ST MARY'S SCIENCE DEPARTMENT:

PHYSICS

A LEVEL PHYSICS YEAR 1
STUDENT PREPARATORY READING
3.1: MEASUREMENTS AND ERRORS

NAME	
PHYSICS CLASS	
MODULE TEACHER	
ALPS GRADE	

A-LEVEL PHYSICS

TOPIC 1

READING BOOK

THIS MUST BE BROUGHT TO ALL PHYSICS LESSONS.

Contents

OVERVIEW

This booklet provides the basic information and knowledge needed to access the A Level Physics course.
Over the course of the year, you will be directed to read this information in both lessons and in your spare time.
Ensure all information in this booklet is understood and remembered before examinations.

IMPORTANT NOTE

This book, along with the student workbook and independent work book, must be brought to all Physics lessons with the appropriate teacher.

This booklet may be used as a learning resource in lessons, you are not fully equipped to learn if this is not used in lesson.

This book may also be used as a revision resource for intervention, internal assessments and external assessments.
Please keep this in your student file.

The Language of Measurement

The following subject specific vocabulary provides definitions of key terms used in the A-level Science specifications.

Accuracy
A measurement result is considered accurate if it is judged to be close to the true value.

Calibration
Marking a scale on a measuring instrument.
This involves establishing the relationship between indications of a measuring instrument and standard or reference quantity values, which must be applied.
For example, placing a thermometer in melting ice to see whether it reads 0 °C, to check if it has been calibrated correctly.

Data
Information, either qualitative or quantitative, that has been collected.

Errors
See also uncertainties.

Measurement error
The difference between a measured value and the true value.
anomalies
These are values in a set of results which are judged not to be part of the variation caused by random uncertainty.

Random error
These cause readings to be spread about the true value, due to results varying in an unpredictable way from one measurement to the next.
Random errors are present when any measurement is made and cannot be corrected. The effect of random errors can be reduced by making more measurements and calculating a new mean.

Systematic error
These cause readings to differ from the true value by a consistent amount each time a measurement is made.
Sources of systematic error can include the environment, methods of observation or instruments used.
Systematic errors cannot be dealt with by simple repeats. If a systematic error is suspected, the data collection should be repeated using a different technique or a different set of equipment, and the results compared.

Zero error
Any indication that a measuring system gives a false reading when the true value of a measured quantity is zero, e.g. the needle on an ammeter failing to return to zero when no current flows.
A zero error may result in a systematic uncertainty.

Evidence
Data which has been shown to be valid.

Fair test
A fair test is one in which only the independent variable has been allowed to affect the dependent variable.

Hypothesis
A proposal intended to explain certain facts or observations.

Interval
The quantity between readings, e.g. a set of 11 readings equally spaced over a distance of 1 metre would give an interval of 10 centimetres.

Precision
Precise measurements are ones in which there is very little spread about the mean value.
Precision depends only on the extent of random errors – it gives no indication of how close results are to the true value.

Prediction
A prediction is a statement suggesting what will happen in the future, based on observation, experience or a hypothesis.

Range
The maximum and minimum values of the independent or dependent variables; important in ensuring that any pattern is detected.
For example, a range of distances may be quoted as either:
'From 10 cm to 50 cm'
or
'From 50 cm to 10 cm'

Repeatable
A measurement is repeatable if the original experimenter repeats the investigation using same method and equipment and obtains the same results.

Reproducible
A measurement is reproducible if the investigation is repeated by another person, or by using different equipment or techniques, and the same results are obtained.

Resolution
This is the smallest change in the quantity being measured (input) of a measuring instrument that gives a perceptible change in the reading.

Sketch graph
A line graph, not necessarily on a grid, that shows the general shape of the relationship between two variables. It will not have any points plotted and although the axes should be labelled they may not be scaled.

True value
This is the value that would be obtained in an ideal measurement.

Uncertainty
The interval within which the true value can be expected to lie, with a given level of confidence or probability, e.g. "the temperature is 20 °C ± 2 °C, at a level of confidence of 95%.

Validity
Suitability of the investigative procedure to answer the question being asked. For example, an investigation to find out if the rate of a chemical reaction depended upon the concentration of one of the reactants would not be a valid procedure if the temperature of the reactants was not controlled.

Valid conclusion
A conclusion supported by valid data, obtained from an appropriate experimental design and based on sound reasoning.

Variables
These are physical, chemical or biological quantities or characteristics.

Categoric variables
Categoric variables have values that are labels. E.g. names of plants or types of material.

Continuous variables
Continuous variables can have values (called a quantity) that can be given a magnitude either by counting (as in the case of the number of shrimp) or by measurement (e.g. light intensity, flow rate etc.).

Control variables
A control variable is one which may, in addition to the independent variable, affect the outcome of the investigation and therefore must be kept constant or at least monitored.

Dependent variables
The dependent variable is the variable of which the value is measured for each change in the independent variable.

Independent variables
The independent variable is the variable for which values are changed or selected by the investigator.

IMPORTANT NOTE
These definitions must be memorised by students.

You will be tested on your knowledge of these

TOPIC 1: SI UNITS AND PREFIXES SPEC CHECK

Specification	Completed?
Fundamental (base) units.	
Use of mass, length, time, quantity of matter, temperature, electric current and their associated SI units.	
SI units derived.	
Knowledge and use of the SI prefixes, values and standard form.	
Students should be able to use the prefixes: T, G, M, k, c, m, μ, n, p, f,	
Students should be able to convert between different units of the same quantity, e.g. J and eV, J and kW h.	

Student Checklist

Have I…….	Yes or No?
Read through the notes of this section?	
Highlighted/underlined the key concepts of this section?	
Made my own notes based on the notes of this section?	
Brought the notes to be used in lesson?	

NOTES
Base Quantities and SI Units

In 1971 world's scientists agreed on a common system of units for those 7 base quantities.

They are the SI Units. These are the fundamental quantities and units of the Universe.

These are….

Base Quantity	SI Unit & Abbreviation
Length	metre, m
Time	second, s
Mass	kilogram, kg
Temperature	kelvin, K
Electric current	ampere, A
Amount of substance	mole, mol
Luminous Intensity	candela, cd

Study Tip

Memorise the SI base quantities and their units.

All the other physical quantities used in Physics are **DERIVED** from these base quantities.

These called the SI derived units.

You must be able to express all units in SI unit terms.

Example 1: $Units\ of\ volume = unit\ of\ (length \times length \times length)$

$$= m \times m \times m$$

$$= m^3$$

Example 2: $Units\ of\ density = \frac{unit\ of\ mass}{unit\ of\ volume}$

$$= \frac{kg}{m^3}$$

$$= kgm^{-3}$$

Example 3: $$Units\ of\ velocity = \frac{unit\ of\ displacement}{unit\ of\ time}$$

$$= \frac{m}{s}$$

$$= ms^{-1}$$

Below are examples of SI derived units.

Examples of SI derived units		
Derived quantity	**SI derived unit**	
	Name	**Symbol**
Area (A)	square meter	m^2
Volume (V)	cubic meter	m^3
speed, velocity (v)	meter per second	m/s
Acceleration (a)	meter per second squared	m/s^2
mass density (ρ)	kilogram per cubic meter	kg/m^3
amount-of-substance concentration (n)	mole per cubic meter	mol/m^3

Below are examples of SI derived units which have special names.

Derived quantity	Name	Symbol	Expression in terms of other SI units	Expression in terms of SI base units
plane angle (θ)	radian	rad	-	$m\ m^{-1} = 1$
Frequency (f)	Hertz	Hz	-	s^{-1}
Force (F)	Newton	N	-	$mkgs^{-2}$
pressure, stress (P)	Pascal	Pa	N/m^2	$m^{-1}kgs^{-2}$
energy, work, quantity of heat (E or w)	Joule	J	N·m	m^2kgs^{-2}
Power (P)	Watt	W	J/s	m^2kgs^{-3}
electric charge, quantity of electricity (Q)	Coulomb	C	-	sA
electric potential difference (V), electromotive force (ε)	Volt	V	W/A	$m^2kgs^{-3}A^{-1}$
Capacitance (C)	Farad	F	C/V	$m^{-2}kg^{-1}s^4A^2$
electric resistance (R)	Ohm	Ω	V/A	$m^2kgs^{-3}A^{-2}$
magnetic flux (φ)	Weber	Wb	V·s	$m^2kgs^{-2}A^{-1}$
magnetic flux density (B)	Tesla	T	Wb/m^2	$kgs^{-2}A^{-1}$
Celsius temperature (T)	degree Celsius	°C	-	K
activity (of a radionuclide) (A)	Becquerel	Bq	-	s^{-1}

