A LEVEL PHYSICS YEAR 1
STUDENT CLASS BOOK
3.1: MEASUREMENTS AND ERRORS

NAME	
PHYSICS CLASS	
MODULE TEACHER	
ALPS GRADE	

A-LEVEL PHYSICS

TOPIC 1

CLASS WORKBOOK

THIS MUST BE BROUGHT TO ALL PHYSICS LESSONS.

Contents

OVERVIEW

Content in this section is a continuing study for a student of physics. A working knowledge of the specified fundamental (base) units of measurement is vital.

Likewise, practical work in the subject needs to be underpinned by an awareness of the nature of measurement errors and of their numerical treatment. The ability to carry through reasonable estimations is a skill that is required throughout the course and beyond.

IMPORTANT NOTE

This booklet, along with the preparatory reading notes, must be brought to all Physics lessons with the appropriate teacher.

This booklet may be used as a learning resource in lessons, you are not fully equipped to learn if this is not used in lesson.

This booklet may also be used as a revision resource for intervention, internal assessments and external assessments.

Please keep this in your student file.

There are several activities in this book which may not be covered in lessons.

It is advised that students complete these activities outside of lessons as revision aides.

The Language of Measurement

The following subject specific vocabulary provides definitions of key terms used in the A-level Science specifications.

Accuracy
A measurement result is considered accurate if it is judged to be close to the true value.

Calibration
Marking a scale on a measuring instrument.
This involves establishing the relationship between indications of a measuring instrument and standard or reference quantity values, which must be applied.
For example, placing a thermometer in melting ice to see whether it reads 0 °C, to check if it has been calibrated correctly.

Data
Information, either qualitative or quantitative, that has been collected.

Errors
See also uncertainties.

Measurement error
The difference between a measured value and the true value.
anomalies
These are values in a set of results which are judged not to be part of the variation caused by random uncertainty.

Random error
These cause readings to be spread about the true value, due to results varying in an unpredictable way from one measurement to the next.
Random errors are present when any measurement is made and cannot be corrected. The effect of random errors can be reduced by making more measurements and calculating a new mean.

Systematic error
These cause readings to differ from the true value by a consistent amount each time a measurement is made.
Sources of systematic error can include the environment, methods of observation or instruments used.
Systematic errors cannot be dealt with by simple repeats. If a systematic error is suspected, the data collection should be repeated using a different technique or a different set of equipment, and the results compared.

Zero error
Any indication that a measuring system gives a false reading when the true value of a measured quantity is zero, e.g. the needle on an ammeter failing to return to zero when no current flows.
A zero error may result in a systematic uncertainty.

Evidence
Data which has been shown to be valid.

Fair test
A fair test is one in which only the independent variable has been allowed to affect the dependent variable.

Hypothesis
A proposal intended to explain certain facts or observations.

Interval
The quantity between readings, e.g. a set of 11 readings equally spaced over a distance of 1 metre would give an interval of 10 centimetres.

Precision
Precise measurements are ones in which there is very little spread about the mean value.
Precision depends only on the extent of random errors – it gives no indication of how close results are to the true value.

Prediction
A prediction is a statement suggesting what will happen in the future, based on observation, experience or a hypothesis.

Range
The maximum and minimum values of the independent or dependent variables; important in ensuring that any pattern is detected.
For example, a range of distances may be quoted as either:
'From 10 cm to 50 cm'
or
'From 50 cm to 10 cm'

Repeatable
A measurement is repeatable if the original experimenter repeats the investigation using same method and equipment and obtains the same results.

Reproducible
A measurement is reproducible if the investigation is repeated by another person, or by using different equipment or techniques, and the same results are obtained.

Resolution
This is the smallest change in the quantity being measured (input) of a measuring instrument that gives a perceptible change in the reading.

Sketch graph
A line graph, not necessarily on a grid, that shows the general shape of the relationship between two variables. It will not have any points plotted and although the axes should be labelled they may not be scaled.

True value
This is the value that would be obtained in an ideal measurement.

Uncertainty
The interval within which the true value can be expected to lie, with a given level of confidence or probability, e.g. "the temperature is 20 °C ± 2 °C, at a level of confidence of 95%.

Validity
Suitability of the investigative procedure to answer the question being asked. For example, an investigation to find out if the rate of a chemical reaction depended upon the concentration of one of the reactants would not be a valid procedure if the temperature of the reactants was not controlled.

Valid conclusion
A conclusion supported by valid data, obtained from an appropriate experimental design and based on sound reasoning.

Variables
These are physical, chemical or biological quantities or characteristics.

Categoric variables
Categoric variables have values that are labels. E.g. names of plants or types of material.

Continuous variables
Continuous variables can have values (called a quantity) that can be given a magnitude either by counting (as in the case of the number of shrimp) or by measurement (e.g. light intensity, flow rate etc.).

Control variables
A control variable is one which may, in addition to the independent variable, affect the outcome of the investigation and therefore must be kept constant or at least monitored.

Dependent variables
The dependent variable is the variable of which the value is measured for each change in the independent variable.

Independent variables
The independent variable is the variable for which values are changed or selected by the investigator.

IMPORTANT NOTE
These definitions must be memorised by students.

You will be tested on your knowledge of these definitions.

TOPIC 1: SI UNITS AND PREFIXES
SPEC CHECK

Specification	Completed?
Fundamental (base) units.	
Use of mass, length, time, quantity of matter, temperature, electric current and their associated SI units.	
SI units derived.	
Knowledge and use of the SI prefixes, values and standard form.	
Students should be able to use the prefixes: T, G, M, k, c, m, μ, n, p, f,	
Students should be able to convert between different units of the same quantity, e.g. J and eV, J and kW h.	

NOTES
Base Quantities and SI Units

In 1971 world's scientists agreed on a common system of units for those 7 base quantities.

They are the SI Units. These are the fundamental quantities and units of the Universe.

These are....

Base Quantity	SI Unit & Abbreviation
Length	metre, m
Time	second, s
Mass	kilogram, kg
Temperature	kelvin, K
Electric current	ampere, A
Amount of substance	mole, mol
Luminous Intensity	candela, cd

All the other physical quantities used in Physics are **DERIVED** from these base quantities.

These called the **SI derived units.**

You must be able to express all units in SI unit terms.

Example 1: $Units\ of\ volume = unit\ of\ (length \times length \times length)$

$$= m \times m \times m$$

$$= m^3$$

This is the unit of volume in SI base units.

Example 2: $Units\ of\ density = \frac{unit\ of\ mass}{unit\ of\ volume}$

$$= \frac{kg}{m^3}$$

$$= kgm^{-3}$$

This is the unit of density in SI base units.

Example 3: $Units\ of\ velocity = \frac{unit\ of\ displacement}{unit\ of\ time}$

$$= \frac{m}{s}$$

$$= ms^{-1}$$

This is the unit of velocity in SI base units.

Below are examples of SI derived units.

Examples of SI derived units		
Derived quantity	**SI derived unit**	
	Name	**Symbol**
Area (A)	square meter	m^2
Volume (V)	cubic meter	m^3
speed, velocity (v)	meter per second	m/s
Acceleration (a)	meter per second squared	m/s^2
mass density (ρ)	kilogram per cubic meter	kg/m^3
amount-of-substance concentration (n)	mole per cubic meter	mol/m^3

These are all units which are composed of SI base units.

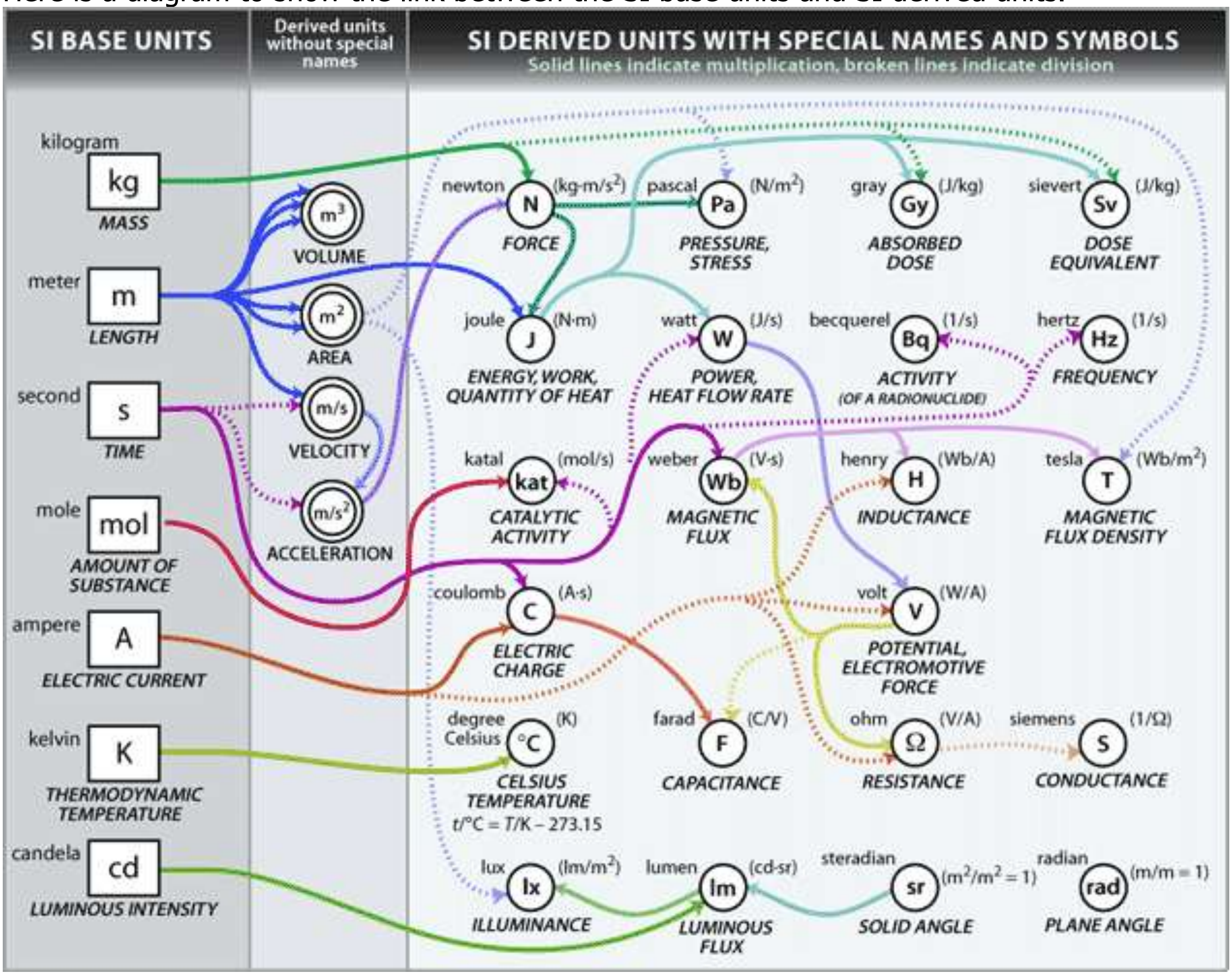
Below are examples of SI derived units which have special names.

Derived quantity	**Name**	**Symbol**	**Expression in terms of other SI units**	**Expression in terms of SI base units**
plane angle (θ)	radian	rad	-	$m\ m^{-1} = 1$
Frequency (f)	Hertz	Hz	-	s^{-1}
Force (F)	Newton	N	-	$mkgs^{-2}$
pressure, stress (P)	Pascal	Pa	N/m^2	$m^{-1}kgs^{-2}$
energy, work, quantity of heat (E or w)	Joule	J	N·m	m^2kgs^{-2}
Power (P)	Watt	W	J/s	m^2kgs^{-3}
electric charge, quantity of electricity (Q)	Coulomb	C	-	sA
electric potential difference (V), electromotive force (ε)	Volt	V	W/A	$m^2kgs^{-3}A^{-1}$
Capacitance (C)	Farad	F	C/V	$m^{-2}kg^{-1}s^4A^2$
electric resistance (R)	Ohm	Ω	V/A	$m^2kgs^{-3}A^{-2}$
magnetic flux (φ)	Weber	Wb	V·s	$m^2kgs^{-2}A^{-1}$
magnetic flux density (B)	Tesla	T	Wb/m^2	$kgs^{-2}A^{-1}$
Celsius temperature (T)	degree Celsius	°C	-	K
activity (of a radionuclide) (A)	Becquerel	Bq	-	s^{-1}

Below are examples of SI derived units based on other SI derived units.

Examples of SI derived units whose names and symbols include SI derived units with special names and symbols		
	SI derived unit	
Derived quantity	**Name**	**Symbol**
moment of force (M)	newton meter	Nm
angular velocity	radian per second	rad/s
angular acceleration	radian per second squared	rad/s^2
heat flux density, irradiance	watt per square meter	W/m^2
heat capacity, entropy	joule per kelvin	J/K
specific heat capacity, specific entropy	joule per kilogram kelvin	J/(kgK)
electric field strength	volt per meter	V/m
permittivity	farad per meter	F/m
molar energy	joule per mole	J/mol
molar entropy, molar heat capacity	joule per mole kelvin	J/(molK)

Here is a diagram to show the link between the SI base units and SI derived units.



Standard Prefixes

One of the strengths of the SI is that absolutely any measurement can be expressed in terms of the seven base units (and angle if needed).

Thanks to the SI and a well-organised set of agreements and procedures to make it work, everyone all over the world can not only agree just how fast their cars are, they can make sure their components fit together properly too.

It would be inconvenient to measure everything using only the pure base or derived units - motorists don't want road distance in metres or astronomers do not want stellar distances in metres. A shorthand system of prefixes was agreed as part of the SI system.

For example:

10 metres =1 decametre 10 decametres = 1 hectometre 10 hectometres = 1 kilometre.

All the prefixes are related to each other by numbers like 10, 100 or 1,000; which are called powers of 10.

Throughout the SI system, the same prefixes are used for the same multiples, no matter what the unit - except for the kilogram. Because the kilogram already includes a prefix in its name we don't refer to a thousandth of a kilogram as a millikilogram.

Some of these units aren't used much – decimetres, for example, are uncommon, and megametres are practically unheard of.

And the prefixes aren't used for time, where there was already an internationally agreed system in place long before the SI came along.

Here is the conversion for time units.

Name	Symbol	Quantity	Equivalent SI unit
minute	min	time	1 min = 60 s
hour	h	time	1 h = 3600 s
day	d	time	1 d = 86400 s

Time is the one part of the SI system not considered for prefixes.

Here are the conversion prefixes used in Physics.
You must be able to use these prefixes in any Physics question.

Prefix	Symbol	Decimal	Power of 10
yotta	Y	1000000000000000000000000	10^{24}
zetta	Z	1000000000000000000000	10^{21}
exa	E	1000000000000000000	10^{18}
peta	P	1000000000000000	10^{15}
tera	T	1000000000000	10^{12}
giga	G	1000000000	10^{9}
mega	M	1000000	10^{6}
kilo	k	1000	10^{3}
hecto	h	100	10^{2}
deca	da	10	10^{1}
deci	d	0.1	10^{-1}
centi	c	0.01	10^{-2}
milli	m	0.001	10^{-3}
micro	µ	0.000001	10^{-6}
nano	n	0.000000001	10^{-9}
pico	p	0.000000000001	10^{-12}
femto	f	0.000000000000001	10^{-15}
atto	a	0.000000000000000001	10^{-18}
zepto	z	0.000000000000000000001	10^{-21}
yocto	y	0.000000000000000000000001	10^{-24}

Exam Tip

In any examination question, you should substitute the prefix for the standard form conversion given in the table above.

For example, 20km = 20 x 10^3 m

Here are the conversion for angular measurements.

Name	Symbol	Quantity	Equivalent SI unit
degree of arc	°	angle	1° = (π/180) rad
minute of arc	′	angle	1π = (π/10800) rad
second of arc	″	angle	1π = (π/648000) rad
hectare	ha	area	1 ha = 10000 m^2
litre	l or L	volume	1 l = 0.001 m^3
tonne	t	mass	1 t = 1000 kg

Significant Figures

In Physics, you cannot write an answer to a problem with all the numbers given on the calculator.

A method of giving an approximated answer is to round off using significant figures.

The word significant means: having meaning.

With the number 368249, the 3 is the most significant digit, because it tells us that the number is 3 hundred thousand and something. It follows that the 6 is the next most significant, and so on.

With the number 0.0000058763, the 5 is the most significant digit, because it tells us that the number is 5 millionths and something. The 8 is the next most significant, and so on.

Be careful however with numbers such as 30245, the 3 is the first significant figure and 0 the second, because of its value as a place holder.

We round off a number using a certain number of significant figures. The most common are 1, 2 or 3 significant figures.

Remember the rules for rounding up are:

If the next number is 5 or more, we round up. If the next number is 4 or less, we do not round up.

Examples

E1. What would you get if you wrote the number 368249 correct to 1 significant figure?

400000

3 is the first significant figure, and the digit after it is more than 5, so you round up.

E2. What would you get if you wrote the number 0.00245 correct to 1 significant figure?

0.002

2 is the first significant figure and the digit after this is less than 5, so you do not round up.

E3. What would you get if you wrote 0.0000058763 correct to 2 significant figures?

0.0000059

You had to round up the 8 to 9.

E4. What is 7.994 to two significant figures?

8.0

You had to round up.

The 2 first significant figures are 7 and 9. The digit after 9 is 9 again, so we have to round up, 7.99 rounds up to 8.00.

Examination Questions

In an examination question, you should give your answer to the same number of significant figures as the value given in the question with the lowest number of significant figures.

You do not include constants given in the data booklet as a value.

If it is vague as to the number of significant figures in the question – there will be allowance in the mark scheme for this.

Example

0 2 . 3 Calculate the maximum kinetic energy, in J, of the electrons emitted from a zinc plate when illuminated with ultraviolet light.

work function of zinc = 4.3 eV
frequency of ultraviolet light = 1.2×10^{15} Hz

[3 marks]

maximum kinetic energy ________________ J

In this question, as both values are given to two significant figures, you should give your answer to two significant figures.

If you do not do this, you are penalised one mark.

Practical Work

Data should be written in tables to the same number of significant figures. This number should be determined by the resolution of the device being used to measure the data or the uncertainty in measurement. For example, a length of string measured to be 60 cm using a ruler with mm graduations should be recorded as 600 mm, 60.0 cm or 0.600 m, and **not** just 60 cm. Similarly, a resistor value quoted by the manufacturer as 56 kΩ, 5% tolerance should **not** be recorded as 56.0 kΩ.

There is sometimes confusion over the number of significant figures when readings cross multiples of 10. Changing the number of decimal places across a power of ten retains the number of significant figures **but changes the accuracy.** The same number of decimal places should therefore generally be used, as illustrated below.

0.97
0.98
0.99
1.00
1.10

99.7
99.8
99.9
100.0
101.0

It is good practice to write down all digits showing on a digital meter.

Calculated quantities should be shown to the number of significant figures of the data with the least number of significant figures.

Example:

Calculate the size of an object if the magnification of a photo is ×25 and it is measured to be 24.6 mm on the photo.

$$size\ of\ real\ object = \frac{size\ of\ image}{magnification}$$

$$size\ of\ real\ object = \frac{24.6 \times 10^{-3}}{25}$$

$$size\ of\ real\ object = 9.8 \times 10^{-4}$$

Note that the size of the real object can only be quoted to two significant figures as the magnification is only quoted to two significant figures.

Equipment measuring to half a unit (e.g. a thermometer measuring to 0.5 °C) should have measurements recorded to one decimal place (e.g. 1.0 °C, 2.5 °C). The uncertainty in these measurements would be ±0.25, but this would be rounded to the same number of decimal places (giving measurements quoted with uncertainty of (1.0 ± 0.3) °C etc).

Dealing with anomalous results

At GCSE, you are often taught automatically to ignore anomalous results.

