Valley Gardens Middle School

Calculation Policy

Guidance on progression of written and mental methods.

Concrete and visual supports for key concepts.

Ideas for activities, questioning, and challenge.

Main Menu

- Aims
- Expectations
- Place value
- Addition & Subtraction
- Multiplication & Division
- Fractions
- Geometry & Measures
- Algebra

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Aims

- To provide pupil with a coherent and consistent progression through the school.
- To support non-specialist colleagues in delivering the New Curriculum.
- To provide parents with a clear guidance on methods and ideas so they can confidently support their children at home.

Expectations

The layout of the **written methods** shown in this document are to be followed carefully to ensure consistency across the school.

The **progression of methods** within an area is intended to clarify what should be mastered (as far as possible) at each stage *before moving on*.

Concrete, visual, abstract: The "concrete" and "visual" supports *(provided in the "mental methods sections" and the "fractions" section)* should be the basis on which understanding is built for more formal written methods. Where possible, pupils should be given sufficient time working with the concrete apparatus *before* moving onto visual models and finally to abstract methods (using formal algorithms etc.) By aiming for mastery at each stage, time and energy will be saved in subsequent stages / years.

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Place Value

- Understanding the Number System
- Roman Numerals
- Working with the Number Line - Counting
- Working with the Number Line - Negative Numbers
- Rounding and Estimating Answers
- Multiplying and Dividing by Powers of 10

Understanding the Number System

Notes:

"U" as *units* rather than "O" for *ones*.

Emphasise the repeating pattern of "…HTU,HTU".

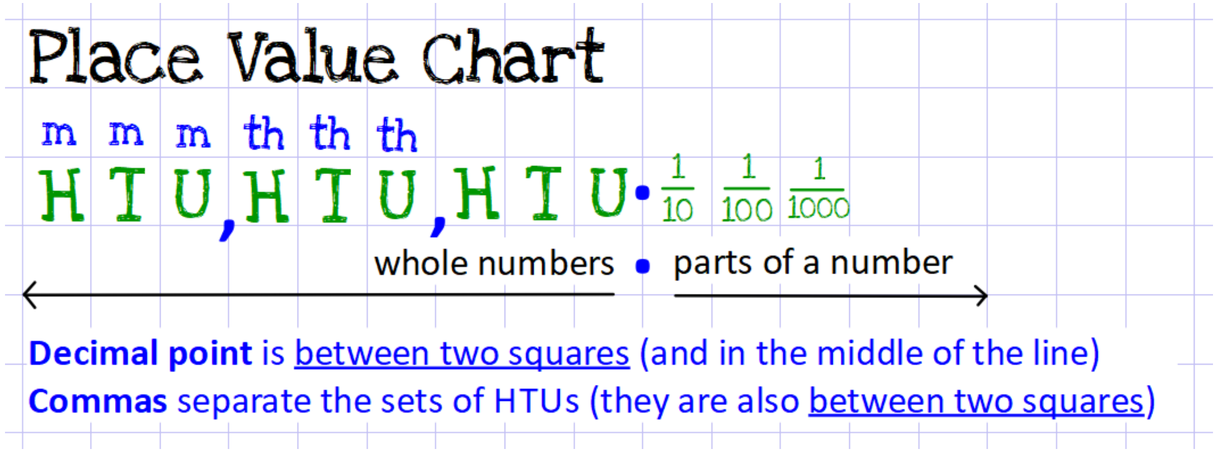
Encourage pupils to read numbers one "section" of three digits at a time (millions, thousands, units)

Encourage precise vocabulary.

Encourage pupils to use a place value chart when:

Reading / writing large numbers; ordering numbers; performing column methods; multiplying / dividing by powers of ten; converting metric units, working with standard index form etc.

Pupils should be able to draw their own PV chart in their books rather than rely on a printed version.



(the above image can be copied and pasted for demonstration on an interactive whiteboard or printed and stuck in pupils' books as reference)

S6PV1	Read, write, order and compare numbers up to 10 000 000 and determine the value of each digit.
S5PV1	Read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit.
S4PV5	Order and compare numbers beyond 1000.
S4PV4	Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones).
S3PV5	Read and write numbers up to 1000 in numerals and in words.
S3PV3	Compare and order numbers up to 1000.
S3PV2	Recognise the place value of each digit in a three-digit number (hundreds, tens, ones).

Roman Numerals

Notes:

Arabic (or *Indo-Arabic*) *numerals* refers to the digits 0-9 used within our place value system.

Introduce in sets of five to emphasise repeating patterns (see diagram).

When converting Arabic numbers into Roman numerals, use a **place value chart** - this helps to avoid the misconception of IC being 99 (it should be XCIX i.e. 90 + 9).

When reading Roman numerals, encourage pupils to work out where each place value section is.

N.B: Try googling "Roman clock faces" – what do you notice about the number 4?

1	I	6	VI	11	XI	16	XVI
2	II	7	VII	12	XII	17	XVII
3	III	8	VIII	13	XIII	18	XVIII
4	IV	9	IX	14	XIV	19	XIX
5	V	10	X	15	XV	20	XX

10	X	60	LX	100	C	600	DC
20	XX	70	LXX	200	CC	700	DCC
30	XXX	80	LXXX	300	CCC	800	DCCC
40	XL	90	XC	400	CD	900	CM
50	L	100	C	500	D	1000	M

Extension: to write larger numbers, Romans used a "vinculum" – a line drawn above a Roman numeral to show 1000 times the value. e.g: $\overline{\text{VII}}$ = 7,000 or $\overline{\text{VII}}\text{CXX}$ = 7,120

S5PV6	Read Roman numerals to 1000 (M) and recognise years written in Roman numerals.
S4PV9	Read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value.

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Working with the Number Line - Counting

Notes:

Needs examples of counting on and back in different multiples on a number-line, relating to place value, number bonds, patterns in digits etc.

S5PV2	Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000.
S4PV2	Find 1000 more or less than a given number.
S4PV1	Count in multiples of 6, 7, 9, 25 and 1000.
S3PV1	Count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number.

Working with the Number Line – Negative Numbers

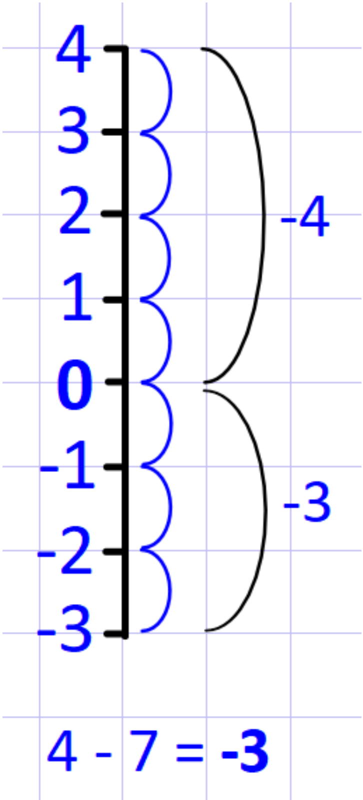
S6PV3	Use negative numbers in context, and calculate intervals across zero.
S5PV3	Interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero.
S4PV3	Count backwards through zero to include negative numbers.

Notes:

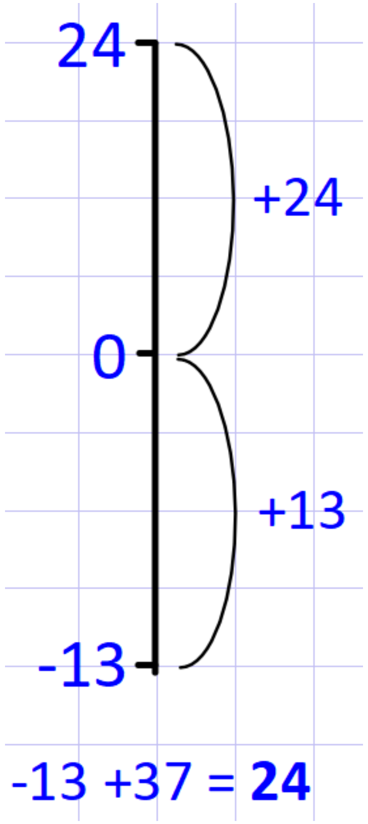
Begin with counting through zero.