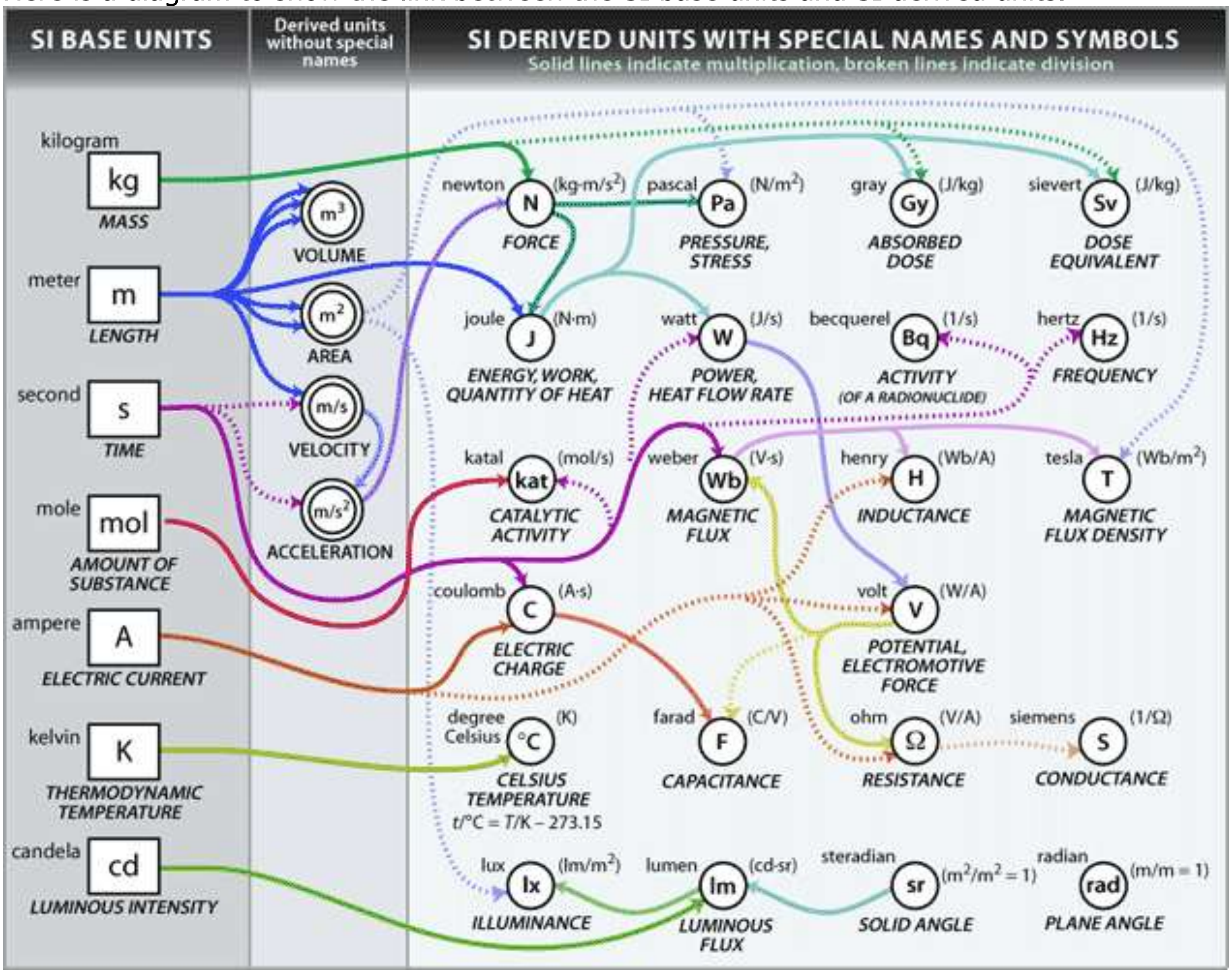
Study Tip

Do not memorise the expression of quantities in SI base units – memorise the method of working this out.

Below are examples of SI derived units based on other SI derived units.

Examples of SI derived units whose names and symbols include SI derived units with special names and symbols		
	SI derived unit	
Derived quantity	**Name**	**Symbol**
moment of force (M)	newton meter	Nm
angular velocity	radian per second	rad/s
angular acceleration	radian per second squared	rad/s^2
heat flux density, irradiance	watt per square meter	W/m^2
heat capacity, entropy	joule per kelvin	J/K
specific heat capacity, specific entropy	joule per kilogram kelvin	J/(kgK)
electric field strength	volt per meter	V/m
permittivity	farad per meter	F/m
molar energy	joule per mole	J/mol
molar entropy, molar heat capacity	joule per mole kelvin	J/(molK)

Here is a diagram to show the link between the SI base units and SI derived units.



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Standard Prefixes

One of the strengths of the SI is that absolutely any measurement can be expressed in terms of the seven base units (and angle if needed).

Thanks to the SI and a well-organised set of agreements and procedures to make it work, everyone all over the world can not only agree just how fast their cars are, they can make sure their components fit together properly too.

It would be inconvenient to measure everything using only the pure base or derived units - motorists don't want road distance in metres or astronomers do not want stellar distances in metres. A shorthand system of prefixes was agreed as part of the SI system.

For example:

10 metres =1 decametre 10 decametres = 1 hectometre 10 hectometres = 1 kilometre.

All the prefixes are related to each other by numbers like 10, 100 or 1,000; which are called powers of 10.

Throughout the SI system, the same prefixes are used for the same multiples, no matter what the unit - except for the kilogram. Because the kilogram already includes a prefix in its name we don't refer to a thousandth of a kilogram as a millikilogram.

Some of these units aren't used much – decimetres, for example, are uncommon, and megametres are practically unheard of.

And the prefixes aren't used for time, where there was already an internationally agreed system in place long before the SI came along.

Here is the conversion for time units.

Name	Symbol	Quantity	Equivalent SI unit
minute	min	time	1 min = 60 s
hour	h	time	1 h = 3600 s
day	d	time	1 d = 86400 s

Time is the one part of the SI system not considered for prefixes.

Examination Tip

Always ensure values are given in SI Units – if they are not convert them.

Always remove the prefixes from answers before calculating.

Here are the conversion prefixes used in Physics.
You must be able to use these prefixes in any Physics question.

Prefix	Symbol	Decimal	Power of 10
yotta	Y	1000000000000000000000000	10^{24}
zetta	Z	1000000000000000000000	10^{21}
exa	E	1000000000000000000	10^{18}
peta	P	1000000000000000	10^{15}
tera	T	1000000000000	10^{12}
giga	G	1000000000	10^{9}
mega	M	1000000	10^{6}
kilo	k	1000	10^{3}
hecto	h	100	10^{2}
deca	da	10	10^{1}
deci	d	0.1	10^{-1}
centi	c	0.01	10^{-2}
milli	m	0.001	10^{-3}
micro	µ	0.000001	10^{-6}
nano	n	0.000000001	10^{-9}
pico	p	0.000000000001	10^{-12}
femto	f	0.000000000000001	10^{-15}
atto	a	0.000000000000000001	10^{-18}
zepto	z	0.000000000000000000001	10^{-21}
yocto	y	0.000000000000000000000001	10^{-24}

Exam Tip

In any examination question, you should substitute the prefix for the standard form conversion given in the table above.

For example, 20km = 20×10^3 m

Here are the conversion for angular measurements.

Name	Symbol	Quantity	Equivalent SI unit
degree of arc	°	angle	1° = (π/180) rad
minute of arc	′	angle	1π = (π/10800) rad
second of arc	″	angle	1π = (π/648000) rad
hectare	ha	area	1 ha = 10000 m^2
litre	l or L	volume	1 l = 0.001 m^3
tonne	t	mass	1 t = 1000 kg

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Significant Figures

In Physics, you cannot write an answer to a problem with all the numbers given on the calculator.

A method of giving an approximated answer is to round off using significant figures.

The word significant means: having meaning.

With the number 368249, the 3 is the most significant digit, because it tells us that the number is 3 hundred thousand and something. It follows that the 6 is the next most significant, and so on.

With the number 0.0000058763, the 5 is the most significant digit, because it tells us that the number is 5 millionths and something. The 8 is the next most significant, and so on.

Be careful however with numbers such as 30245, the 3 is the first significant figure and 0 the second, because of its value as a place holder.

We round off a number using a certain number of significant figures. The most common are 1, 2 or 3 significant figures.

Remember the rules for rounding up are:

If the next number is 5 or more, we round up. If the next number is 4 or less, we do not round up.

Study Tip

Always check the number of significant figures in a question and give your answer to the lowest value.

Examples

E1. What would you get if you wrote the number 368249 correct to 1 significant figure?

400000

3 is the first significant figure, and the digit after it is more than 5, so you round up.

E2. What would you get if you wrote the number 0.00245 correct to 1 significant figure?

0.002

2 is the first significant figure and the digit after this is less than 5, so you do not round up.

E3. What would you get if you wrote 0.0000058763 correct to 2 significant figures?

0.0000059

You had to round up the 8 to 9.

Examination Tip

If you are unsure about the number of significant figures in a value e.g. is 100 1 sig fig or 3 sig figs? Use the smallest number of significant figures in your answer.

E4. What is 7.994 to two significant figures?

8.0

You had to round up.

The 2 first significant figures are 7 and 9. The digit after 9 is 9 again, so we have to round up, 7.99 rounds up to 8.00.

Examination Questions

In an examination question, you should give your answer to the same number of significant figures as the value given in the question with the lowest number of significant figures.

You do not include constants given in the data booklet as a value.

If it is vague as to the number of significant figures in the question – there will be allowance in the mark scheme for this.

Example

0 2 . 3 Calculate the maximum kinetic energy, in J, of the electrons emitted from a zinc plate when illuminated with ultraviolet light.

work function of zinc = 4.3 eV
frequency of ultraviolet light = 1.2×10^{15} Hz

[3 marks]

maximum kinetic energy ________________ J

In this question, as both values are given to two significant figures, you should give your answer to two significant figures.

If you do not do this, you are penalised one mark.

Examination Tip

Always check your answers for the correct number of significant figures.

It is important that the significant figures are respected – it shows your confidence in your experimental work.

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Practical Work

Data should be written in tables to the same number of significant figures. This number should be determined by the resolution of the device being used to measure the data or the uncertainty in measurement. For example, a length of string measured to be 60 cm using a ruler with mm graduations should be recorded as 600 mm, 60.0 cm or 0.600 m, and **not** just 60 cm. Similarly, a resistor value quoted by the manufacturer as 56 kΩ, 5% tolerance should **not** be recorded as 56.0 kΩ.

There is sometimes confusion over the number of significant figures when readings cross multiples of 10. Changing the number of decimal places across a power of ten retains the number of significant figures **but changes the accuracy.** The same number of decimal places should therefore generally be used, as illustrated below.

0.97
0.98
0.99
1.00
1.10

99.7
99.8
99.9
100.0
101.0

It is good practice to write down all digits showing on a digital meter.

Calculated quantities should be shown to the number of significant figures of the data with the least number of significant figures.

Example:

Calculate the size of an object if the magnification of a photo is ×25 and it is measured to be 24.6 mm on the photo.

$$size\ of\ real\ object = \frac{size\ of\ image}{magnification}$$

$$size\ of\ real\ object = \frac{24.6 \times 10^{-3}}{25}$$

$$size\ of\ real\ object = 9.8 \times 10^{-4}$$

Note that the size of the real object can only be quoted to two significant figures as the magnification is only quoted to two significant figures.

Equipment measuring to half a unit (e.g. a thermometer measuring to 0.5 °C) should have measurements recorded to one decimal place (e.g. 1.0 °C, 2.5 °C).

The uncertainty in these measurements would be ±0.25, but this would be rounded to the same number of decimal places (giving measurements quoted with uncertainty of (1.0 ± 0.3) °C etc).

Dealing with anomalous results

At GCSE, you are often taught automatically to ignore anomalous results.

At A-level, you should think carefully about what could have caused the unexpected result and therefore whether it is anomalous.

You might be able to identify a reason for the unexpected result and so validly regard it as an anomaly. For example, an anomalous result might be explained by a different experimenter making the measurement, a different solution or a different measuring device being used. In the case where the reason for an anomalous result occurring can be identified, the result should be recorded and plotted but may then be ignored.

Anomalous results should also be ignored where results are expected to be the same.

Where there is no obvious error and no expectation that results should be the same, anomalous results should be included. This will reduce the possibility that a key point is being overlooked.

Please note: when recording results, it is important that all data are included. Anomalous results should only be ignored at the data analysis stage.

It is best practice whenever an anomalous result is identified for the experiment to be repeated. This highlights the need to tabulate and even graph results as an experiment is carried out.

Mean Average

When calculating the mean average of a value in Physics, you must remove the anomalous result before the calculation.

The mean average should be given to the same number of significant figures as the value with the least number of significant figures in the calculation.

Example

1. Four results were taken.

12cm, 13cm, 12cm, 11cm.

