internal resistance' or  $r$ ) in series with the cell or battery

### Combining cells in series

If the cells are connected in the same direction, the net emf is the sum of the individual emf values of the cells, and the total internal resistance is the sum of the individual resistance values of the cells.

If the cells are connected in the opposite direction, the net emf is the *difference* between the individual emf values of the cells. However, the total internal resistance is still the *sum* of the individual resistance values of the cells.

net emf = 4 V	net emf = 0 V	net emf = 2 V
total $r = 2 \Omega$	total $r = 2 \Omega$	total $r = 3 \Omega$

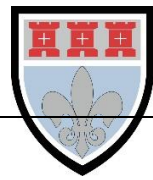
**Figure 2** Combining cells in series; each cell has an emf of 2 V and an internal resistance of 1  $\Omega$ .

### Combining cells in parallel

For  $n$  identical cells connected in parallel, each with emf  $\varepsilon$  and internal resistance  $r$ :

The net emf is the same as the individual emf of each cell,  $\varepsilon$

The total internal resistance is equal to  $\frac{r}{n}$



## Worked example

### Question

A cell of emf  $1.5\text{ V}$  and internal resistance  $1.0\ \Omega$  is connected with a  $5.0\ \Omega$  resistor to form a complete circuit as shown in Figure 3.

Calculate

- 1.1 the current in the circuit
- 1.2 the terminal pd
- 1.3 the power dissipated in the external circuit
- 1.4 the power dissipated in the cell.

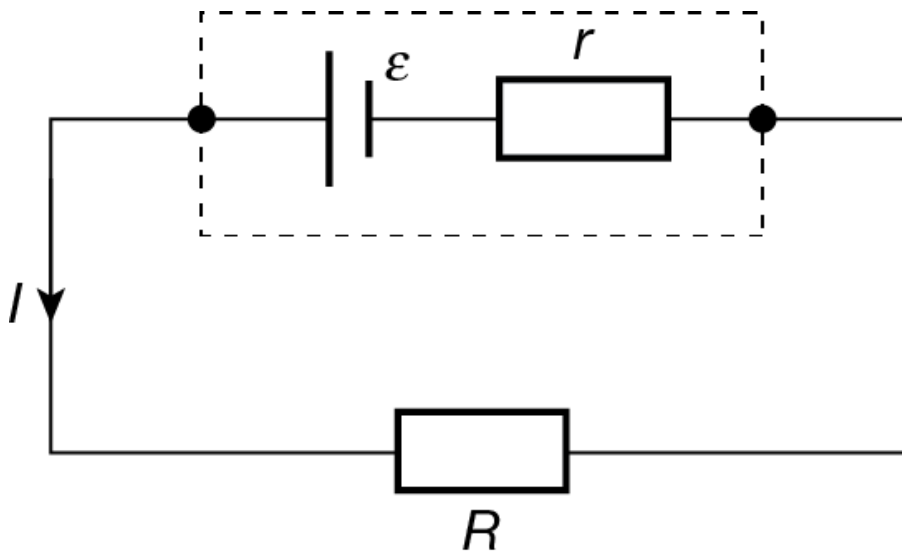


Figure 3

### Answer

#### Step 1

Write down the values given in the question.

$$\epsilon = 1.5\text{ V}; r = 1.0\ \Omega; R = 5.0\ \Omega$$

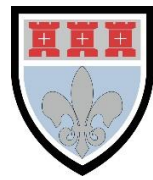
#### Step 2

1.1 Use  $\epsilon = I(R + r)$  to find the current in the circuit,  $I$ .

$$\epsilon = I(R + r)$$

$$1.5 = I(5.0 + 1.0) = 6.0 I$$

$$I = \frac{1.5}{6.0} = 0.25\text{ A}$$

**Step 3**

The terminal pd is equal to the pd in external circuit.

**Method 1** – rearrange  $\varepsilon = V + Ir$  for  $V$  and substitute the values.

$$\mathbf{1.2} \quad V = \varepsilon - Ir$$

$$= 1.5 - 0.25 \times 1.0$$

$$= 1.5 - 0.25$$

$$= 1.25 \text{ V} = 1.3 \text{ V (to two significant figures)}$$

**Method 2** – use  $V = IR$  in the external circuit to find  $V$ .

$$V = IR$$

$$= 0.25 \times 5.0$$

$$= 1.25 \text{ V} = 1.3 \text{ V (to two significant figures)}$$

**Step 4**

Use  $P = I^2 R$  to find the power dissipated.

**1.3** For the external circuit:

$$P = I^2 R$$

$$= 0.25^2 \times 5.0$$

$$= 0.31 \text{ W}$$

**1.4** For the cell:

$$P = I^2 R$$

$$= 0.25^2 \times 1.0$$

$$= 0.0625 \text{ W} = 0.063 \text{ W (to two significant figures)}$$



## Questions

**A1.** A cell has an emf of 2.0 V and an internal resistance of 0.1  $\Omega$ .

Calculate

**A1.1** the pd across the terminals when it is supplying a current of 5.0 A

[2 Marks]

.....  
.....

**A1.2** the external resistance in the circuit.

[2 Marks]

.....  
.....

**A2.** The emf of a cell is 2.0 V. When a resistor is connected across the terminals of the cell, the pd across the terminals drops to 1.5 V and a current of 2.0 A flows.

Calculate

**A2.1** the internal resistance of the cell

[2 Marks]

.....  
.....

**A2.2** the resistance of the resistor

[2 Marks]

.....  
.....

**A2.3** the energy wasted in the cell when 1.0 C of charge passes round the circuit.

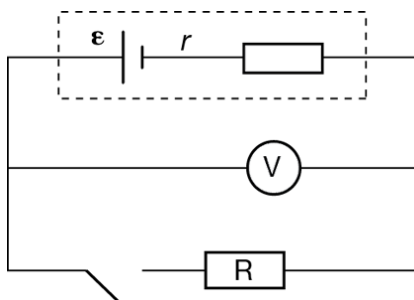
[2 Marks]

.....  
.....



**A3.** For the circuit shown in Figure 4, when the switch is open the voltmeter reads 10.0 V and when it is closed the voltmeter reads 8.0 V. The resistance  $R = 4.0 \Omega$ .

Calculate the internal resistance,  $r$ .



**Figure 4**

**[3 Marks]**

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**A4.** A bulb in a torch is powered by two identical cells connected in series, each of emf 1.5 V. The bulb dissipates power at the rate of 625 mW and the pd across the bulb is 2.5 V. Calculate

**A4.1** the internal resistance of each cell

**[2 Marks]**

.....

.....

**A4.2** the energy dissipated in each cell in one minute.

**[2 Marks]**

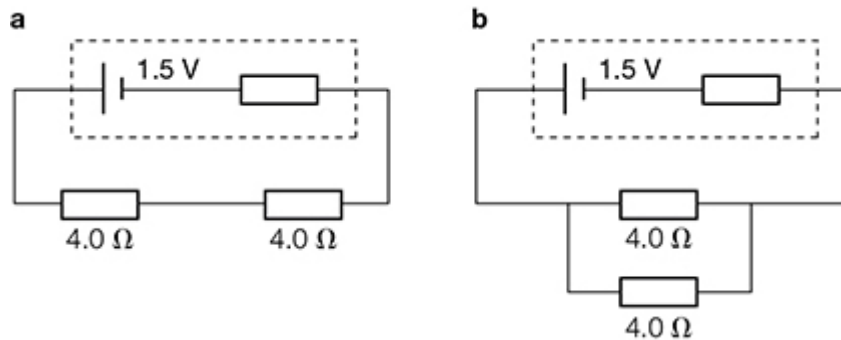
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**Exam-style question**

**A5.** A 1.5 V cell of internal resistance  $2.0\ \Omega$  is connected in two different ways to two  $4.0\ \Omega$  resistors, as shown in Figure 5.



**Figure 5**

For the series combination calculate  
The current through both resistors

**[2 Marks]**

.....

.....

The terminal pd.

**[2 Marks]**

.....

.....

For the parallel combination calculate  
The current from the cell

**[2 Marks]**

.....

.....

The current through each resistor.

**[2 Marks]**

.....

.....



## ANSWERS

**A1.1** 1.5 V

*(2 marks)*

**A1.2** 0.30  $\Omega$

*(2 marks)*

**A2.1** 0.25  $\Omega$

*(2 marks)*

**A2.2** 0.75  $\Omega$

*(2 marks)*

**A2.3** 0.50 J

*(2 marks)*

**A3.** 1.0  $\Omega$

*(3 marks)*

**A4.1** 1.0  $\Omega$

*(2 marks)*

**A4.2** 3.8 J

*(2 marks)*

**A5.** 0.15 A

*(2 marks)*

1.2 V

*(2 marks)*

0.38 A (to two significant figures) (0.375 A)

*(2 marks)*

0.19 A

*(2 marks)*



## REQUIRED PRACTICAL FOLLOW UP

Complete the following practical skills-based task to improve your experimental understanding of the required practical 'internal resistance of a power pack'.

The internal resistance of a cell causes dissipation of energy as heat within the cell. This loss of energy causes a reduction in the terminal potential difference (pd) of the cell when compared to the electromotive force (emf) of the cell. Energy loss is most significant when the current in the circuit is large.