Encourage thinking ahead (“will I pass through zero?” “will my answer be positive or negative” “which direction will I be moving in”)

Move on to “open number lines” and using the distance to zero to break up the calculation.



Beginning with discrete counting, emphasise the distance to zero.



Move on to “open number lines”

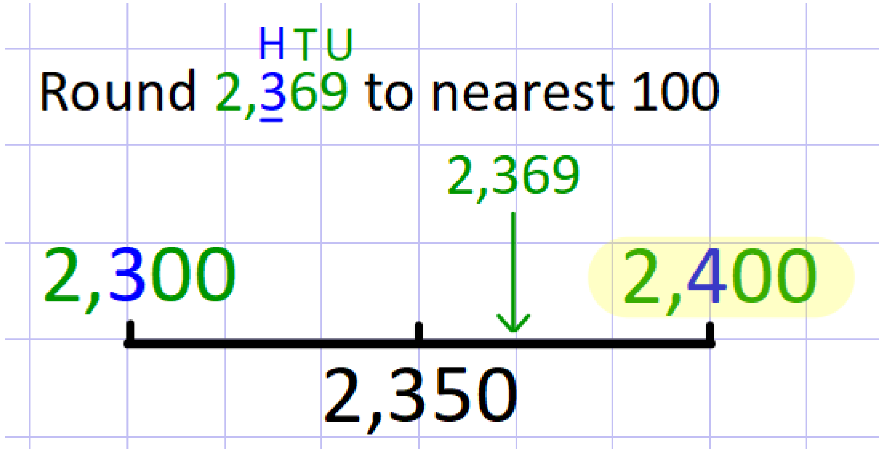
Rounding and Estimating Answers (1)

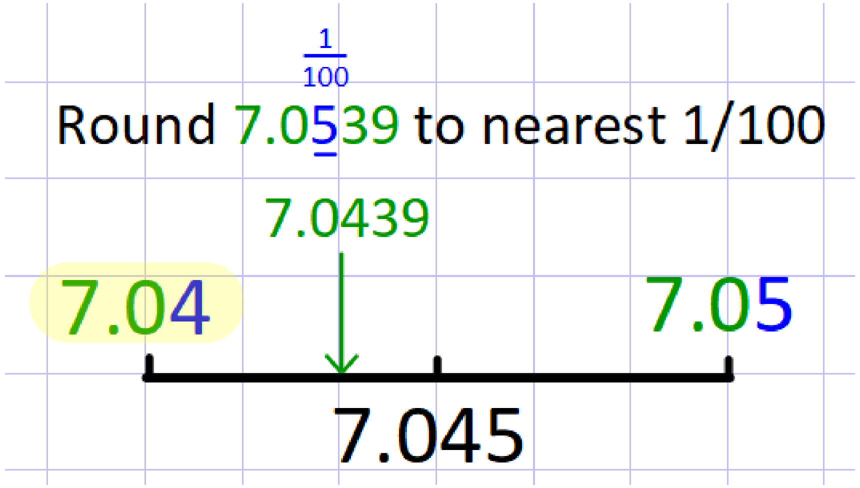
Notes:

A support to rounding:

Use a number-line with the relevant lower and upper possibilities marked, then find the half-way point.

This is useful for general number sense and understanding.





S6PV6	Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.	S5PV4	Round any number up to 1,000,000 to the nearest 10, 100, 1000, 10,000 and 100,000.
S6PV5	Solve problems which require answers to be rounded to specified degrees of accuracy.	S4FR7	Round decimals with one decimal place to the nearest whole number.
S6PV2	Round any whole number to a required degree of accuracy.	S4PV7	Round any number to the nearest 10, 100 or 1000.
S5PV8	Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.	S4PV6	Identify, represent and estimate numbers using different representations.
S5PV7	Round decimals with two decimal places to the nearest whole number and to one decimal place.	S3PV4	Identify, represent and estimate numbers using different representations.

Note

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Rounding and Estimating Answers (2)

Notes:

Approximating calculations:
Round the numbers first *before* calculating.

Rounding is often to 1 or 2 sig.fig. or to a number which makes the calculation simple to perform.

e.g: $329 \div 8$ would be better approximated as $\approx 320 \div 8$, rather than (the more accurate) $\approx 330 \div 8$.

S8PV2	Use approximation through rounding to estimate answers and calculate possible resulting errors expressed using inequality notation $a<x\leq b$.
S7PV4	Use approximation through rounding to estimate answers.
S7PV3	Round numbers and measures to an appropriate degree of accuracy [for example, to a number of decimal places or significant figures].

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Multiplying and Dividing by Powers of 10

Notes:

Use a place value chart to show clearly that the digits move rather than the decimal point.

Emphasise correct vocabulary (e.g: “What column is the 2 digit now in?” “The hundreds column” etc.)

Different base systems

Binary / ternary etc. can be introduced as an enrichment task (e.g: multiplying by 2 in binary is equivalent to multiplying by 10 in decimal)

thousands			(units)					
H	T	U,	H	T	U ·	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
				1	2			
		1	2	0	0			

$12 \times 100 = \underline{1200}$

thousands			(units)					
H	T	U,	H	T	U ·	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
				4	7			
					0 ·	4	7	

$47 \div 100 = \underline{0.47}$

16	8	4	2	1
		1	0	1
	1	0	1	0

(in base 10)

$5 \times 2 = \underline{10}$

$(101 \times 10 = \underline{1010})$

(in base 2)

Note

S8PV1	Interpret and compare numbers in standard form A x 10n 1≤A<10, where n is a positive or negative integer or zero.
S7PV2	Interpret numbers in standard form A x 10n 1≤A<10, where n is a positive integer or zero.
S6PV7	Identify the value of each digit in numbers given to three decimal places and x & ÷ numbers by 10, 100 and 1000 giving answers to 3 decimal places.
S5PV9	Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
S4PV10	Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as 1s, 10ths & 100ths.

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Standard Index Form / Scientific Notation

Notes:

Link to multiplying / dividing by powers of 10. Use a place value chart in the same way to show the digits moving.

Emphasise that 1000 = 10^3 so that $\times 10^3$ moves the number 3 columns to the left.

Similarly; $10^{-3} = 1/10^3$. This means $\times 10^{-3}$ is equivalent to dividing by 1000 (moving 3 columns to the right).

Power	10^{20}	10^{19}	10^{18}		10^{17}	10^{16}	10^{15}		10^{14}	10^{13}	10^{12}		10^{11}	10^{10}	10^{9}		10^{8}	10^{7}	10^{6}		10^{5}	10^{4}	10^{3}		10^{2}	10^{1}	10^{0}
Name	Hundred Quintillions	Ten Quintillions	Quintillions		Hundred Quadrillions	Ten Quadrillions	Quadrillions		Hundred Trillions	Ten Trillions	Trillions		Hundred Billions	Ten Billions	Billions		Hundred Millions	Ten Millions	Millions		Hundred Thousands	Ten Thousands	Thousands		Hundreds	Tens	Units
Digits	0	0	0	,	0	0	0	,	0	0	0	,	0	0	0	,	0	0	0	,	0	0	0	,	0	0	0
				,				,				,				,				,				,			

In Figures:		In Words:		Standard Form:
9,000	would read	"nine thousand"	or	9.0×10^3
6,024	would read	"six thousand and twenty-four"	or	6.024×10^3
102,000	would read	"one hundred and two thousand"	or	1.02×10^5
7,000,000,000	would read	"seven billion"	or	7.0×10^9
73,550,000,000	would read	"seventy-three billion, five hundred and fifty million"	or	7.355×10^{10}
184,000,000,000,000	would read	"one hundred and eighty-four trillion"	or	1.84×10^{14}

S8PV1	Interpret and compare numbers in standard form A x 10n 1≤A<10, where n is a positive or negative integer or zero.
S7PV2	Interpret numbers in standard form A x 10n 1≤A<10, where n is a positive integer or zero.
S6PV7	Identify the value of each digit in numbers given to three decimal places and x & ÷ numbers by 10, 100 and 1000 giving answers to 3 decimal places.
S5PV9	Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
S4PV10	Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as 1s, 10ths & 100ths.