At A-level, you should think carefully about what could have caused the unexpected result and therefore whether it is anomalous.

You might be able to identify a reason for the unexpected result and so validly regard it as an anomaly. For example, an anomalous result might be explained by a different experimenter making the measurement, a different solution or a different measuring device being used. In the case where the reason for an anomalous result occurring can be identified, the result should be recorded and plotted but may then be ignored.

Anomalous results should also be ignored where results are expected to be the same.

Where there is no obvious error and no expectation that results should be the same, anomalous results should be included. This will reduce the possibility that a key point is being overlooked.

Please note: when recording results, it is important that all data are included. Anomalous results should only be ignored at the data analysis stage.

It is best practice whenever an anomalous result is identified for the experiment to be repeated. This highlights the need to tabulate and even graph results as an experiment is carried out.

Mean Average

When calculating the mean average of a value in Physics, you must remove the anomalous result before the calculation.

The mean average should be given to the same number of significant figures as the value with the least number of significant figures in the calculation.

Example

1. Four results were taken.

12cm, 13cm, 12cm, 11cm.

What is the mean?

$$\text{Mean} = \frac{\text{Sum of Values}}{\text{Number of Values}} = \frac{(12 + 13 + 12 + 11)}{4} = 12\text{cm}$$

2. Four results were taken.

12cm, 13cm, 19cm, 11cm.

What is the mean?

$$\text{Mean} = \frac{\text{Sum of Values}}{\text{Number of Values}} = \frac{(12 + 13 + 11)}{3} = 12\text{cm}$$

Additional Note Space

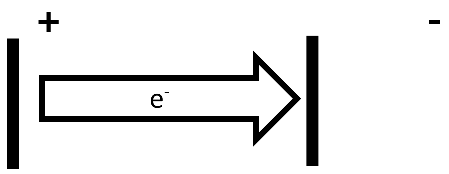
Additional Note Space

SAMPLE QUESTION

The Electron Volt (eV)

The electron volt is a unit of work (or energy), much smaller than a Joule.

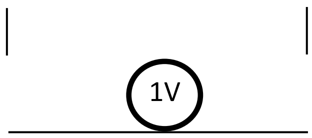
Imagine that 1 electron is moved across a potential difference of 1 Volt:



The work done to move the e^- to the negative plate can be calculated using

$$\boldsymbol{w} = \boldsymbol{VQ} = 1V \times 1.60 \times 10^{-19}C$$

$$= 1.60 \times 10^{-19}J$$



Thus if it takes 1J of work to move 1C across 1V

Then it takes 1.6×10^{-19}J to move $1e^-$ across 1V

This unit of energy is given the name **1eV**

$$\mathbf{1eV = 1.60 \times 10^{-19}J}$$

To convert energy in J to energy in eV, use the following:

$$\boldsymbol{energy\ in\ eV} = \frac{\boldsymbol{energy\ in\ Joules}}{\mathbf{1.60 \times 10^{-19}}}$$

The Kilowatt-Hour (kWh)

The kilowatt-hour (symbolized kWh) is a unit of energy equivalent to one kilowatt (1 kW) of power expended for one hour (1 h) of time. The kilowatt-hour is not a standard unit in any formal system, but it is commonly used in electrical applications and for buying electricity for our homes.

We have seen that $energy(J) = power(W) \times time(s)$

Since $1kW = 1{,}000W \ and \ 1h = 3{,}600s$

Then $energy(kWh) = power(kW) \times time(h)$

$$1kWh = 1{,}000Js^{-1} \times 3600s$$

$$\mathbf{1kWh = 3{,}600{,}000J}$$

REVISION SHEET

Highlight the key information.

Significant Figures Can Show *Uncertainty*

1) You always have to assume the **largest** amount of uncertainty in data.
2) Whether you're looking at experimental results or just doing a calculation question in an exam, you must round your results to have the **same number** of significant figures as the given data value with the **least** significant figures. Otherwise you'd be saying there is less uncertainty in your result than in the data used to calculate it.
3) If no uncertainty is given for a value, the number of **significant figures** a value has gives you an estimate of the **uncertainty**. For example, 2 N only has **1 significant figure**, so without any other information you know this value must be 2 **± 0.5 N** — if the value was less than 1.5 N it would have been rounded to 1 N (to 1 s.f.), if it was greater than 2.5 N it would have been rounded to 3 N (to 1 s.f.).

Reference: CGP Revision Guide

SKILLS TEST
PREFIX TEST

In Physics, we have to deal with quantities from the very large to the very small. A prefix is something that goes in front of a unit and acts as a multiplier.

Symbol	Name	What it means		How to convert	
P	peta	10^{15}	1000000000000000		↓ x1000
T	tera	10^{12}	1000000000000	↑ ÷ 1000	↓ x1000
G	giga	10^{9}	1000000000	↑ ÷ 1000	↓ x1000
M	mega	10^{6}	1000000	↑ ÷ 1000	↓ x1000
k	kilo	10^{3}	1000	↑ ÷ 1000	↓ x1000
			1	↑ ÷ 1000	↓ x1000
m	milli	10^{-3}	0.001	↑ ÷ 1000	↓ x1000
μ	micro	10^{-6}	0.000001	↑ ÷ 1000	↓ x1000
n	nano	10^{-9}	0.000000001	↑ ÷ 1000	↓ x1000
p	pico	10^{-12}	0.000000000001	↑ ÷ 1000	↓ x1000
f	femto	10^{-15}	0.000000000000001	↑ ÷ 1000	

Convert the figures into the prefixes required.

s	ms	μs	ns	ps
134.6				
96.21				
0.773				

m	km	mm	Mm	Gm
12873				
0.295				
57.23				

kg	Mg	mg	g	Gg
94.76				
0.000765				
823.46				

A	mA	μA	nA	kA
0.000000678				
3.56				
0.00092				

MEAN TEST

For each set of values calculate the mean and then calculate the mean ignoring any anomalous results.

1	2	3	Mean	
4152	2996	4018		
935.5	925.8	926.7		
16.2	19.1	17.4		
80.1316	80.1324	80.1466		
2229	2011	1610		
127.664	127.416	127.489		
55.88	11.97	37.59		
3.767	3.763	3.751		
375.5	511.5	463.4		
1048	888	1655		
0.507	0.415	0.230		
27145	25157	26017		
1450	1014	2238		
9104.32	10529.45	9160.97		

1	2	3	4	Mean	
63.10	62.97	62.53	62.99		
465.98	463.40	466.96	155.56		
3.61	7.39	3.55	3.64		
73.71	70.98	74.19	72.38		
2.058	1.566	2.078	1.787		
416	402	189	986		
700653	739762	742471	726161		
2670887	2670901	2669942	2670733		
110.4	260.1	1044.2	488.8		

1	2	3	4	5	Mean	
140	220	90	180	140		
56300	41200	58600	48300	53800		
0.186	0.341	0.276	0.216	0.314		
1.427	0.235	0.488	1.922	1.620		
34	62	46	12	39		
326.19	360.22	314.20	352.22	400.18		
1.4	5.3	2.7	3.9	2.6		

SIGNIFICANT FIGURES TEST

For each value state, how many significant figures it is stated to.

Value	Sig Figs	Value	Sig Figs	Value	Sig Figs	Value	Sig Figs
2		1066		1800.45		0.07	
2.0		82.42		2.483×10^4		69324.8	
2.00		750000		2.483		0.0063	
0.136		310		5906.4291		9.81×10^4	
0.34		3.10×10^2		200000		6717	
54.1		3.1×10^2		12.711		0.91	

Add the values below then write the answer to the appropriate number of significant figures

Value 1	Value 2	Value 3	Total Value	Total to correct sig figs
51.4	1.67	3.23		
7146	–32.54	12.8		
20.8	18.72	0.851		
1.4693	10.18	–1.062		
9.07	0.56	3.14		
739762	26017	2.058		
8.15	0.002	106		
132.303	4.123	53800		
152	0.8	0.55		
0.1142	4922388	132000		

Multiply the values below then write the answer to the appropriate number of significant figures

Value 1	Value 2	Total Value	Total to correct sig figs
0.91	1.23		
8.764	7.63		
2.6	31.7		
937	40.01		
0.722	634.23		

Divide value1 by value 2 then write the answer to the appropriate number of significant figures

Value 1	Value 2	Total Value	Total to correct sig figs
5.3	748		
3781	6.434		
91×10^2	180		
5.56	22×10^{-3}		
3.142	8.314		

For each value state how many significant figures it is stated to.

Value	Sig Figs	Value	Sig Figs	Value	Sig Figs	Value	Sig Figs
2.863		689671.49		100000		6.4981×10^{7}	
100		356865		8.5×10^{-3}		7.85	
24.92		13		6400		17.99	
5.18×10^{27}		182.15		875.4		3.189×10^{6}	
2.8		4.267		94		0.053	
2.9970		0.02		94.0		0.422	

Calculate the mean of the values below then write the answer to the appropriate number of significant figures

Value 1	Value 2	Value 3	Mean Value	Mean to correct sig figs
1	1	2		
435	299	4130		
500	600	900		
3.038	4.925	3.6		
720	498	168		
1655	2996	140		
0.230	925.8	56300		
26017	19.1	0.186		
2238	80.1324	1.427		
9160.97	2011	34		
62.99	127.416	326.19		
155.56	11.97	1.4		
3.64	3.763	700653		
72.38	511.5	2670887		
1.787	888	110.4		
986	0.415	62.97		
726161	25157	463.40		
2670733	1014	7.39		
488.8	10529.45	70.98		
0.186	140	1.566		
1.427	53800	402		
34	0.314	739762		
326.19	1.620	2670901		
1.4	39	260.1		

SI UNITS TEST

Write down the standard (or commonly used) units for the following quantities.

Energy
Wavelength
Frequency
Current
Potential Difference
Resistance
EMF
Resistivity
Power
Moment
Velocity
Acceleration
Mass
Weight
Force
Work Done
Density
Tensile Strength
Tensile Stress
Refractive Index
Momentum
Impulse
Angular Displacement
Angular Speed
Time Period
Gravitational Field Strength
Gravitational Potential
Electric Field Strength
Electric Potential
Capacitance
Time Constant
Energy Stored
Force on a Moving Charge
Magnetic Flux
Magnetic Flux Density
Magnetic Flux Linkage
Induced EMF
Intensity
Activity
Half Life
Pressure
Volume
Temperature
Specific Heat Capacity
Specific Latent

Write down the SI derived units for the following quantities.

Energy
Wavelength
Frequency
Current
Potential Difference
Resistance
EMF
Resistivity
Power
Moment
Velocity
Acceleration
Mass
Weight
Force
Work Done
Density
Tensile Strength
Tensile Stress
Refractive Index
Momentum
Impulse
Angular Displacement
Angular Speed
Time Period
Gravitational Field Strength
Gravitational Potential
Electric Field Strength
Electric Potential
Capacitance
Time Constant
Energy Stored
Force on a Moving Charge
Magnetic Flux
Magnetic Flux Density
Magnetic Flux Linkage
Induced EMF
Intensity
Activity
Half Life
Pressure
Volume
Temperature
Specific Heat Capacity
Specific Latent

TOPIC 2: LIMITS OF MEASUREMENTS
SPEC CHECK

Specification	Completed?
Random and systematic errors.	
Precision, repeatability, reproducibility, resolution and accuracy.	
Absolute, fractional and percentage uncertainties represent uncertainty in the final answer for a quantity.	
Combination of absolute and percentage uncertainties.	
Represent uncertainty in a data point on a graph using error bars.	
Determine the uncertainties in the gradient and intercept of a straight-line graph.	
Individual points on the graph may or may not have associated error bars.	
Students should be able to identify random and systematic errors and suggest ways to reduce or remove them.	
Students should understand the link between the number of significant figures in the value of a quantity and its associated uncertainty.	
Students should be able to combine uncertainties in cases where the measurements that give rise to the uncertainties are added, subtracted, multiplied, divided, or raised to powers.	

NOTES

Sources of uncertainties

Every measurement has some inherent uncertainty.

The important question to ask is whether an experimenter can be confident that the true value lies in the range that is predicted by the uncertainty that is quoted. Good experimental design will attempt to reduce the uncertainty in the outcome of an experiment. The experimenter will design experiments and procedures that produce the least uncertainty and to provide a realistic uncertainty for the outcome.

In assessing uncertainty, there are many issues that have to be considered. These include

- the resolution of the instrument used
- the manufacturer's tolerance on instruments
- the judgments that are made by the experimenter
- the procedures adopted (e.g. repeated readings)
- the size of increments available (e.g. the size of drops from a pipette).

Often, the resolution of the equipment will be the guiding factor in assessing a numerical uncertainty.

There may be further questions that would require you to evaluate arrangements and procedures. You could be asked how particular procedures would affect uncertainties and how they could be reduced by different apparatus design or procedure – for example using the human reaction has a larger uncertainty than the stopwatch.

A combination of the above factors means that there can be no hard and fast rules about the actual uncertainty in a measurement **– it is normally the discretion of the experimenter**. What we can assess from an instrument's resolution is the **minimum** possible uncertainty.

Only the experimenter can assess the other factors based on the arrangement and use of the apparatus and a rigorous experimenter would draw attention to these factors and take them into account.

Readings and measurements

It is useful, when discussing uncertainties, to separate measurements into two forms:

Readings
the values found from a single judgement when using a piece of equipment

Measurements
the values taken as the difference between the judgements of two values.

Examples:

When using a thermometer, a student only needs to make one judgement (the height of the liquid). This is a reading. It can be assumed that the zero value has been correctly set.

For protractors and rulers, both the starting point and the end point of the measurement must be judged, leading to two uncertainties.

The following list is not exhaustive, and the way that the instrument is used will determine whether the student is taking a reading or a measurement.

Reading (one judgement only)	Measurement (two judgements required)
thermometer	ruler
top pan balance	vernier calliper
measuring cylinder	micrometer
digital voltmeter	protractor
Geiger counter	stopwatch
pressure gauge	analogue meter

Readings

The uncertainty in a **reading** when using an instrument is **no smaller** than plus or minus half of the smallest division or greater.

For example, a temperature measured with a thermometer is likely to have an uncertainty of ±0.5 °C if the graduations are 1 °C apart.

You should be aware that readings are often written with the uncertainty.

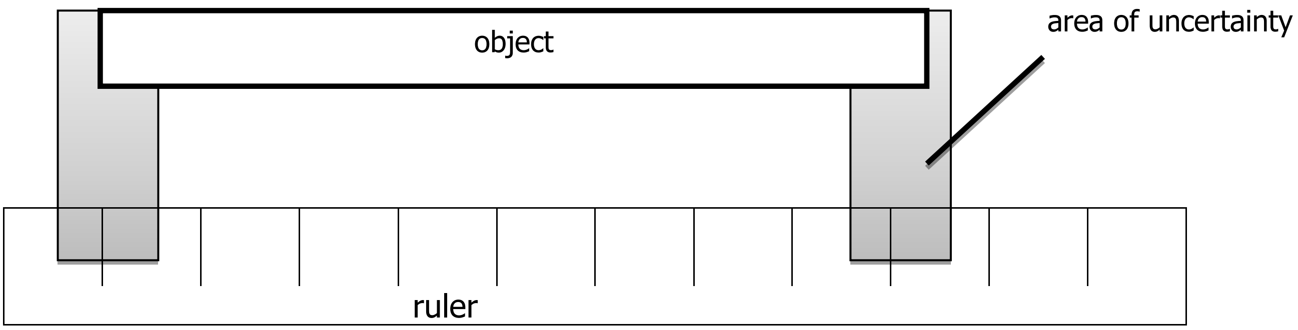
An example of this would be to write a voltage as (2.40 ± 0.01) V. It is usual for the uncertainty quoted to be the same number of decimal places as the value. Unless there are good reasons otherwise (e.g. an advanced statistical analysis), **you should quote the uncertainty in a measurement to the same number of decimal places as the value.**

Measurements

The uncertainty in a **measurement** when using an instrument is **no smaller** than plus or minus the smallest division or greater. For example, length.

When measuring length, **two** uncertainties must be included: the uncertainty of the placement of the zero of the ruler and the uncertainty of the point the measurement is taken from.

As both ends of the ruler have a ±0.5 scale division uncertainty, the measurement will have an uncertainty of ±1 division.



For most rulers, this will mean that the uncertainty in a measurement of length will be ±1 mm.

This "initial value uncertainty" will apply to any instrument where the user can set the zero (incorrectly), but would not apply to equipment such as balances or thermometers where the zero is set at the point of manufacture.

In summary

The uncertainty of a reading (one judgement) is at least ±0.5 of the smallest scale reading.
The uncertainty of a measurement (two judgements) is at least ±1 of the smallest scale reading.

The way measurements are taken can also affect the uncertainty.

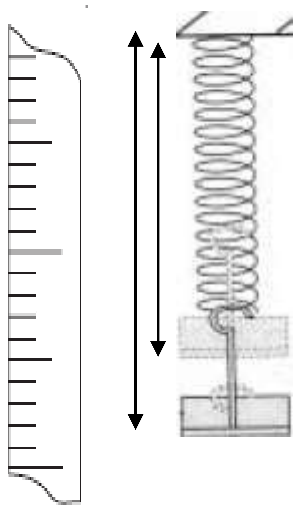
Measurement example: the extension of a spring

Measuring the extension of a spring using a metre ruler can be achieved in two ways.

1. Measuring the total length unloaded and then loaded.

Four readings must be taken for this: The start and end point of the unloaded spring's length and the start and end point of the loaded spring's length.

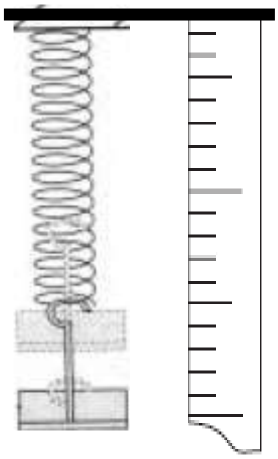
The minimum uncertainty in each measured length is ±1 mm using a meter ruler with 1 mm divisions (the actual uncertainty is likely to be larger due to parallax in this instance). The extension would be the difference between the two readings so the minimum uncertainty would be ±2 mm.



2. Fixing one end and taking a scale reading of the lower end.

Two readings must be taken for this: the end point of the unloaded spring's length and the end point of the loaded spring's length. The start point is assumed to have zero uncertainty as it is fixed.

The minimum uncertainty in each reading would be ±0.5 mm, so the minimum extension uncertainty would be ± 1 mm.



Even with other practical uncertainties this second approach would be better.

Realistically, the uncertainty would be larger than this and an uncertainty in each reading of 1 mm or would be more sensible. This depends on factors such as how close the ruler can be mounted to the point as at which the reading is to be taken.

Other factors

There are some occasions where the resolution of the instrument is not the limiting factor in the uncertainty in a measurement.

Best practice is to write down the full reading and then to write to fewer significant figures when the uncertainty has been estimated.

Examples:

A stopwatch has a resolution of hundredths of a second, but the uncertainty in the measurement is more likely to be due to the reaction time of the experimenter.

Here, you should write the full reading on the stopwatch (e.g. 12.20 s), carry the significant figures through for all repeats, and reduce this to a more appropriate number of significant figures after an averaging process later.

If you measure the length of a piece of wire, it is very difficult to hold the wire completely straight against the ruler. The uncertainty in the measurement is likely to be higher than the ±1 mm uncertainty of the ruler.

Depending on the number of "kinks" in the wire, the uncertainty could be reasonably judged to be nearer ± 2 or 3 mm.

The uncertainty of the reading from digital voltmeters and ammeters depends on the electronics and is not strictly the last figure in the readout.

Manufacturers usually quote the percentage uncertainties for the different ranges. Unless otherwise stated it may be assumed that ±0.5 in the least significant digit is to be the uncertainty in the measurement. This would generally be rounded up to ±1 of the least significant digit when quoting the value and the uncertainty together. For example (5.21 ±0.01) V. If the reading fluctuates, then it may be necessary to take many readings and do a mean and range calculation.