The terminal pd ( $V$ ), emf ( $\mathcal{E}$ ), current ( $I$ ), and internal resistance ( $r$ ) are related by the equation:  
 $V = \mathcal{E} - Ir$ .

### Questions

The following set of questions is designed to develop your ability to apply your understanding of internal resistance and emf to a range of circumstances.

**P1.** A new battery is labelled 3.0 V but, when connected in a circuit containing a lamp, the terminal pd is measured to be 2.80 V.

**P1.1** Explain why the terminal pd of the battery is less than 3.0 V when connected in a circuit.

[2 Marks]

.....

.....

**P1.2** Explain what would happen to the terminal pd of the battery if a second lamp were connected in series with the first. Assume the resistances of the lamps remain constant.

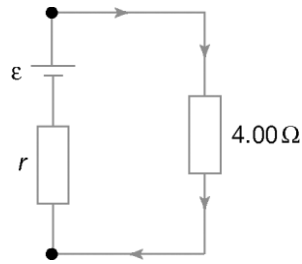
[2 Marks]

.....

.....



**P2.**



**Figure 1**

The circuit in **Figure 1** shows a battery connected to a single  $4.00\ \Omega$  resistor. The terminal pd of the battery is  $1.60\ \text{V}$ . The cell has an internal resistance of  $0.60\ \Omega$ .

**P2.1** Calculate the current through the circuit.

**[1 Mark]**

.....

.....

**P2.2** Calculate the pd drop due to the internal resistance of the cell.

**[1 Mark]**

.....

.....

**P2.3** Calculate the emf of the cell.

**[1 Mark]**

.....

.....

**P2.4** The resistor is replaced with an  $8.0\ \Omega$  resistor. Calculate the current through the resistor.

**[2 Marks]**

.....

.....



**P3.** A battery in a torch has an emf of 6.0 V and internal resistance of 1.00  $\Omega$ . It is used to power a lamp of resistance 12.0  $\Omega$  for 1 hour.

**P3.1** Calculate the current in the torch circuit when it operates.

[1 Mark]

.....  
.....

**P3.2** Calculate the pd across the lamp.

[1 Mark]

.....  
.....

**P3.3** Calculate the energy transferred by the lamp in 1 hour.

[2 Marks]

.....  
.....  
.....

**P3.4** Calculate the energy transferred by the internal resistance of the battery in 1 hour.

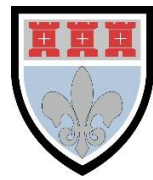
[2 Marks]

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.....

**P3.5** Calculate the efficiency of the energy transfer to the lamp, if the resistance in the connecting leads is negligible.

[2 Marks]

.....  
.....  
.....



**P3.6** The lamp is replaced by one with a resistance of  $15.0 \Omega$ . Discuss the effect this has on the energy transfer to the lamp.

[2 Marks]

.....

.....

.....

**P4.** During an investigation, a student attempts to measure the internal resistance and emf of a reference cell. Their results are shown in **Table 1**.

**Table 1**

Terminal pd / V	Current / A
1.92	1.92
2.35	1.11
2.43	0.87
2.58	0.68
2.66	0.56
2.69	0.45
2.72	0.41

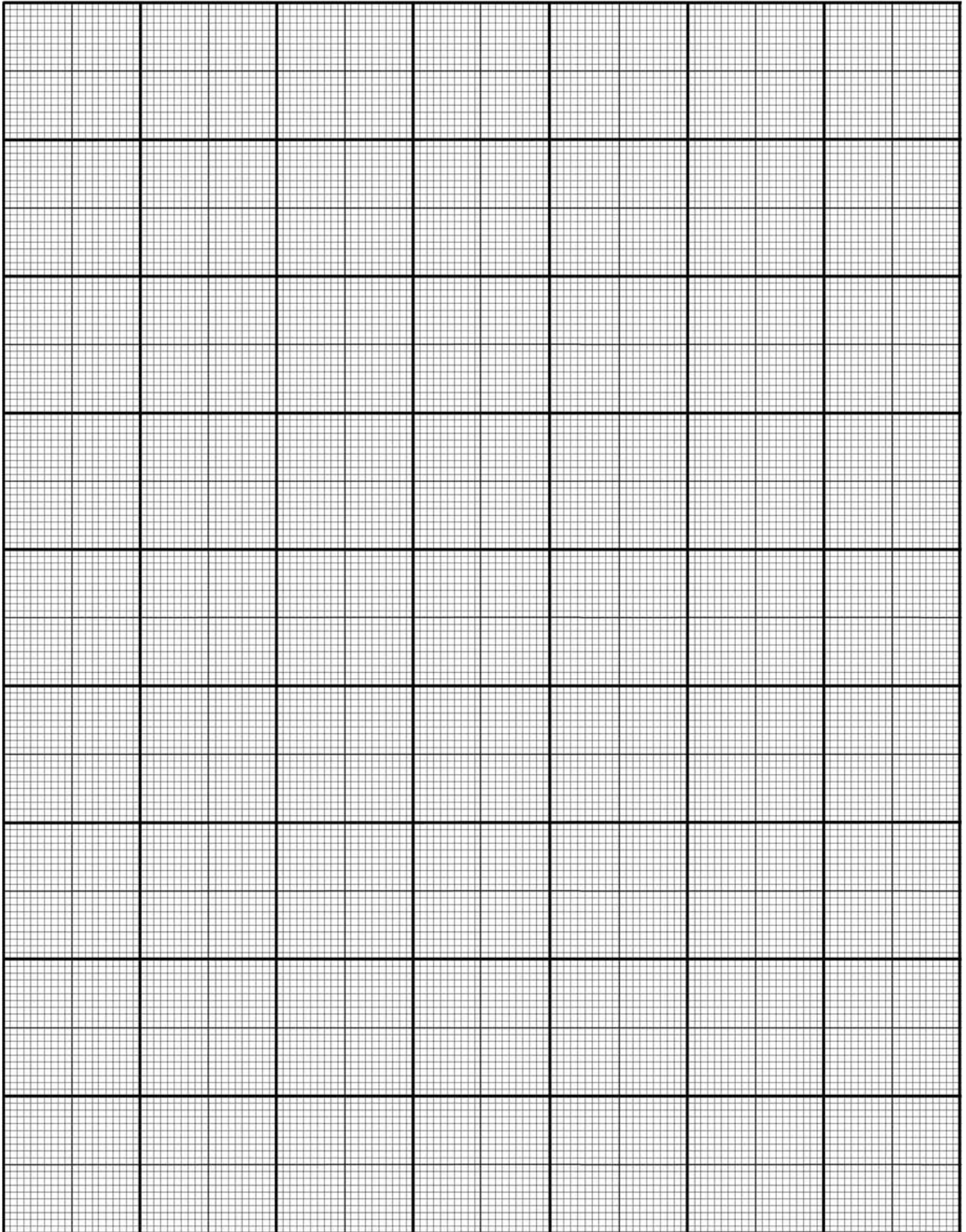
**P4.1** Draw a circuit suitable for collecting this experimental data.

[3 Marks]



**P4.2** Plot a graph suitable for determining the resistance and emf of the cell using the data provided.

**[4 Marks]**





**P4.3** Determine the emf of the cell from the graph.

[1 Mark]

.....  
.....

**P4.4** Determine the internal resistance from the graph.

[1 Mark]

.....  
.....

**P4.5** The cell manufacturer states that the emf of the cell is 3.05 V. Calculate the percentage difference between the manufacturer's specification and the experimental result.

[1 Mark]

.....  
.....



## ANSWERS

**P1.1** The cell has an internal resistance and this results in a drop in terminal pd (1 mark) when there is a current in the cell (1 mark).

**P1.2** The current would be lower (1 mark) and so the pd drop due to internal resistance would be less (1 mark).

**P2.1**  $I = \frac{V}{R} = \frac{1.60}{4.00} = 0.40 \text{ A}$  (1 mark)

**P2.2**  $V = Ir = 0.40 \times 0.60 = 0.24 \text{ V}$  (1 mark)

**P2.3**  $1.60 + 0.24 = 1.84 \text{ V}$  (1 mark)

**P2.4**  $I = \frac{\varepsilon}{(R+r)}$  (1 mark)

$$= \frac{1.84}{(0.6 + 8.0)}$$

$$= 0.21 \text{ A}$$
 (1 mark)

**P3.1**  $I = \frac{V}{R+r} = \frac{6.0}{13} = 0.46 \text{ A}$  (1 mark)

**P3.2**  $V = IR = 0.46 \times 12 = 5.54 \text{ V}$  (1 mark)

**P3.3**  $W = IVt$  (1 mark)

$$= 0.46 \times 5.54 \times (60 \times 60)$$

$$= 9.2 \text{ kJ}$$
 (1 mark)

**P3.4** pd across internal resistance =  $Ir = 0.46 \times 1.00 = 0.46 \text{ V}$  (1 mark)

$$W = IVt = 0.46 \times 0.28 \times (60 \times 60) = 762 \text{ J}$$
 (1 mark)

**P3.5** Efficiency of transfer =  $\frac{\text{useful energy provided}}{\text{total energy dissipated}}$  (1 mark)

$$= \frac{9200}{9200 + 464} = 0.95 \text{ (95\%)}$$
 (1 mark)