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Addition & Subtraction

- Mental Calculations: Addition and Subtraction
- Addition: Formal written methods
- Subtraction: Formal written methods
- Bar Modelling – Addition and Subtraction

Mental Calculations: Addition and Subtraction

Notes:

Useful strategies / skills:

- **Number bonds** to 10, 100 etc. (e.g: 7 + ? = 10 or 100 – 38 = ? etc.)
- **Bridging through ten** (e.g: 7 + 8 = 7 + 3 + 5)
- **Near doubles** (e.g: 7 + 8 = 7 + 7 + 1 or 8 + 8 - 1)
- **Compensation for addition** (e.g: 7 + 8 = 7 + 10 – 2)
- **Compensation for subtraction** (e.g: 23 – 9 = 23 – 10 + 1)
- **Compensation for addition** - *alternative* (e.g: 27 + 9 = 26 + 10)
- **Compensation for subtraction** - *alternative* (e.g: 27 - 9 = 28 + 10)

  This can be shown clearly on a number-line, where the difference (distance) between 9 and 27 is the same as between 10 and 28

- **Counting on** and **taking away** for subtraction (the former being useful when the numbers (minuend and subtrahend) are close together e.g: 394 - 389, the later when the difference subtrahend is small e.g: 394 – 8
- **Partitioning for addition** (e.g: 39 + 48 = (30 + 40) + (9 + 8)

(Needs examples of each strategy shown on a number-line etc.)

S5AS2	Add and subtract numbers mentally with increasingly large numbers.
S3AS1	Add and subtract numbers mentally, including: a three-digit number and ones, tens and hundreds.
S4AS2	Estimate and use inverse operations to check answers to a calculation.
S3AS3	Estimate the answer to a calculation and use inverse operations to check answers.

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Addition: Formal written methods (1)

Notes:

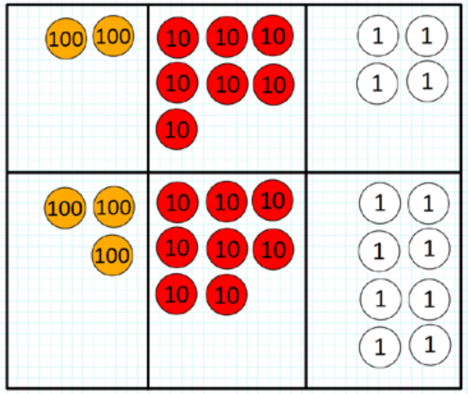
Step 1 may be used for lower ability pupils. It may seem un-necessary for addition, but provides a very helpful lead into the (more challenging) expanded column method for subtraction.

All three methods can be supported using place value counters or Dienes (multi-base).

Place value headings written above (at least when introducing method).

Expanded column method (step 1)

	H			T		U				
	2	0	0	7	0	4				
+	3	0	0	8	0	7				
	6	0	0	6	0	1	=	6	6	1
	1	0	0	1	0					



S5AS1	Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction).
S4AS1	Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.
S3AS2	Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.

Expanded column method (step 2)

	H	T	U
	2	7	4
+	3	8	7
		1	1
	1	5	0
	5	0	0
	6	6	1

Compact column method (step 3)

	H	T	U
	2	7	4
+	3	8	7
	6	6	1
	1	1	

	T	U	$\frac{1}{10}$	$\frac{1}{100}$
		2·	7	4
+	3	8·	7	0
	4	1·	4	4
	1	1		

Compact method: all exchanges written at the bottom and in the **correct column** (not in-between)

Decimals in line & fill any spaces with “0”s.

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Addition: Formal written methods (2)

Notes:

All three methods can be supported using place value counters or Dienes (multi-base).

S5AS1	Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction).
S4AS1	Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.
S3AS2	Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.

Subtraction: Formal written methods (1)

Notes:

“Compact” method: All exchanges written carefully **next to** the number (e.g. since the 1 in the units column actually represents 10 units, not 1 unit).

Emphasise vocabulary:

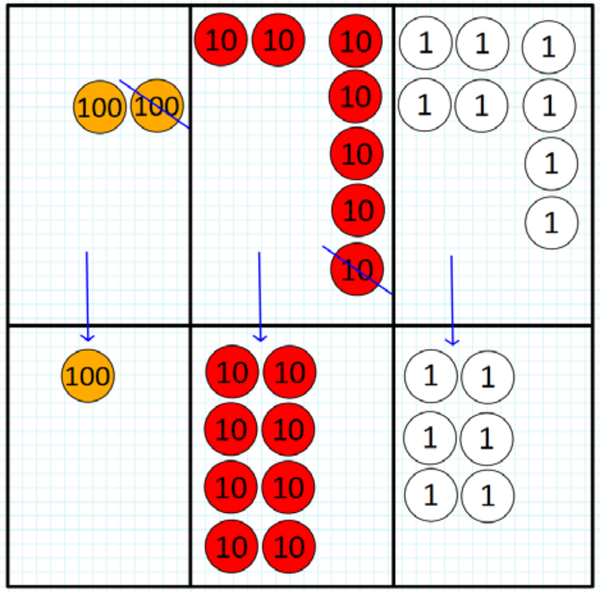
Minuend: The number we begin with.

Subtrahend: what is being taken away.

Difference: the answer.

Expanded column method (step 1)

	H			T		U
	2	0	0	1 4	0	1
	~~3~~	~~0~~	~~0~~	~~5~~	~~0~~	4
-	1	0	0	8	0	6
	1	0	0	6	0	8



N.B: the difference ends up at the top, the subtrahend is at the bottom.

Compact column method (step 2)

	H	T	U
	2	14	1
	~~3~~	~~5~~	4
-	1	8	6
	1	6	8

S5AS1	Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction).
S4AS1	Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.
S3AS2	Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.

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Subtraction: Formal written methods (2)

Notes:

Both methods can be supported using place value counters or Dienes (multi-base).

S5AS1	Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction).
S4AS1	Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.
S3AS2	Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.

Bar Modelling – Addition & Subtraction

S5AS2	Add and subtract numbers mentally with increasingly large numbers.
S3AS1	Add and subtract numbers mentally, including: a three-digit number and ones, tens and hundreds.
S4AS2	Estimate and use inverse operations to check answers to a calculation.
S3AS3	Estimate the answer to a calculation and use inverse operations to check answers.

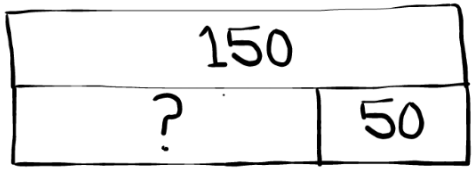
Notes:

Lower ability / younger pupils will need to begin with "discrete" bar models, writing number sentences like: 5 + 3 = 8 , 8 - 3 = 5 etc.

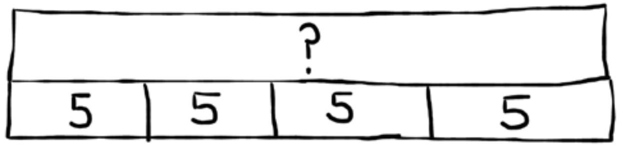
Bar models work well for calculating

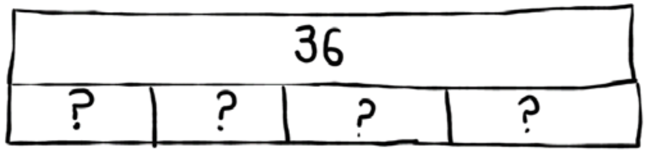
Writing / speaking answers in full sentences helps with being clear on specifically what has been calculated, as well as helping with clearer thinking in problem solving.

a "discrete" bar model representing: 5 + 3 = 8 etc.



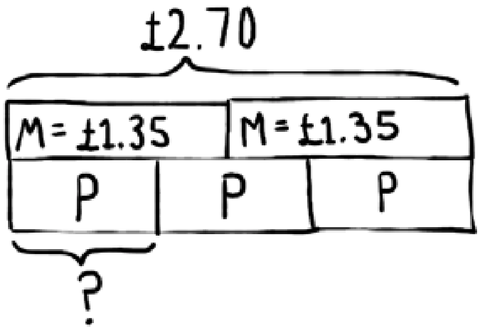
a "continuous" bar model representing 150 – 50 = ? or ? + 50 = 150





Bar models leading to multiplication and division:

e.g: 5 + 5 + 5 + 5 = ? is equivalent to: 5 x 4 = ? etc.