Uncertainties in given values

The value of the charge on an electron is given in the data sheet as 1.60×10^{-19} C.

In all such cases assume the uncertainty to be ±1 in the last significant digit. In this case the uncertainty $\pm 0.01 \times 10^{-19}$ C. The uncertainty may be lower than this but without knowing the details of the experiment and procedure that lead to this value there is no evidence to assume otherwise.

Example:

If the number of lines per m is quoted as 3.5×10^3 (as in AS Physics Specimen Paper 2 (set 1) Q1.1) then it is usual to assume that the uncertainty is ±1 in the last significant figure, $\pm 0.1 \times 10^3$ since there is no indication of the uncertainties in the measurements from which that figure came.

Multiple instances of measurements

Some methods of measuring involve the use of multiple instances to reduce the uncertainty. For example, measuring the thickness of several sheets of paper together rather than one sheet, or timing several swings of a pendulum.

The uncertainty of each measurement will be the uncertainty of the whole measurement divided by the number of sheets or swings.

This method works because the absolute uncertainty on the time for a single swing is the same as the absolute uncertainty for the time taken for multiple swings, but there is a lower percentage in the time taken for multiple swings.

For example:

Time taken for a pendulum to swing 10 times: (5.1 ± 0.1) s

Mean time taken for one swing: (0.51 ± 0.01) s

Repeated measurements

Repeating a measurement is a method for reducing the uncertainty.

With many readings, one can also identify those that are exceptional (that are far away from a significant number of other measurements). Sometimes it will be appropriate to remove outliers from measurements before calculating a mean.

If measurements are repeated, the uncertainty can be calculated by finding half the range of the measured values.

For example:

Repeat	1	2	3	4
Distance / m	1.23	1.32	1.27	1.22

1.32 – 1.22 = 0.10 therefore

Mean distance: (1.26 ± 0.05) m

Percentage uncertainties

The percentage uncertainty in a measurement can be calculated using:

$$percentage\ uncertainty = \frac{uncertainty}{value}\ x\ 100\%$$

The percentage uncertainty in a repeated measurement can also be calculated using:

$$percentage\ uncertainty = \frac{uncertainty}{mean\ value}\ x\ 100\%$$

Further examples:

Example 1. Some values for diameter of a wire

Repeat	1	2	3	4
Diameter / mm	0.35	0.37	0.36	0.34

The exact values for the mean is 0.355 mm and for the uncertainty is 0.015 mm

This could be quoted as such or recorded as 0.36 ±0.02 mm given that there is a wide range and only 4 readings. Given the simplistic nature of the analysis then giving the percentage uncertainty as 5% or 6% would be acceptable.

Example 2. Different values for the diameter of a wire

Repeat	1	2	3
Diameter / mm	0.35	0.36	0.35

The mean here is 0.3533 mm with uncertainty of 0.005 mm

The percentage uncertainty is 1.41% so may be quoted as 1% but really it would be better to obtain further data.

Uncertainties in exams

Wherever possible, questions in exams will be clear on whether you are being asked to calculate the uncertainty of a reading, a measurement, or given data.

Where there is ambiguity, mark schemes will allow alternative sensible answers and credit clear thinking.

If the examination states 'uncertainty' it refers to absolute uncertainty. A percentage uncertainty is only referred to by that name.

Uncertainties in practical work

You are expected to develop an understanding of uncertainties in measurements through their practical work.

Examples:

CPAC 2: You should be attempting to reduce the uncertainties in experiments. This could be by choosing appropriate equipment (CPAC 2d), or by choosing procedures such as repeating readings that reduce overall uncertainties (CPAC 2c).

CPAC 4: You should consider uncertainties. For example, you should be making sensible decisions about the number of significant figures to include, particularly in calculated values.

CPAC 5: You could comment on the uncertainties in their measurements. For example, you could comment on whether the true value (e.g. for a concentration, or the acceleration due to gravity) lies within their calculated range of uncertainty. With some measurements, you may compare their value with those from secondary sources, contributing evidence for CPAC 5b.

Error bars in Physics

There are many ways to draw error bars.

The following simple method of plotting error bars would therefore be acceptable:

- Plot the data point at the mean value
- Calculate the range of the data, ignoring any anomalies
- Add error bars with lengths equal to half the range on either side of the data point.

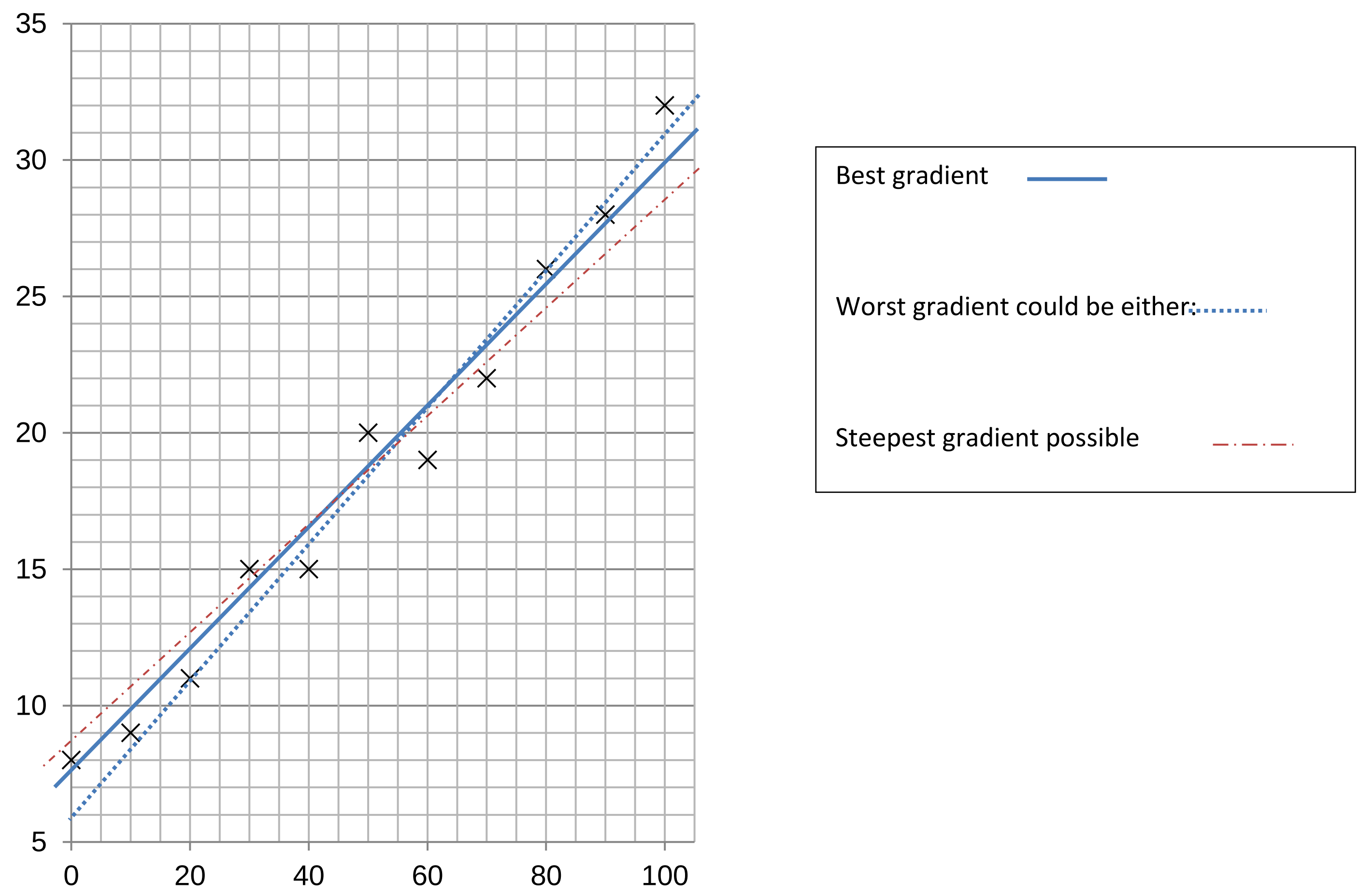
Uncertainties from gradients

To find the uncertainty in a gradient, two lines should be drawn on the graph. One should be the "best" line of best fit. The second line should be the steepest or shallowest gradient line of best fit possible from the data. The gradient of each line should then be found.

The uncertainty in the gradient is found by:

$$\text{percentage uncertainty} = \frac{|\text{best gradient} - \text{worst gradient}|}{\text{best gradient}} \times 100\%$$

Note the modulus bars meaning that this percentage will always be positive.



In the same way, the percentage uncertainty in the y-intercept can be found:

$$\text{percentage uncertainty} = \frac{|\text{best } y \text{ } intercept - \text{worst } y \text{ } intercept|}{\text{best } y \text{ } intercept} \times 100\%$$

Combining uncertainties

Percentage uncertainties should be combined using the following rules:

Combination	Operation	Example
Adding or subtracting values $\boldsymbol{a = b + c}$	Add the absolute uncertainties Δa = Δb + Δc	Object distance, *u* = (5.0 ± 0.1) cm Image distance, *v* = (7.2 ± 0.1) cm Difference (*v* – *u*) = (2.2 ± 0.2) cm
Multiplying values $\boldsymbol{a = b \times c}$	Add the percentage uncertainties εa = εb + εc	Voltage = (15.20 ± 0.1) V Current = (0.51 ± 0.01) A Percentage uncertainty in voltage = 0.7% Percentage uncertainty in current = 1.96% Power = Voltage × current = 7.75 W Percentage uncertainty in power = 2.66% Absolute uncertainty in power = ± 0.21 W
Dividing values $\boldsymbol{a = \frac{b}{c}}$	Add the percentage uncertainties εa = εb + εc	Mass of object = (30.2 ± 0.1) g Volume of object = (18.0 ± 0.5) cm^3 Percentage uncertainty in mass of object = 0.3 % Percentage uncertainty in volume = 2.8% Density = $\frac{30.2}{18.0}$ = 1.68 g cm^{-3} Percentage uncertainty in density = 3.1% Absolute uncertainty in density = ± 0.05 g cm^{-3}
Power rules $\boldsymbol{a = b^c}$	Multiply the percentage uncertainty by the power εa = c × εb	Radius of circle = (6.0 ± 0.1) cm Percentage uncertainty in radius = 1.6% Area of circle = πr^2 = 113.1 cm^2 Percentage uncertainty in area = 3.2% Absolute uncertainty = ± 3.6 cm^2 (Note – the uncertainty in π is taken to be zero)

Note: Absolute uncertainties (denoted by Δ) have the same units as the quantity.

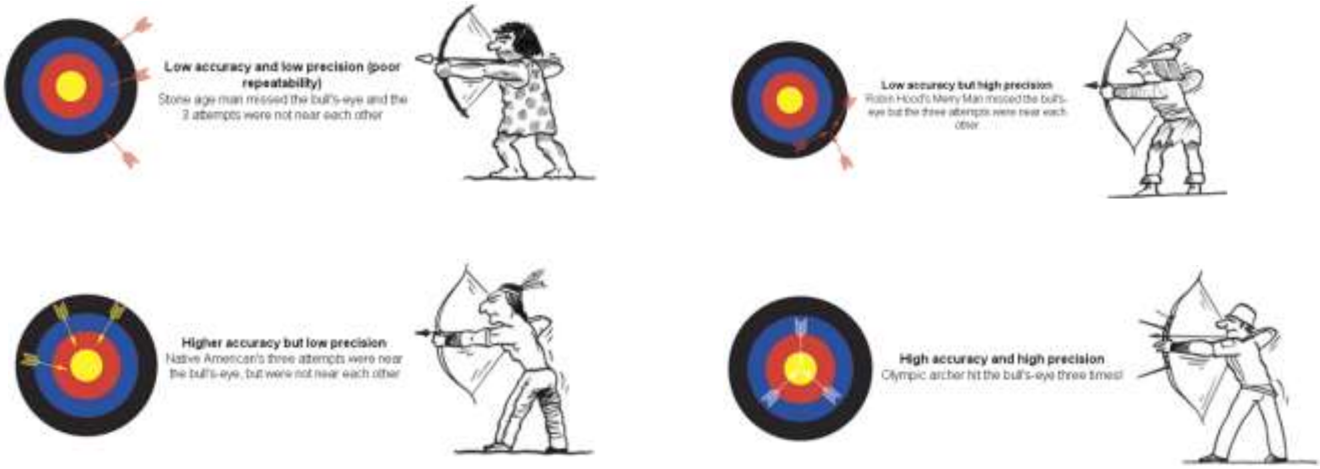
Percentage uncertainties (denoted by ε) have no units.

Uncertainties in trigonometric and logarithmic functions will not be tested in A-level exams.

Commonly Asked Questions.

What is the difference between Accuracy and Precision?

The difference between accuracy and precision is illustrated below by four different archers, each with varying degree of ability. The bull's-eye in the target represents the true value of a measurement.



Accuracy: A measurement result is considered accurate if it is judged to be close to the true value.

The precision of a measurement is the degree of exactness (sometimes the number of significant figures) to which the measurement of a quantity or value can be obtained and reproduced consistently.

- If a reading is constant when repeated, the precision of the measurement will be the precision of the instrument. Precision
- If a reading fluctuates the precision of the measurement (its mean value) is given by half the maximum range of the readings.

The precision of an instrument is the smallest non-zero reading that can be measured using the instrument.

What is the difference between Repeatable and Reproducible?

Repeatable: A measurement is repeatable if the original experimenter repeats the investigation using same method and equipment and obtains the same results.
Reproducible: A measurement is reproducible if the investigation is repeated by another person, or by using different equipment or techniques, and the same results are obtained.

Additional Note Space

Additional Note Space

REVISION SHEET

Highlight the key information.

Uncertainties Come in Absolute Amounts, Fractions and Percentages

Absolute uncertainty is the uncertainty of a measurement given as certain fixed quantity.

Fractional uncertainty is the uncertainty given as a **fraction** of the measurement taken.

Percentage uncertainty is the uncertainty given as a **percentage** of the measurement.

An uncertainty should also include a **level of confidence** or **probability**, to indicate how **likely** the true value is to lie in the interval. E.g. '5.0 ± 0.4 Ω at a level of confidence of 80%' means you're **80% sure** that the true value is **within** 0.4 Ω of 5.0 Ω. (Don't worry, you **don't need** to calculate the level of confidence.)

Example: The resistance of a filament lamp is given as 5.0 ± 0.4 Ω. Give the absolute, fractional and percentage uncertainties for this measurement.

1) The absolute uncertainty is given in the question — it's **0.4 Ω.**
2) To calculate fractional uncertainty, divide the uncertainty by the measurement and simplify. The fractional uncertainty is $\frac{0.4}{5.0} = \frac{4}{50} = \mathbf{\frac{2}{25}}$
3) To calculate percentage uncertainty, divide the uncertainty by the measurement and multiply by 100. The percentage uncertainty is $\frac{2}{25} \times 100 = \mathbf{8\%}$

You can **decrease** the **percentage uncertainty** in your data by taking measurements of **large** quantities. Say you take measurements with a ruler which measures to the nearest **± 0.5 mm**. The **percentage error** in measuring a length of **10 mm** will be **± 5%**, but using the same ruler to measure a distance of **20 cm** will give a percentage error of only **± 0.25%**.

The uncertainty on a **mean** of repeated results is equal to **half the range** of the results.
E.g. say the repeated measurement of a current gives the results 0.5 A, 0.3 A, 0.3 A, 0.3 A and 0.4 A.
The range of these results is 0.5 – 0.3 = 0.2 A, so the uncertainty on the mean current would be **± 0.1 A**.

Sometimes You Need to Combine Uncertainties

When you do calculations involving values that have an uncertainty, you have to **combine** the uncertainties to get the **overall** uncertainty for your result.

Adding or Subtracting Data — ADD the Absolute Uncertainties

Example: A wire is stretched from 4.3 ± 0.1 cm to 5.5 ± 0.1 cm. Calculate the extension of the wire.

1) First subtract the lengths without the uncertainty values: 5.5 – 4.3 = 1.2 cm
2) Then find the total uncertainty by adding the individual absolute uncertainties: 0.1 + 0.1 = 0.2 cm
   So, the wire has been stretched **1.2 ± 0.2 cm.**

Multiplying or Dividing Data — ADD the Percentage Uncertainties

Example: A force of 15 ± 3% N is applied to a stationary object which has a mass of 6.0 ± 0.3 kg. Calculate the acceleration of the object and state the percentage uncertainty in this value.

1) First calculate the acceleration without uncertainty: $a = F \div m = 15 \div 6.0 = 2.5\ \text{ms}^{-2}$
2) Next, calculate the percentage uncertainty in the mass: % uncertainty in $m = \frac{0.3}{6} \times 100 = 5\%$
3) Add the percentage uncertainties in the force and mass values to find the total uncertainty in the acceleration: Total uncertainty = 3% + 5% = 8%
   So, the acceleration = $\mathbf{2.5 \pm 8\%\ ms^{-2}}$

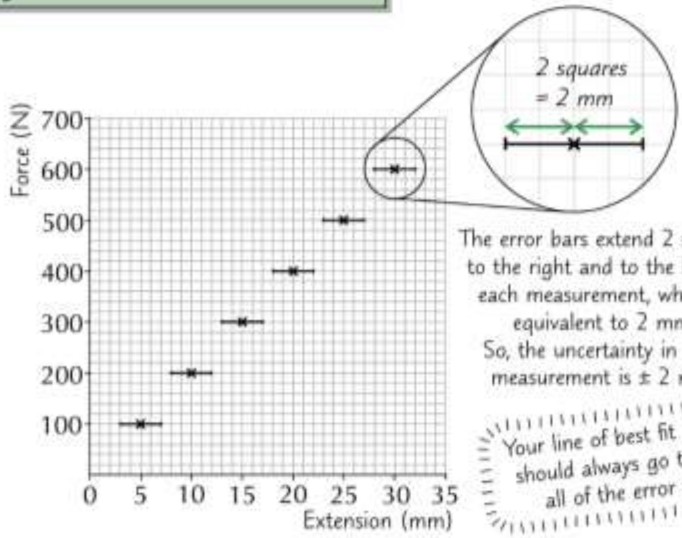
Raising to a Power — MULTIPLY the Percentage Uncertainty by the Power

Example: The radius of a circle is r = 40 ± 2.5% cm. What will the percentage uncertainty be in the area of this circle, i.e. πr^2?

The radius will be raised to the power of **2** to calculate the area.
So, the percentage uncertainty will be 2.5% × 2 = **5%**

Error Bars Show the **Uncertainty** of **Individual Points**

1) Most of the time, you work out the **uncertainty** in your **final** result using the uncertainty in **each measurement** you make.
2) When you're plotting a **graph**, you show the uncertainty in **each measurement** by using **error bars** to show the **range** the point is likely to lie in.
3) In exams, you might have to **analyse data** from graphs **with** and **without** error bars — so make sure you really understand what error bars are showing.
4) The **error** of **each measurement** when measuring the extension of a material is shown by the **error bars** in the graph to the right.



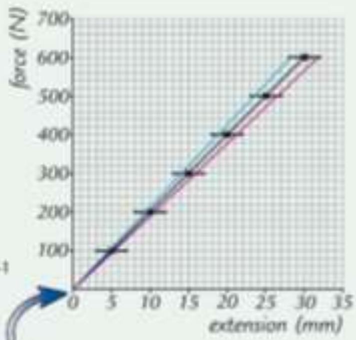
The error bars extend 2 squares to the right and to the left for each measurement, which is equivalent to 2 mm. So, the uncertainty in each measurement is ± 2 mm.

Your line of best fit (p.240) should always go through all of the error bars.