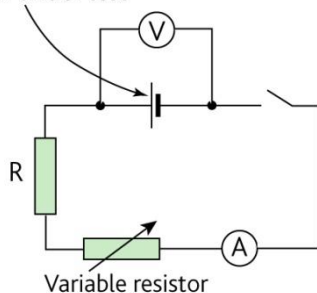
**P3.6** The current will reduce. (1 mark)

The pd across the internal resistance decreases. (1 mark)

A smaller proportion of energy is wasted by internal resistance. (1 mark)

### P4

Rechargeable battery  
or cell under test



**P4.1** Diagram has ammeter and voltmeter in correct places (1 mark)

Method to control current (1 mark)

Resistor to prevent short circuit (1 mark)



- P4.2** Suitable axes for  $V$  against  $I$  (1 mark)  
Accurate plotting (over half of points plotted accurately) (1 mark)  
Straight line of best fit with negative gradient (1 mark)  
Line of best fit extended to determine intercept on  $y$ -axis (1 mark)
- P4.3** 2.93 V (**Accept:** intercept if graph is incorrect) (1 mark)
- P4.4** 0.53  $\Omega$  (**Accept:** gradient of graph if graph is incorrect) (1 mark)
- P4.5** Calculation of difference (0.12 V) (1 mark)  
Percentage difference =  $\frac{0.12}{3.05} \times 100$   
= 4% (1 mark)

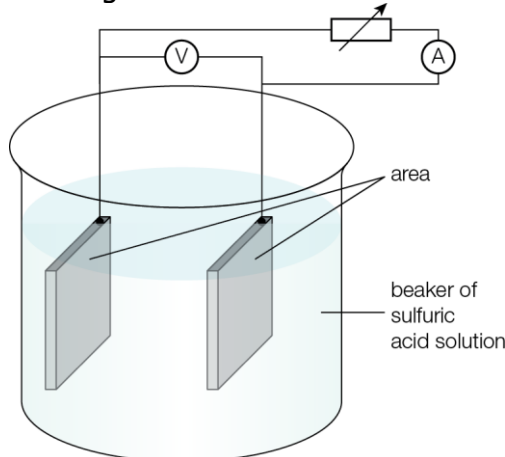


# PRACTICAL SKILLS

Complete the following practical skills-based task to improve your experimental understanding of this part of the course in preparation for the **Physics Paper 3 questions**.

A student carried out an investigation into the hypothesis 'the greater the surface area of the lead plates in a cell, the lower the internal resistance of the cell'.

To explore this potential relationship, they built a simple lead–acid cell by placing two plates in a solution of 4.0 molar sulfuric acid, as shown in **Figure 1** (note that the figure shows the test set-up only). These plates were then connected into a circuit to measure the current provided, and the terminal potential difference when a range of external resistances were attached to the cell.



**Figure 1**

The data was collected for four sets of metal plates with different surface areas. The results are shown in **Table 1**.

**Table 1**

Cross-sectional area of 12.5 cm <sup>2</sup>		Cross-sectional area of 14.6 cm <sup>2</sup>		Cross-sectional area of 17.5 cm <sup>2</sup>		Cross-sectional area of 22.4 cm <sup>2</sup>	
Terminal pd / V	Current / A	Terminal pd / V	Current / A	Terminal pd / V	Current / A	Terminal pd / V	Current / A
0.79	1.58	1.12	1.43	1.33	1.18	1.45	1.10
1.03	1.28	1.26	1.23	1.38	1.09	1.47	1.07
1.19	1.08	1.33	1.12	1.40	1.05	1.47	1.05
1.30	0.93	1.38	1.06	1.41	1.03	1.48	1.05
1.39	0.82	1.41	1.01	1.42	1.01	1.48	1.04
1.46	0.73	1.43	0.98	1.43	1.00	1.48	1.04

**Q1.1** Which factors would you expect the student to control in this experiment to ensure that the data and conclusions are valid?

**[3 Marks]**

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**Q1.2** This experiment has a range of potential hazards. Research the materials and procedures involved and perform an appropriate risk assessment.

Describe any control measures you would expect to take place.

**[4 Marks]**

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**Q2.1** Plot an appropriate graph (or graphs) that will allow you to determine the internal resistance for each of the four different cells.

You will be awarded *3 marks* for each cell.

**Q2.2** Use your graphs to determine the internal resistance of each of the four cells.

**[4 Marks]**

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**Q2.3** Determine the emf of the **four** different cells.

**[4 Marks]**

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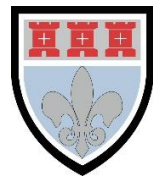
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**Q3.** Considering the values you found for internal resistance, state whether the initial hypothesis is correct or incorrect.

**[1 Mark]**

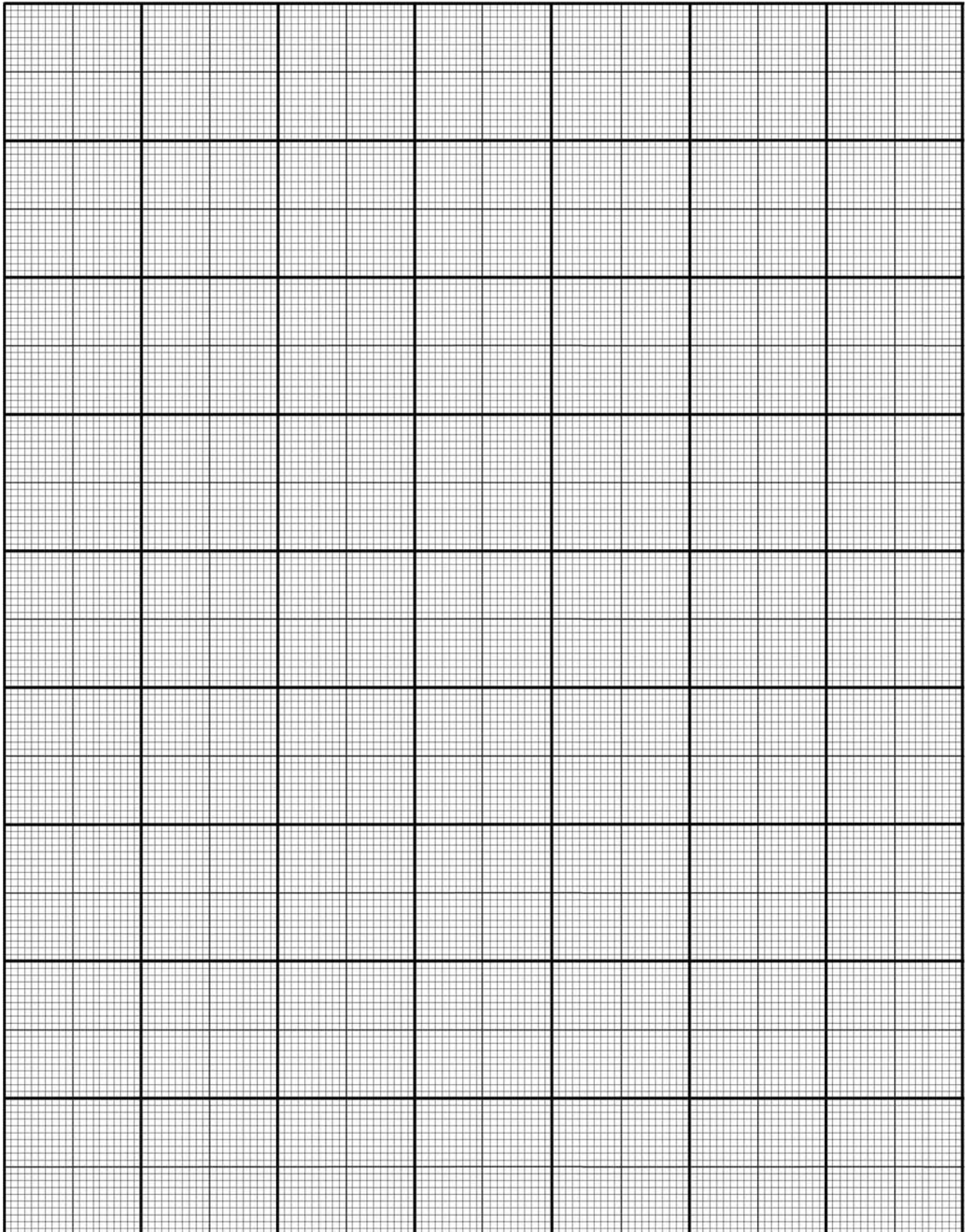
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**Q4.** Plot a graph that will allow you to determine if there is a quantitative relationship between the cross-sectional area and the internal resistance of the cells. State your conclusion.

**[4 Marks]**





**Q5.** Using your answers to Question **Q4**, discuss the validity of your conclusion. You may also make reference to any other factors which may affect the validity of this experiment.

**[2 Marks]**

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## ANSWERS

**1.1** Award *1 mark* for each of the following possible factors.

Control of separation of the metal plates

Control of temperature

Control of concentration / molarity of the solution (which may reduce if the cell is used over a period of time)

**1.2** The student should identify the following hazards and potential control measures. Award *1 mark* for each up to a maximum of *4 marks*.

The sulfuric acid concentration used is classed as corrosive.

Lead compounds are toxic (lead metal is only low hazard, but the reactions will produce compounds such as lead oxide and lead sulfate).