Bar model representing:

"3 pineapples cost the same as 2 mangoes. One mango costs £1.35.
How much does one pineapple cost?"

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Multiplication & Division

- Multiplication & Division - Mental methods
- Multiplication - Formal written methods
- Division - Formal written methods
- Other relevant topics

Multiplication - Mental methods

Notes:

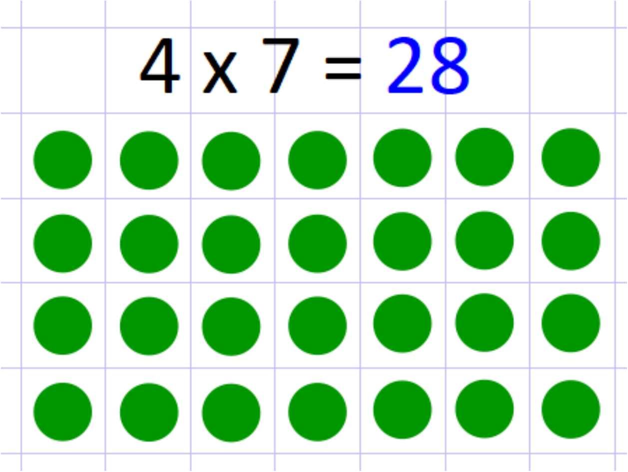
Arrays are a powerful way of representing multiplication (and division), clearly showing:

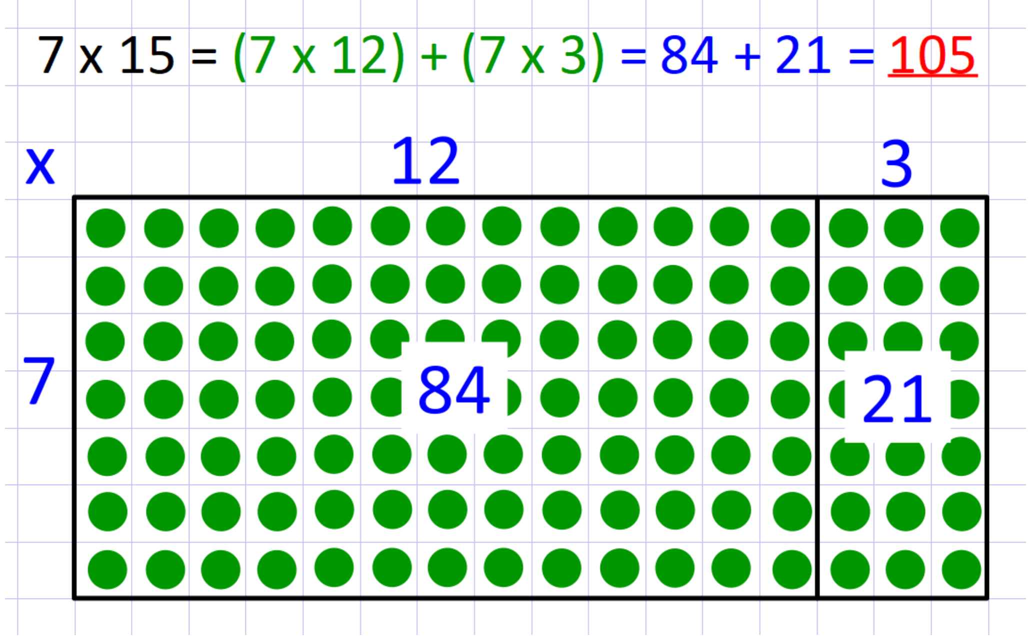
- the commutative property of multiplication

  (4 x 7 = 7 x 4)
- The distributive property of multiplication

  (4 x 7 is the same as: 4 x [2+5] = 4x2 + 4x5)
- division being the inverse of multiplication
- the connection / equivalence between: "sharing into groups of 4" (there are *7 groups*) and "dividing into 4 groups" (there are 7 *in each group*) (in the calculation 28 ÷ 4 = 7)

Arrays also lead clearly into **grid multiplication,** where partitioning is literally dividing the array up into manageable / convenient pieces.





A non-standard partition of 7 x 15 *(distributive property)*

S6FO3	Perform mental calculations, including with mixed operations and large numbers.
S5MD1	Multiply and divide numbers mentally drawing upon known facts.
S4MD3	Recognise and use factor pairs and commutativity in mental calculations.
S4MD2	Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers.
S4MD1	Recall multiplication and division facts for multiplication tables up to 12 × 12.
S3MD1	Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.
S3MD2	Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.

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Division - Mental methods

Notes:

“Chunking” division with the support of a number line.

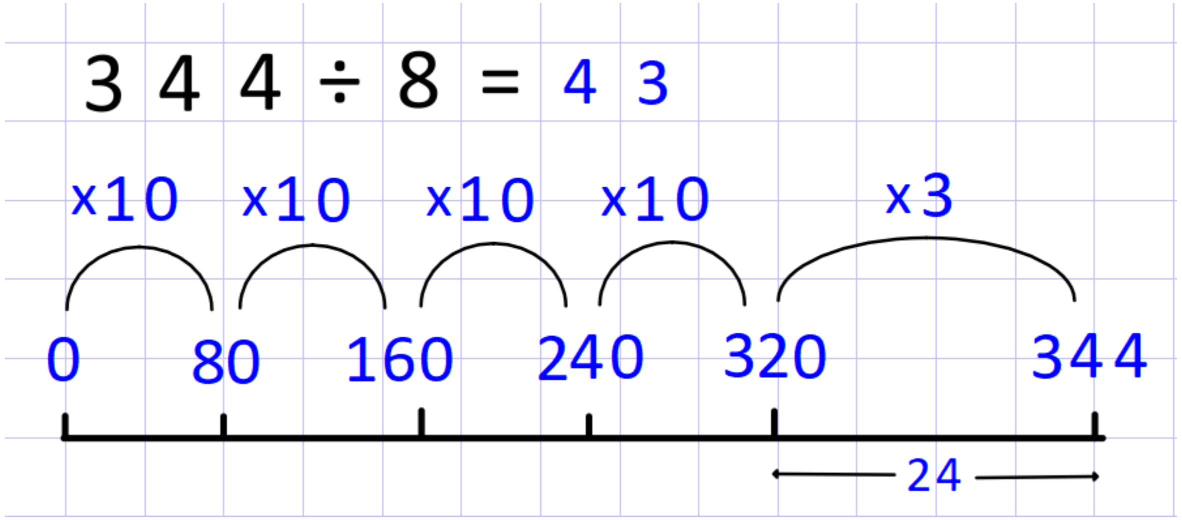
This is supports mental methods, including “overshooting” the target then compensating.

E.g:

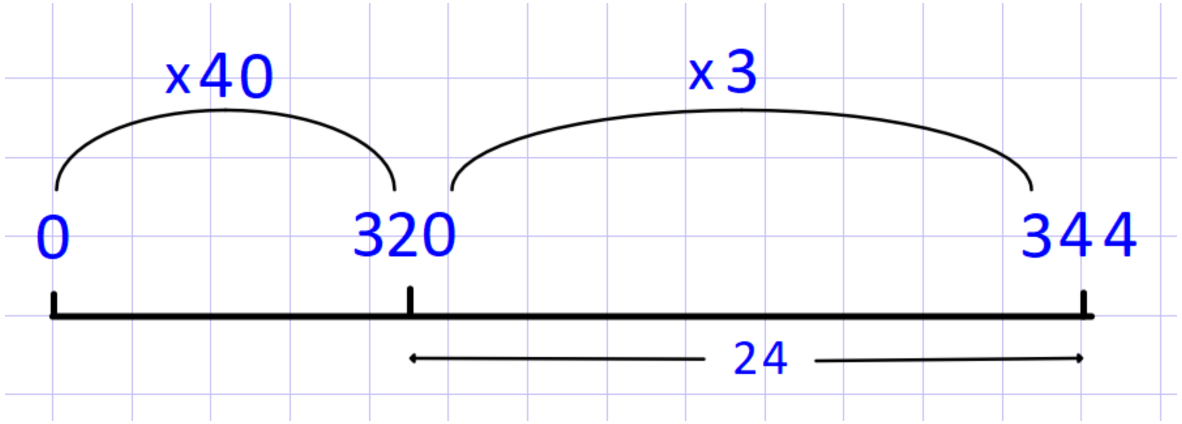
177 ÷ 3 = (60 x 3) - (1 x 3)

= 59 x 3

When the maximum amount of hundreds, tens then units are jumped each step, this leads neatly onto short division.