What is the mean?

$$\text{Mean} = \frac{\text{Sum of Values}}{\text{Number of Values}} = \frac{(12 + 13 + 12 + 11)}{4} = 12\text{cm}$$

2. Four results were taken.

12cm, 13cm, 19cm, 11cm.

What is the mean?

$$\text{Mean} = \frac{\text{Sum of Values}}{\text{Number of Values}} = \frac{(12 + 13 + 11)}{3} = 12\text{cm}$$

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TOPIC 2: LIMITS OF MEASUREMENTS SPEC CHECK

Specification	Completed?
Random and systematic errors.	
Precision, repeatability, reproducibility, resolution and accuracy.	
Absolute, fractional and percentage uncertainties represent uncertainty in the final answer for a quantity.	
Combination of absolute and percentage uncertainties.	
Represent uncertainty in a data point on a graph using error bars.	
Determine the uncertainties in the gradient and intercept of a straight-line graph.	
Individual points on the graph may or may not have associated error bars.	
Students should be able to identify random and systematic errors and suggest ways to reduce or remove them.	
Students should understand the link between the number of significant figures in the value of a quantity and its associated uncertainty.	
Students should be able to combine uncertainties in cases where the measurements that give rise to the uncertainties are added, subtracted, multiplied, divided, or raised to powers.	

Student Checklist

Have I…….	Yes or No?
Read through the notes of this section?	
Highlighted/underlined the key concepts of this section?	
Made my own notes based on the notes of this section?	
Brought the notes to be used in lesson?	

NOTES

Sources of uncertainties

Every measurement has some inherent uncertainty.

The important question to ask is whether an experimenter can be confident that the true value lies in the range that is predicted by the uncertainty that is quoted. Good experimental design will attempt to reduce the uncertainty in the outcome of an experiment. The experimenter will design experiments and procedures that produce the least uncertainty and to provide a realistic uncertainty for the outcome.

In assessing uncertainty, there are many issues that must be considered. These include

The resolution of the instrument used
The manufacturer’s tolerance on instruments
The judgments that are made by the experimenter
The procedures adopted (e.g. repeated readings)
The size of increments available (e.g. the size of drops from a pipette).

Often, the resolution of the equipment will be the guiding factor in assessing a numerical uncertainty.

There may be further questions that would require you to evaluate arrangements and procedures. You could be asked how particular procedures would affect uncertainties and how they could be reduced by different apparatus design or procedure – for example using the human reaction has a larger uncertainty than the stopwatch.

A combination of the above factors means that there can be no hard and fast rules about the actual uncertainty in a measurement **– it is normally the discretion of the experimenter**. What we can assess from an instrument’s resolution is the **minimum** possible uncertainty.

Only the experimenter can assess the other factors based on the arrangement and use of the apparatus and a rigorous experimenter would draw attention to these factors and take them into account.

Readings and measurements

Study Tip

Learn the difference between a reading and a measurement.

It is useful, when discussing uncertainties, to separate measurements into two forms:

Readings
the values found from a single judgement when using a piece of equipment

Measurements
the values taken as the difference between the judgements of two values.

Examples:

When using a thermometer, a student only needs to make one judgement (the height of the liquid). This is a reading. It can be assumed that the zero value has been correctly set.

For protractors and rulers, both the starting point and the end point of the measurement must be judged, leading to two uncertainties.

The following list is not exhaustive, and the way that the instrument is used will determine whether the student is taking a reading or a measurement.

Reading (one judgement only)	**Measurement (two judgements required)**
thermometer	ruler
top pan balance	vernier calliper
measuring cylinder	micrometer
digital voltmeter	protractor
Geiger counter	stopwatch
pressure gauge	analogue meter

Readings

The uncertainty in a **reading** when using an instrument is **no smaller** than plus or minus half of the smallest division or greater.

For example, a temperature measured with a thermometer is likely to have an uncertainty of ±0.5 °C if the graduations are 1 °C apart.

You should be aware that readings are often written with the uncertainty.

An example of this would be to write a voltage as (2.40 ± 0.01) V. It is usual for the uncertainty quoted to be the same number of decimal places as the value. Unless there are good reasons otherwise (e.g. an advanced statistical analysis), **you should quote the uncertainty in a measurement to the same number of decimal places as the value.**

Study Tip

Values experimentally recorded are always given in terms of decimal places.

Values worked out in questions are always given in terms of significant figures.

Physics Tip

To calibrate a set of scales, you could measure a 10.0g mass and check that it reads 10.0g.

If these scales are precise to the nearest 0.1g, then you can only calibrate to within 0.05g.

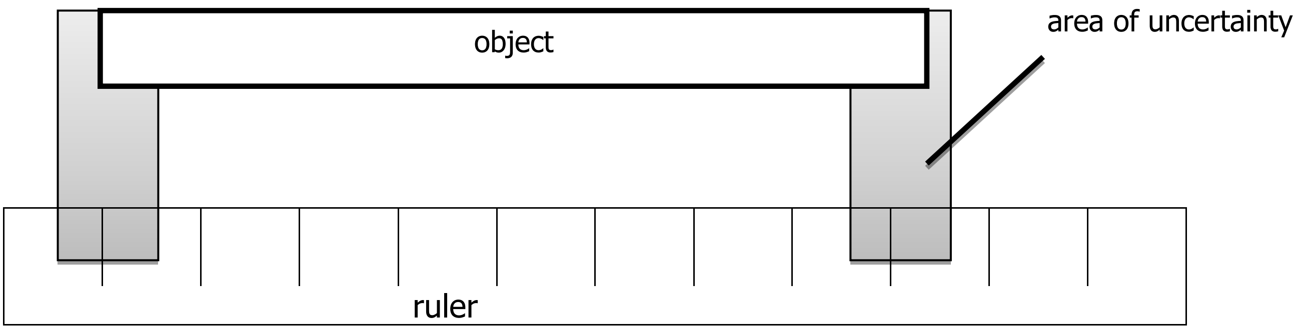
Any reading has an uncertainty which is half of the smallest division of scale.

Measurements

The uncertainty in a **measurement** when using an instrument is **no smaller** than plus or minus the smallest division or greater. For example, length.

When measuring length, **two** uncertainties must be included: the uncertainty of the placement of the zero of the ruler and the uncertainty of the point the measurement is taken from.

As both ends of the ruler have a ±0.5 scale division uncertainty, the measurement will have an uncertainty of ±1 division.



For most rulers, this will mean that the uncertainty in a measurement of length will be ±1 mm.

This "initial value uncertainty" will apply to any instrument where the user can set the zero (incorrectly), but would not apply to equipment such as balances or thermometers where the zero is set at the point of manufacture.

Physics Tip

It is not just rulers which have two errors in each measurement – lots of devices do, including protractors, callipers, stop watches and anything where you take a reading using a swing arm.

In summary

The uncertainty of a reading (one judgement) is at least ±0.5 of the smallest scale reading.
The uncertainty of a measurement (two judgements) is at least ±1 of the smallest scale reading.

Examination Tip

To get the lowest possible value, subtract the value after the '±' sign from your number, and to get the highest possible value, add it to the number.

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The way measurements are taken can also affect the uncertainty.

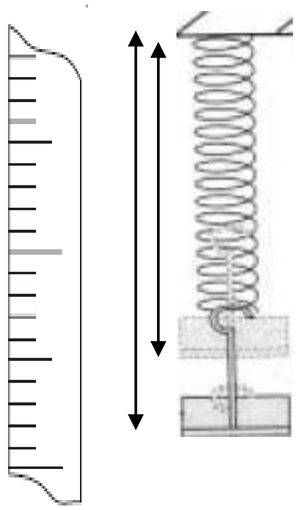
Measurement example: the extension of a spring

Measuring the extension of a spring using a metre ruler can be achieved in two ways.

1. Measuring the total length unloaded and then loaded.

Four readings must be taken for this: The start and end point of the unloaded spring's length and the start and end point of the loaded spring's length.

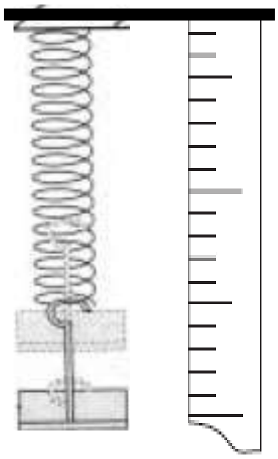
The minimum uncertainty in each measured length is ± 1 mm using a meter ruler with 1 mm divisions (the actual uncertainty is likely to be larger due to parallax in this instance). The extension would be the difference between the two readings so the minimum uncertainty would be ± 2 mm.



2. Fixing one end and taking a scale reading of the lower end.

Two readings must be taken for this: the end point of the unloaded spring's length and the end point of the loaded spring's length. The start point is assumed to have zero uncertainty as it is fixed.

The minimum uncertainty in each reading would be ± 0.5 mm, so the minimum extension uncertainty would be $\pm$ 1 mm.



Even with other practical uncertainties this second approach would be better.

Realistically, the uncertainty would be larger than this and an uncertainty in each reading of 1 mm or would be more sensible. This depends on factors such as how close the ruler can be mounted to the point as at which the reading is to be taken.

Other factors

There are some occasions where the resolution of the instrument is not the limiting factor in the uncertainty in a measurement.

Best practice is to write down the full reading and then to write to fewer significant figures when the uncertainty has been estimated.

Examples:

A stopwatch has a resolution of hundredths of a second, but the uncertainty in the measurement is more likely to be due to the reaction time of the experimenter.

Here, you should write the full reading on the stopwatch (e.g. 12.20 s), carry the significant figures through for all repeats, and reduce this to a more appropriate number of significant figures after an averaging process later.

If you measure the length of a piece of wire, it is very difficult to hold the wire completely straight against the ruler. The uncertainty in the measurement is likely to be higher than the ±1 mm uncertainty of the ruler.

Depending on the number of "kinks" in the wire, the uncertainty could be reasonably judged to be nearer ± 2 or 3 mm.

The uncertainty of the reading from digital voltmeters and ammeters depends on the electronics and is not strictly the last figure in the readout.

Manufacturers usually quote the percentage uncertainties for the different ranges. Unless otherwise stated it may be assumed that ±0.5 in the least significant digit is to be the uncertainty in the measurement. This would generally be rounded up to ±1 of the least significant digit when quoting the value and the uncertainty together. For example (5.21 ±0.01) V. If the reading fluctuates, then it may be necessary to take many readings and do a mean and range calculation.

Physics Tip

A common source of random error is changes in temperature – all sorts of things are affected by temperature, from the properties of a material to the current flowing in a circuit.

Uncertainties in given values

The value of the charge on an electron is given in the data sheet as 1.60×10^{-19} C.

In all such cases assume the uncertainty to be ±1 in the last significant digit. In this case the uncertainty $\pm 0.01 \times 10^{-19}$ C. The uncertainty may be lower than this but without knowing the details of the experiment and procedure that lead to this value there is no evidence to assume otherwise.