A relevant control measure would be to reduce the concentration of the acid (to below 1.5 molar solution (irritant)) **or** use a small volume

Wear gloves.

Wear eye protection.

Lead compounds are *harmful to the environment* and must not be disposed of down a drain.

**2.1** The students should find the internal resistance of the four cells by plotting graphs of terminal pd against current and finding the gradients. Award *1 mark* for each the following, for each graph/line (student can plot all data on the same set of axes).

suitable scales

accurate plotting of data points

appropriate line of best fit for the gradient.

**2.2** Award *1 mark* for each.

Internal resistances of  $0.80 \Omega$ ,  $0.69 \Omega$ ,  $0.57 \Omega$ ,  $0.45 \Omega$  (allow  $\pm 0.05 \Omega$ ).

**2.3** Award *1 mark* for each (allow e.c.f. if inaccurate plotting results in incorrect answer, as long as correct method is used).

Students use the intercepts on the pd axis to find the pd, giving 2.05 V, 2.10 V, 2.00 V, 1.95 V.

**3.** The initial hypothesis is correct.

(*1 mark*)

**4.** Student plots a graph showing internal resistance against  $1/\text{area}$  (*1 mark*) with accurate plotting (*1 mark*), and a line of best fit (*1 mark*), leading to the conclusion that the internal resistance is inversely proportional to the cross-sectional area (*1 mark*).

Allow *2 marks* for a graph of resistance against area showing a curve of best fit.

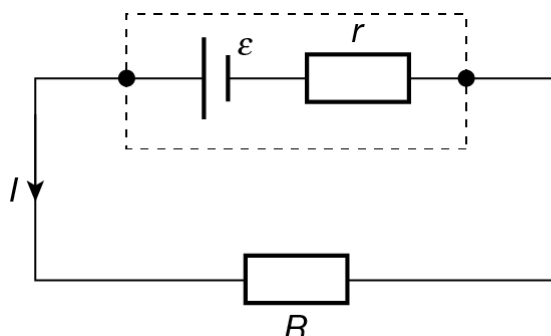
**5.** Student notes that there is some variation in emf (*1 mark*) which lowers the validity of the conclusion (*1 mark*).



## STRETCH AND CHALLENGE

### Emf and internal resistance

When a cell of emf  $\varepsilon$  and internal resistance  $r$  is connected to an external resistor of resistance  $R$ , as shown in **Figure 1**, then  $\varepsilon = I(R + r)$  or  $\varepsilon = V + Ir$ .



**Figure 1** The internal resistance of a source is often shown as a resistor (labelled 'internal resistance' or  $r$ ) in series with the cell or battery

### Series circuits

The current is the same all the way around a series circuit.

- $V = V_1 + V_2 + V_3 + \dots$
- $R = R_1 + R_2 + R_3 + \dots$

If cells are connected in the same direction, the net emf is the sum of the individual emf values of the cells and the total internal resistance is the sum of the individual resistance values of the cells.

If cells are connected in the opposite direction, the net emf is the difference between the individual emf values of the cells. However, the total internal resistance is still the sum of the individual resistance values of the cells.

### Parallel circuits

The pd is the same across each component in a parallel circuit.

- $I = I_1 + I_2 + I_3 + \dots$
- $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$

For  $n$  identical cells connected in parallel, each with emf  $\varepsilon$  and internal resistance  $r$ ,  
The net emf is the same as the individual emf of each cell,  $\varepsilon$

The total internal resistance is equal to  $\frac{r}{n}$ .

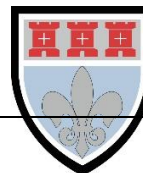
### Power

Power can be calculated using one of the following equations.

$$P = IV$$

$$P = I^2 R$$

$$P = \frac{V^2}{R t}$$



## Worked example

### Question

A  $4.0\ \Omega$  resistor is connected to the terminals of two cells in parallel, both with internal resistances as shown in Figure 2. Calculate the current supplied by each cell.

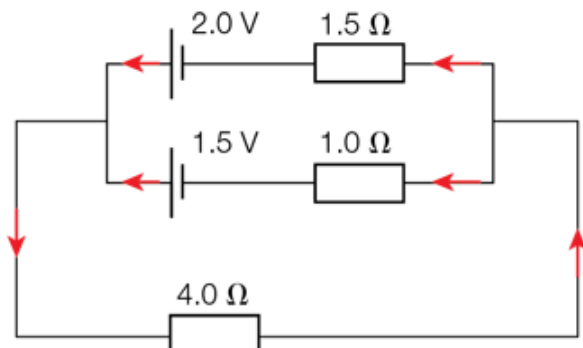


Figure 2

### Answer

#### Step 1

Let  $I_1$  be the current supplied by the 2.0 V cell.

Let  $I_2$  be the current supplied by the 1.5 V cell.

Mark these on the diagram (Figure 3).

Since the two cells are connected in parallel, they supply current  $I_1 + I_2$  to the  $4.0\ \Omega$  resistor.

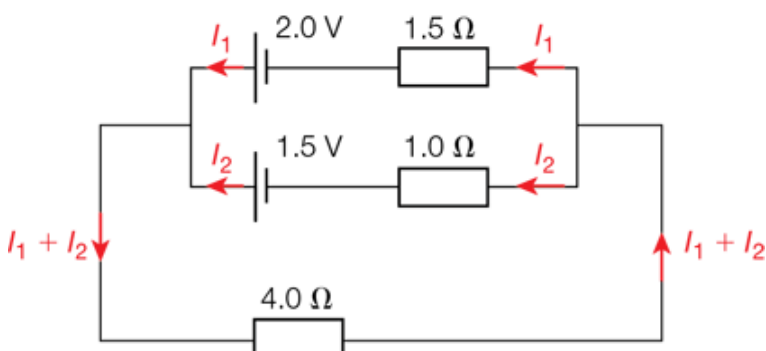


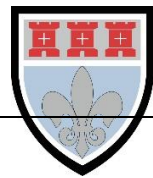
Figure 3

#### Step 2

To find  $I_1$  and  $I_2$  we need two equations as we have two unknowns. We first identify two loops.

Let loop 1 be the 'inside' loop of the circuit containing the 1.5 V cell and the  $4.0\ \Omega$  resistor.

Let loop 2 be the 'outside' loop of the circuit containing the 2 V cell and the  $4.0\ \Omega$  resistor

**Step 3**

Now write down a separate equation for each loop.

Since  $V = IR$  and the net emf in a loop is the sum of pds round the loop, you get:

*For loop 1*

net emf = pd across the internal resistor = pd across the  $4.0 \Omega$  resistor

$$1.5 = (I_2 \times 1.0) = (I_1 + I_2) \times 4.0$$

$$1.5 = 5.0 I_2 + 4.0 I_1 \quad (\text{Equation 1})$$

*For loop 2*

net emf = pd across the internal resistor = pd across the  $4.0 \Omega$  resistor

$$2 = (I_1 \times 1.5) + (I_1 + I_2) \times 4.0$$

$$2 = 5.5 I_1 + 4.0 I_2 \quad (\text{Equation 2})$$

**Step 4**

Solve the simultaneous equations to find  $I_1$  and  $I_2$ .

Rearrange Equation 1 for  $I_1$ .

$$I_1 = \frac{(1.5 - 5.0 I_2)}{4.0}$$

Substitute into Equation 2.

$$2 = 5.5 \frac{(1.5 - 5.0 I_2)}{4.0} + 4.0 I_2$$

$$2 = 2.0625 - 6.875 I_2 = 4.0 I_2$$

Rearrange to calculate  $I_2$ .

$$2.875 I_2 = 0.0625$$

$$I_2 = 0.022 \text{ A}$$

Substitute the value for  $I_2$  into Equation 1 to find  $I_1$ .

$$I_1 = \frac{1.5 - (5.0 \times 0.022)}{4.0}$$

$$= 0.348 \text{ A}$$

$$= 0.35 \text{ A (to two significant figures)}$$

**Tip**

When solving circuit problems, do not worry if one of your current values comes out as a negative number – this simply means you marked it on the diagram in the wrong direction. Remember to substitute the negative answer back into any further equations to calculate the other values.



## Questions

**A1.** Two cells are connected in parallel by joining the terminals together so that they both face in the same direction. One of the cells has emf 6.0 V and internal resistance 3.0  $\Omega$ , and the other cell has emf 4.0 V and internal resistance 2.0  $\Omega$ . An 8.0  $\Omega$  resistor is connected between the terminals.

**A1.1** Draw a circuit diagram.

[2 Marks]

**A1.2** Calculate the current passing through each cell.

[6 Marks]

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**A1.3** Calculate the current passing through the 8  $\Omega$  resistor.

[2 Marks]

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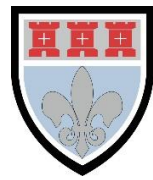
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**A1.4** Calculate the pd across the 8  $\Omega$  resistor.

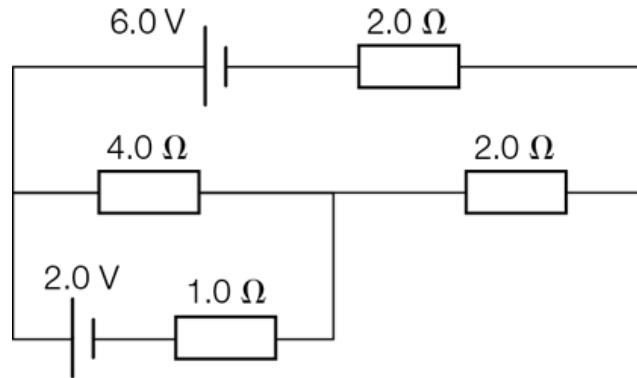
[2 Marks]

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**A2.** Calculate the currents passing through the two cells in the circuit shown in **Figure 4**.