Pupils may begin by counting in multiples of ten, gradually they are encouraged to do x 40 instead.



’24’: Showing what is left when we get near the ‘target’.

S6FO3	Perform mental calculations, including with mixed operations and large numbers.
S5MD1	Multiply and divide numbers mentally drawing upon known facts.
S4MD3	Recognise and use factor pairs and commutativity in mental calculations.
S4MD2	Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers.
S4MD1	Recall multiplication and division facts for multiplication tables up to 12 × 12.
S3MD1	Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.
S3MD2	Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.

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Multiplication - Formal written methods (1)

Notes:

Grid method: Set out the grid as a rectangle split into sections (this links in with previous work on arrays – see mental methods).

Link column methods to previous work on grid method.

Expanded method:
Calculation labels (right hand side) *may* be included when first learning, but dropped when confident.

Grid method (step 1)

7 x 1 5 8 = 1 1 0 6

x	100	50	8
7	700	350	56

	H	T	U
	7	0	0
	3	5	0
+		5	6
1	1	0	6
	1		

This is a *perfectly acceptable* alternative, especially when place value is not secure. It also has the added benefit of giving extra practice of place value multiplication (e.g. 50 x 400)

Expanded column method (step 2)

	H	T	U	
	1	5	8	
x			7	
		5	6	7 x 8
	3	5	0	7 x 50
	7	0	0	7 x 100
1	1	0	6	
1	4	5		

Compact column method (step 3)

	H	T	U
	1	5	8
x			7
1	1	0	6
1	4	5	

S6FR7	Multiply one-digit numbers with up to two decimal places by whole numbers.
S6FO1	Multiply numbers up to 4 digits by a 2-digit whole number using the formal written method of long multiplication.
S5MD2	Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for 2-digit no.s.
S4MD4	Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.

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Multiplication - Formal written methods (2)

Notes:

All exchanges written at the bottom and in the **correct column** (not in-between columns).

Expanded method:

Calculation labels (right hand side) *may* be included when first learning, but *this can cause confusion with place value* (since the column multiplication algorithm deliberately 'hides' place value in exchange for ease of calculation).

Grid method (step 1)

3 2 x 2 4 7 = 7 9 0 4

x	200	40	7
30	6000	1200	210
2	400	80	14

Th	H	T	U
6	0	0	0
1	2	0	0
	2	1	0
	4	0	0
		8	0
		1	4
7	9	0	4
	1		

The link between column method and grid method should be made clear. Again, grid is a *perfectly acceptable* method.

Teaching 40 x 500 as "4 x 5 = 20 then add three zeros" is an acceptable *mental strategy* as long as you also keep focusing pupils what that means (e.g. "adding three zeros is really doing what?" … "multiplying by 1000")

Long multiplication – expanded (step 2)

		H	T	U	
		2	4	7	
x			3	2	
			1	4	2 x 7
			8	0	2 x 40
		4	0	0	2 x 200
		2	1	0	30 x 7
	1	2	0	0	30 x 40
	6	0	0	0	30 x 200
	7	9	0	4	
		1			

Long multiplication – compact (step 3)

		H	T	U	
		2	4	7	
x			3	2	
		4	9 (1)	4	2 x 247
	7 (1)	4 (2)	1	0	30 x 247
	7	9	0	4	
		1			

Code	Objective
S6FR7	Multiply one-digit numbers with up to two decimal places by whole numbers.
S6FO1	Multiply numbers up to 4 digits by a 2-digit whole number using the formal written method of long multiplication.
S5MD2	Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for 2-digit no.s.
S4MD4	Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.

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Multiplication - Formal written methods (3)

Notes:

Use of correct place value should be stressed here (as opposed to counting the number of digits after the decimal point, which misses the understanding of what is really happening).

Alternative approach:

- Estimate the answer first (e.g $2 \times 3 = 6$)
- Next perform the calculation *without any decimals* (e.g: $247 \times 32 = 7904$).
- Finally place the decimal in to match the estimation (e.g: 7.904) *making the answer "7. something" not "70 something" or "700 and something")*

$$2{\cdot}47 \times 3{\cdot}2 = 7{\cdot}904$$

x 100   x 10   ÷ 1000

$$247 \times 32 = 7904$$

	H	T	U	
	2	4	7	
x		3	2	
	4	9	4	2 x 247
7	4	1	0	30 x 247
7	9	0	4	

Code	Objective
S6FR7	Multiply one-digit numbers with up to two decimal places by whole numbers.
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S4MD4	Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.

Division - Formal written methods (1)

S6FR8	Use written division methods in cases where the answer has up to two decimal places.
S6FO2	Divide numbers up to 4 digits by a two-digit whole number using formal methods, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate to the context.
S5MD3	Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Notes:

This method can be built up using place value counters or Dienes (video to follow).

Short division (step 1)

$600 \div 5 = \underline{\underline{120}}$

$$\begin{array}{r|l} & 1\ \ 2\ \ 0 \\ \hline 5 & 6\ {}^{1}0\ \ 0 \end{array}$$

Remainders: discuss whether to round final answer up or down or simply state the remainder (depending on the context of the question).

Decimal remainders: focus on money answers at first since this is a familiar context.

Fractional remainders: this is effectively shown through converting improper fractions to mixed numbers first – this makes it clear that the fraction bar also represents division. The quotient (answer) is the whole number and the remainder is the fraction left over).

With remainder

$7605 \div 8 = \underline{\underline{950}}$ r 5

$$\begin{array}{r|l} & 0\ \ 9\ \ 5\ \ 0 \quad \text{r } 5 \\ \hline 8 & 7\ {}^{7}6\ {}^{4}0\ \ 5 \end{array}$$

As a decimal answer

$7605 \div 8 = \underline{\underline{950{\cdot}625}}$

$$\begin{array}{r|l} & 0\ \ 9\ \ 5\ \ 0\ {\cdot}\ 6\ \ 2\ \ 5 \\ \hline 8 & 7\ {}^{7}6\ {}^{4}0\ \ 5\ {\cdot}\ {}^{5}0\ {}^{2}0\ {}^{4}0\ \ 0 \end{array}$$

With fractional remainder

$7605 \div 8 = 950\frac{5}{8}$

$$\begin{array}{r|l} & 0\ \ 9\ \ 5\ \ 0\ \frac{5}{8} \\ \hline 8 & 7\ {}^{7}6\ {}^{4}0\ \ 5 \end{array}$$

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Division - Formal written methods (2)

Notes:

Links should be made with the short division method.

Long division for a single digit divisor is presented as an intermediate step towards long division with a 2 (or more) digit divisor.

Short division for 2-digit divisors should be considered. Supported by a list of multiples and other working, it is much less complicated and means only one formal method needs teaching.

"Maths Antics" video provides a good demonstration.

Long division – single digit

7 6 0 5 ÷ 8 = 9 5 0 r 5

```
      0 9 5 0   r 5
  8 | 7 6 0 5
   -  0           0 x 8
      7 6
   -  7 2         9 x 8
        4 0
     -  4 0       5 x 8
          0 5     remainder
```

Short division – double digit (alternative method)

3 8 5 5 ÷ 1 8 = 2 1 4 r 3

```
        0  2  1  4   r 3
  1 8 | 3 ³8 ²5 ⁷5
```

1 x 18 = 18
2 x 18 = 36
3 x 18 = 54
4 x 18 = 72
5 x 18 = 90
....

Long division – double digit

3 8 5 5 ÷ 1 8 = 2 1 4 r 3

```
        0 2 1 4   r 3
  1 8 | 3 8 5 5
      - 0           0 x 18
        3 8
      - 3 6         2 x 18
          2 5
        - 1 8       1 x 18
            7 5
            7 2     4 x 18
              3     remainder
```

1 x 18 = 18
2 x 18 = 36
3 x 18 = 54
4 x 18 = 72
5 x 18 = 90
6 x 18 = 108
7 x 18 = 136
8 x 18 = 144
9 x 18 = 162
10 x 18 = 180

Writing out multiples to 10 (see mental methods) helps to avoid mistakes in an already complicated method.