You Can **Calculate** the **Uncertainty** of **Final Results** from a **Line of Best Fit**

Normally when you draw a graph you'll want to find the **gradient** or **intercept**. For example, you can calculate *k*, the **spring constant** of the object being stretched, from the **gradient** of the graph on the right — here it's about 20 000 Nm^{-1}. You can find the **uncertainty** in that value by using **worst lines**:

1) Draw lines of best fit which have the **maximum** and **minimum** possible slopes for the data and which should go through all of the **error bars** (see the pink and blue lines on the right). These are the **worst lines** for your data.
2) Calculate the **worst gradient** — the gradient of the slope that is **furthest** from the gradient of the line of best fit. The blue line's gradient is about 21 000 Nm^{-1} and the pink line's gradient is about 19 000 Nm^{-1}, so you can use either here.
3) The **uncertainty** in the gradient is given by the difference between the **best gradient** (of the line of best fit) and the **worst gradient** — here it's 1000 Nm^{-1}. So this is the uncertainty in the value of the spring constant. For this object, the spring constant is 20 000 ± 1000 Nm^{-1} (or 20 000 Nm^{-1} ± 5%).
4) Similarly, the uncertainty in the **y-intercept** is just the **difference** between the **best** and **worst** intercepts (although there's no uncertainty here since the best and worst lines both go through the origin).



When the force is 0 N the extension is 0 mm — this is a measurement with no uncertainty.

Reference: CGP Revision Guide

Data processing

For a single measurement, the precision of the measuring instrument determines the precision of the measurement. A micrometer with a precision of 0.01 mm gives readings that each have a precision of 0.01 mm.

For several readings, the number of significant figures of the mean value should be the same as the precision of each reading. For example, consider the following measurements of the diameter of a wire: 0.34 mm, 0.33 mm, 0.36 mm, 0.33 mm, 0.35 mm. The mean value of the diameter readings works out at 0.342 mm, but the third significant figure cannot be justified as the precision of each reading is 0.01 mm. Therefore the mean value is rounded down to 0.34 mm.

Note:
The uncertainty in the mean value is ±0.02 mm (i.e., half the range) as explained in Topic 14.2.

Using error estimates

How confident can you be in your measurements and any results or conclusions you draw from your measurements? If you work out what each uncertainty is, as a percentage of the measurement (the percentage uncertainty), you can then see which measurement is least accurate. You can then think about how that measurement could be made more accurately.

Worked example

The mass and diameter of a ball bearing were measured and the uncertainty of each measurement was estimated.

The mass, m, of a ball bearing = $4.85 \times 10^{-3} \pm 0.02 \times 10^{-3}$ kg

The diameter, d, of the ball bearing = $1.05 \times 10^{-2} \pm 0.01 \times 10^{-2}$ m

Calculate and compare the percentage uncertainty of these two measurements.

Solution

The percentage uncertainty of the mass $m = \frac{0.02}{4.85} \times 100\% = 0.4\%$

The percentage uncertainty of the diameter $d = \frac{0.01}{1.05} \times 100\% = 1.0\%$

The diameter measurement is therefore more than twice as inaccurate as the mass measurement.

SKILLS TEST
UNCERTAINTY TEST

Complete the table.

Variable	Reading 1	Reading 2	Reading 3	Mean Value	Uncertainty	% Uncertainty
A	121	118	119			
B	599	623	593			
C	3.3	3.6	3.2			

What would be the percentage uncertainty in the following quantities?

A^2		CB	
AB		ABC	
$\frac{C}{B}$		$\frac{A^2C}{B}$	

Complete the table.

Variable	Reading 1	Reading 2	Reading 3	Mean Value	Uncertainty	% Uncertainty
D	17	17	17			
E	42.5	42.8	42.1			
F	3.60	3.28	3.73			
G	757	714	739			

What would be the percentage uncertainty in the following quantities?

D^3F		EFG^3	
GE^2F		EGD^2	
$\frac{G^2}{DE}$		$\frac{DG}{FE}$	
AFD		F^2B^2G	

Complete the table.

Variable	Reading 1	Reading 2	Reading 3	Mean Value	Uncertainty	% Uncertainty
H	58205	58309	58193			
I	82.3	81.4	82.8			
J	1985	1988	1980			
K	43	19	27			

What would be the percentage uncertainty in the following quantities?

$\frac{H^2K^4}{AEI}$		$J^3\frac{HI}{K}$	
KFC		JFK	
K^4I		I^2JK	
$\frac{ABCDEF}{GHIJK}$		$ADH\frac{BEI}{CFJ^2}$	

Complete the table.

Variable	1	2	3	4	Mean Value	Uncertainty	% Uncertainty
L	11.49	11.56	11.63	10.53			
M	385	322	408	328			
N	2736	2729	2743	2643			
O	5101	5108	5003	5098			
P	125	137	167	142			
Q	6124	6118	6510	6123			
R	3.29	3.29	3.29	3.29			
S	4589	4606	4644	4596			
T	417	488	460	456			
U	1.506	3.061	3.085	1.513			
V	274	333	338	277			
W	33.46	33.45	33.96	33.65			

What would be the percentage uncertainty in the following quantities?

MO		MO^2N	
$OMLM$		N^3O	
$\frac{L}{M}$		$\frac{NO^2}{L}$	
NML		$LMON$	
P^2R		QPR	
SNO^2P		PMT	
$\frac{SR}{PM}$		$\frac{R^2S}{N^2}$	
$(QR)^2S$		$TROL^2$	
$QP\frac{VR}{ST}$		$\frac{PO^2}{RUT}$	
$SWOT$		$OWLS$	
$N^4\frac{O^4P^2}{W^2}S^2$		$\frac{TUW^2PN}{MS^2R}$	
$RUST$		WO^2L	

For each of the measurements listed below identify the most likely source of error what type of error this is and one method of reducing it.

Measurement	Source	Type
A range of values are obtained for the length of a copper wire		
The reading for the current through a wire is 0.74A higher for one group in the class		
A beaker of hot water left on the desk appears to have gained temperature		
A mass of a beaker shows different values on different balances		
A range of values are obtained for the bounce back height of a dropped ball		
A few groups obtain different graphs of resistance vs light intensity for an LDR		
The time period (time of one oscillation) of a pendulum		
A range of values are obtained for the time a parachute takes to reach the ground from 0.5m		

TOPIC 3: ESTIMATION OF QUANTITIES
SPEC CHECK

Specification	Completed?
Orders of magnitude.	
Estimation of approximate values of physical quantities.	
Students should be able to estimate approximate values of physical quantities to the nearest order of magnitude.	
Students should be able to use these estimates together with their knowledge of physics to produce further derived estimates also to the nearest order of magnitude.	

NOTES
An order of magnitude is a multiple of 10.

An example of this is 10^3 or 10^{-7}.

Such differences in order of magnitude can be measured on the logarithmic scale in "decades," or factors of ten.

It is common among scientists and technologists to say that a parameter whose value is not accurately known or is known only within a range is "on the order of" some value. The order of magnitude of a physical quantity is its magnitude in powers of ten when the physical quantity is expressed in powers of ten with one digit to the left of the decimal.

Orders of magnitude are generally used to make very approximate comparisons and reflect very large differences. If two numbers differ by one order of magnitude, one is about ten times larger than the other. If they differ by two orders of magnitude, they differ by a factor of about 100. Two numbers of the same order of magnitude have roughly the same scale - the larger value is less than ten times the smaller value.

It is important in the field of science that estimates be at least in the right ballpark. In many situations, it is often sufficient for an estimate to be within an order of magnitude of the value in question. Although making order-of-magnitude estimates seems simple and natural to experienced scientists, it may be completely unfamiliar to the less experienced.

Answering Examination Questions

We can use significant figures to get an approximate answer to a problem.

We can round off all the numbers in a problem to 1 significant figure to make 'easier' numbers.

It is often possible to do this in your head.

This should give you answer to the correct order of magnitude.

Examples

E1. Find a rough answer to 19.4 over 0.0437

We first round off both numbers to 1 significant figure (s.f.):

19.4 = 20 (1 s.f.)

0.0437 = 0.04 (1 s.f.)

So, we now need to make the denominator a whole number. We can do this by multiplying both 20 and 0.04 by 100. $\frac{20}{0.04} = \frac{20 \times 100}{0.04 \times 100} = \frac{2000}{4}$

Divide everything by 4.

$$= \frac{2000}{4} = 500$$

The real answer to 19.4 ÷ 0.0437 = 443.9359... So, this was a good estimate.

E2. How would you get an approximate answer for 386062 × 0.007243?

Did you get the answer **400000 × 0.007 = 2800**?

Rounding to 1 s.f.

386062 = 400000

0.007243 = 0.007

So, **400000 × 0.007 = 2800**

Examination Tip

It is good practice to estimate answers in your examination, this checks to see if you answer seems sensible.

Additional Note Space

Additional Note Space

SKILLS TEST
ESTIMATION TEST
SECTION A

1) What is the approximate thickness of this piece of paper?
A. 10^{1} m
B. 10^{0} m
C. 10^{-2} m
D. 10^{-4} m

2) Which is the most likely mass of a high school student?
A. 1 kg
B. 5 kg
C. 60 kg
D. 250 kg

3) The weight of an apple is closest to…
A. 10^{-2} N
B. 10^{0} N
C. 10^{2} N
D. 10^{4} N

4) The length of a high school physics classroom is probably closest to…
A. 10^{-2} m
B. 10^{-1} m
C. 10^{1} m
D. 10^{4} m

5) What is the approximate diameter of a dinner plate?
A. 0.0025 m
B. 0.025 m
C. 0.25 m
D. 2.5 m

6) The mass of a physics textbook is closest to…
A. 10^{3} kg
B. 10^{1} kg
C. 10^{0} kg
D. 10^{-2} kg

7) Which object weighs approximately 1 newton?
A. a pound coin
B. paper clip
C. physics student
D. golf ball?

8) The maximum time allowed for the completion of this worksheet is approximately…
A. 10^2 s
B. 10^3 s
C. 10^4 s
D. 10^5 s

9) Which measurement of an average classroom door is closest to 1 meter?
A. thickness
B. width
C. height
D. surface area?

10) What is the approximate mass of a pencil?
A. 5.0×10^{-3} kg
B. 5.0×10^{-1} kg
C. 5.0×10^{0} kg
D. 5.0×10^{1} kg

11) What is the approximate width of a person's little finger?
A. 1 m
B. 0.1 m
C. 0.01 m
D. 0.001 m

12) A high school physics student is sitting in a seat reading this question. The magnitude of the force with which the seat is pushing up on the student to support him is closest to…
A. 0 N
B. 60 N
C. 600 N
D. 6,000 N

13) What is the approximate mass of a car?
A. 10^1 kg
B. 10^2 kg
C. 10^3 kg
D. 10^6 kg

14) The length of a £20 note is approximately…
A. 1.5×10^{-2} m
B. 1.5×10^{-1} m
C. 1.5×10^{1} m
D. 1.5×10^{2} m

15) Which wavelength is in the infrared range of the electromagnetic spectrum?
A. 100 nm
B. 100 mm
C. 100 m
D. 100 μm

16) What is the approximate length of a tennis racket?
A. 10^{-1} m
B. 10^{0} m
C. 10^{1} m
D. 10^{2} m

17) The weight of a chicken egg is most nearly equal to…
A. 10^{-3} N
B. 10^{-2} N
C. 10^{0} N
D. 10^{2} N

18) The mass of a paper clip is approximately…
A. 1×10^{6} kg
B. 1×10^{3} kg
C. 1×10^{-3} kg
D. 1×10^{-6} kg

19) The work done in lifting an apple one meter near Earth's surface is approximately…
A. 1 J
B. 0.01 J
C. 100 J
D. 1,000 J

20) The height of a 30 story building is approximately…
A. 10^{0} m
B. 10^{1} m
C. 10^{2} m
D. 10^{3} m

SECTION B

Perform an order of magnitude calculation to determine the approximate value of the following quantities in SI units.

1) surface area of a door

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2) lifetime of an average human

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3) The volume of air in a room

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4) The volume of a sparrow

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SECTION C

Given that each copper atom has 1 delocalised electron, estimate the number of delocalised atoms in 10g of copper wire.

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CHALLENGE

Use the idea of estimations to estimate how many aliens civilisations there are in the Universe.

The Drake Equation: $N = N^* \times f_s \times N_p \times f_e \times f_l \times f_i \times f_c \times f_t$

N = the number of technical civilizations in the Galaxy at present

Factors	My Guess
N^* = the number of stars in our Galaxy	N^* = ____________
f_s = the fraction of all stars that are similar to the Sun	f_s = ______
N_p = the average number of planets per Sun-like star	N_p = ______
f_e = the fraction of planets about Sun-like stars that are similar to the Earth	f_e = ______
f_l = the fraction of such planets on which life has actually developed	f_l = ______
f_i = the fraction of these planets on which intelligent life has evolved	f_i = ______
f_c = the fraction of these planets on which a technical civilization has developed	f_c = ______

f_t = the fraction of time since the development of a planet's initial technical civilization that a technical civilization has existed on the planet; $f_t = L / t$ where L is the lifetime of an average technical civilization and t is the average time elapsed since the inception of a technical civilization on each planet -- estimated at about 3 billion years. ($f_t = L / 3$ billion yrs)

L = _________ yrs

N = the number of Earth-like planets (orbiting Sun-like stars in the Galaxy) on which a technical civilization currently exists, which equals the number of technical civilizations in the Galaxy at present.

$$N = N^* \times f_s \times N_p \times f_e \times f_l \times f_i \times f_c \times f_t$$

My Guess: ___________ x ____ x ____ x ____ x ____ x ____ x ____ x _______ / 3 billion = _________

TOPIC 4: WRITING AN INVESTIGATION

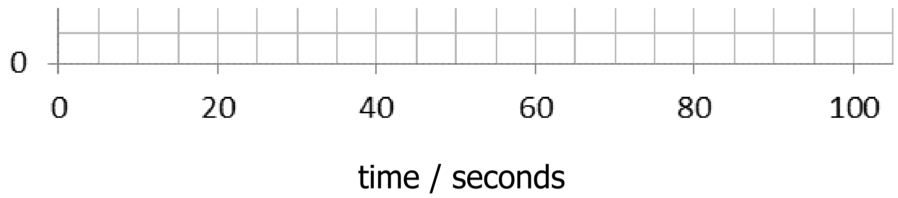
NOTES

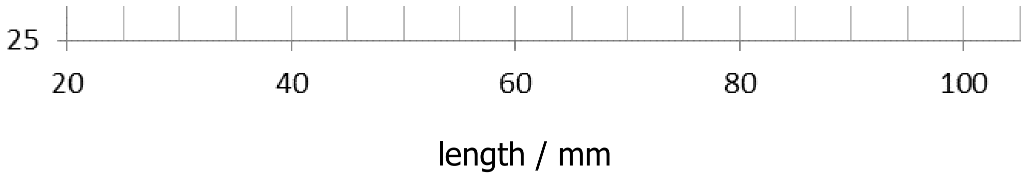
You should recognise that the type of graph that they draw should be based on an understanding of the type of data they are using and the intended analysis of the data.

The rules below are guidelines which will vary according to the specific circumstances.

Labelling axes

Axes should always be labelled with the quantity being measured and the units. These should be separated with a forward slash (solidus):





Axes should not be labelled with the units on each scale marking.

Data points

Data points should be marked with a cross. Both × and + marks are acceptable, but care should be taken that data points can be seen against the grid.

Error bars can take the place of data points where appropriate.

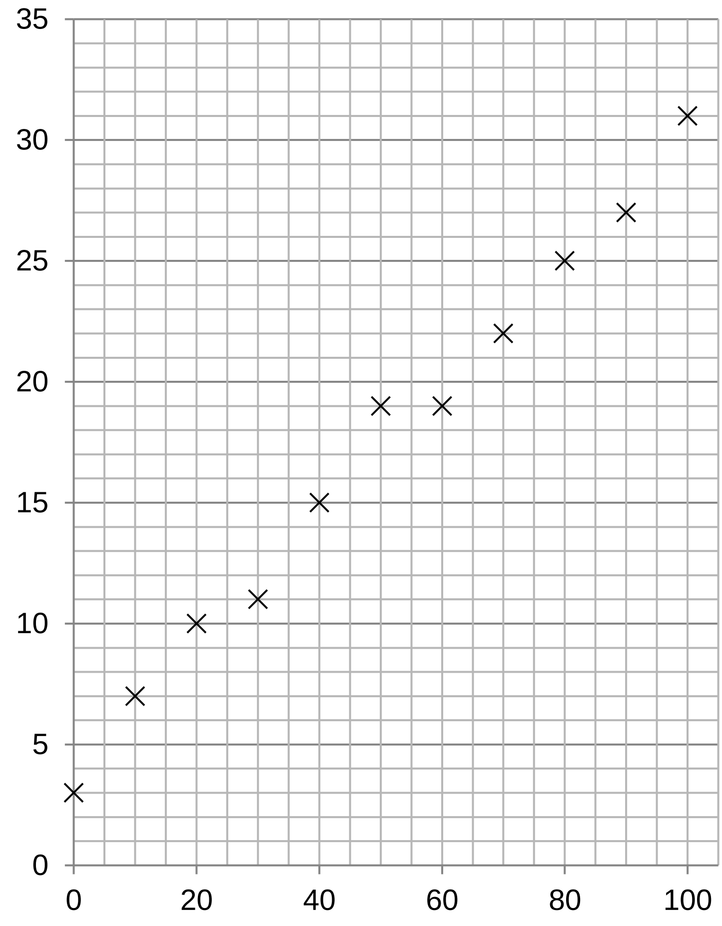
Scales and origins

You should attempt to spread the data points on a graph as far as possible without resorting to scales that are difficult to deal with. You should consider:

- the maximum and minimum values of each variable
- the size of the graph paper
- whether 0.0 should be included as a data point
- whether they will be attempting to calculate the equation of a line, therefore needing the y intercept (Physics only)
- how to draw the axes without using difficult scale markings (e.g. multiples of 3, 7, 11 etc)
- In exams, the plots should cover **at least half** of the grid supplied for the graph.

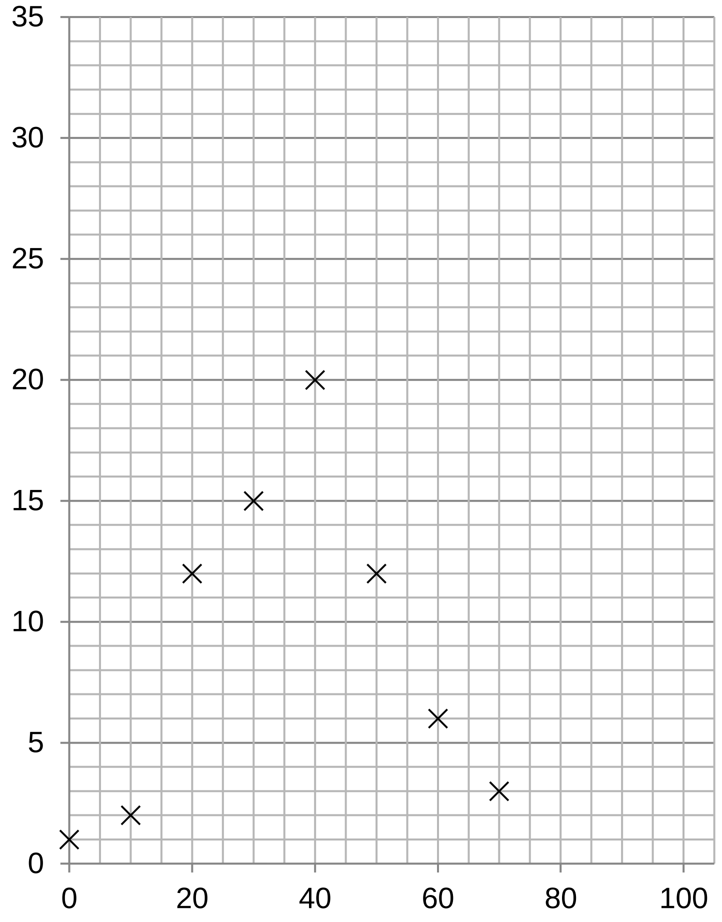
For example, the following three graphs are intended to illustrate the information above relating to the spread of data points on a graph.