**Figure 4**

**Hint:** the 2.0 Ω resistors are connected in series so have a combined resistance of 4.0 Ω. Compare this with the circuit in the worked example.

**[6 Marks]**

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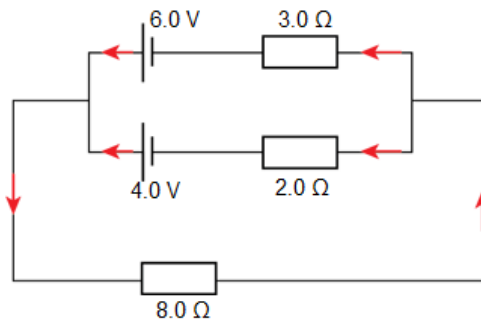
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## ANSWERS

**A1.1**



(2 marks)

**A1.2** current through the 6.0 V cell = 0.61 A to two significant figures (0.609 A)

current through the 4.0 V cell = 0.09 A to two significant figures (0.087 A)

(6 marks)

**A1.3** 0.70 A to two significant figures (0.696 A)

(2 marks)

**A1.4** 5.6 V to two significant figures (5.568 V)

(2 marks)

**A2.** current through 6 V cell = 0.92 A to two significant figures (0.915 A)

current through 2 V cell = (-)0.33 A to two significant figures

(6 marks)



## EXTENDED WRITING

To enhance your skills in answering examination questions for the A-Level examinations, complete the following task.

Describe how you would use a voltmeter, ammeter and other standard laboratory equipment to obtain accurate and reliable measurement of the e.m.f. and internal resistance of a battery. Your description should include:

- A labelled circuit diagram.
- Details of measurements you would take
- An account of how you would use your measurements to determine the values
- Details of how to improve the precision of your measurements and values.

## ANSWERING SPACE

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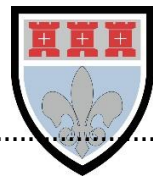
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# MARKING

<b>P</b>	<b>Praise. What were the positive aspects of the work? What did they do well? What skills did they demonstrate?</b>				
<b>I</b>	<b>Improvements. What were the literacy issues in the piece of work?</b>	<i>Write in ink.</i>		<i>Draw in Pencil.</i>	<i>Use a ruler.</i>
	Always use capital letters at the beginning of a sentence.			Learn the spellings identified in your work.	
	Always use capital letters for proper nouns.			Ensure sentences make sense.	
	Make sure you write on the line and not above or below it.			Use correct punctuation.	
	Use scientific vocabulary appropriate to the task.			Vary your sentences to demonstrate your understanding.	
<b>D answer</b>		<b>B answer</b>		<b>A* answer</b>	
The circuit diagram contains a voltmeter, ammeter, battery and external resistor...		...the voltmeter is connected in parallel <u>to the battery</u> and the ammeter in series...		...there is a switch and a variable resistor in series with the battery and ammeter.	
There is a description of how the external resistance will be changed either by replacing the external resistor or altering the variable resistor...		...with (between 5 and 10) values of external resistance (R)...		...to clearly show the relationship between voltmeter reading and ammeter reading.	
		The battery must be disconnected or switched off between readings...		...as this will change the value of the e.m.f./the battery will run down.	
For each value of external resistance (R) the current and voltage is measured (at least) three times...		...anomalous results are removed and a mean is calculated...		...accuracy is improved further by using a wide(r) range of values of R so a more detailed line (of best fit) can be drawn.	
The voltmeter reading is plotted (on the y axis) against the ammeter reading (on the x axis)...		...a (straight) line of best fit is drawn...		...and extrapolated (owtte).	
The e.m.f. is obtained from where the line (of best fit) intercepts (or cuts) the y axis...		...the internal resistance is given by the *gradient of the line (of best fit)		*some reference to 'negative' or the gradient giving $-r$ .	
The precision of the measurements could be improved by using a probe with smaller scale divisions connected to a data logger...		...the values of e.m.f. and internal resistance could be more precise by having smaller scale divisions on the graph...		...or using a computer program to draw the graph (previous statements needed for this to be awarded).	
<b>N</b>	<b>Next Steps. How can they move their work onto the next grade? What didn't they include?</b>			<b>Grade</b>	<b>Effort</b>



## PRACTICAL SKILLS

Complete the following practical skills-based task to improve your experimental understanding of this part of the course in preparation for the **Physics Paper 3 questions**.

### Experimental Data

Here is the experimental data collected from using different external resistors.

Resistance, $R$ ( $\Omega$ )	Terminal p.d., $V$ (V)			Current $I$ (A)			Mean p.d., $V$ (V)	Mean Current, $I$ (A)
1	1.23	1.18	1.22	1.07	1.14	1.09		
1.5	1.26	1.35	1.26	0.84	0.76	0.84		
2.2	1.33	1.34	1.31	0.57	0.55	0.59		
3.3	1.38	1.37	1.38	0.39	0.43	0.41		
4.7	1.39	1.40	1.40	0.32	0.29	0.29		
6.8	1.42	1.42	1.41	0.21	0.21	0.22		
10	1.43	1.45	1.44	0.14	0.14	0.14		
22	1.46	1.45	1.46	0.07	0.08	0.07		

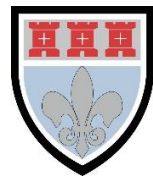
### Analysis

**A1.** Draw the circuit used in this investigation.

**A2.** Calculate the mean p.d. and current for each of the resistance values.

**A3.** What is the resolution of the voltmeter and ammeter?

.....  
 .....



**A4.** What is the independent variable in this investigation?

.....  
.....

**A5.** What is the dependent variable in this investigation?

.....  
.....

**A6.** Calculate the uncertainty in the largest mean value of p.d.

.....  
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**A7.** What is this as a percentage of the mean?

.....  
.....

**A8.** Calculate the uncertainty in the p.d. when the resistance was  $3.3\Omega$

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**A9.** What is this as a percentage of the mean?

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.....

**A10.** Calculate the uncertainty in the smallest mean value of current.

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**A11.** What is this as a percentage of the mean?

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.....

**A12.** Calculate the uncertainty in the current when the resistance was  $6.8\Omega$

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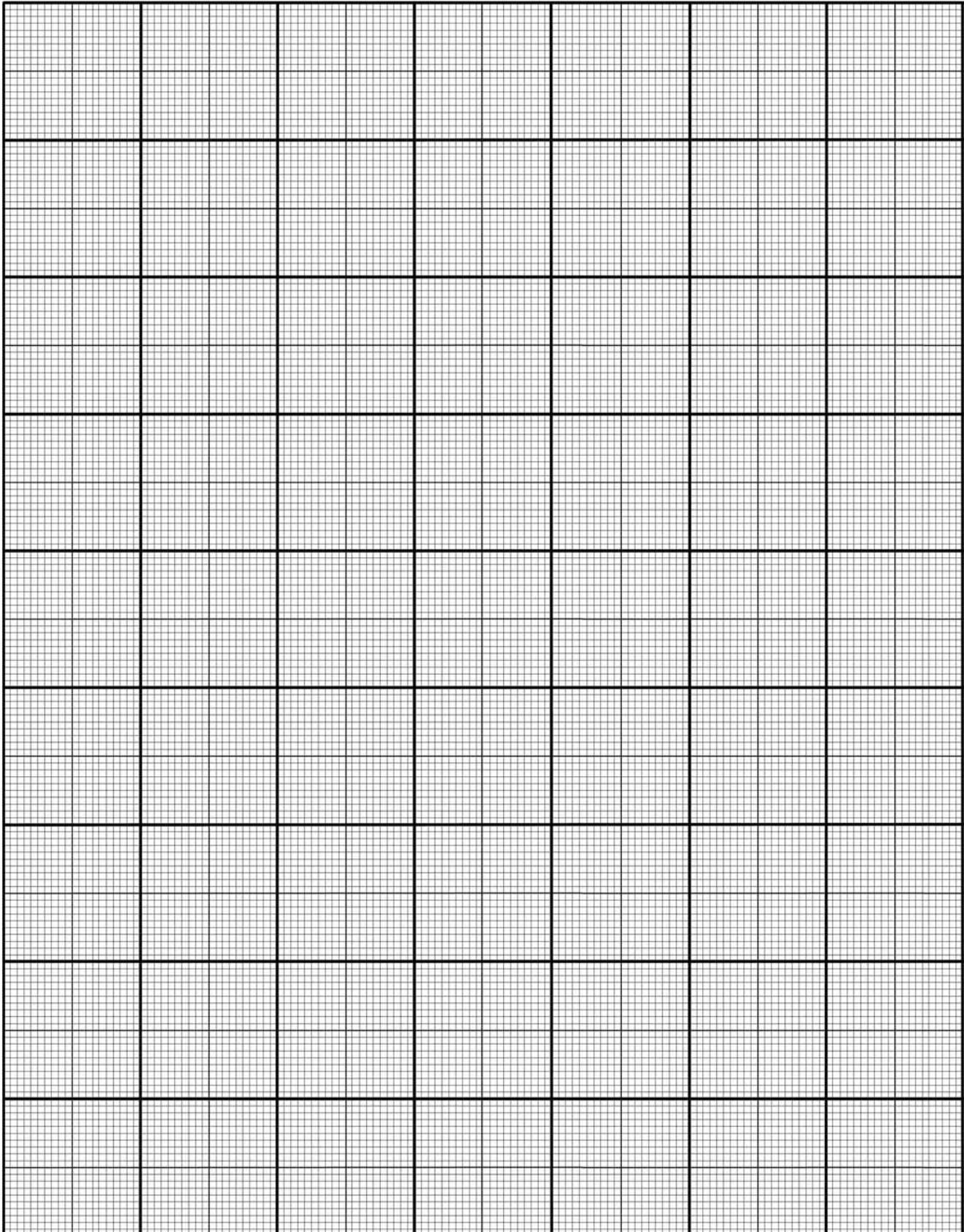


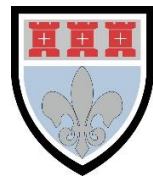
**A13.** What is this as a percentage of the mean?