S6FR8	Use written division methods in cases where the answer has up to two decimal places.
S6FO2	Divide numbers up to 4 digits by a two-digit whole number using formal methods, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate to the context.
S5MD3	Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

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Division - Formal written methods (3)

Notes:

Dividing decimals:

(Where the dividend is a decimal, but the divisor is a whole number).

This is straight-forward using short / long division.

Dividing by decimals:

(Where the divisor is a decimal)

a) By estimation: 24.9 ÷ 5.9 = 4 (roughly)

Now perform the division without decimals and place the decimal according the estimate.

b) By equivalent fractions: 24.9/5.9 = 249/59

Perform the whole-number division. The answer will not need adjusting since the two fractions give the same answer (decimal expansion).

S6FR8	Use written division methods in cases where the answer has up to two decimal places.
S6FO2	Divide numbers up to 4 digits by a two-digit whole number using formal methods, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate to the context.
S5MD3	Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Other Relevant Topics - Number theory (factors)

Notes:

Year 5 use "Fergus" as a memorable diagram.

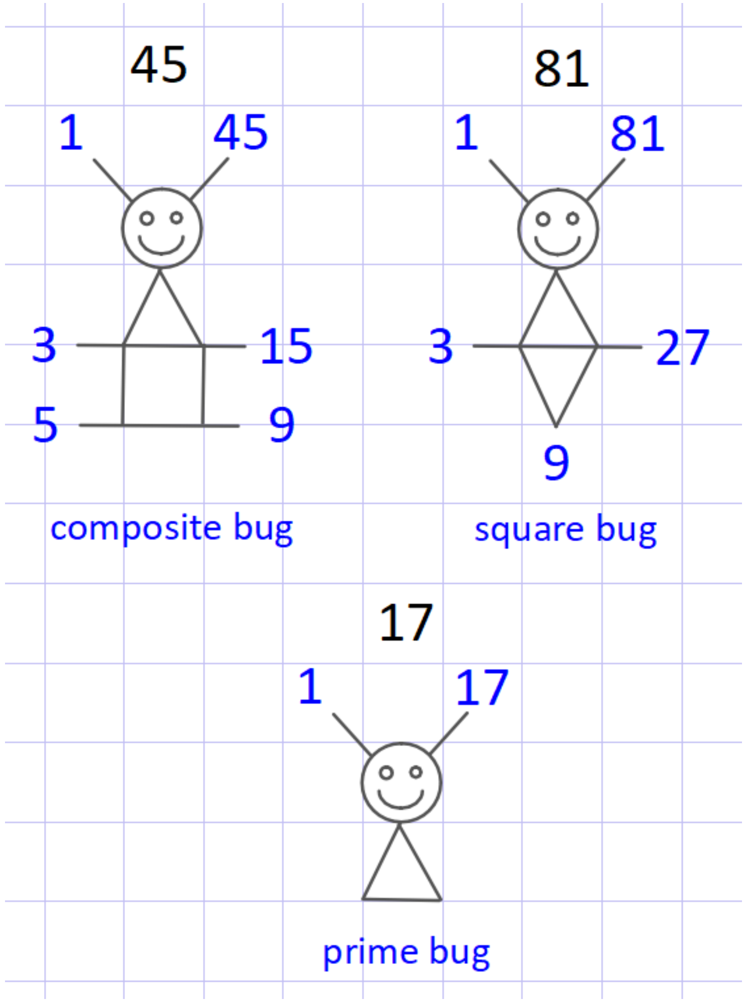
Head: is the number.
Antennae: first pair of factors.
Arms: other factor pairs.
Square numbers end with a **tail**, since it is only one factor (not two).

Year 6 and beyond use factor pairs. This builds a systematic approach, emphasises multiplication and division, and helps pupils know when they have found all the factors (they arrive at a number already in the list).

N.B: prime factor trees are used in KS3 – they don't appear in KS2 (see exemplification for KS2).

Year 5:

Fergus the Friendly Factor Bug



Year 6 (and on):

Factor pairs

45
1 x 45
3 x 15
5 x 9
(composite numbers have an even number of factors)

81
1 x 81
3 x 27
9 x 9
(Squares have an odd number of factors)

17
1 x 17
(primes have 2 factors)

S6FO4	Identify common factors, common multiples and prime numbers.
S5MD7	Establish whether a number up to 100 is prime and recall prime numbers up to 19.
S5MD6	Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
S5MD5	Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.
S5MD4	Recognise and use square numbers and cube numbers, and the notation for squared and cubed.

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Other Relevant Topics - Number theory (2)

Notes:

Square and cube numbers – link to area and volume

Model for listing multiples (to differentiate from factors) – perhaps a "multiples millipede" which is very long, like a long list of multiples. Discuss with maths department.

S6FO4	Identify common factors, common multiples and prime numbers.
S5MD7	Establish whether a number up to 100 is prime and recall prime numbers up to 19.
S5MD6	Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
S5MD5	Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.
S5MD4	Recognise and use square numbers and cube numbers, and the notation for squared and cubed.

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Other Relevant Topics - Number theory (prime factors)

Notes:

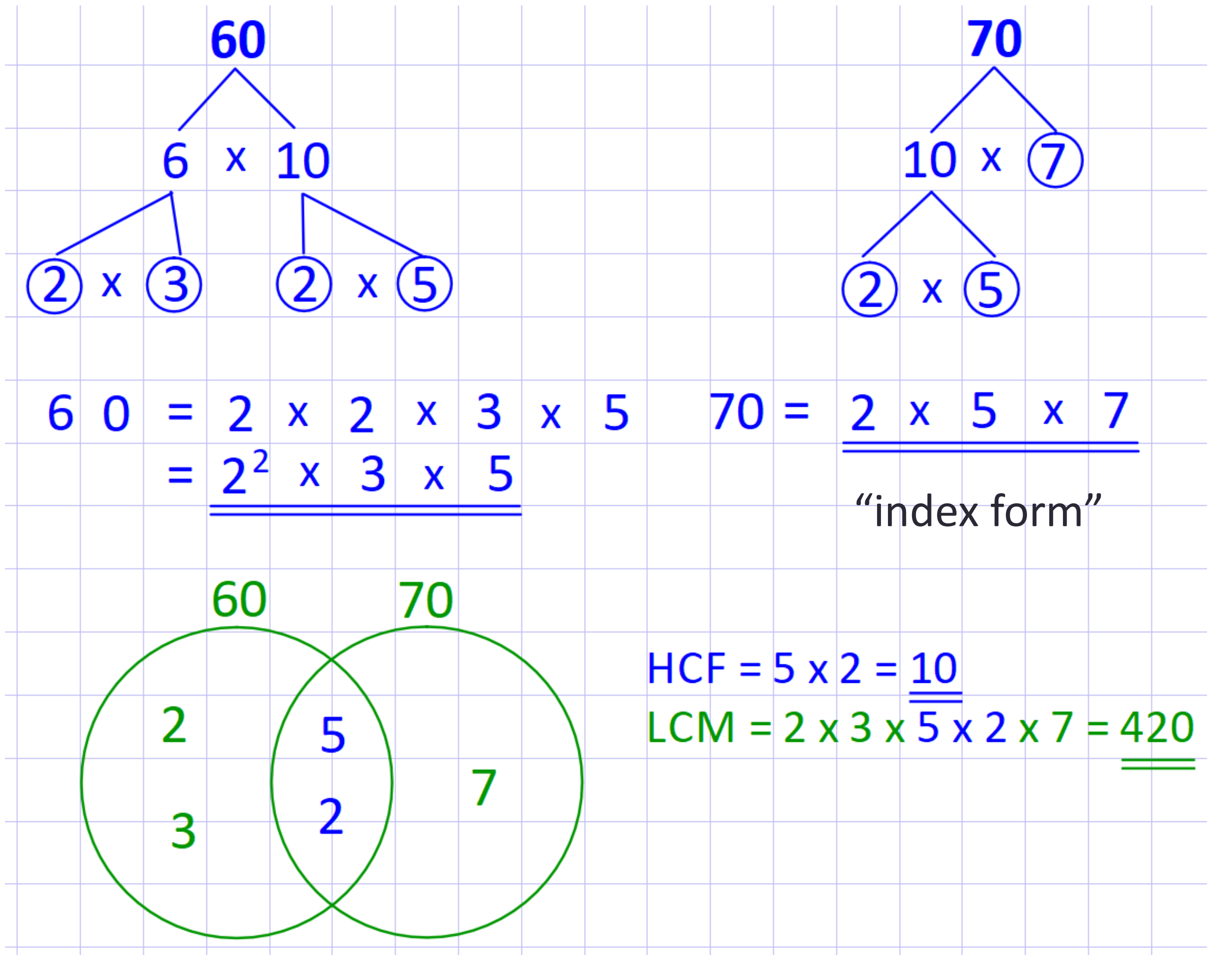
Circle primes, write in the multiplication signs. Finish off by *writing in "index form"* (bases in order from smallest to largest)

Emphasise that the prime factors multiply to give the number, and that each number has a unique prime factorisation.