Example:

If the number of lines per m is quoted as 3.5×10^3 (as in AS Physics Specimen Paper 2 (set 1) Q1.1) then it is usual to assume that the uncertainty is ±1 in the last significant figure, $\pm 0.1 \times 10^3$ since there is no indication of the uncertainties in the measurements from which that figure came.

Examination Tip

Always check the uncertainties in experimental constants given in questions.

Multiple instances of measurements

Some methods of measuring involve the use of multiple instances to reduce the uncertainty. For example, measuring the thickness of several sheets of paper together rather than one sheet, or timing several swings of a pendulum.

The uncertainty of each measurement will be the uncertainty of the whole measurement divided by the number of sheets or swings.

This method works because the absolute uncertainty on the time for a single swing is the same as the absolute uncertainty for the time taken for multiple swings, but there is a lower percentage in the time taken for multiple swings.

For example:

Time taken for a pendulum to swing 10 times: (5.1 ± 0.1) s

Mean time taken for one swing: (0.51 ± 0.01) s

Repeated measurements

Repeating a measurement is a method for reducing the uncertainty.

With many readings, one can also identify those that are exceptional (that are far away from a significant number of other measurements). Sometimes it will be appropriate to remove outliers from measurements before calculating a mean.

If measurements are repeated, the uncertainty can be calculated by finding half the range of the measured values.

For example:

Repeat	1	2	3	4
Distance / m	1.23	1.32	1.27	1.22

1.32 – 1.22 = 0.10 therefore

Mean distance: (1.26 ± 0.05) m

> **Study Tip**
>
> Learn the method of calculating absolute uncertainties from repeated values.

Percentage uncertainties

The percentage uncertainty in a measurement can be calculated using:

$$percentage\ uncertainty = \frac{uncertainty}{value}\ x\ 100\%$$

The percentage uncertainty in a repeated measurement can also be calculated using:

$$percentage\ uncertainty = \frac{uncertainty}{mean\ value}\ x\ 100\%$$

> **Study Tip**
>
> Learn the method of calculating percentage uncertainties from absolute uncertainties.

Further examples:

Example 1. Some values for diameter of a wire

Repeat	1	2	3	4
Diameter / mm	0.35	0.37	0.36	0.34

The exact values for the mean is 0.355 mm and for the uncertainty is 0.015 mm

This could be quoted as such or recorded as 0.36 ±0.02 mm given that there is a wide range and only 4 readings. Given the simplistic nature of the analysis then giving the percentage uncertainty as 5% or 6% would be acceptable.

Example 2. Different values for the diameter of a wire

Repeat	1	2	3
Diameter / mm	0.35	0.36	0.35

The mean here is 0.3533 mm with uncertainty of 0.005 mm

The percentage uncertainty is 1.41% so may be quoted as 1% but really it would be better to obtain further data.

Uncertainties in exams

Wherever possible, questions in exams will be clear on whether you are being asked to calculate the uncertainty of a reading, a measurement, or given data.

Where there is ambiguity, mark schemes will allow alternative sensible answers and credit clear thinking.

If the examination states 'uncertainty' it refers to absolute uncertainty. A percentage uncertainty is only referred to by that name.

Uncertainties in practical work

You are expected to develop an understanding of uncertainties in measurements through their practical work.

Examples:

CPAC 2: You should be attempting to reduce the uncertainties in experiments. This could be by choosing appropriate equipment (CPAC 2d), or by choosing procedures such as repeating readings that reduce overall uncertainties (CPAC 2c).

CPAC 4: You should consider uncertainties. For example, you should be making sensible decisions about the number of significant figures to include, particularly in calculated values.

CPAC 5: You could comment on the uncertainties in their measurements. For example, you could comment on whether the true value (e.g. for a concentration, or the acceleration due to gravity) lies within their calculated range of uncertainty. With some measurements, you may compare their value with those from secondary sources, contributing evidence for CPAC 5b.

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Error bars in Physics

There are many ways to draw error bars.

The following simple method of plotting error bars would therefore be acceptable:

- Plot the data point at the mean value
- Calculate the range of the data, ignoring any anomalies
- Add error bars with lengths equal to half the range on either side of the data point.

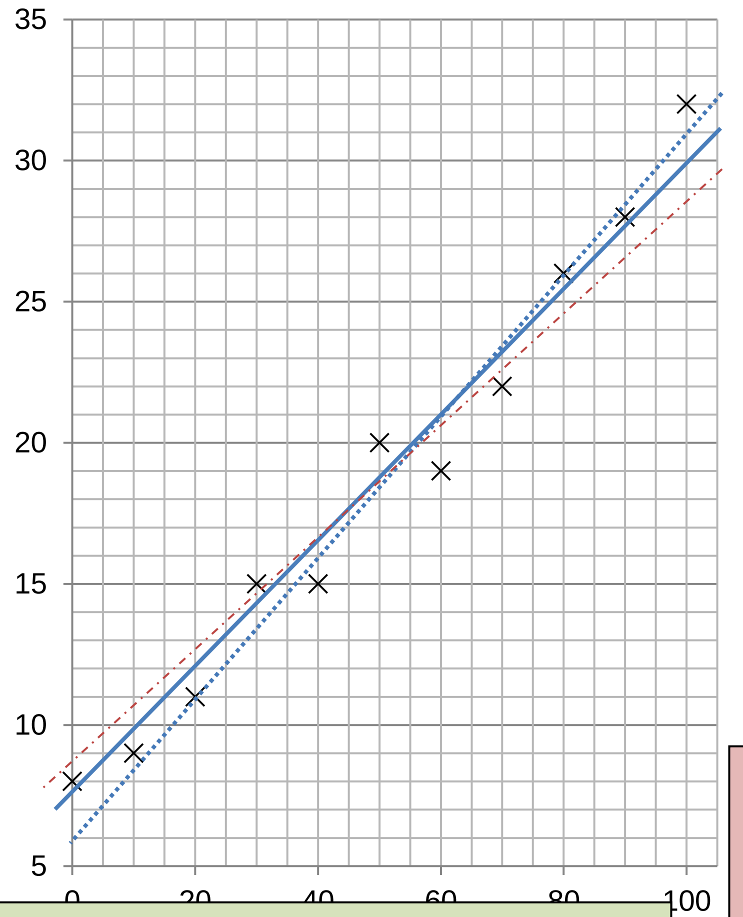
Uncertainties from gradients

To find the uncertainty in a gradient, two lines should be drawn on the graph. One should be the "best" line of best fit. The second line should be the steepest or shallowest gradient line of best fit possible from the data. The gradient of each line should then be found.

The uncertainty in the gradient is found by:

$$\text{percentage uncertainty} = \frac{|\text{best gradient} - \text{worst gradient}|}{\text{best gradient}} \times 100\%$$

Note the modulus bars meaning that this percentage will always be positive.



Study Tip

Learn the method of calculating percentage uncertainties from graphical methods.

Examination Tip

Be careful if error bars are based on percentage uncertainties, they change will change for each measurement.

Physics Tip

If you have a graph showing the relationship between two variables that are directly proportional, you must treat the origin as a normal point whilst drawing the line of best fit.

The origin will be a measurement with no uncertainty and so all lines of best fit must pass through it.

In the same way, the percentage uncertainty in the y-intercept can be found:

$$\text{percentage uncertainty} = \frac{|\text{best } y\ intercept - \text{worst } y\ intercept|}{\text{best } y\ intercept} \times 100\%$$

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Combining uncertainties

Percentage uncertainties should be combined using the following rules:

Combination	Operation	Example
Adding or subtracting values $a = b + c$	Add the absolute uncertainties Δa = Δb + Δc	Object distance, *u* = (5.0 ± 0.1) cm Image distance, *v* = (7.2 ± 0.1) cm Difference (*v* – *u*) = (2.2 ± 0.2) cm
Multiplying values $a = b \times c$ **Examination Tip** Remember to convert all uncertainties to percentages before you combine them.	Add the percentage uncertainties εa = εb + εc	Voltage = (15.20 ± 0.1) V Current = (0.51 ± 0.01) A Percentage uncertainty in voltage = 0.7% Percentage uncertainty in current = 1.96% Power = Voltage × current = 7.75 W Percentage uncertainty in power = 2.66% Absolute uncertainty in power = ± 0.21 W
Dividing values $a = \frac{b}{c}$ **Examination Tip** Remember to convert all uncertainties to percentages before you combine them.	Add the percentage uncertainties εa = εb + εc	Mass of object = (30.2 ± 0.1) g Volume of object = (18.0 ± 0.5) cm^3 Percentage uncertainty in mass of object = 0.3 % Percentage uncertainty in volume = 2.8% Density = $\frac{30.2}{18.0}$ = 1.68 g cm^{-3} Percentage uncertainty in density = 3.1% Absolute uncertainty in density = ± 0.05 g cm^{-3}
Power rules $a = b^c$ **Examination Tip** Remember to convert all uncertainties to percentages before you combine them.	Multiply the percentage uncertainty by the power εa = c × εb	Radius of circle = (6.0 ± 0.1) cm Percentage uncertainty in radius = 1.6% Area of circle = πr^2 = 113.1 cm^2 Percentage uncertainty in area = 3.2% Absolute uncertainty = ± 3.6 cm^2 (Note – the uncertainty in π is taken to be zero)

Note: Absolute uncertainties (denoted by Δ) have the same units as the quantity.

Percentage uncertainties (denoted by ε) have no units.

Uncertainties in trigonometric and logarithmic functions will not be tested in A-level exams.

Study Tip

Learn the method of combining uncertainties in equations.

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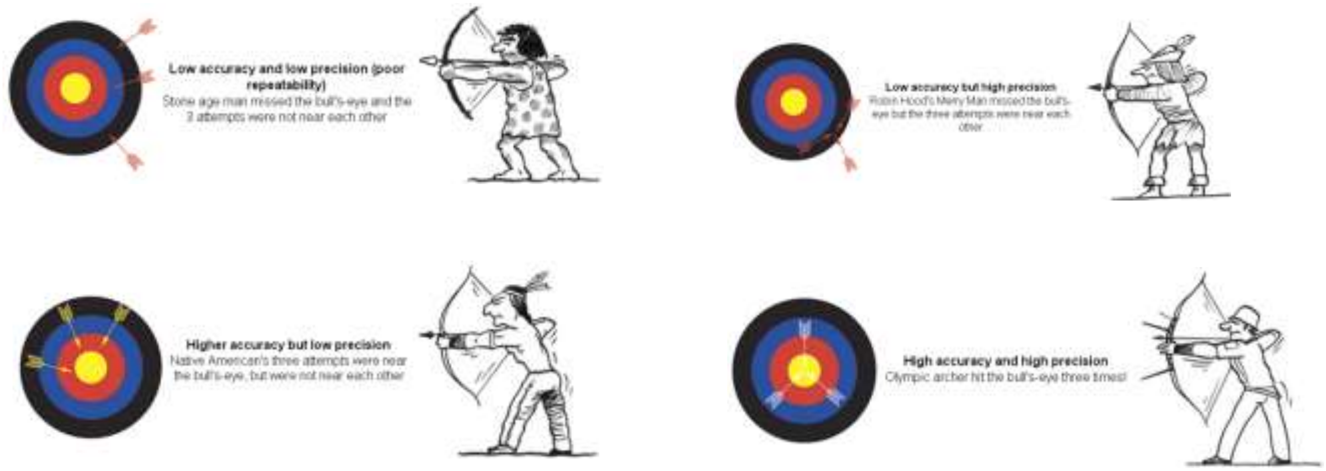
Commonly Asked Questions.