.....

.....

**A14.** Plot a graph of  $V$  against  $I$  (on the x axis) and draw a line of best fit.





**A15.** If  $V$  and  $I$  are connected by the equation  $\varepsilon = V + Ir$ , what does the gradient of the line represent?

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**A16.** Calculate the gradient of your line of best fit.

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**A17.** What does the y-intercept of your line represent?

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**A18.** Write down the value of the y-intercept from your graph.

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**A19.** If the internal resistance was increased what effect would this have on your graph?

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## CHALLENGE QUESTION

To assess your understanding, answer the following higher-level question on this topic.

(a) (i) State Kirchhoff's first law.

.....  
 ..... [1]

(ii) Kirchhoff's first law is linked to the conservation of a certain quantity. State this quantity.

..... [1]

(b) A variable resistor of resistance  $R$  is used to control the current in a circuit, as shown in Fig. 5.1.

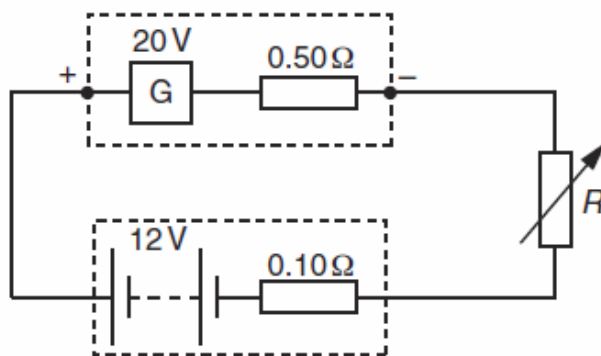


Fig. 5.1

The generator  $G$  has e.m.f.  $20\text{V}$  and internal resistance  $0.50\Omega$ . The battery has e.m.f.  $12\text{V}$  and internal resistance  $0.10\Omega$ . The current in the circuit is  $2.0\text{A}$ .

(i) Apply Kirchhoff's second law to the circuit to determine the resistance  $R$ .

$R = \dots\dots\dots \Omega$  [2]

(ii) Calculate the total power generated by  $G$ .

power =  $\dots\dots\dots \text{W}$  [2]



(iii) Calculate the power loss in the total resistance of the circuit.

power = ..... W [2]

(iv) The circuit is used to supply energy to the battery from the generator. Determine the efficiency of the circuit.

efficiency = ..... [2]

**Reference:** Cambridge International A Level Physics AS Unit 2 June 2012



## MARK SCHEME

- (a) (i) sum of currents into a junction = sum of currents out of junction B1 [1]
- (ii) charge B1 [1]
- (b) (i)  $\Sigma E = \Sigma IR$   
 $20 - 12 = 2.0(0.6 + R)$  (not used 3 resistors 0/2) C1  
 $R = 3.4 \Omega$  A1 [2]
- (ii)  $P = EI$  C1  
 $= 20 \times 2$   
 $= 40 \text{ W}$  A1 [2]
- (iii)  $P = I^2 R$  C1  
 $P = (2)^2 \times (0.1 + 0.5 + 3.4)$   
 $= 16 \text{ W}$  A1 [2]
- (iv) efficiency = useful power / output power C1  
 $24 / 40 = 0.6$  or  $12 \times 2 / 20 \times 2$  or 60% A1 [2]

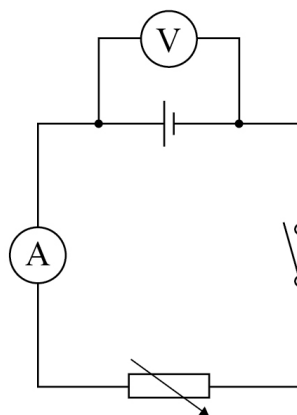


## CHALLENGE QUESTION

To assess your understanding, answer the following higher-level question on this topic.

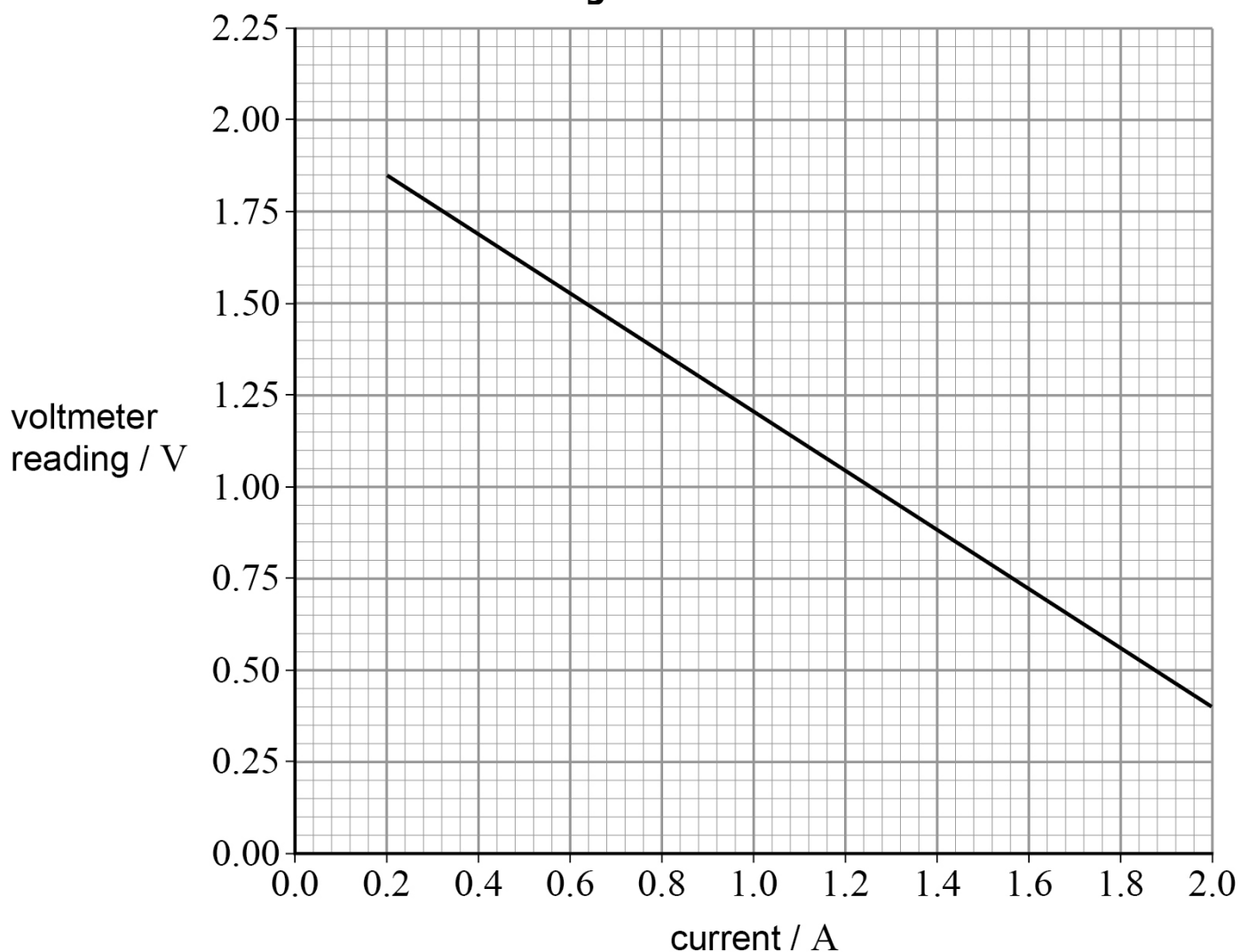
1. **Figure 3** shows a circuit used to determine the emf and internal resistance of a cell.

**Figure 3**



**Figure 4** shows the variation of the voltmeter reading with current in the circuit as the variable resistor is adjusted.

**Figure 4**





**1.1** The circuit contains an ideal voltmeter and an ideal ammeter.  
State the resistance of an ideal voltmeter.

**[1 mark]**

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**1.2** Show that the internal resistance of the cell is approximately  $0.8 \Omega$ .