When producing such graphs on the basis of real practical work or in examination questions would be expected to add in axes labels and units.

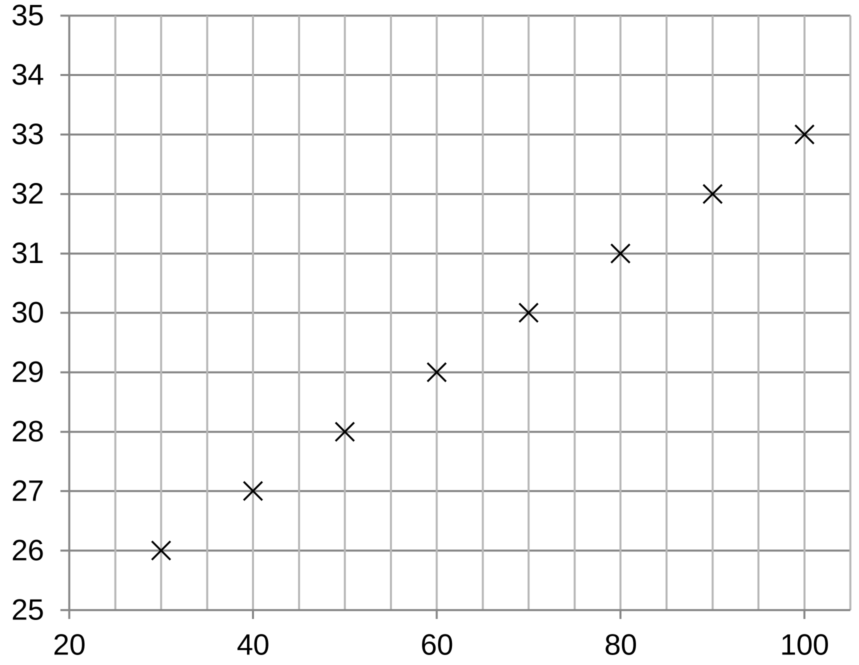


This graph has well-spaced marking points and the data fills the paper.

Each point is marked with a cross (so points can be seen even when a line of best fit is drawn).



This graph is on the limit of acceptability. The points do not quite fill the page, but to spread them further would result in the use of awkward scales.



At first glance, this graph is well drawn and has spread the data out sensibly.

However, if the graph were to later be used to calculate the equation of the line, the lack of a y-intercept could cause problems. Increasing the axes to ensure all points are spread out but the y-intercept is also included is a skill that requires practice and may take a couple of attempts.

Lines of best fit

Lines of best fit should be drawn when appropriate. You should consider the following when deciding where to draw a line of best fit:

- **Are the data likely to be following an underlying equation (for example, a relationship governed by a physical law)? This will help decide if the line should be straight or curved.**
- **Are there any anomalous results?**
- **Are there uncertainties in the measurements? The line of best fit should fall within error bars if drawn.**

There is no definitive way of determining where a line of best fit should be drawn. A good rule of thumb is to make sure that there are as many points on one side of the line as the other. Often the line should pass through, or very close to, the majority of plotted points.

Graphing programs can sometimes help but tend to use algorithms that make assumptions about the data that may not be appropriate.

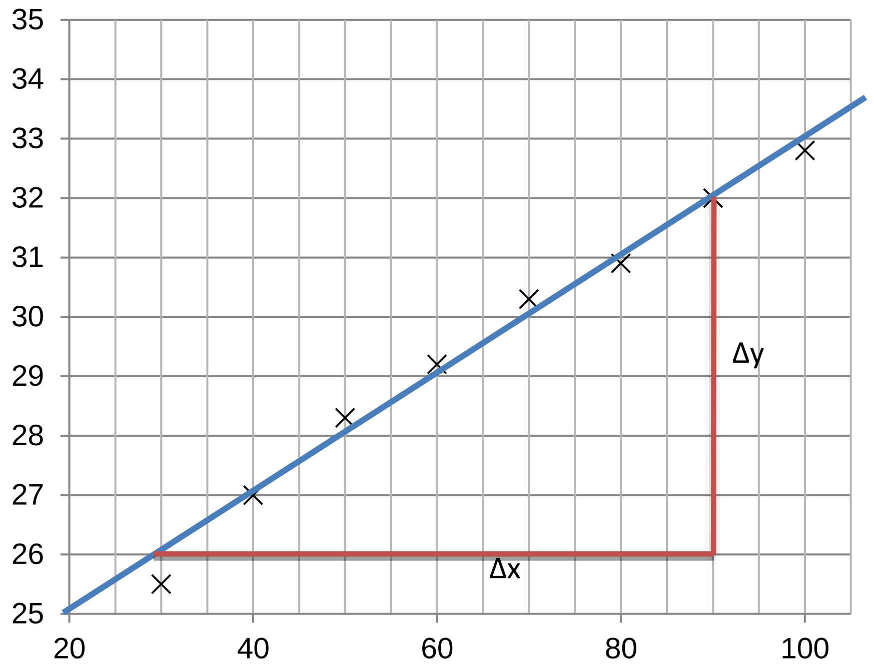
Lines of best fit should be continuous and drawn as a thin pencil that does not obscure the points below and does not add uncertainty to the measurement of gradient of the line.

Not all lines of best fit go through the origin. You should ask whether a 0 in the independent variable is likely to produce a 0 in the dependent variable. This can provide an extra and more certain point through which a line must pass.

A line of best fit that is expected to pass through (0,0), but does not, would imply some systematic error in the experiment. This would be a thorough source of discussion in an evaluation.

Measuring gradients

When finding the gradient of a line of best fit, students should show their working by drawing a triangle on the line. The hypotenuse of the triangle should be at least half as big as the line of best fit.



The line of best fit here has an equal number of points on both sides. It is not too wide so points can be seen under it.

The gradient triangle has been drawn so the hypotenuse includes more than half of the line.

In addition, it starts and ends on points where the line of best fit crosses grid lines so the points can be read easily (this is not always possible).

$$gradient = \frac{\Delta y}{\Delta x}$$

When finding the gradient of a curve, e.g., the rate of reaction at a time that was not sampled, students should draw a tangent to the curve at the relevant value of the independent variable (x-axis).

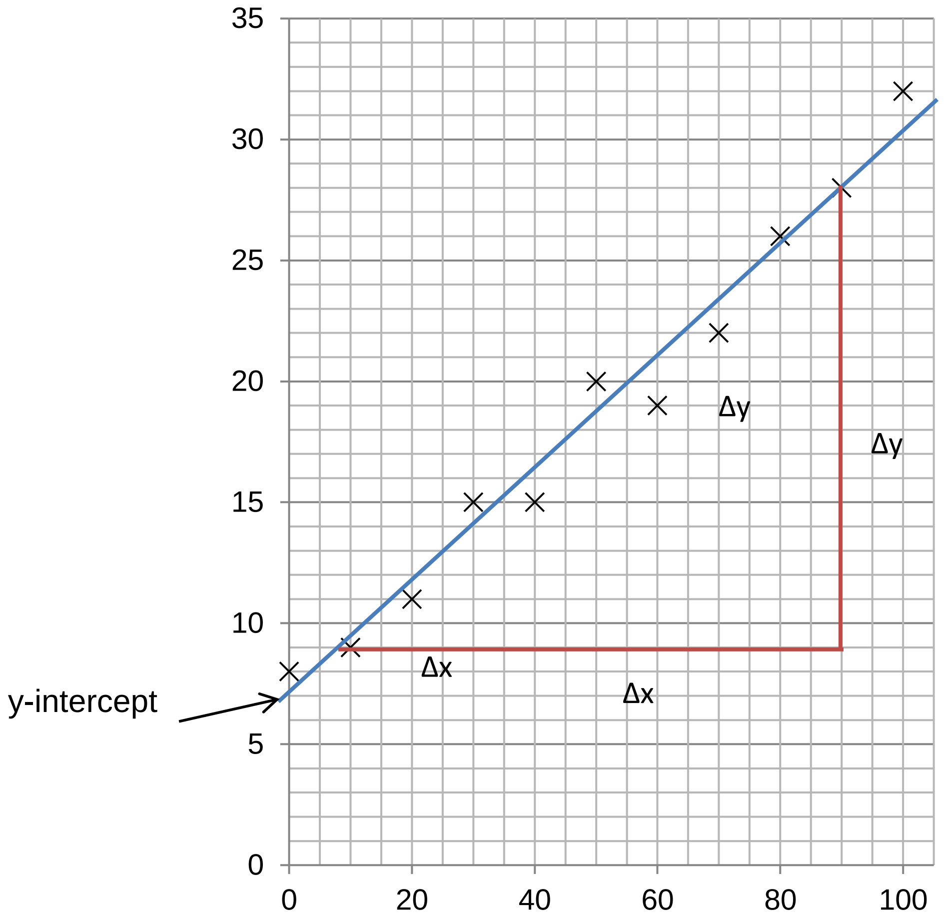
Use of a set square to draw a triangle over this point on the curve can be helpful in drawing an appropriate tangent.

The equation of a straight line

Students should be able to translate graphical data into the equation of a straight line.

$$y = mx + c$$

Where y is the dependent variable, m is the gradient, x is the independent variable and c is the y-intercept.



Δy = 28 – 9 = 19 Δx = 90 – 10 = 80 gradient = 19 / 80 = 0.24 (2 sf) y-intercept = 7.0 **equation of line:** **y = 0.24 x + 7.0**

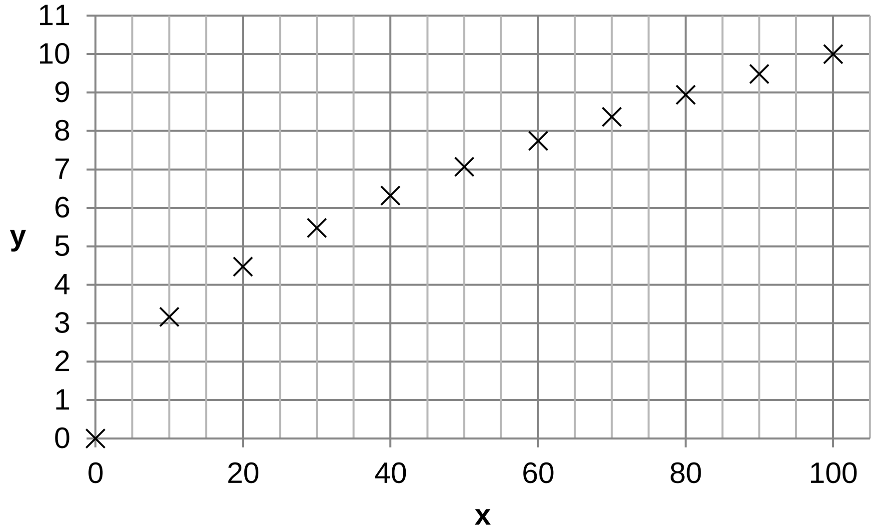
Testing relationships

Sometimes it is not clear what the relationship between two variables is. A quick way to find a possible relationship is to manipulate the data to form a straight-line graph from the data by changing the variable plotted on each axis.

For example:

Raw data and graph

x	y
0	0.00
10	3.16
20	4.47
30	5.48
40	6.32
50	7.07
60	7.75
70	8.37
80	8.94
90	9.49
100	10.00

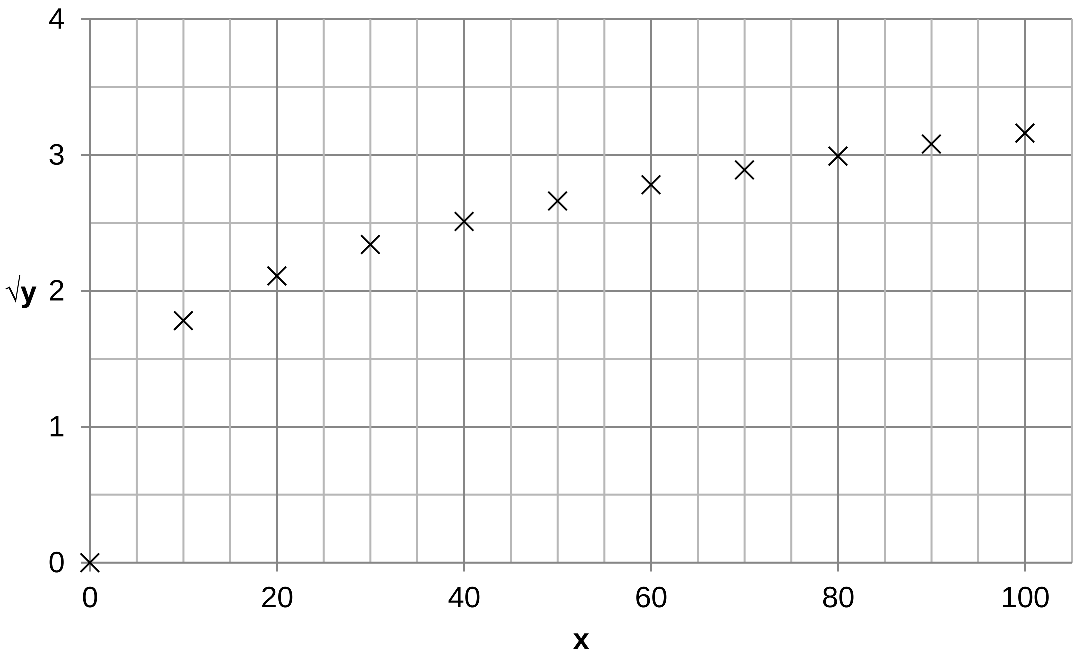


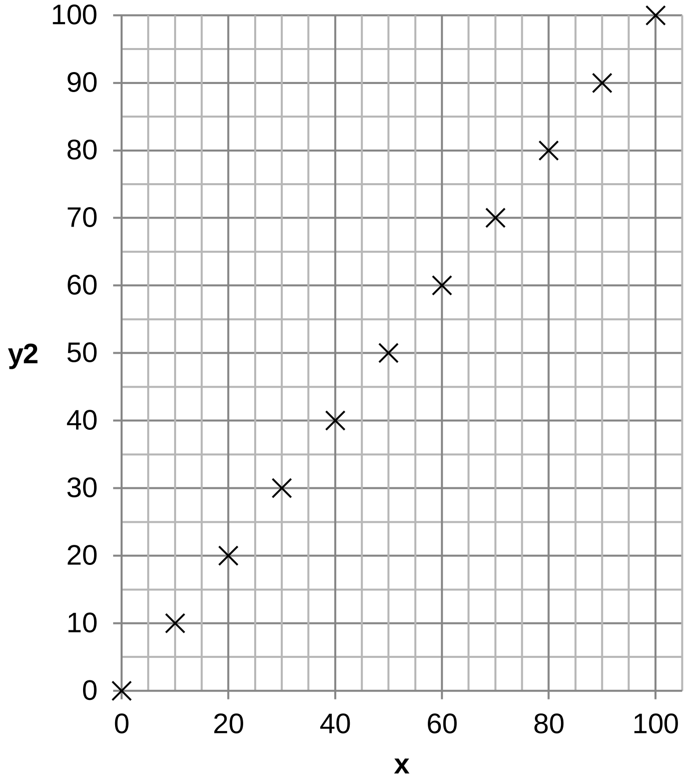
This is clearly not a straight-line graph. The relationship between x and y is not clear.

Manipulated data and graphs

A series of different graphs can be drawn from these data. The one that is closest to a straight line is a good candidate for the relationship between x and y.

x	**y**	**$\sqrt{y}$**	**y^2**	**y^3**
0	0.00	0.00	0.00	0.00
10	3.16	1.78	10.00	32
20	4.47	2.11	20.00	89
30	5.48	2.34	30.00	160
40	6.32	2.51	40.00	250
50	7.07	2.66	50.00	350
60	7.75	2.78	60.00	470
70	8.37	2.89	70.00	590
80	8.94	2.99	80.00	720
90	9.49	3.08	90.00	850
100	10.00	3.16	100.00	1000

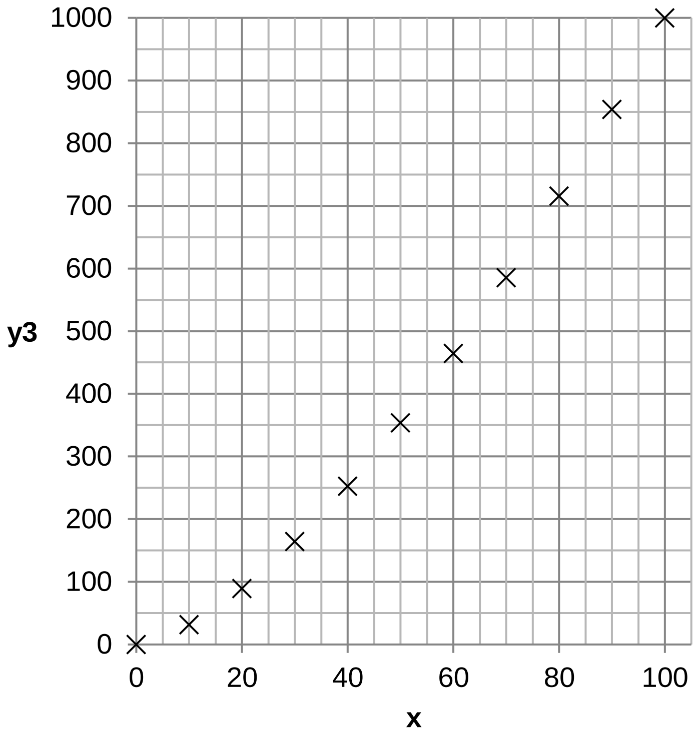




This is an idealised set of data to illustrate the point.

The straightest graph is y^2 against x, suggesting that the relationship between x and y is

$$y^2 \propto x$$



REVISION SHEET

Highlight the key information.

Data can be **Discrete**, **Continuous**, **Categoric** or **Ordered**

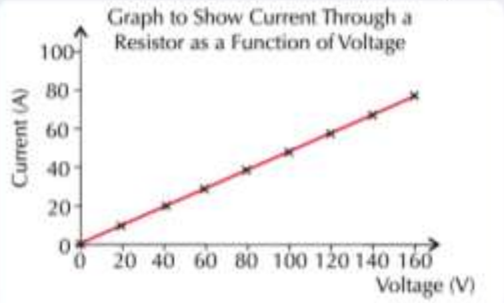
Experiments always involve some sort of measurement to provide data.
There are different types of data — and you need to know what they are.

1) **Discrete data** — you get discrete data by **counting**. E.g. the number of weights added to the end of a spring would be discrete. You can't have 1.25 weights.
2) **Continuous data** — a continuous variable can have **any value** on a scale. For example, the extension of a spring or the current through a circuit. You can never measure the exact value of a continuous variable.
3) **Categoric data** — a categoric variable has values that can be **sorted** into **categories**. For example, types of material might be brass, wood, glass, steel.
4) **Ordered (ordinal) data** — ordered data is similar to **categoric**, but the categories can be put **in order**. For example, if you classify frequencies of light as 'low', 'fairly high' and 'very high' you'd have ordered data.