What is the difference between Accuracy and Precision?

The difference between accuracy and precision is illustrated below by four different archers, each with varying degree of ability. The bull's-eye in the target represents the true value of a measurement.



Accuracy: A measurement result is considered accurate if it is judged to be close to the true value.

The precision of a measurement is the degree of exactness (sometimes the number of significant figures) to which the measurement of a quantity or value can be obtained and reproduced consistently.

- If a reading is constant when repeated, the precision of the measurement will be the precision of the instrument. Precision
- If a reading fluctuates the precision of the measurement (its mean value) is given by half the maximum range of the readings.

The precision of an instrument is the smallest non-zero reading that can be measured using the instrument.

What is the difference between Repeatable and Reproducible?

Repeatable: A measurement is repeatable if the original experimenter repeats the investigation using same method and equipment and obtains the same results.
Reproducible: A measurement is reproducible if the investigation is repeated by another person, or by using different equipment or techniques, and the same results are obtained.

TOPIC 3: ESTIMATION OF QUANTITIES SPEC CHECK

Specification	Completed?
Orders of magnitude.	
Estimation of approximate values of physical quantities.	
Students should be able to estimate approximate values of physical quantities to the nearest order of magnitude.	
Students should be able to use these estimates together with their knowledge of physics to produce further derived estimates also to the nearest order of magnitude.	

Student Checklist

Have I…….	Yes or No?
Read through the notes of this section?	
Highlighted/underlined the key concepts of this section?	
Made my own notes based on the notes of this section?	
Brought the notes to be used in lesson?	

NOTES

An order of magnitude is a multiple of 10.

An example of this is 10^3 or 10^{-7}.

Such differences in order of magnitude can be measured on the logarithmic scale in "decades," or factors of ten.

It is common among scientists and technologists to say that a parameter whose value is not accurately known or is known only within a range is "on the order of" some value. The order of magnitude of a physical quantity is its magnitude in powers of ten when the physical quantity is expressed in powers of ten with one digit to the left of the decimal.

Orders of magnitude are generally used to make very approximate comparisons and reflect very large differences. If two numbers differ by one order of magnitude, one is about ten times larger than the other. If they differ by two orders of magnitude, they differ by a factor of about 100. Two numbers of the same order of magnitude have roughly the same scale - the larger value is less than ten times the smaller value.

It is important in the field of science that estimates be at least in the right ballpark. In many situations, it is often sufficient for an estimate to be within an order of magnitude of the value in question. Although making order-of-magnitude estimates seems simple and natural to experienced scientists, it may be completely unfamiliar to the less experienced.

Answering Examination Questions

We can use significant figures to get an approximate answer to a problem.

We can round off all the numbers in a problem to 1 significant figure to make 'easier' numbers.

It is often possible to do this in your head.

This should give you answer to the correct order of magnitude.

Examples

E1. Find a rough answer to 19.4 over 0.0437

We first round off both numbers to 1 significant figure (s.f.):

19.4 = 20 (1 s.f.)

0.0437 = 0.04 (1 s.f.)

So, we now need to make the denominator a whole number. We can do this by multiplying both 20 and 0.04 by 100. $\frac{20}{0.04} = \frac{20 \times 100}{0.04 \times 100} = \frac{2000}{4}$

Divide everything by 4.

$= \frac{2000}{4} = 500$

The real answer to 19.4 ÷ 0.0437 = 443.9359... So, this was a good estimate.

E2. How would you get an approximate answer for 386062 × 0.007243?

Did you get the answer **400000 × 0.007 = 2800**?

Rounding to 1 s.f.

386062 = 400000

0.007243 = 0.007

So, **400000 × 0.007 = 2800**

Examination Tip

It is good practice to estimate answers in your examination, this checks to see if you answer seems sensible.

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TOPIC 4: WRITING AN INVESTIGATION

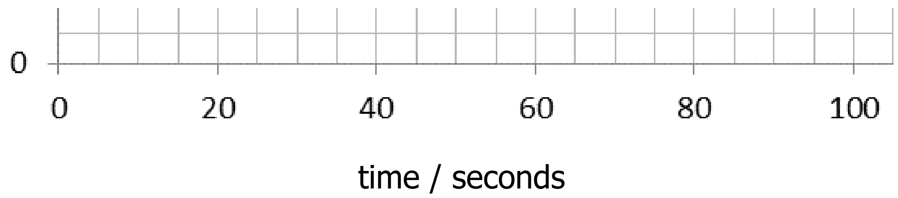
NOTES

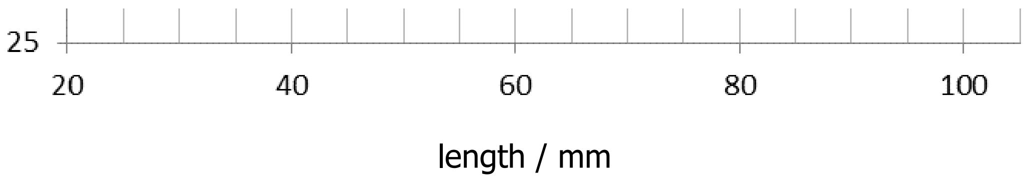
You should recognise that the type of graph that they draw should be based on an understanding of the type of data they are using and the intended analysis of the data.

The rules below are guidelines which will vary according to the specific circumstances.

Labelling axes

Axes should always be labelled with the quantity being measured and the units. These should be separated with a forward slash (solidus):





Axes should not be labelled with the units on each scale marking.

Data points

Data points should be marked with a cross. Both × and + marks are acceptable, but care should be taken that data points can be seen against the grid.

Error bars can take the place of data points where appropriate.

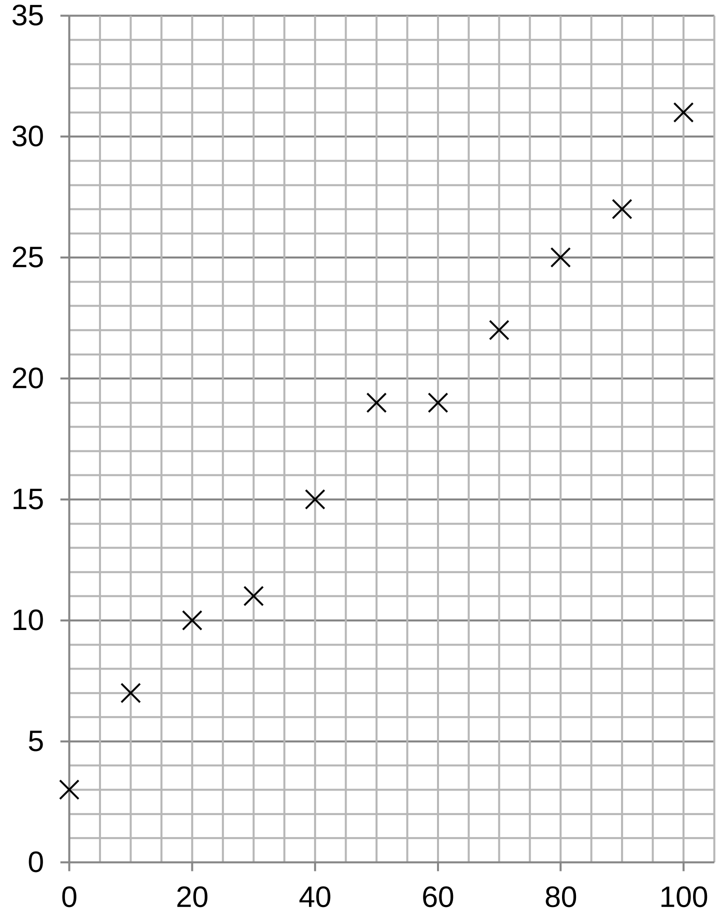
Scales and origins

You should attempt to spread the data points on a graph as far as possible without resorting to scales that are difficult to deal with. You should consider:

- the maximum and minimum values of each variable
- the size of the graph paper
- whether 0.0 should be included as a data point
- whether they will be attempting to calculate the equation of a line, therefore needing the y intercept (Physics only)
- how to draw the axes without using difficult scale markings (e.g. multiples of 3, 7, 11 etc)
- In exams, the plots should cover **at least half** of the grid supplied for the graph.

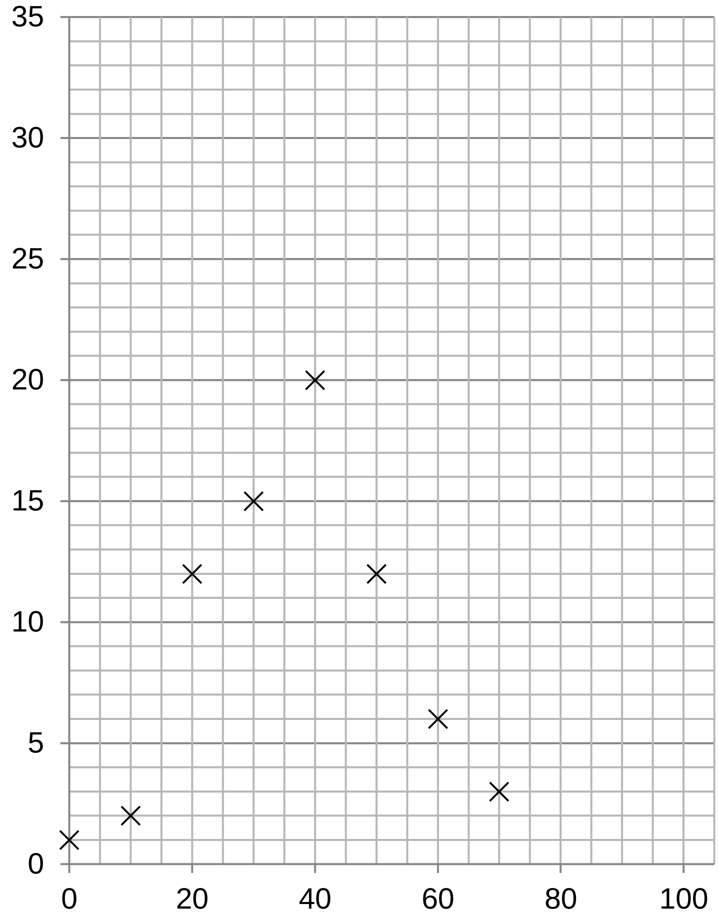
For example, the following three graphs are intended to illustrate the information above relating to the spread of data points on a graph.