**[1 mark]**

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**1.3** The variable resistor is adjusted until the current in the circuit is  $2.10 \text{ A}$ .  
Calculate the resistance of the variable resistor.

**[3 marks]**

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Resistance = .....  $\Omega$



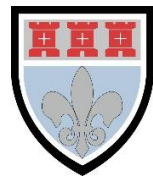
# MARK SCHEME

Question	Marking guidance	Mark	Comments
<b>1.1</b>	infinite ✓	1	Condone "very large"
<b>1.2</b>	Evidence of using graph to get gradient OR use of intercept (2 V) and a data point eg (1, 1.2) ✓	1	Must be a correct pair of coordinates.
<b>1.3</b>	$E = 2.00 \text{ (V)}$ ✓ Substitution into $E = I(R + r)$ (eg candidate's $E = 2.1(R + 0.8)$ ) ✓ $0.15(2) \text{ (}\Omega\text{)}$ ✓ OR Total $R = 2.00/2.1 = 0.95$ ✓ $R = 0.95 - 0.8$ ✓ $0.15(2) \text{ (}\Omega\text{)}$ ✓	3	Alternative method: Deduce pd correctly (0.32 V) and substitute into $R = V/I = 0.32/2.1$ Allow reasonable range for data extraction Allow 1 max for any extrapolation done by extending the grid and line but reward a "mathematical" extrapolation such as using similar triangles.



## REVISION CHECKLIST

Specification reference	Checklist questions	
3.5.1.4	Can you carry out calculations for resistors in series and in parallel?	<input type="checkbox"/>
3.5.1.4	Can you explain and use the energy and power equations: $E = Ivt$ and $P = IV = I^2R = \frac{V^2}{R}$ ?	<input type="checkbox"/>
3.5.1.4	Can you explain the relationships between currents, voltages and resistances in series and parallel circuits, including cells in series and identical cells in parallel?	<input type="checkbox"/>
3.5.1.4	Can you explain conservation of charge and conservation of energy in dc circuits?	<input type="checkbox"/>
3.5.1.5	Can you describe how the potential divider is used to supply constant or variable potential difference from a power supply?	<input type="checkbox"/>
3.5.1.5	Can you explain the use of variable resistors, thermistors, and light dependent resistors in the potential divider?	<input type="checkbox"/>
3.5.1.6	Can you use the formulae $\varepsilon = \frac{E}{Q}$ and $\varepsilon = I(R + r)$ ?	<input type="checkbox"/>
3.5.1.6	Can you explain terminal pd and emf?	<input type="checkbox"/>
3.5.1.6	Can you understand and perform calculations for circuits in which the internal resistance of the supply is not negligible?	<input type="checkbox"/>
3.5.1.6	Have you carried out an investigation into the emf and internal resistance of electric cells and batteries by measuring the variation of the terminal pd across the cell with the current in it?	<input type="checkbox"/>



## DATASHEET

### DATA - FUNDAMENTAL CONSTANTS AND VALUES

Quantity	Symbol	Value	Units
speed of light in vacuo	$c$	$3.00 \times 10^8$	$\text{m s}^{-1}$
permeability of free space	$\mu_0$	$4\pi \times 10^{-7}$	$\text{H m}^{-1}$
permittivity of free space	$\epsilon_0$	$8.85 \times 10^{-12}$	$\text{F m}^{-1}$
magnitude of the charge of electron	$e$	$1.60 \times 10^{-19}$	C
the Planck constant	$h$	$6.63 \times 10^{-34}$	J s
gravitational constant	$G$	$6.67 \times 10^{-11}$	$\text{N m}^2 \text{kg}^{-2}$
the Avogadro constant	$N_A$	$6.02 \times 10^{23}$	$\text{mol}^{-1}$
molar gas constant	$R$	8.31	$\text{J K}^{-1} \text{mol}^{-1}$
the Boltzmann constant	$k$	$1.38 \times 10^{-23}$	$\text{J K}^{-1}$
the Stefan constant	$\sigma$	$5.67 \times 10^{-8}$	$\text{W m}^{-2} \text{K}^{-4}$
the Wien constant	$\alpha$	$2.90 \times 10^{-3}$	m K
electron rest mass (equivalent to $5.5 \times 10^{-4}$ u)	$m_e$	$9.11 \times 10^{-31}$	kg
electron charge/mass ratio	$\frac{e}{m_e}$	$1.76 \times 10^{11}$	$\text{C kg}^{-1}$
proton rest mass (equivalent to 1.00728 u)	$m_p$	$1.67(3) \times 10^{-27}$	kg
proton charge/mass ratio	$\frac{e}{m_p}$	$9.58 \times 10^7$	$\text{C kg}^{-1}$
neutron rest mass (equivalent to 1.00867 u)	$m_n$	$1.67(5) \times 10^{-27}$	kg
gravitational field strength	$g$	9.81	$\text{N kg}^{-1}$
acceleration due to gravity	$g$	9.81	$\text{m s}^{-2}$
atomic mass unit (1u is equivalent to 931.5 MeV)	u	$1.661 \times 10^{-27}$	kg

### ALGEBRAIC EQUATION

quadratic equation  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

### ASTRONOMICAL DATA

Body	Mass/kg	Mean radius/m
Sun	$1.99 \times 10^{30}$	$6.96 \times 10^8$
Earth	$5.97 \times 10^{24}$	$6.37 \times 10^6$

### GEOMETRICAL EQUATIONS

arc length =  $r\theta$

circumference of circle =  $2\pi r$

area of circle =  $\pi r^2$

curved surface area of cylinder =  $2\pi r h$

area of sphere =  $4\pi r^2$

volume of sphere =  $\frac{4}{3}\pi r^3$



### Particle Physics

Class	Name	Symbol	Rest energy/MeV
photon	photon	$\gamma$	0
lepton	neutrino	$\nu_e$	0
		$\nu_\mu$	0
	electron	$e^\pm$	0.510999
	muon	$\mu^\pm$	105.659
mesons	$\pi$ meson	$\pi^\pm$	139.576
		$\pi^0$	134.972
	K meson	$K^\pm$	493.821
		$K^0$	497.762
baryons	proton	p	938.257
	neutron	n	939.551

### Properties of quarks

antiquarks have opposite signs

Type	Charge	Baryon number	Strangeness
<b>u</b>	$+\frac{2}{3}e$	$+\frac{1}{3}$	0
<b>d</b>	$-\frac{1}{3}e$	$+\frac{1}{3}$	0
<b>s</b>	$-\frac{1}{3}e$	$+\frac{1}{3}$	-1

### Properties of Leptons

		Lepton number
Particles:	$e^-, \nu_e; \mu^-, \nu_\mu$	+1
Antiparticles:	$e^+, \bar{\nu}_e, \mu^+, \bar{\nu}_\mu$	-1

### Photons and energy levels

*photon energy*  $E = hf = hc / \lambda$   
*photoelectricity*  $hf = \phi + E_{k(\max)}$   
*energy levels*  $hf = E_1 - E_2$   
*de Broglie wavelength*  $\lambda = \frac{h}{p} = \frac{h}{mv}$

### Waves

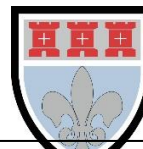
*wave speed*  $c = f\lambda$     *period*  $f = \frac{1}{T}$   
*first harmonic*  $f = \frac{1}{2l} \sqrt{\frac{T}{\mu}}$   
*fringe spacing*  $w = \frac{\lambda D}{s}$     *diffraction grating*  $d \sin \theta = n\lambda$   
*refractive index of a substance s*,  $n = \frac{c}{c_s}$   
*for two different substances of refractive indices  $n_1$  and  $n_2$* ,  
*law of refraction*  $n_1 \sin \theta_1 = n_2 \sin \theta_2$   
*critical angle*  $\sin \theta_c = \frac{n_2}{n_1}$  for  $n_1 > n_2$

### Mechanics

*moments*    **moment =  $Fd$**   
*velocity and acceleration*  $v = \frac{\Delta s}{\Delta t}$      $a = \frac{\Delta v}{\Delta t}$   
*equations of motion*  $v = u + at$      $s = \left(\frac{u+v}{2}\right)t$   
 $v^2 = u^2 + 2as$      $s = ut + \frac{at^2}{2}$   
*force*     $F = ma$   
*force*     $F = \frac{\Delta(mv)}{\Delta t}$   
*impulse*     $F \Delta t = \Delta(mv)$   
*work, energy and power*  $W = F s \cos \theta$   
 $E_k = \frac{1}{2} m v^2$      $\Delta E_p = mg\Delta h$   
 $P = \frac{\Delta W}{\Delta t}, P = Fv$   
*efficiency* =  $\frac{\text{useful output power}}{\text{input power}}$

### Materials

*density*  $\rho = \frac{m}{V}$     *Hooke's law*  $F = k \Delta L$   
*Young modulus* =  $\frac{\text{tensile stress}}{\text{tensile strain}}$     *tensile stress* =  $\frac{F}{A}$   
*tensile strain* =  $\frac{\Delta L}{L}$   
*energy stored*  $E = \frac{1}{2} F \Delta L$