Graphs — Use the **Best Type** for the Data You've Got

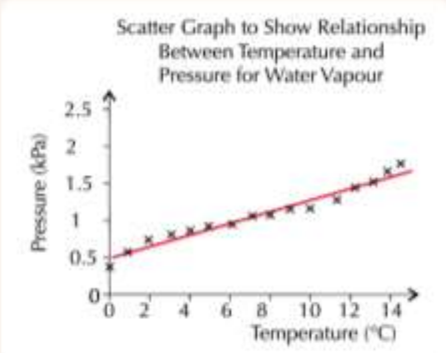
You'll usually be expected to make a **graph** of your results. Not only are graphs **pretty**, they make your data **easier to understand** — so long as you choose the right type. No matter what the type though, make sure you always **label your axes** — including **units**. Choose a **sensible scale** for your axes and **plot points accurately** using a sharp pencil.

Line graphs are best when you have **two sets of continuous data**. For example:



You can also make bar and pie charts — they're normally used to display categoric data.

Scatter plots are great for showing how two sets of data are related (or **correlated**). Don't try to join all the points — draw a **line of best fit** to show the **trend**.



When drawing graphs, the dependent variable should go on the y-axis, the independent on the x-axis.

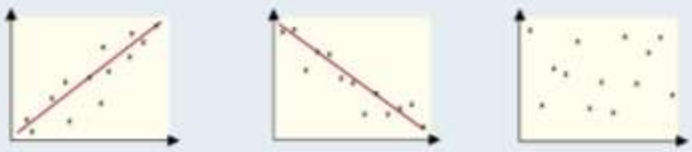
Correlation Shows **Trends** in Data

1) **Correlation** describes the relationship between **two variables** — usually the **independent** and **dependent** ones.
2) Data can show **positive**, **negative** or **no correlation**. An easy way to see correlation is to plot a **scatter graph** of your data. If you can, draw a **line of best fit** to help show the **trend**.

Positive correlation — as one variable **increases**, the other also **increases**.

Negative correlation — as one variable **increases**, the other **decreases**.

No correlation — there is **no relationship** between the variables.



Positive correlation **Negative correlation** **No correlation**

3) If you've done a **controlled** experiment in a lab and can see **correlation** in your results, you can be fairly certain there's a **causal relationship** between the **independent** and **dependent variables**. This means that a **change** in one **causes** a change in the other.
4) But in experiments or studies **outside** the lab, you can't usually control all the variables. So even if two variables are **correlated**, the change in one may not be causing the change in the other. Both changes might be caused by a **third** variable.

Evaluating Your Data

Once an experiment's over, you have to **explain** what the data shows.
There are some key words you need to know about (and use) when evaluating data:

1) **Precision** — the **smaller** the amount your data spreads from the **mean**, the more precise it is. Precision only depends on the amount of **random error** in your readings.

Precision is sometimes called reliability.

2) **Repeatable** — **you** can **repeat** an experiment multiple times and get the **same results**.
3) **Reproducible** — if **someone else** can recreate your experiment using different equipment or methods, and gets the **same results** you do, the results are reproducible.
4) **Valid** — the **original question** is **answered** using **precise** data. If you don't keep all variables apart from the one you're testing **constant**, you haven't **only** tested the variable you're investigating and so the results **aren't valid**.
5) **Accurate** — the result is really close to the **true answer**. You can only comment on how accurate a result is if you know the true value of the result.

There's normally loads of stuff to say when you're looking at data. Have a think about...

1) What **patterns** or trends, if any, the results show.
2) Whether the experiment managed to **answer** the question it set out to answer. If it did, is this a **valid** experiment and if not, why not? How **precise** was the data?
3) How close the results are to the **true value**.
4) Did the measuring instruments have enough **resolution**?
5) Any **anomalies** in the results and the possible causes of them.
6) How **large** the **uncertainties** are. If the percentage uncertainty is large, this suggests the data is not precise and a strong conclusion cannot be made.

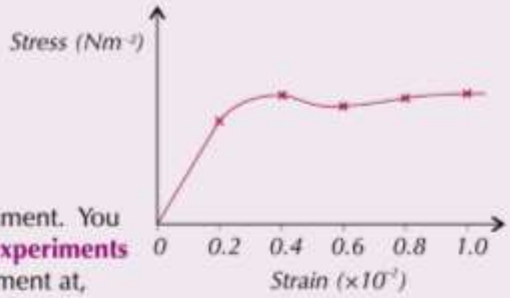
If you're asked to analyse data in the exam, look at how many marks the question is worth — the more **marks** allocated to the question in the exam, the **more detail** you have to go into.

Drawing Conclusions From Your Data

You need to make sure your conclusion is **specific** to the data you have and is **supported** by the data — don't go making any sweeping generalisations.
Your conclusion is only **valid** if it is supported by **valid data**, known as **evidence**.

Example: The stress of a material X was measured at strains of 0.002, 0.004, 0.006, 0.008 and 0.010. Each strain reading had an error of 0.001. All other variables were kept constant. A science magazine concluded from a graph of this data that material X's yield point is at a strain of 0.005. Explain whether or not you agree with this conclusion.

Their conclusion **could** be true — but the **data doesn't support this**. You can't tell **exactly** where the yield point is from the data because strain increases of 0.002 at a time were used. The stress at in-between strains wasn't measured — so all you know is that the yield point is somewhere **between** 0.004 and 0.006, as the stress drops between these values.



Also, the graph only gives information about this particular experiment. You can't conclude that the yield point would be in this range for **all experiments** — only this one. And you can't say for sure that doing the experiment at, say, a **different constant temperature** wouldn't give a different yield point.

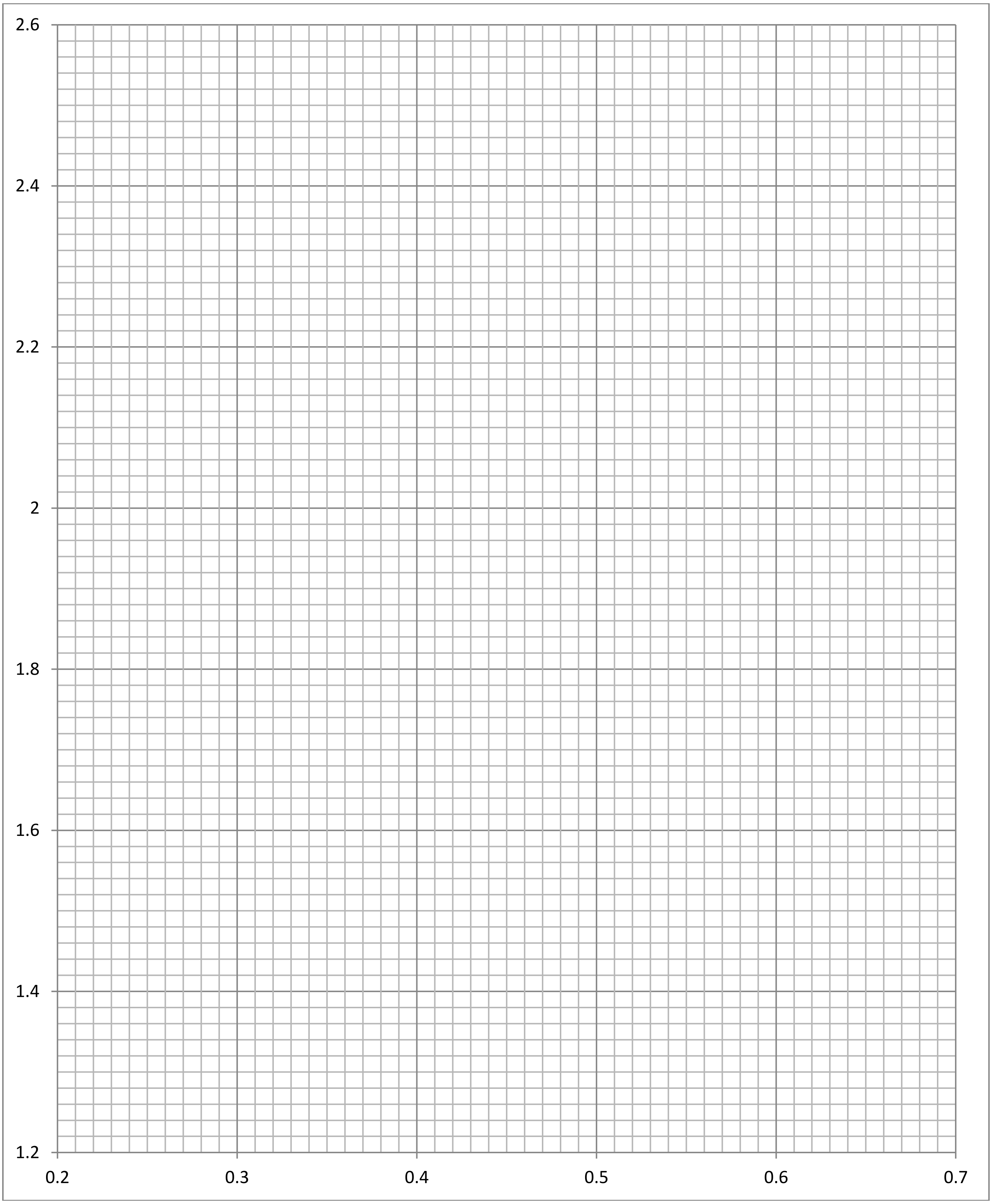
The error in each reading is 0.001, which gives a **percentage uncertainty** of 50% for the lowest strain reading. This is very large and could mean the results are not valid, so no definite conclusions can be drawn from them.

Reference: CGP Revision Guide

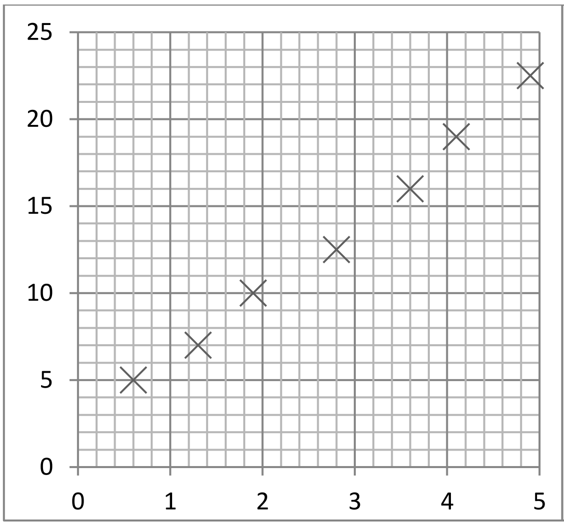
SKILLS TEST
GRAPH TEST

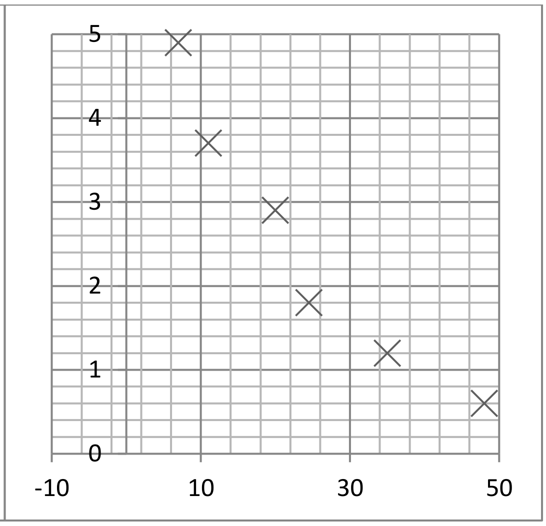
You are going to practice plotting points on a graph.

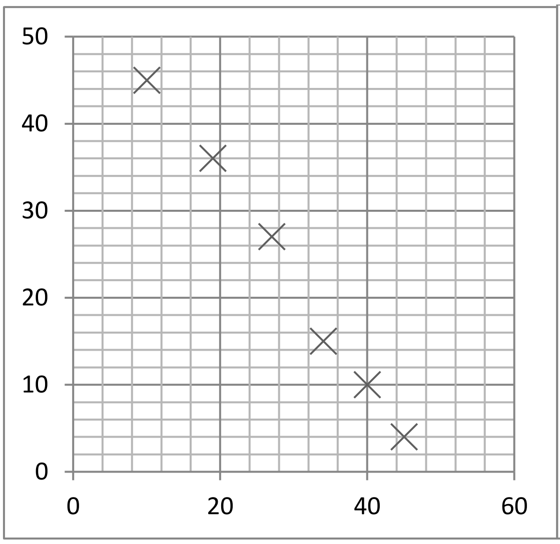
x axis	y axis	x axis	y axis	x axis	y axis
0.44	2.44	0.34	1.75	0.67	2.12
0.27	1.39	0.49	1.99	0.58	1.64
0.39	2.13	0.26	2.22	0.65	2.52
0.62	1.23	0.31	2.49	0.29	1.92
0.37	1.52	0.52	2.36	0.45	1.47
0.22	2.56	0.61	2.23	0.53	1.27
0.42	1.84	0.64	1.83	0.24	1.71
0.48	1.70	0.55	2.15	0.67	1.45

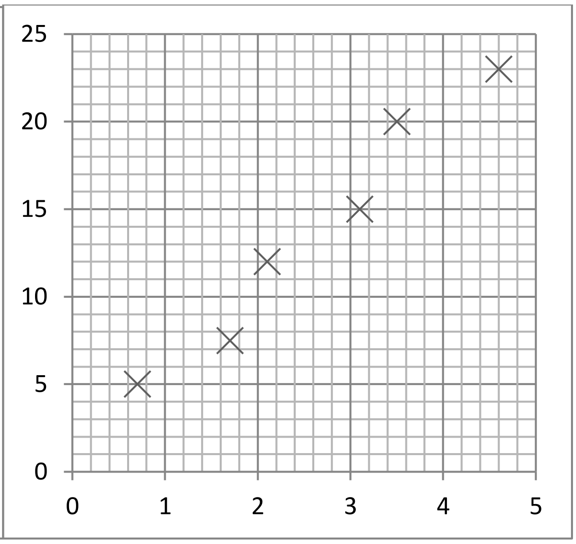

2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
0.2
0.3
0.4
0.5
0.6
0.7

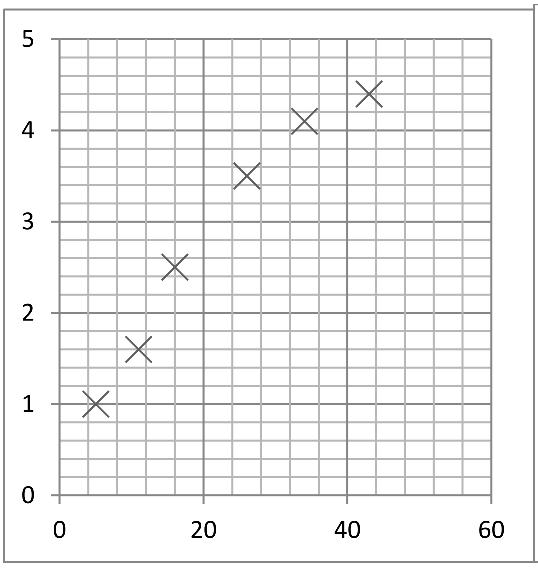
Draw a line of best fit for each of the graphs.

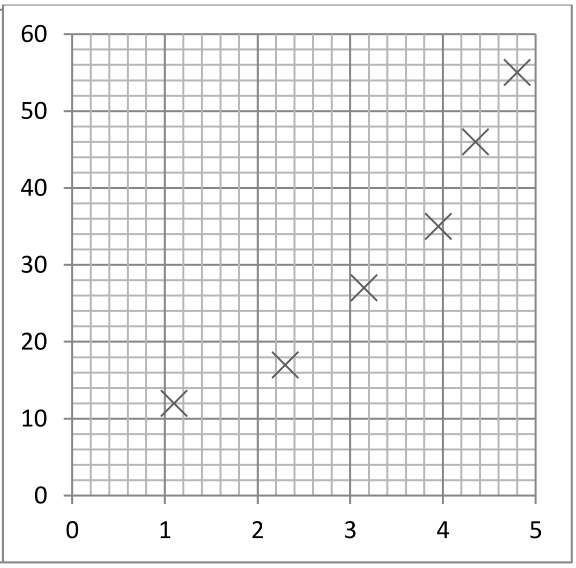




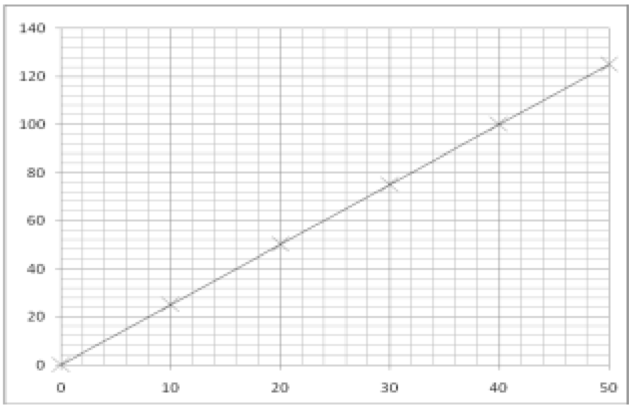


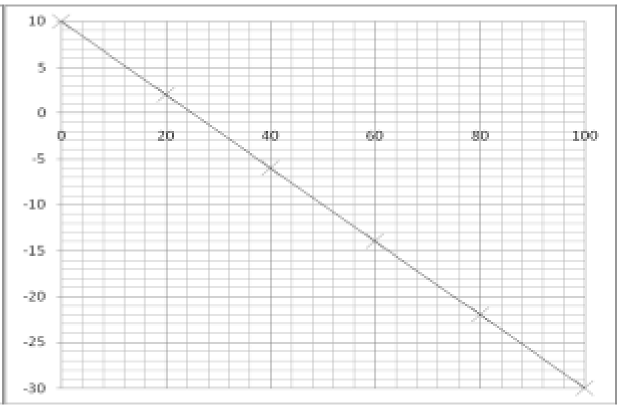


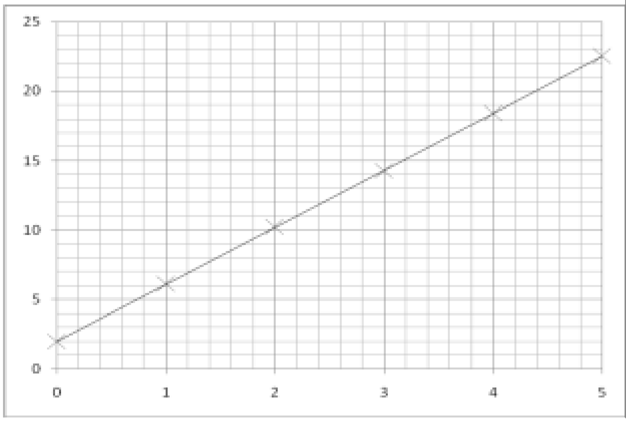


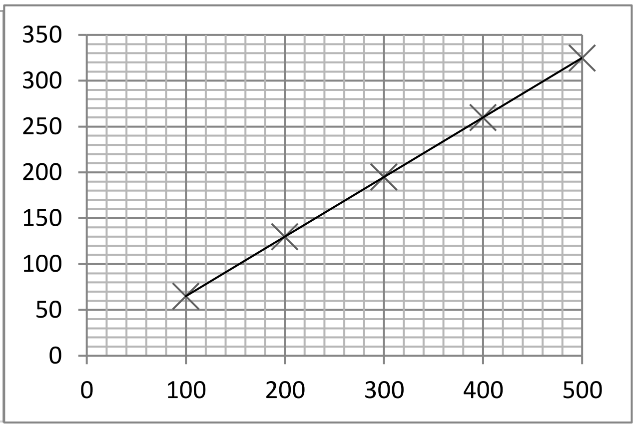


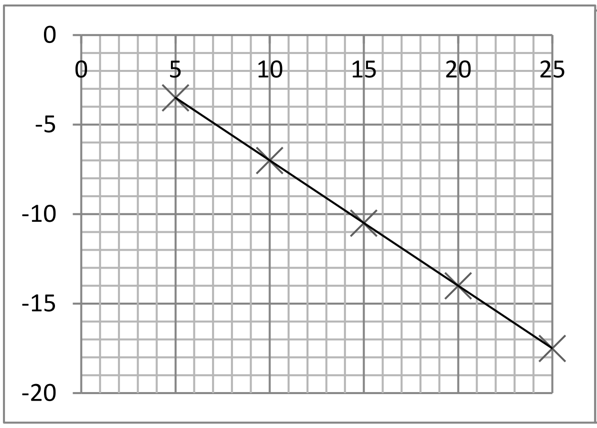
Calculate the gradients of the graphs below. Work out the equation for the line.