All the factors of a number (apart from 1) can be derived from the prime factors (e.g: factors of 70: 2x5=10 , 2x7=14 , 5x7=35 , 2x5x7=70)

Use a Venn diagram for HCF and LCM.

Prime factor trees



S8FO2	Use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5 and distinguish between exact representations of roots and their decimal approximations.
S8FO1	Use the concepts of prime factorisation, including using product notation and the unique factorisation property to assist with finding HCF and LCM.
S7FO4	Recognise and use relationships between operations including inverse operations.
S7FO3	Use integer powers and associated real roots (square and cube), recognise powers of 2, 3, 4, 5.
S7FO2	Use the concepts of prime factorisation, including using product notation and the unique factorisation property.
S7FO1	Use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple.

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Fractions

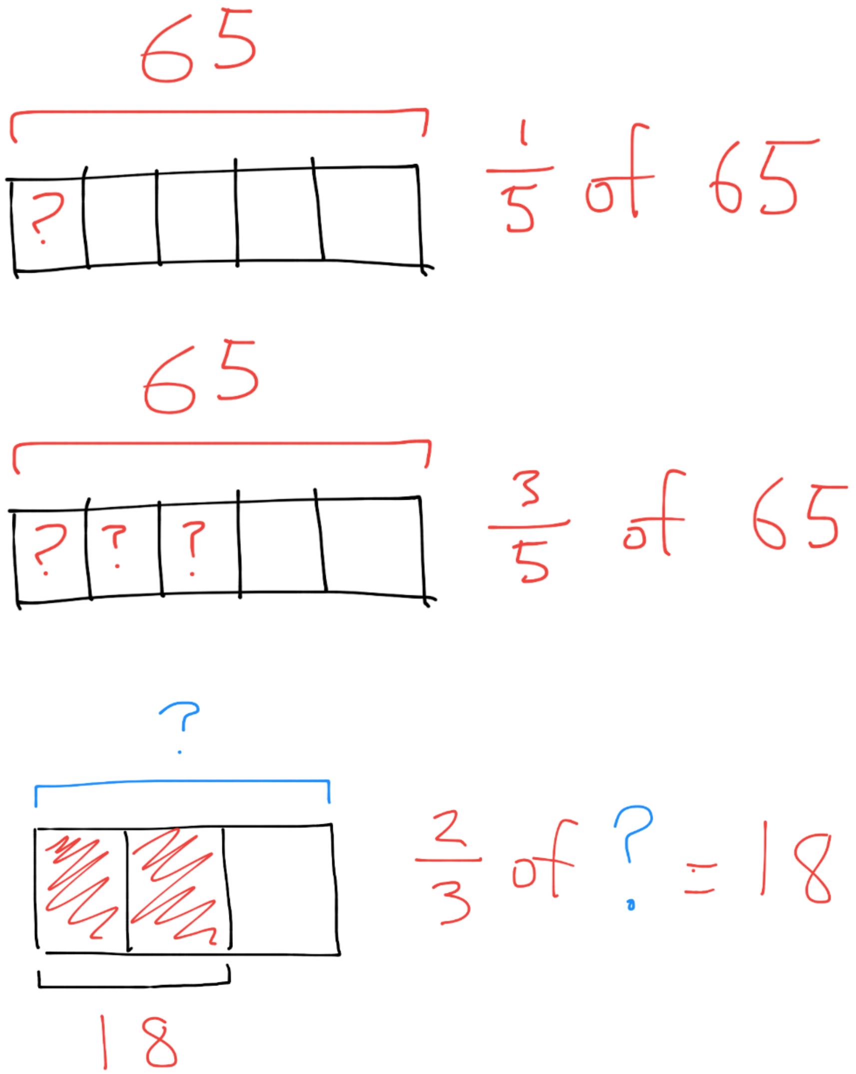
- Fractions of an amount
- Improper fractions
- Equivalent fractions
- Adding & subtracting fractions
- Multiplying fractions
- Dividing fractions

Fractions of an amount

Notes:

“Bar modelling” approach shown here. The “bracket” at the top represents the whole.

This bar model for fractions is almost identical to that used when working with ratio.



S4FR3	Solve problems involving harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole no.
S3FR2	Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators.

Improper / top heavy fractions

S5FR3

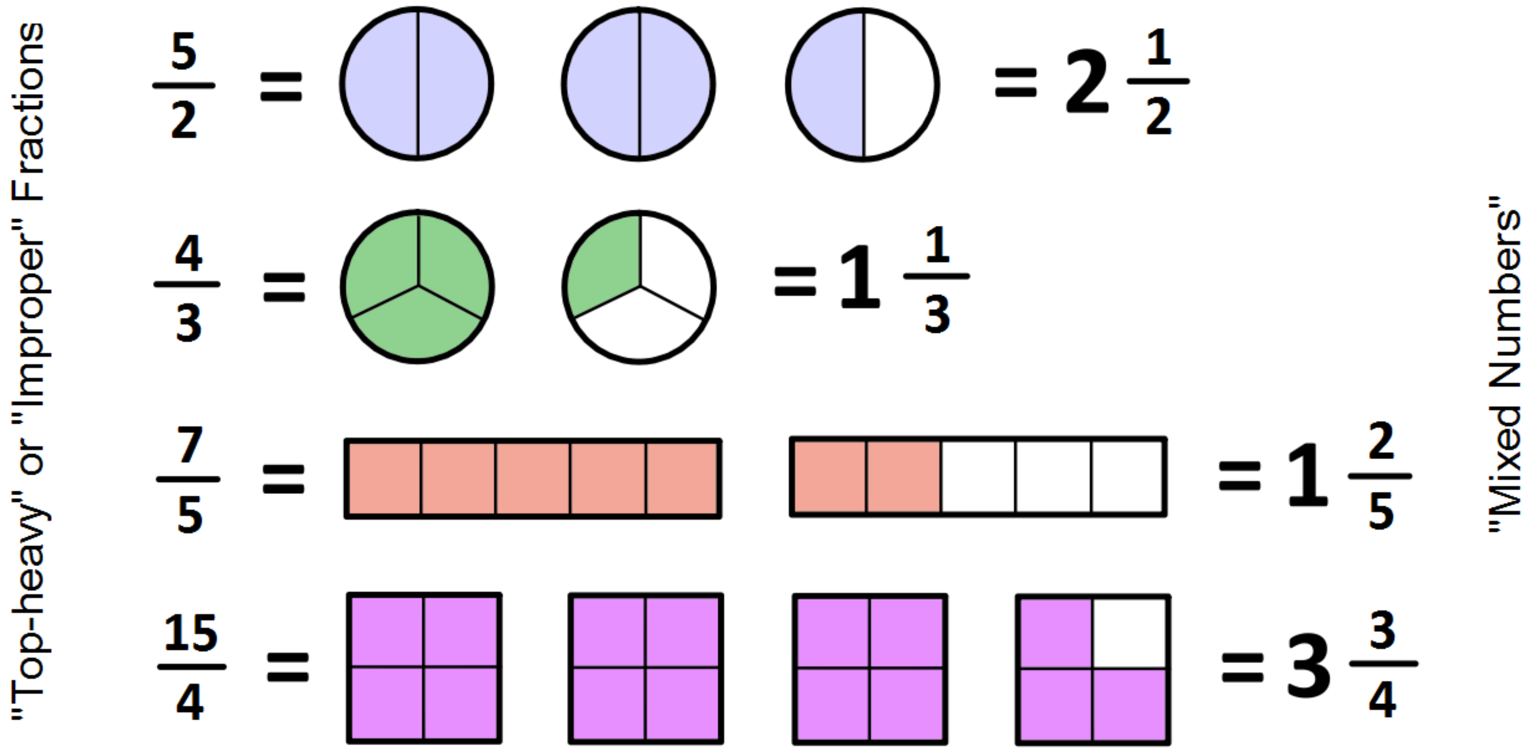
Recognise mixed numbers and improper fractions and convert from one form to the other and write statements > 1 as a mixed number [e.g 2/5 + 4/5 = 1 1/5].