## Electricity

current and pd  $I = \frac{\Delta Q}{\Delta t}$   $V = \frac{W}{Q}$   $R = \frac{V}{I}$

resistivity  $\rho = \frac{RA}{L}$

resistors in series  $R_T = R_1 + R_2 + R_3 + \dots$

resistors in parallel  $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$

power  $P = VI = I^2R = \frac{V^2}{R}$

emf  $\varepsilon = \frac{E}{Q}$   $\varepsilon = I(R + r)$

## Circular motion

magnitude of angular speed  $\omega = \frac{v}{r}$

$$\omega = 2\pi f$$

centripetal acceleration  $a = \frac{v^2}{r} = \omega^2 r$

centripetal force  $F = \frac{mv^2}{r} = m\omega^2 r$

## Simple harmonic motion

acceleration  $a = -\omega^2 x$

displacement  $x = A \cos(\omega t)$

speed  $v = \pm \omega \sqrt{(A^2 - x^2)}$

maximum speed  $v_{\max} = \omega A$

maximum acceleration  $a_{\max} = \omega^2 A$

for a mass-spring system  $T = 2\pi \sqrt{\frac{m}{k}}$

for a simple pendulum  $T = 2\pi \sqrt{\frac{l}{g}}$

## Thermal physics

energy to change temperature  $Q = mc\Delta\theta$

energy to change state  $Q = ml$

gas law  $pV = nRT$   
 $pV = NkT$

kinetic theory model  $pV = \frac{1}{3} N m (c_{\text{rms}})^2$

kinetic energy of gas molecule  $\frac{1}{2} m (c_{\text{rms}})^2 = \frac{3}{2} kT = \frac{3RT}{2N_A}$

## Gravitational fields

force between two masses  $F = \frac{Gm_1m_2}{r^2}$

gravitational field strength  $g = \frac{F}{m}$

magnitude of gravitational field strength in a radial field  $g = \frac{GM}{r^2}$

work done  $\Delta W = m\Delta V$

gravitational potential  $V = -\frac{GM}{r}$

$$g = -\frac{\Delta V}{\Delta r}$$

## Electric fields and capacitors

force between two point charges  $F = \frac{1}{4\pi\epsilon_0} \frac{Q_1Q_2}{r^2}$

force on a charge  $F = EQ$

field strength for a uniform field  $E = \frac{V}{d}$

work done  $\Delta W = Q\Delta V$

field strength for a radial field  $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$

electric potential  $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$

$$E = \frac{\Delta V}{\Delta r}$$

capacitance  $C = \frac{Q}{V}$

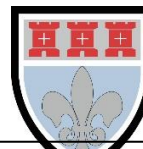
$$C = \frac{A\epsilon_0\epsilon_r}{d}$$

capacitor energy stored  $E = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C}$

capacitor charging  $Q = Q_0(1 - e^{-t/RC})$

decay of charge  $Q = Q_0 e^{-t/RC}$

time constant  $RC$



## Magnetic fields

<i>force on a current</i>	$F = BIl$
<i>force on a moving charge</i>	$F = BQv$
<i>magnetic flux</i>	$\Phi = BA$
<i>magnetic flux linkage</i>	$N\Phi = BAN \cos \theta$
<i>magnitude of induced emf</i>	$\varepsilon = N \frac{\Delta \Phi}{\Delta t}$
	$N\Phi = BAN \cos \theta$
<i>emf induced in a rotating coil</i>	$\varepsilon = BAN\omega \sin \omega t$
<i>alternating current</i>	$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \quad V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$
<i>transformer equations</i>	$\frac{N_s}{N_p} = \frac{V_s}{V_p}$
	$\text{efficiency} = \frac{I_s V_s}{I_p V_p}$

## Nuclear physics

<i>the inverse square law for <math>\gamma</math> radiation</i>	$I = \frac{k}{x^2}$
<i>radioactive decay</i>	$\frac{\Delta N}{\Delta t} = -\lambda N, N = N_0 e^{-\lambda t}$
<i>activity</i>	$A = \lambda N$
<i>half-life</i>	$T_{1/2} = \frac{\ln 2}{\lambda}$
<i>nuclear radius</i>	$R = R_0 A^{1/3}$
<i>energy-mass equation</i>	$E = mc^2$

## OPTIONS

### Astrophysics

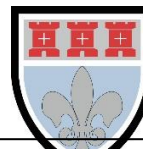
1 astronomical unit	$= 1.50 \times 10^{11} \text{ m}$
1 light year	$= 9.46 \times 10^{15} \text{ m}$
1 parsec	$= 206265 \text{ AU} = 3.08 \times 10^{16} \text{ m}$ $= 3.26 \text{ light year}$
Hubble constant, $H$	$= 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$

$$M = \frac{\text{angle subtended by image at eye}}{\text{angle subtended by object at unaided eye}}$$

<i>in normal adjustment</i>	$M = \frac{f_0}{f_e}$
<i>Rayleigh criterion</i>	$\theta \approx \frac{\lambda}{D}$
<i>magnitude equation</i>	$m - M = 5 \log \frac{d}{10}$
<i>Wien's law</i>	$\lambda_{\text{max}} T = 2.9 \times 10^{-3} \text{ m K}$
<i>Stefan's law</i>	$P = \sigma AT^4$
<i>Schwarzschild radius</i>	$R_s \approx \frac{2GM}{c^2}$
<i>Doppler shift for <math>v \ll c</math></i>	$\frac{\Delta f}{f} = -\frac{\Delta \lambda}{\lambda} = \frac{v}{c}$
<i>red shift</i>	$z = -\frac{v}{c}$
<i>Hubble's law</i>	$v = Hd$

### Medical physics

<i>lens equations</i>	$P = \frac{1}{f}$ $m = \frac{v}{u}$ $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$
<i>threshold of hearing</i>	$I_0 = 1.0 \times 10^{-12} \text{ W m}^{-2}$
<i>intensity level</i>	$\text{intensity level} = 10 \log \frac{I}{I_0}$
<i>absorption</i>	$I = I_0 e^{-\mu x}$ $\mu_m = \frac{\mu}{\rho}$
<i>ultrasound imaging</i>	$Z = p c$ $\frac{I_r}{I_i} = \left( \frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2$
<i>half-lives</i>	$\frac{1}{T_B} = \frac{1}{T_B} + \frac{1}{T_P}$



## Engineering physics

<i>moment of inertia</i>	$I = \Sigma mr^2$
<i>angular kinetic energy</i>	$E_k = \frac{1}{2} I \omega^2$
<i>equations of angular motion</i>	$\omega_2 = \omega_1 + \alpha t$ $\omega_2^2 = \omega_1^2 + 2\alpha\theta$ $\theta = \omega_1 t + \frac{\alpha t^2}{2}$ $\theta = \frac{(\omega_1 + \omega_2) t}{2}$
<i>torque</i>	$T = I \alpha$ $T = F r$
<i>angular momentum</i>	<i>angular momentum</i> = $I\omega$
<i>angular impulse</i>	$T\Delta t = \Delta(I\omega)$
<i>work done</i>	$W = T\theta$
<i>power</i>	$P = T\omega$
<i>thermodynamics</i>	$Q = \Delta U + W$ $W = p\Delta V$
<i>adiabatic change</i>	$pV^\gamma = \text{constant}$
<i>isothermal change</i>	$pV = \text{constant}$
<i>heat engines</i>	$\text{efficiency} = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H}$ $\text{maximum theoretical efficiency} = \frac{T_H - T_C}{T_H}$
<i>work done per cycle</i>	= <i>area of loop</i>
<i>input power</i>	= <i>calorific value</i> × <i>fuel flow rate</i>
<i>indicated power</i>	= ( <i>area of p - V loop</i> ) × ( <i>number of cycles per second</i> ) × ( <i>number of cylinders</i> )
<i>output or brake power</i>	$P = T\omega$
<i>friction power</i>	= <i>indicated power</i> - <i>brake power</i>
<i>heat pumps and refrigerators</i>	
<i>refrigerator:</i>	$COP_{\text{ref}} = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C}$
<i>heat pump:</i>	$COP_{\text{hp}} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_C}$

## Turning points in physics

<i>electrons in fields</i>	$F = \frac{eV}{d}$ $F = Bev$ $r = \frac{mv}{Be}$ $\frac{1}{2} mv^2 = eV$
<i>Millikan's experiment</i>	$\frac{QV}{d} = mg$ $F = 6\pi\eta r v$
<i>Maxwell's formula</i>	$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$
<i>special relativity</i>	$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ $l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$ $E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$

## Electronics

<i>resonant frequency</i>	$f_0 = \frac{1}{2\pi\sqrt{LC}}$
<i>Q-factor</i>	$Q = \frac{f_0}{f_B}$
<i>operational amplifiers: open loop</i>	$V_{\text{out}} = A_{\text{OL}}(V_+ - V_-)$
<i>inverting amplifier</i>	$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_f}{R_{\text{in}}}$
<i>non-inverting amplifier</i>	$\frac{V_{\text{out}}}{V_{\text{in}}} = 1 + \frac{R_f}{R_1}$
<i>summing amplifier</i>	$V_{\text{out}} = -R_f \left( \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \dots \right)$
<i>difference amplifier</i>	$V_{\text{out}} = (V_+ - V_-) \frac{R_f}{R_1}$
<i>Bandwidth requirement:</i>	
<i>for AM</i>	<i>bandwidth</i> = $2f_M$
<i>for FM</i>	<i>bandwidth</i> = $2(\Delta f + f_M)$



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