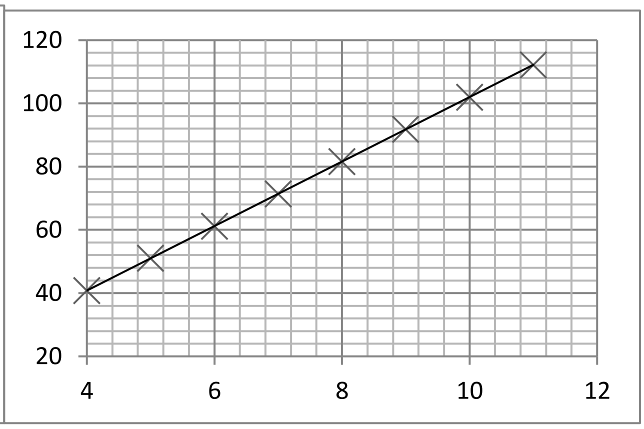


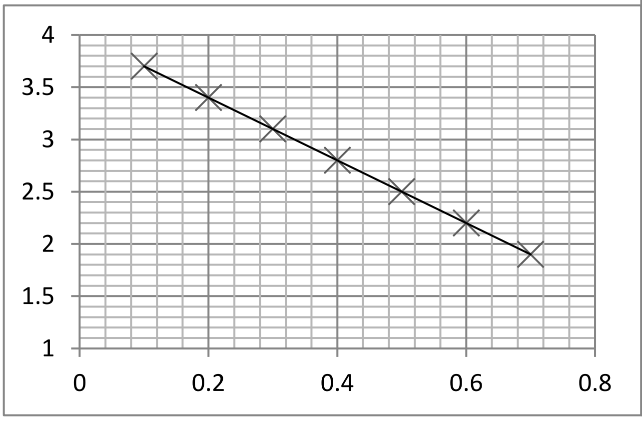


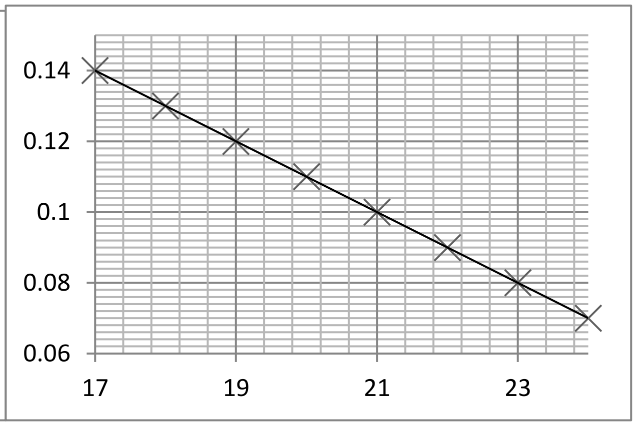








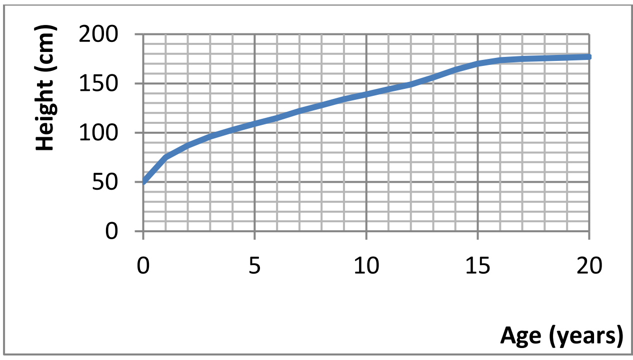


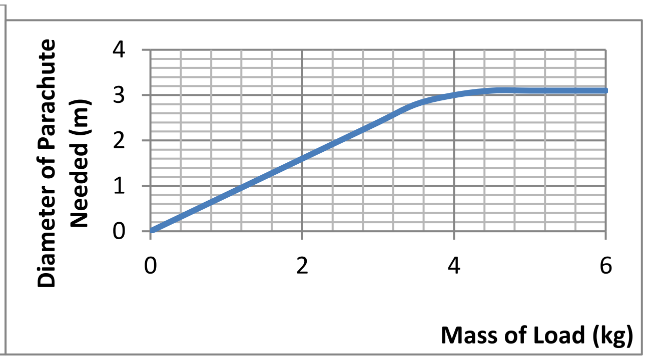


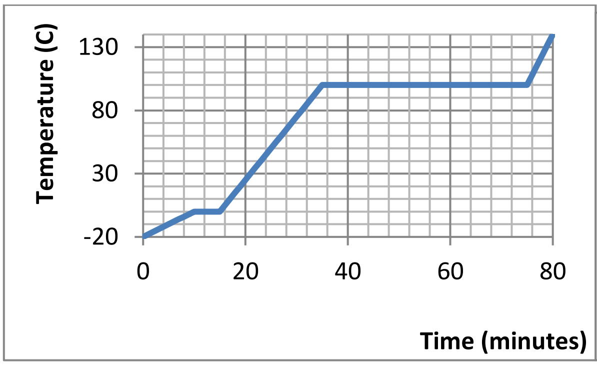
Complete the table below about graphs and gradients

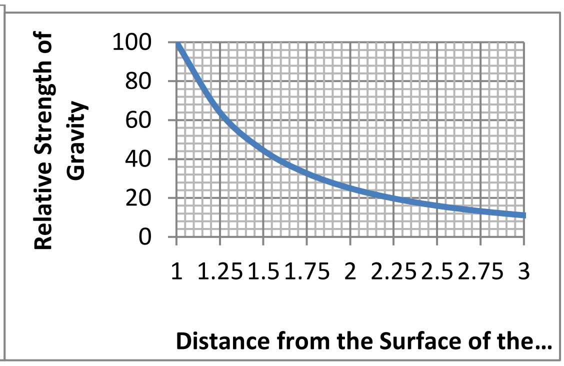
Equation	Graph	Rearrange Equation	Gradient	Intercept
$y = mx + c$	y plotted on the y axis x plotted on the x axis	$y = mx + c$	m	c
$V = IR$	y axis = V x axis = I	$V = RI$	R	0
$I = \frac{Q}{t}$	y axis = t x axis = Q			
$\rho = \frac{RA}{l}$	y axis = l x axis = R			
$\varepsilon = V + Ir$	y axis = V x axis = I			
$E = VIt$	y axis = E/t x axis = V			
$hf = \phi + E_K$	y axis = E_K x axis = f			
$\lambda = \frac{h}{mv}$	y axis = $1/v$ x axis = m			
$E_P = mgh$	y axis = mg x axis = E_P			
$E = \frac{1}{2}Fe$	y axis = e x axis = $1/F$			
$c = f\lambda$	y axis = $1/\lambda$ x axis = f			
$v = u + at$	y axis = a x axis = $1/t$			
$v^2 = u^2 + 2as$	y axis = v^2 x axis = s			
$s = \frac{(u+v)}{2}t$	y axis = v x axis = s			

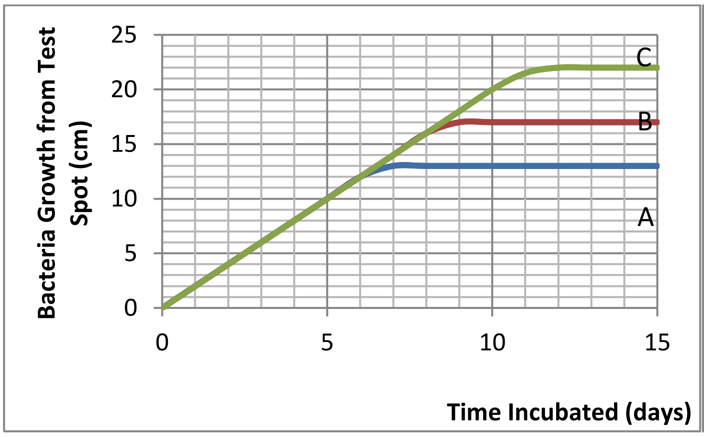
Explain the relationship between the two variables shown in the graphs below.
Describe the general trend/relationship. Identify sections of highest/lowest gradient
Quote any significant numerical values. Calculate any gradients you can

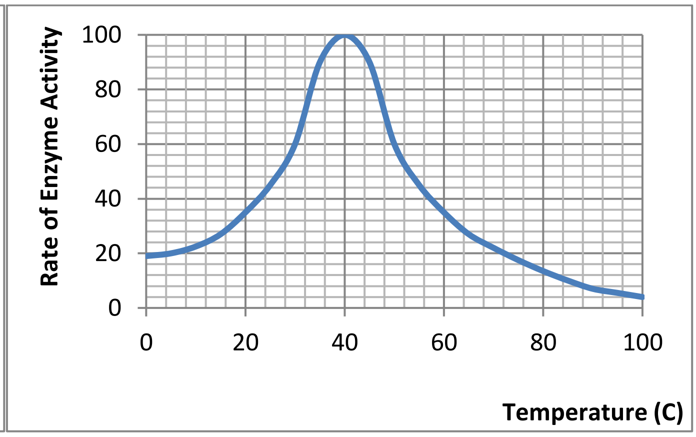


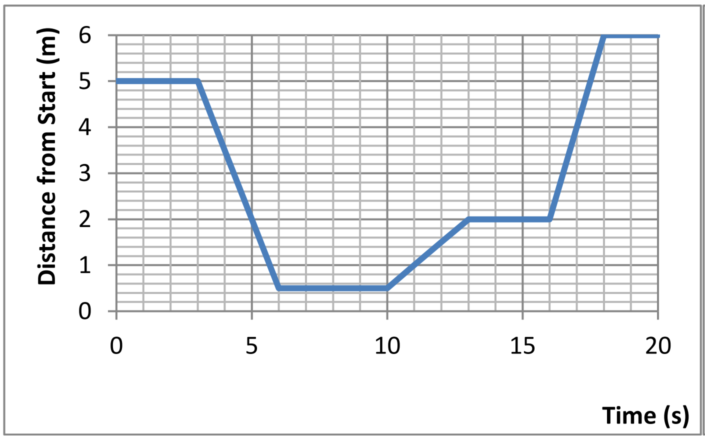


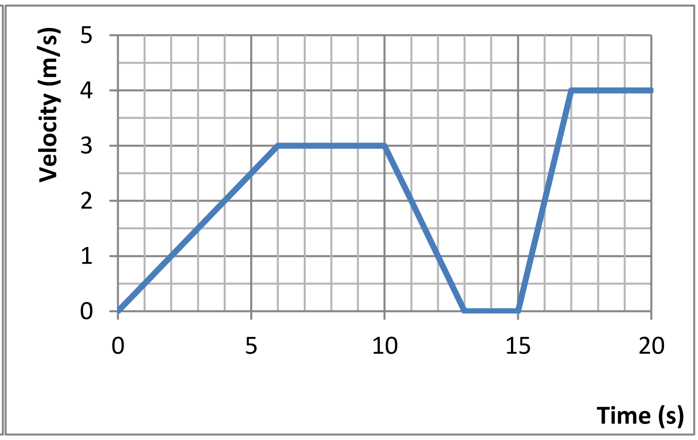












SKILLS TEST
ALGEBRAIC MANIPULATION

In your Physics A-Level, you must be able to re-arrange equations into different forms. This is called algebraic manipulation.

ACTIVITY 1

Rearrange $v^2 = u^2 + 2as$ to make a the subject	
Substitute this into $F = ma$	
Substitute this into the equation $P = Fv$	
Substitute this into the equation $E = Pt$	
Use $v = \frac{s}{t}$ to simplify the equation	

Substitute $v = u + at$ into the equation $\lambda = \frac{h}{mv}$	
Multiply out the brackets	
Substitute this into the equation $d \sin\theta = n\lambda$	
Rearrange the equation to find the angle to the 3rd maxima	

Substitute $R = \frac{V}{I}$ into the equation $\rho = \frac{RA}{l}$	
Substitute $V = \frac{E}{Q}$ into the equation	
Substitute $E = Pt$ into the equation	
Use $I = \frac{Q}{t}$ to remove t from the equation	
Simplify this	

Elastic energy is converted into kinetic	
Make v^2 the subject	
Replace this with $v^2 = u^2 + 2as$	
If u=0 rearrange the equation to make a the subject	

Use $F = ma$ to make F the subject	
Calculate the Energy stored using $E = Fs$ if the distance moved is e	

The centripetal force of a satellite is due to the gravitational force between it and the planet	$\frac{mv^2}{r} = \frac{GMm}{r^2}$
Simplify the equation and much as you can and rearrange to make r the subject	
Use $\omega = \frac{v}{r}$ to remove v from the equation	
Simplify the equation and use $\omega = 2\pi f$ to replace ω	
Use $f = \frac{1}{T}$ to remove f from the equation	

Rearrange $Q = Q_0 e^{-t/RC}$ to make $e^{-t/RC}$ the subject	
Make $-\frac{t}{RC}$ the subject	
Rearrange the equation to make t the subject	

The centripetal force of a charged particle is due to the magnetic force acting on it	$\frac{mv^2}{r} = BQv$
Rearrange the equation to make Q the subject	
Substitute $\frac{Q}{t} = I$ into the equation	
Use $\phi = BA$ to remove B from the equation	

Rearrange $N = N_0 e^{-\lambda t}$ to make $e^{-\lambda t}$ the subject	
Make $-\lambda t$ the subject	
Substitute $T_{½} = \frac{\ln 2}{\lambda}$ into the equation	

Rearrange $E = mcT$ to make T the subject	
Substitute this into the equation $pV = nRT$	
Substitute $p = \frac{F}{A}$ into the equation	
If the V is volume of a cube of side l, rearrange the equation to make F the subject	

ACTIVITY 2

1. Simplify the following as far as possible

a) $3ab + 2a - 3b - ab$

b) $2x^2 + 3x - 4x + 5 + 6x^2$

c) $2a \times 3a$

d) $e^2 \times 4e^3$

e) $3ab \times 2a$

f) $\frac{2x^2}{x}$

g) $\frac{6ab}{2ac}$

h) $\frac{a+b}{c-b}$

2. Multiply out the following brackets, and simplify the answer as far as possible

a) $3a(2 - b)$

b) $x(3x - 4)$

c) $-4y(2 + 5y)$

d) $-3a^2(4b - a)$

e) $(x + 2)(x - 3)$

f) $(2x - 5)(x - 7)$

g) $(1 - 4x)(2 + 7x)$

h) $(x^2 + 2)(2x + 5)$

3. Rearrange each of the following to make the letter indicated the subject

a) $s = ut + \frac{1}{2}at^2$ — u

b) $F = \frac{GMm}{r^2}$ — m

c) $F = \frac{GMm}{r^2}$ — r

d) $I = \frac{Q}{t}$ — t

e) $T = 2\pi\sqrt{\frac{L}{g}}$ — L

f) $y = \frac{2-x}{3+x}$ — x

g) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ — R_1

h) $2as = v^2 - u^2$ — v

4. In the following examples, the equation is given to you. Rearrange the equation for each of the other quantities. The first line shows an example of this.

Equation	First Rearrangement	Second Rearrangement
(Power of lens) $P = \frac{1}{f}$	1 = Pf	$f = \frac{1}{P}$
(Magnification of lens) $m = \frac{v}{u}$	$v =$	$u =$
(refractive index) $n = \frac{c}{v}$	$c =$	$v =$
(current) $I = \frac{\Delta Q}{\Delta t}$		
(electric potential) $V = \frac{\Delta E}{\Delta Q}$		
(power) $P = \frac{\Delta E}{\Delta t}$		
(power) $P = VI$		
(power) $P = I^2 R$		
(power) $P = \frac{V^2}{R}$		
(stress) $\sigma = \frac{F}{A}$	$F =$	$A =$
(strain) $\varepsilon = \frac{x}{l}$	$x =$	$l =$
(conductance) $G = \frac{\sigma A}{L}$		
(resistance) $R = \frac{\rho L}{A}$		

(phase angle) $\theta = 2\pi ft$	$f =$	$t =$	
(displacement) $y = a\sin\theta$	$a =$	θ =	
(Young's interference) $x = \frac{\lambda L}{d}$			
(electron wavelength) $\lambda = \frac{h}{mv}$			

MARK SCHEME

1. a) $2ab + 2a - 3b$ b) $8x^2 - x + 5$ c) $6a^2$
   d) $4e^5$ e) $6a^2b$ f) $2x$
   g) $\frac{3b}{c}$ h) $\frac{a+b}{c-b}$

2. a) $6a - 3ab$ b) $3x^2 - 4x$ c) $-8y - 20y^2$
   d) $-12a^2b + 3a^3$ e) $x^2 - x - 6$ f) $2x^2 - 19x + 35$
   g) $-28x^2 - x + 2$ h) $2x^3 + 5x^2 + 4x + 10$

3 a) $u = \frac{s}{t} - \frac{1}{2}at$ b) $m = \frac{Fr^2}{GM}$ c) $r = \sqrt{\frac{GMm}{F}}$
d) $t = \frac{Q}{I}$ e) $t = \frac{gT^2}{4\pi^2}$ f) $x = \frac{2 - 3y}{y+1}$
g) $R_1 = \frac{R_2R}{R_2 - R}$ h) $v = \sqrt{(u^2 + 2as)}$

SKILLS TEST
PRACTICAL GLOSSARY

In your Physics A-Level, you must be able to use the key words concerning practical investigations.
Complete the following practical glossary.

Accuracy	
Anomalies	
Calibration	
Measurement Error	
Random Error	
Systematic Error	
Zero Error	
Interval	
Precision	
Range	
Repeatable	
Reproducible	
Resolution	
Uncertainty	
Validity	
Categoric Variable	
Continuous Variable	
Control Variable	
Dependent Variable	
Independent Variable	

ANSWERS

Accuracy	A measurement result is considered accurate if it is judged to be close to the true value.
Anomalies	These are values in a set of results which are judged not to be part of the variation caused by random uncertainty.
Calibration	Marking a scale on a measuring instrument. This involves establishing the relationship between indications of a measuring instrument and standard or reference quantity values, which must be applied. For example, placing a thermometer in melting ice to see whether it reads 0ºC, in order to check if it has been calibrated correctly.
Measurement Error	The difference between a measured value and the true value.
Random Error	These cause readings to be spread about the true value, due to results varying in an unpredictable way from one measurement to the next. Random errors are present when any measurement is made and cannot be corrected. The effect of random errors can be reduced by making more measurements and calculating a new mean.
Systematic Error	These cause readings to differ from the true value by a consistent amount each time a measurement is made. Sources of systematic error can include the environment, methods of observation or instruments used. Systematic errors cannot be dealt with by simple repeats. If a systematic error is suspected, the data collection should be repeated using a different technique or a different set of equipment, and the results compared.
Zero Error	Any indication that a measuring system gives a false reading when the true value of a measured quantity is zero, eg the needle on an ammeter failing to return to zero when no current flows. A zero error may result in a systematic uncertainty.
Interval	The quantity between readings, eg a set of 11 readings equally spaced over a distance of 1 metre would give an interval of 10 centimetres.
Precision	Precise measurements are ones in which there is very little spread about the mean value. Precision depends only on the extent of random errors – it gives no indication of how close results are to the true value.
Range	The maximum and minimum values of the independent or dependent variables; important in ensuring that any pattern is detected. For example a range of distances may be quoted as either: 'From 10cm to 50 cm'
Repeatable	A measurement is repeatable if the original experimenter repeats the investigation using same method and equipment and obtains the same results.
Reproducible	A measurement is reproducible if the investigation is repeated by another person, or by using different equipment or techniques, and the same results are obtained.
Resolution	This is the smallest change in the quantity being measured (input) of a measuring instrument that gives a perceptible change in the reading.
Uncertainty	The interval within which the true value can be expected to lie, with a given level of confidence or probability, e.g. "the temperature is 20 °C ± 2 °C, at a level of confidence of 95 %.
Validity	Suitability of the investigative procedure to answer the question being asked. For example, an investigation to find out if the rate of a chemical reaction depended upon the concentration of one of the reactants would not be a valid procedure if the temperature of the reactants was not controlled.
Categoric Variable	Categoric variables have values that are labels. E.g. names of plants or types of material.