When producing such graphs on the basis of real practical work or in examination questions would be expected to add in axes labels and units.



This graph has well-spaced marking points and the data fills the paper.

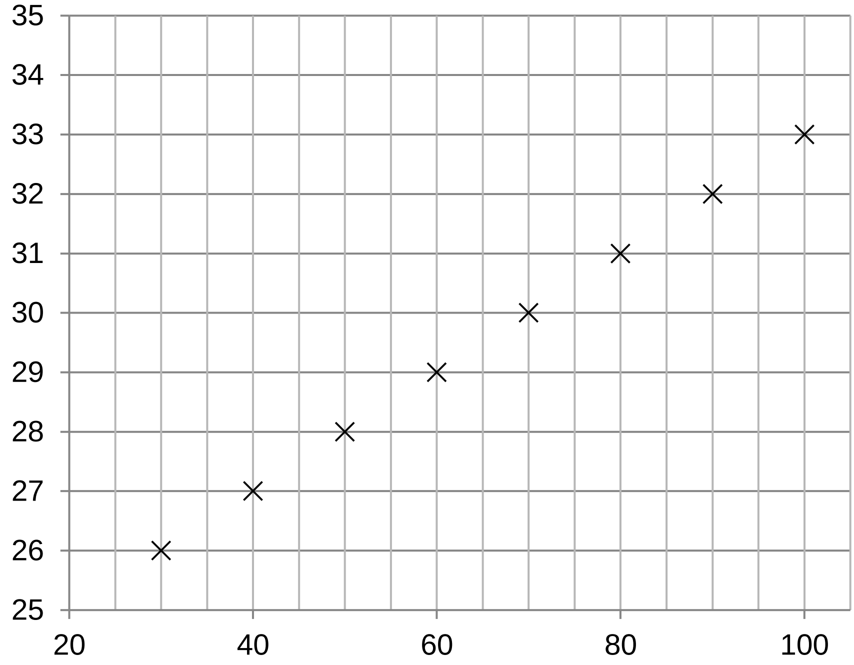
Each point is marked with a cross (so points can be seen even when a line of best fit is drawn).



This graph is on the limit of acceptability. The points do not quite fill the page, but to spread them further would result in the use of awkward scales.

Examination Tip

A common examination question is draw in a line of best fit and then determine either the gradient, the maximum value, minimum value or the y-intercept.



At first glance, this graph is well drawn and has spread the data out sensibly.

However, if the graph were to later be used to calculate the equation of the line, the lack of a y-intercept could cause problems. Increasing the axes to ensure all points are spread out but the y-intercept is also included is a skill that requires practice and may take a couple of attempts.

Lines of best fit

Lines of best fit should be drawn when appropriate. You should consider the following when deciding where to draw a line of best fit:

- **Are the data likely to be following an underlying equation (for example, a relationship governed by a physical law)? This will help decide if the line should be straight or curved.**
- **Are there any anomalous results?**
- **Are there uncertainties in the measurements? The line of best fit should fall within error bars if drawn.**

There is no definitive way of determining where a line of best fit should be drawn. A good rule of thumb is to make sure that there are as many points on one side of the line as the other. Often the line should pass through, or very close to, the majority of plotted points.

Graphing programs can sometimes help, but tend to use algorithms that make assumptions about the data that may not be appropriate.

Lines of best fit should be continuous and drawn as a thin pencil that does not obscure the points below and does not add uncertainty to the measurement of gradient of the line.

Not all lines of best fit go through the origin. You should ask whether a 0 in the independent variable is likely to produce a 0 in the dependent variable. This can provide an extra and more certain point through which a line must pass.

A line of best fit that is expected to pass through (0,0), but does not, would imply some systematic error in the experiment. This would be a thorough source of discussion in an evaluation.

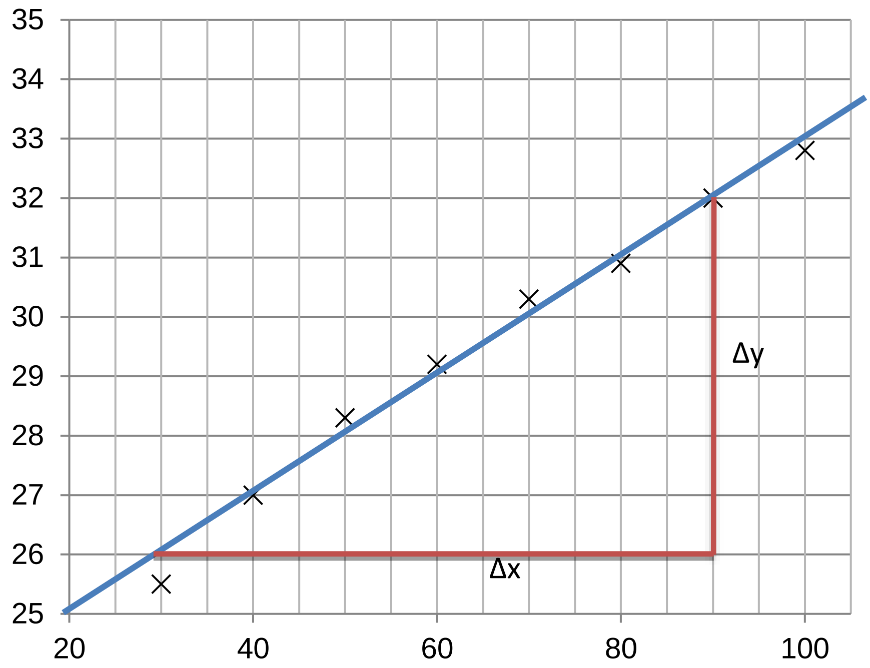
Examination Tip

You must be able to work out values from the equation of a straight-line graph.

This includes using the y-intercept and the gradient to derive answers.

Measuring gradients

When finding the gradient of a line of best fit, students should show their working by drawing a triangle on the line. The hypotenuse of the triangle should be at least half as big as the line of best fit.



The line of best fit here has an equal number of points on both sides. It is not too wide so points can be seen under it.

The gradient triangle has been drawn so the hypotenuse includes more than half of the line.

In addition, it starts and ends on points where the line of best fit crosses grid lines so the points can be read easily (this is not always possible).

$$gradient = \frac{\Delta y}{\Delta x}$$

When finding the gradient of a curve, e.g., the rate of reaction at a time that was not sampled, students should draw a tangent to the curve at the relevant value of the independent variable (x-axis).

Use of a set square to draw a triangle over this point on the curve can be helpful in drawing an appropriate tangent.

Examination Tip

When you are analysing a curve of best fit – a tangent should always be drawn and used.

Physics Tip

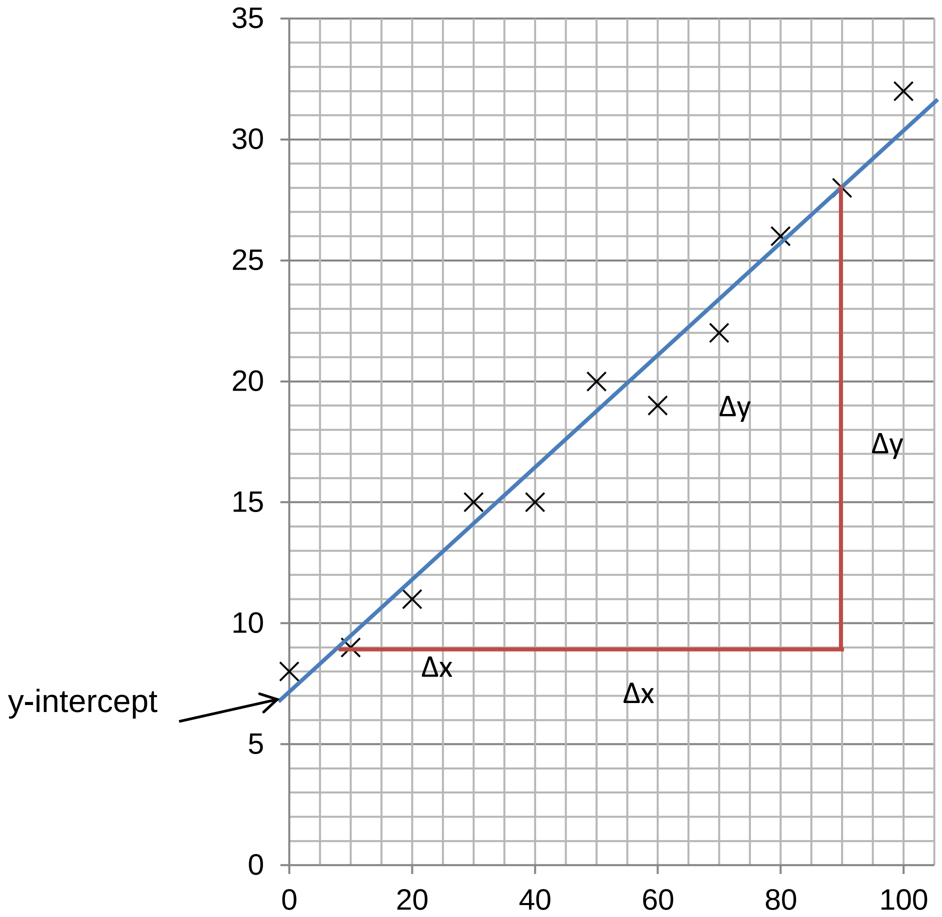
When drawing a tangent, it helps to make it long – it will be easier to draw, and the tangent line will be more likely to intersect some grid lines, making the gradient easier to calculate.

The equation of a straight line

Students should be able to translate graphical data into the equation of a straight line.

$$y = mx + c$$

Where y is the dependent variable, m is the gradient, x is the independent variable and c is the y-intercept.



Δ y = 28 – 9 = 19

Δ x = 90 – 10 = 80

gradient = 19 / 80 = 0.24 (2 sf)

y-intercept = 7.0

equation of line:

y = 0.24 x + 7.0

Examination Tip

You must be able to work out values from the equation of a straight-line graph.

This includes using the y-intercept and the gradient to derive answers.