Continuous Variable	Continuous variables can have values (called a quantity) that can be given a magnitude either by counting (as in the case of the number of shrimp) or by measurement (eg light intensity, flow rate etc).
Control Variable	A control variable is one which may, in addition to the independent variable, affect the outcome of the investigation and therefore has to be kept constant or at least monitored.
Dependent Variable	The dependent variable is the variable of which the value is measured for each and every change in the independent variable.
Independent Variable	The independent variable is the variable for which values are changed or selected by the investigator.

REVISION QUIZ

To assess your understanding on the concepts covered in measurements and errors, complete the following quiz.

Circle the correct answer.

1.
Which of the following is a base unit of the SI?
A. ampere
B. coulomb
C. ohm
D. volt

2.
Which statement is correct?
A. Charge is a base quantity.
B. Current is a derived quantity.
C. Energy is a base quantity
D. Resistance is a derived quantity

3.
Which of the following lists contains only base quantities of the SI?
A. amount of substance, current, velocity
B. current, mass, time
C. mass, time, weight
D. time, weight, work

4.
Which of the following lists shows prefixes in the correct (ascending) order?
A. femto, pico, micro, nano, milli
B. femto, nano, pico, micro, milli
C. pico, nano, milli, micro, kilo
D. pico, nano, micro, milli, kilo

5.
Which of the following is not a base quantity of the SI?
A. amount of substance
B. current
C. length
D. weight

6.
Which of the following is equal to a volume of 100 cm^3?
A. $1.00 \times 10^{-6} m^3$
B. $1.00 \times 10^{-4} m^3$
C. $1.00 \times 10^{-2} m^3$
D. $1.00\ m^3$

7.
Which unit is equivalent to 1 kg $m^2 s^{-2}$?
A. 1 J
B. 1 N
C. 1 Pa
D. 1 W

8.
Which line in the table is correct?

	Prefix	**Meaning**
A	femto	x 10^{-18}
B	micro	x 10^{-9}
C	nano	x 10^{-6}
D	pico	x 10^{-12}

9.
If 1 electronvolt = 1.6 x 1^{0-19}J, what is the equivalent value of 3.2 MeV?
A.5.1 x 10^{-19}J
B. 5.1 x 10^{-13}J
C. 2.0 x 10^{19}J
D. 2.0 x 10^{25}J

Questions 10 and 11
A student conducted an experiment to determine the time period for different lengths of a simple pendulum by timing 10oscillations. For each length of
pendulum, the student started the stopwatch on the count of 'one' and stopped it on the count of 'ten'.
10.
What type of error did the student make?
A. parallax error
B. random error
C. systematic error
D. zero error

11.
In what way did this error affect the student's determination of the time periods?
A. The periods were too low because the numbers of oscillations used in the calculation were too low.
B. The periods were too low because the numbers of oscillations used in the calculation were too high.
C. The periods were too high because the numbers of oscillations used in the calculation were too low.
D. The periods were too high because the numbers of oscillations used in the calculation were too high.

12.
A student used the following values to calculate the pressure at the bottom of a deep tank of water.
depth of water = 2.4m
density of water = 1052kg m^{-3}
g = 9.81 N kg^{-1}
How many significant figures should the student use in her final answer?
A. 1
B. 2
C. 3
D. 4

13
In order to determine the momentum of a model railway locomotive, a teacher made the following measurements of mass and the time taken to travel a measured distance along a track.
mass = 160 ± 2 g
distance = 100 ± 2cm
time taken= 12.5 ± 0.1 s

Which measurement had the largest percentage uncertainty?
A. mass
B. distance
C. time taken
D. All 3 measurements had the same percentage uncertainty

14.
The diameter (d) of a length of copper wire was measured at different points.

d/ mm
0.84, 0.88, 0.78, 0.80, 0.86, 0.80, 0.87, 0.83. 0.90, 0.84

What is the mean and uncertainty in the value of d?
A. 0.84 ±0.06 mm
B. 0.84 ±0.12 mm
C. 0.840 ±0.060 mm
D. 0.840 ±0.012 mm

15.
A student made some measurements to determine the density of a solid glass sphere. The measurements and their absolute uncertainties are shown below.
mass = 5.0 ± 0.1 g
diameter = 1.00 ± 0.03 cm
What is the percentage uncertainty in the density of the glass sphere?
A. 1%
B. 5%
C. 8%
D. 11%

16.
A nucleus contains 7 nucleons. What is the best estimate for the mass of the nucleus?
A. 10^{-25}kg
B. 10^{-26}kg
C. 10^{-27}kg
D. 10^{-28}kg

17.
A student makes some measurements to determine the resistivity of constantan.
The measurements and their absolute uncertainties are shown below.
length of wire = 1.80 ± 0.05m
diameter of wire= 0.60 ± 0.03 mm
resistance = 3.00 ± 0.10 Ω
What is the percentage uncertainty in the resistivity of constantan?
A. 5%
B. 8%
C. 11%
D. 16%

18.
A quantity Q can be calculated using the formula $Q = x^2y/z^{1/3}$.
The orders of magnitude of x, y and z are 10^2, 10^4 and 10^6 respectively. What is the order of magnitude of Q?
A. $\sim 10^2$
B. $\sim 10^4$
C. $\sim 10^6$
D. $\sim 10^8$

19.
A student used a measuring cylinder to find the volumes of different liquids but measured to the top of each liquid's meniscus instead of the bottom.
What type of error did the student make?
A. parallax error
B. random error
C. systematic error
D. zero error

20.
Einstein's famous equation can be written as $E = mc^2$.
What is the best estimate for c^2?
A. $10^{15} m^2s^{-2}$
B. $10^{16} m^2s^{-2}$
C. $10^{17} m^2s^{-2}$
D. $10^{18} m^2s^{-2}$

ANSWERS

1. A
2. D
3. B
4. D
5. D
6. B
7. A
8. D
9. B
10. C
11. B
12. B
13. B
14. A
15. D
16. B
17. D
18. C
19. C
20. C

DATASHEET

DATA - FUNDAMENTAL CONSTANTS AND VALUES

Quantity	Symbol	Value	Units
speed of light in vacuo	c	3.00×10^{8}	$\mathrm{m\,s^{-1}}$
permeability of free space	μ_0	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
permittivity of free space	ε_0	8.85×10^{-12}	$\mathrm{F\,m^{-1}}$
magnitude of the charge of electron	e	1.60×10^{-19}	C
the Planck constant	h	6.63×10^{-34}	J s
gravitational constant	G	6.67×10^{-11}	$\mathrm{N\,m^{2}\,kg^{-2}}$
the Avogadro constant	N_A	6.02×10^{23}	$\mathrm{mol^{-1}}$
molar gas constant	R	8.31	$\mathrm{J\,K^{-1}\,mol^{-1}}$
the Boltzmann constant	k	1.38×10^{-23}	$\mathrm{J\,K^{-1}}$
the Stefan constant	σ	5.67×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
the Wien constant	α	2.90×10^{-3}	m K
electron rest mass (equivalent to 5.5×10^{-4} u)	m_e	9.11×10^{-31}	kg
electron charge/mass ratio	$\frac{e}{m_e}$	1.76×10^{11}	$\mathrm{C\,kg^{-1}}$
proton rest mass (equivalent to 1.00728 u)	m_p	$1.67(3) \times 10^{-27}$	kg
proton charge/mass ratio	$\frac{e}{m_p}$	9.58×10^{7}	$\mathrm{C\,kg^{-1}}$
neutron rest mass (equivalent to 1.00867 u)	m_n	$1.67(5) \times 10^{-27}$	kg
gravitational field strength	g	9.81	$\mathrm{N\,kg^{-1}}$
acceleration due to gravity	g	9.81	$\mathrm{m\,s^{-2}}$
atomic mass unit (1u is equivalent to 931.5 MeV)	u	1.661×10^{-27}	kg

ALGEBRAIC EQUATION

quadratic equation $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

ASTRONOMICAL DATA

Body	Mass/kg	Mean radius/m
Sun	1.99×10^{30}	6.96×10^{8}
Earth	5.97×10^{24}	6.37×10^{6}

GEOMETRICAL EQUATIONS

arc length	$= r\theta$
circumference of circle	$= 2\pi r$
area of circle	$= \pi r^2$
curved surface area of cylinder	$= 2\pi rh$
area of sphere	$= 4\pi r^2$
volume of sphere	$= \frac{4}{3}\pi r^3$

Particle Physics

Class	Name	Symbol	Rest energy/MeV
photon	photon	γ	0
lepton	neutrino	ν_e	0
		ν_μ	0
	electron	$e^\pm$	0.510999
	muon	$\mu^\pm$	105.659
mesons	π meson	$\pi^\pm$	139.576
		π^0	134.972
	K meson	$K^\pm$	493.821
		K^0	497.762
baryons	proton	p	938.257
	neutron	n	939.551

Properties of quarks

antiquarks have opposite signs

Type	Charge	Baryon number	Strangeness
u	$+\frac{2}{3}e$	$+\frac{1}{3}$	0
d	$-\frac{1}{3}e$	$+\frac{1}{3}$	0
s	$-\frac{1}{3}e$	$+\frac{1}{3}$	-1

Properties of Leptons

		Lepton number
Particles:	$e^-, \nu_e; \mu^-, \nu_\mu$	$+1$
Antiparticles:	$e^+, \overline{\nu_e}, \mu^+, \overline{\nu_\mu}$	-1

Photons and energy levels

photon energy $E = hf = hc/\lambda$

photoelectricity $hf = \phi + E_{k\,(max)}$

energy levels $hf = E_1 - E_2$

de Broglie wavelength $\lambda = \frac{h}{p} = \frac{h}{mv}$

Waves

wave speed $c = f\lambda$ *period* $f = \frac{1}{T}$

first harmonic $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

fringe spacing $w = \frac{\lambda D}{s}$ *diffraction grating* $d \sin\theta = n\lambda$

refractive index of a substance s, $n = \frac{c}{c_s}$

for two different substances of refractive indices n_1 *and* n_2,

law of refraction $n_1 \sin\theta_1 = n_2 \sin\theta_2$

critical angle $\sin\theta_c = \frac{n_2}{n_1}$ for $n_1 > n_2$

Mechanics

moments moment $= Fd$

velocity and acceleration $v = \frac{\Delta s}{\Delta t}$ $a = \frac{\Delta v}{\Delta t}$

equations of motion $v = u + at$ $s = \left(\frac{u+v}{2}\right)t$

$v^2 = u^2 + 2as$ $s = ut + \frac{at^2}{2}$

force $F = ma$

force $F = \frac{\Delta(mv)}{\Delta t}$

impulse $F\Delta t = \Delta(mv)$

work, energy and power $W = Fs\cos\theta$

$E_k = \frac{1}{2}mv^2$ $\Delta E_p = mg\Delta h$

$P = \frac{\Delta W}{\Delta t}$, $P = Fv$

$efficiency = \frac{useful\ output\ power}{input\ power}$

Materials

density $\rho = \frac{m}{V}$ *Hooke's law* $F = k\Delta L$

Young modulus $= \frac{tensile\ stress}{tensile\ strain}$ *tensile stress* $= \frac{F}{A}$ *tensile strain* $= \frac{\Delta L}{L}$

energy stored $E = \frac{1}{2}F\Delta L$

Electricity

current and pd $I = \frac{\Delta Q}{\Delta t}$ $V = \frac{W}{Q}$ $R = \frac{V}{I}$

resistivity $\rho = \frac{RA}{L}$

resistors in series $R_T = R_1 + R_2 + R_3 + \ldots$

resistors in parallel $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$

power $P = VI = I^2R = \frac{V^2}{R}$

emf $\varepsilon = \frac{E}{Q}$ $\varepsilon = I(R + r)$

Circular motion

magnitude of angular speed $\omega = \frac{v}{r}$

$\omega = 2\pi f$

centripetal acceleration $a = \frac{v^2}{r} = \omega^2 r$

centripetal force $F = \frac{mv^2}{r} = m\omega^2 r$

Simple harmonic motion

acceleration $a = -\omega^2 x$

displacement $x = A \cos(\omega t)$

speed $v = \pm\omega\sqrt{(A^2 - x^2)}$

maximum speed $v_{max} = \omega A$

maximum acceleration $a_{max} = \omega^2 A$

for a mass-spring system $T = 2\pi\sqrt{\frac{m}{k}}$

for a simple pendulum $T = 2\pi\sqrt{\frac{l}{g}}$

Thermal physics

energy to change temperature $Q = mc\Delta\theta$

energy to change state $Q = ml$

gas law $pV = nRT$

$pV = NkT$

kinetic theory model $pV = \frac{1}{3}Nm(c_{rms})^2$

kinetic energy of gas molecule $\frac{1}{2}m(c_{rms})^2 = \frac{3}{2}kT = \frac{3RT}{2N_A}$

Gravitational fields

force between two masses $F = \frac{Gm_1m_2}{r^2}$

gravitational field strength $g = \frac{F}{m}$

magnitude of gravitational field strength in a radial field $g = \frac{GM}{r^2}$

work done $\Delta W = m\Delta V$

gravitational potential $V = -\frac{GM}{r}$

$g = -\frac{\Delta V}{\Delta r}$

Electric fields and capacitors

force between two point charges $F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$

force on a charge $F = EQ$

field strength for a uniform field $E = \frac{V}{d}$

work done $\Delta W = Q\Delta V$

field strength for a radial field $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

electric potential $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

$E = \frac{\Delta V}{\Delta r}$

capacitance $C = \frac{Q}{V}$

$C = \frac{A\varepsilon_0\varepsilon_r}{d}$

capacitor energy stored $E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$

capacitor charging $Q = Q_0(1 - e^{-t/RC})$

decay of charge $Q = Q_0e^{-t/RC}$

time constant RC

Magnetic fields

force on a current $F = BIl$

force on a moving charge $F = BQv$

magnetic flux $\Phi = BA$

magnetic flux linkage $N\Phi = BAN\cos\theta$

magnitude of induced emf $\varepsilon = N\dfrac{\Delta\Phi}{\Delta t}$

$N\Phi = BAN\cos\theta$

emf induced in a rotating coil $\varepsilon = BAN\omega\sin\omega t$

alternating current $I_{\mathrm{rms}} = \dfrac{I_0}{\sqrt{2}}$ $V_{\mathrm{rms}} = \dfrac{V_0}{\sqrt{2}}$

transformer equations $\dfrac{N_s}{N_p} = \dfrac{V_s}{V_p}$

$efficiency = \dfrac{I_s V_s}{I_p V_p}$

Nuclear physics

the inverse square law for γ radiation $I = \dfrac{k}{x^2}$

radioactive decay $\dfrac{\Delta N}{\Delta t} = -\lambda N,\ N = N_0 e^{-\lambda t}$

activity $A = \lambda N$

half-life $T_{½} = \dfrac{\ln 2}{\lambda}$

nuclear radius $R = R_0 A^{1/3}$

energy-mass equation $E = mc^2$

OPTIONS

Astrophysics

1 astronomical unit = 1.50×10^{11} m

1 light year = 9.46×10^{15} m

1 parsec = 206265 AU = 3.08×10^{16} m
= 3.26 light year

Hubble constant, H = $65\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$

$M = \dfrac{\text{angle subtended by image at eye}}{\text{angle subtended by object at unaided eye}}$

in normal adjustment $M = \dfrac{f_o}{f_e}$

Rayleigh criterion $\theta \approx \dfrac{\lambda}{D}$

magnitude equation $m - M = 5\log\dfrac{d}{10}$

Wien's law $\lambda_{\max} T = 2.9 \times 10^{-3}\ \mathrm{m\,K}$

Stefan's law $P = \sigma A T^4$

Schwarzschild radius $R_s \approx \dfrac{2GM}{c^2}$

Doppler shift for v << c $\dfrac{\Delta f}{f} = -\dfrac{\Delta\lambda}{\lambda} = \dfrac{v}{c}$

red shift $z = -\dfrac{v}{c}$

Hubble's law $v = Hd$

Medical physics

lens equations $P = \dfrac{1}{f}$

$m = \dfrac{v}{u}$

$\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$

threshold of hearing $I_0 = 1.0 \times 10^{-12}\ \mathrm{W\,m^{-2}}$

intensity level $intensity\ level = 10\log\dfrac{I}{I_0}$

absorption $I = I_0 e^{-\mu x}$

$\mu_m = \dfrac{\mu}{\rho}$

ultrasound imaging $Z = \rho c$

$\dfrac{I_r}{I_i} = \left(\dfrac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$

half-lives $\dfrac{1}{T_E} = \dfrac{1}{T_B} + \dfrac{1}{T_P}$

Engineering physics

moment of inertia $I = \Sigma mr^2$

angular kinetic energy $E_k = \frac{1}{2}I\omega^2$

equations of angular motion

$$\omega_2 = \omega_1 + \alpha t$$

$$\omega_2^2 = \omega_1^2 + 2\alpha\theta$$

$$\theta = \omega_1 t + \frac{\alpha t^2}{2}$$

$$\theta = \frac{(\omega_1 + \omega_2)t}{2}$$

torque $T = I\alpha$

$T = Fr$

angular momentum angular momentum $= I\omega$

angular impulse $T\Delta t = \Delta(I\omega)$

work done $W = T\theta$

power $P = T\omega$

thermodynamics $Q = \Delta U + W$

$W = p\Delta V$

adiabatic change $pV^\gamma =$ constant

isothermal change $pV =$ constant

heat engines

$$\text{efficiency} = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H}$$

$$\text{maximum theoretical efficiency} = \frac{T_H - T_C}{T_H}$$

work done per cycle = area of loop

input power = calorific value × fuel flow rate

indicated power = (area of $p - V$ loop) × (number of cycles per second) × (number of cylinders)

output or brake power $P = T\omega$

friction power = indicated power – brake power

heat pumps and refrigerators

refrigerator: $COP_{ref} = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C}$

heat pump: $COP_{hp} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_C}$

Turning points in physics

electrons in fields $F = \frac{eV}{d}$

$F = Bev$

$r = \frac{mv}{Be}$

$\frac{1}{2}mv^2 = eV$

Millikan's experiment $\frac{QV}{d} = mg$

$F = 6\pi\eta rv$

Maxwell's formula $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$

special relativity $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

$l = l_0\sqrt{1 - \frac{v^2}{c^2}}$

$E = mc^2 = \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$

Electronics

resonant frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$

Q-factor $Q = \frac{f_0}{f_B}$

operational amplifiers: open loop $V_{out} = A_{OL}(V_+ - V_-)$

inverting amplifier $\frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}}$

non-inverting amplifier $\frac{V_{out}}{V_{in}} = 1 + \frac{R_f}{R_1}$

summing amplifier $V_{out} = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \cdots\right)$

difference amplifier $V_{out} = (V_+ - V_-)\frac{R_f}{R_1}$

Bandwidth requirement:

for AM bandwidth $= 2f_M$

for FM bandwidth $= 2(\Delta f + f_M)$

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This document has been produced for educational purposes only.

This document has been produced for the AQA A Level Physics Specification.

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If you when using this document, you believe there is an improvement to made, please state this in the space below….

Only constructive and reasoned feedback will be considered.