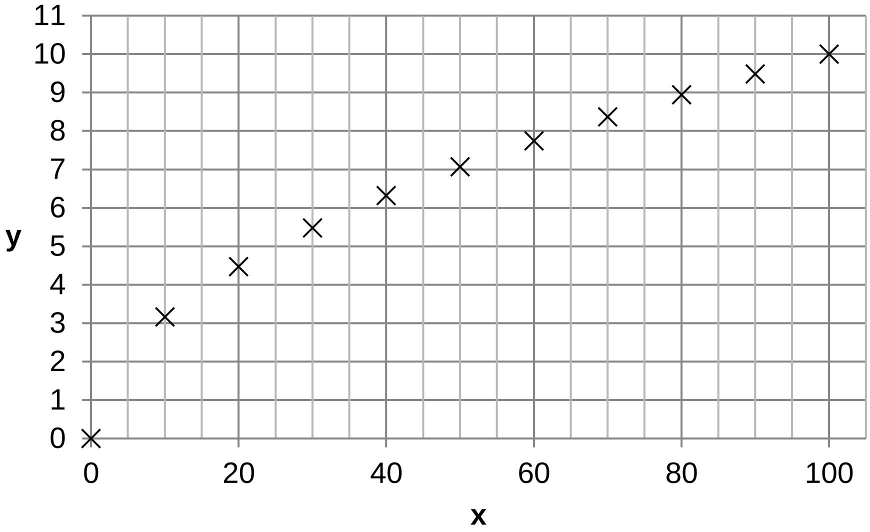
Testing relationships

Sometimes it is not clear what the relationship between two variables is. A quick way to find a possible relationship is to manipulate the data to form a straight-line graph from the data by changing the variable plotted on each axis.

For example:

Raw data and graph

x	y
0	0.00
10	3.16
20	4.47
30	5.48
40	6.32
50	7.07
60	7.75
70	8.37
80	8.94
90	9.49
100	10.00



This is clearly not a straight-line graph. The relationship between x and y is not clear.

Physics Tip

If an experiment really does confirm that changing one variable causes another to change, it is said the variables share a **causal link.**

Examination Tip

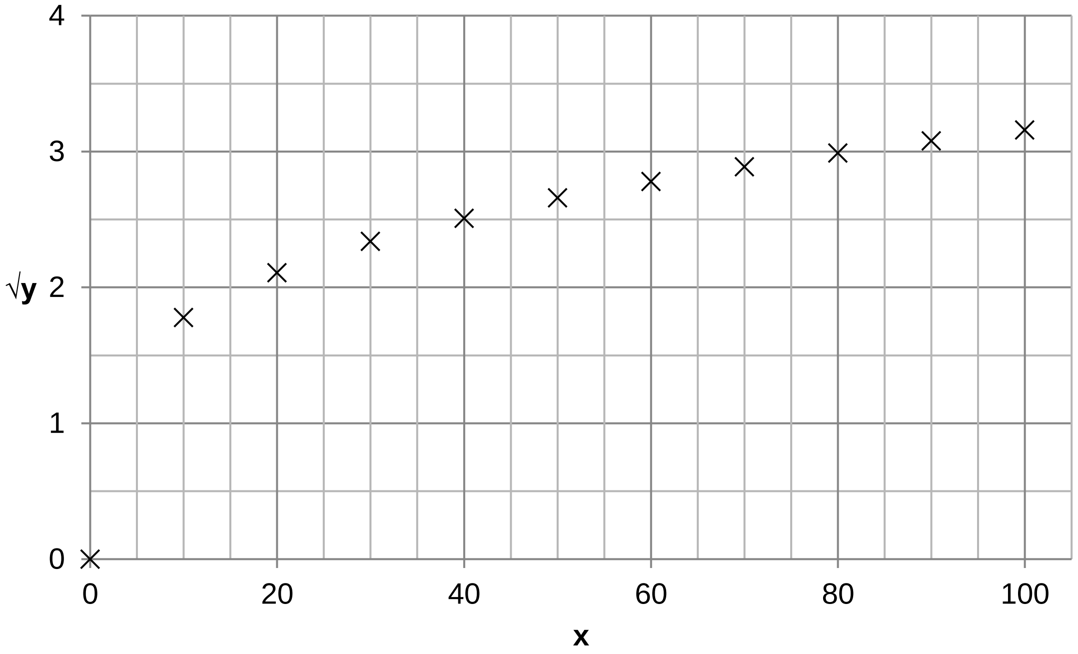
Two variables are directly proportional if

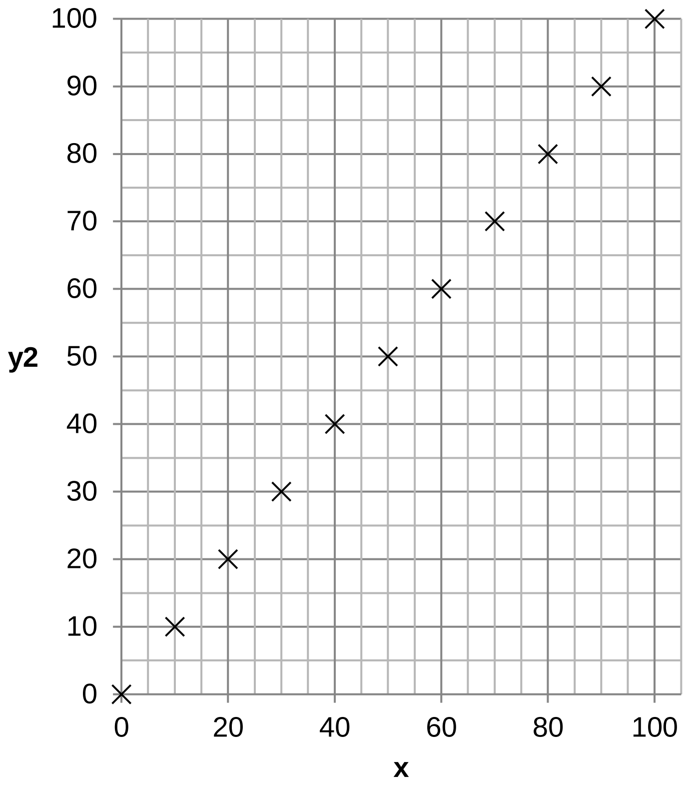
One Variable = Constant x Other Variable

Manipulated data and graphs

A series of different graphs can be drawn from these data. The one that is closest to a straight line is a good candidate for the relationship between x and y.

x	**y**	**√y**	**y^2**	**y^3**
0	0.00	0.00	0.00	0.00
10	3.16	1.78	10.00	32
20	4.47	2.11	20.00	89
30	5.48	2.34	30.00	160
40	6.32	2.51	40.00	250
50	7.07	2.66	50.00	350
60	7.75	2.78	60.00	470
70	8.37	2.89	70.00	590
80	8.94	2.99	80.00	720
90	9.49	3.08	90.00	850
100	10.00	3.16	100.00	1000





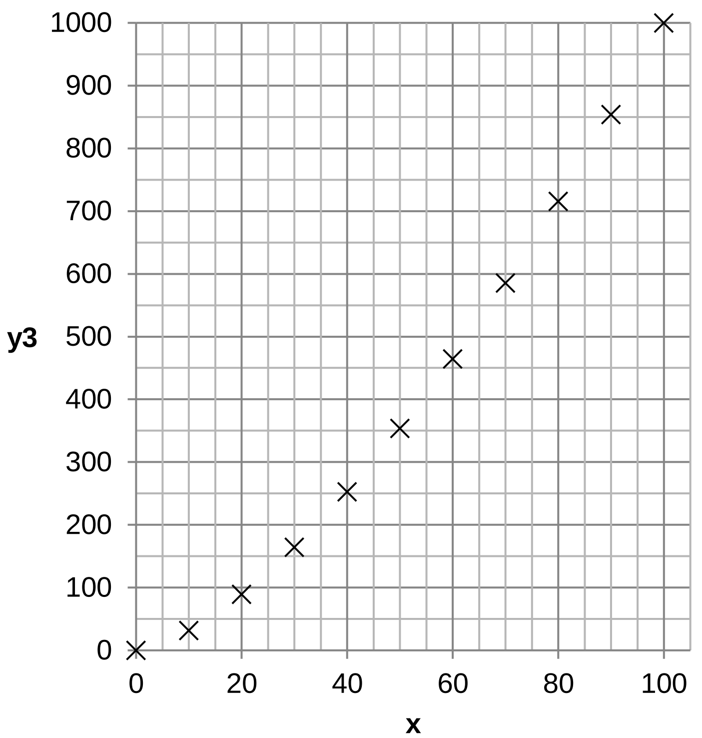
Physics Tip

Your line of best fit should always go through all of the error bars – if it does not – the point must be anomalous.

This is an idealised set of data to illustrate the point.

The straightest graph is y^2 against x, suggesting that the relationship between x and y is

$$y^2 \propto x$$



DATASHEET

DATA - FUNDAMENTAL CONSTANTS AND VALUES

Quantity	Symbol	Value	Units
speed of light in vacuo	c	3.00×10^{8}	$m\,s^{-1}$
permeability of free space	μ_0	$4\pi \times 10^{-7}$	$H\,m^{-1}$
permittivity of free space	ε_0	8.85×10^{-12}	$F\,m^{-1}$
magnitude of the charge of electron	e	1.60×10^{-19}	C
the Planck constant	h	6.63×10^{-34}	J s
gravitational constant	G	6.67×10^{-11}	$N\,m^2\,kg^{-2}$
the Avogadro constant	N_A	6.02×10^{23}	mol^{-1}
molar gas constant	R	8.31	$J\,K^{-1}\,mol^{-1}$
the Boltzmann constant	k	1.38×10^{-23}	$J\,K^{-1}$
the Stefan constant	σ	5.67×10^{-8}	$W\,m^{-2}\,K^{-4}$
the Wien constant	α	2.90×10^{-3}	m K
electron rest mass (equivalent to 5.5×10^{-4} u)	m_e	9.11×10^{-31}	kg
electron charge/mass ratio	$\frac{e}{m_e}$	1.76×10^{11}	$C\,kg^{-1}$
proton rest mass (equivalent to 1.00728 u)	m_p	$1.67(3) \times 10^{-27}$	kg
proton charge/mass ratio	$\frac{e}{m_p}$	9.58×10^{7}	$C\,kg^{-1}$
neutron rest mass (equivalent to 1.00867 u)	m_n	$1.67(5) \times 10^{-27}$	kg
gravitational field strength	g	9.81	$N\,kg^{-1}$
acceleration due to gravity	g	9.81	$m\,s^{-2}$
atomic mass unit (1u is equivalent to 931.5 MeV)	u	1.661×10^{-27}	kg

ALGEBRAIC EQUATION

quadratic equation $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ASTRONOMICAL DATA

Body	Mass/kg	Mean radius/m
Sun	1.99×10^{30}	6.96×10^{8}
Earth	5.97×10^{24}	6.37×10^{6}

GEOMETRICAL EQUATIONS

arc length	$= r\theta$
circumference of circle	$= 2\pi r$
area of circle	$= \pi r^2$
curved surface area of cylinder	$= 2\pi rh$
area of sphere	$= 4\pi r^2$
volume of sphere	$= \frac{4}{3}\pi r^3$

Particle Physics

Class	Name	Symbol	Rest energy/MeV
photon	photon	γ	0
lepton	neutrino	ν_e	0
		ν_μ	0
	electron	$e^\pm$	0.510999
	muon	$\mu^\pm$	105.659
mesons	π meson	$\pi^\pm$	139.576
		π^0	134.972
	K meson	$K^\pm$	493.821
		K^0	497.762
baryons	proton	p	938.257
	neutron	n	939.551

Properties of quarks

antiquarks have opposite signs

Type	Charge	Baryon number	Strangeness
u	$+\frac{2}{3}e$	$+\frac{1}{3}$	0
d	$-\frac{1}{3}e$	$+\frac{1}{3}$	0
s	$-\frac{1}{3}e$	$+\frac{1}{3}$	-1

Properties of Leptons

		Lepton number
Particles:	$e^-, \nu_e; \mu^-, \nu_\mu$	+1
Antiparticles:	$e^+, \overline{\nu_e}, \mu^+, \overline{\nu_\mu}$	−1

Photons and energy levels

photon energy $E = hf = hc/\lambda$

photoelectricity $hf = \phi + E_{k\,(max)}$

energy levels $hf = E_1 - E_2$

de Broglie wavelength $\lambda = \frac{h}{p} = \frac{h}{mv}$

Waves

wave speed $c = f\lambda$ *period* $f = \frac{1}{T}$

first harmonic $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

fringe spacing $w = \frac{\lambda D}{s}$ *diffraction grating* $d\sin\theta = n\lambda$

refractive index of a substance s, $n = \frac{c}{c_s}$

for two different substances of refractive indices n_1 *and* n_2,

law of refraction $n_1 \sin\theta_1 = n_2 \sin\theta_2$

critical angle $\sin\theta_c = \frac{n_2}{n_1}$ for $n_1 > n_2$

Mechanics

moments $\text{moment} = Fd$

velocity and acceleration $v = \frac{\Delta s}{\Delta t}$ $a = \frac{\Delta v}{\Delta t}$

equations of motion $v = u + at$ $s = \left(\frac{u+v}{2}\right)t$

$v^2 = u^2 + 2as$ $s = ut + \frac{at^2}{2}$

force $F = ma$

force $F = \frac{\Delta(mv)}{\Delta t}$

impulse $F\Delta t = \Delta(mv)$

work, energy and power $W = Fs\cos\theta$

$E_k = \frac{1}{2}mv^2$ $\Delta E_p = mg\Delta h$

$P = \frac{\Delta W}{\Delta t}$, $P = Fv$

$efficiency = \frac{useful\ output\ power}{input\ power}$

Materials

density $\rho = \frac{m}{V}$ *Hooke's law* $F = k\Delta L$

Young modulus $= \frac{\text{tensile stress}}{\text{tensile strain}}$ *tensile stress* $= \frac{F}{A}$ *tensile strain* $= \frac{\Delta L}{L}$

energy stored $E = \frac{1}{2}F\Delta L$

Electricity

current and pd	$I = \frac{\Delta Q}{\Delta t}$ $V = \frac{W}{Q}$ $R = \frac{V}{I}$
resistivity	$\rho = \frac{RA}{L}$
resistors in series	$R_T = R_1 + R_2 + R_3 + \ldots$
resistors in parallel	$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$
power	$P = VI = I^2R = \frac{V^2}{R}$
emf	$\varepsilon = \frac{E}{Q}$ $\varepsilon = I(R + r)$

Circular motion

magnitude of angular speed	$\omega = \frac{v}{r}$
	$\omega = 2\pi f$
centripetal acceleration	$a = \frac{v^2}{r} = \omega^2 r$
centripetal force	$F = \frac{mv^2}{r} = m\omega^2 r$

Simple harmonic motion

acceleration	$a = -\omega^2 x$
displacement	$x = A\cos(\omega t)$
speed	$v = \pm\omega\sqrt{(A^2 - x^2)}$
maximum speed	$v_{max} = \omega A$
maximum acceleration	$a_{max} = \omega^2 A$
for a mass-spring system	$T = 2\pi\sqrt{\frac{m}{k}}$
for a simple pendulum	$T = 2\pi\sqrt{\frac{l}{g}}$

Thermal physics

energy to change temperature	$Q = mc\Delta\theta$
energy to change state	$Q = m\,l$
gas law	$pV = nRT$
	$pV = NkT$
kinetic theory model	$pV = \frac{1}{3}N m (c_{rms})^2$
kinetic energy of gas molecule	$\frac{1}{2}m(c_{rms})^2 = \frac{3}{2}kT = \frac{3RT}{2N_A}$

Gravitational fields

force between two masses	$F = \frac{Gm_1m_2}{r^2}$
gravitational field strength	$g = \frac{F}{m}$
magnitude of gravitational field strength in a radial field	$g = \frac{GM}{r^2}$
work done	$\Delta W = m\Delta V$
gravitational potential	$V = -\frac{GM}{r}$
	$g = -\frac{\Delta V}{\Delta r}$

Electric fields and capacitors

force between two point charges	$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$
force on a charge	$F = EQ$
field strength for a uniform field	$E = \frac{V}{d}$
work done	$\Delta W = Q\Delta V$
field strength for a radial field	$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$
electric potential	$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$
	$E = \frac{\Delta V}{\Delta r}$
capacitance	$C = \frac{Q}{V}$
	$C = \frac{A\varepsilon_0\varepsilon_r}{d}$
capacitor energy stored	$E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$
capacitor charging	$Q = Q_0(1 - e^{-t/RC})$
decay of charge	$Q = Q_0 e^{-t/RC}$
time constant	RC

Magnetic fields

force on a current $F = BIl$

force on a moving charge $F = BQv$

magnetic flux $\Phi = BA$

magnetic flux linkage $N\Phi = BAN\cos\theta$

magnitude of induced emf $\varepsilon = N\frac{\Delta\Phi}{\Delta t}$

$N\Phi = BAN\cos\theta$

emf induced in a rotating coil $\varepsilon = BAN\omega\sin\omega t$

alternating current $I_{rms} = \frac{I_0}{\sqrt{2}}$ $V_{rms} = \frac{V_0}{\sqrt{2}}$

transformer equations $\frac{N_s}{N_p} = \frac{V_s}{V_p}$

$efficiency = \frac{I_s V_s}{I_p V_p}$

Nuclear physics

the inverse square law for γ radiation $I = \frac{k}{x^2}$

radioactive decay $\frac{\Delta N}{\Delta t} = -\lambda N,\ N = N_0 e^{-\lambda t}$

activity $A = \lambda N$

half-life $T_{½} = \frac{\ln 2}{\lambda}$

nuclear radius $R = R_0 A^{1/3}$

energy-mass equation $E = mc^2$

OPTIONS

Astrophysics

1 astronomical unit = 1.50×10^{11} m

1 light year = 9.46×10^{15} m

1 parsec = 206265 AU = 3.08×10^{16} m
= 3.26 light year

Hubble constant, H = 65 km s^{-1} Mpc^{-1}

$M = \frac{\text{angle subtended by image at eye}}{\text{angle subtended by object at unaided eye}}$

in normal adjustment $M = \frac{f_o}{f_e}$

Rayleigh criterion $\theta \approx \frac{\lambda}{D}$

magnitude equation $m - M = 5\log\frac{d}{10}$

Wien's law $\lambda_{max} T = 2.9 \times 10^{-3}$ m K

Stefan's law $P = \sigma A T^4$

Schwarzschild radius $R_s \approx \frac{2GM}{c^2}$

Doppler shift for v << c $\frac{\Delta f}{f} = -\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

red shift $z = -\frac{v}{c}$

Hubble's law $v = Hd$

Medical physics

lens equations $P = \frac{1}{f}$

$m = \frac{v}{u}$

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

threshold of hearing $I_0 = 1.0 \times 10^{-12}$ W m^{-2}

intensity level $intensity\ level = 10\log\frac{I}{I_0}$

absorption $I = I_0 e^{-\mu x}$

$\mu_m = \frac{\mu}{\rho}$

ultrasound imaging $Z = \rho c$

$\frac{I_r}{I_i} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$

half-lives $\frac{1}{T_E} = \frac{1}{T_B} + \frac{1}{T_P}$

Engineering physics

moment of inertia	$I = \Sigma mr^2$
angular kinetic energy	$E_k = \frac{1}{2}I\omega^2$
equations of angular motion	$\omega_2 = \omega_1 + \alpha t$
	$\omega_2{}^2 = \omega_1{}^2 + 2\alpha\theta$
	$\theta = \omega_1 t + \frac{\alpha t^2}{2}$
	$\theta = \frac{(\omega_1 + \omega_2)t}{2}$
torque	$T = I\alpha$
	$T = Fr$
angular momentum	$\text{angular momentum} = I\omega$
angular impulse	$T\Delta t = \Delta(I\omega)$
work done	$W = T\theta$
power	$P = T\omega$
thermodynamics	$Q = \Delta U + W$
	$W = p\Delta V$
adiabatic change	$pV^\gamma = \text{constant}$
isothermal change	$pV = \text{constant}$

heat engines

$$\text{efficiency} = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H}$$

$$\text{maximum theoretical efficiency} = \frac{T_H - T_C}{T_H}$$

work done per cycle = area of loop

input power = calorific value × fuel flow rate

$$\text{indicated power} = (\text{area of } p - V \text{ loop}) \times (\text{number of cycles per second}) \times (\text{number of cylinders})$$

output or brake power $P = T\omega$

friction power = indicated power – brake power

heat pumps and refrigerators

$$\text{refrigerator: } COP_{ref} = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C}$$

$$\text{heat pump: } COP_{hp} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_C}$$

Turning points in physics

electrons in fields	$F = \frac{eV}{d}$
	$F = Bev$
	$r = \frac{mv}{Be}$
	$\frac{1}{2}mv^2 = eV$
Millikan's experiment	$\frac{QV}{d} = mg$
	$F = 6\pi\eta rv$
Maxwell's formula	$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$
	$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$
special relativity	$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$
	$l = l_0\sqrt{1 - \frac{v^2}{c^2}}$
	$E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$

Electronics

resonant frequency	$f_0 = \frac{1}{2\pi\sqrt{LC}}$
Q-factor	$Q = \frac{f_0}{f_B}$
operational amplifiers: open loop	$V_{out} = A_{OL}(V_+ - V_-)$
inverting amplifier	$\frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}}$
non-inverting amplifier	$\frac{V_{out}}{V_{in}} = 1 + \frac{R_f}{R_1}$
summing amplifier	$V_{out} = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \cdots\right)$
difference amplifier	$V_{out} = (V_+ - V_-)\frac{R_f}{R_1}$

Bandwidth requirement:

for AM	$\text{bandwidth} = 2f_M$
for FM	$\text{bandwidth} = 2(\Delta f + f_M)$

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Only constructive and reasoned feedback will be considered.