Notes:

May be called “improper”, “top heavy” or even “vulgar” fractions. “Mixed numbers” means part whole number, part fraction.

A mixture of bars, fractions circles and other shapes should be used so pupils remain flexible in their understanding.

Pupils may well see that you can multiply the denominator by the whole number, then add the numerator, but there are plenty of “rules” to remember already in fractions – this one is easy enough to visualise.



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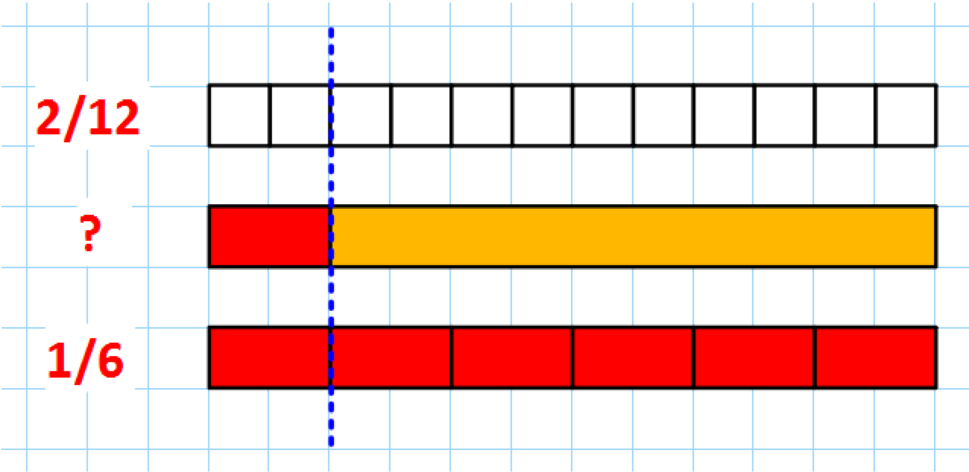
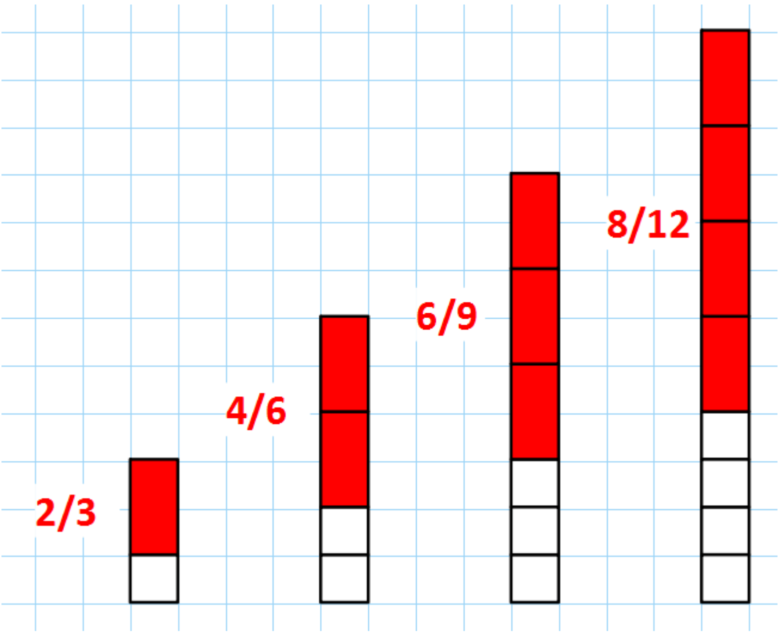
Equivalent fractions

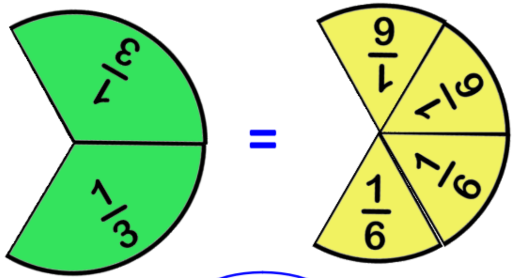
Notes:

Some possible visual representations.

Experience in working with different representations of equivalence will make the leap to the numerical method (multiply / divide numerator and denominator by the same number) more understandable.







x2

$$\frac{2}{3} = \frac{4}{6}$$

x2

S7FR3	Work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and 7/2 or 0.375 and 3/8).
S6FR9	Recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.
S6FR6	Associate a fraction with division and calculate decimal fraction equiv [for example, 0.375] for a simple fraction [e.g 3/8].
S5FR2	Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths.
S4FR1	Recognise and show, using diagrams, families of common equivalent fractions.
S3FR4	Recognise and show, using diagrams, equivalent fractions with small denominators.
S2FR2	Write simple fractions for example, 1/2 of 6 = 3 and recognise the equivalence of 2/4 and 1/2.

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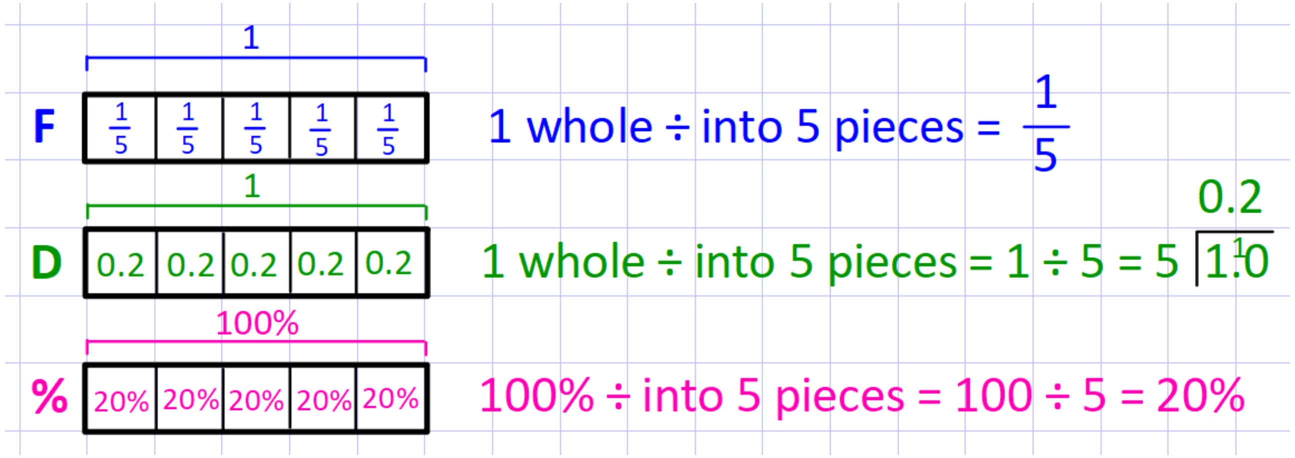
Fractions, Decimals, Percentages

Notes:

Asking pupils to literally divide a whole one (using a bar model) into a certain amount of pieces helps make the link as to why decimal / percentage equivalents are as they are. e.g: 1/5 must = 0.2 since 0.2+0.2+0.2+0.2+0.2 = 1.0 or 0.2 x 5 = 1. Also, since % are "out of 100", their equivalents are 100 x bigger than their decimal equivalent (which are only "out of 1").

This also clearly links "40% of £250" with "2/5 of £250" (or "4/10 of £250") and "0.4 x £250" etc.



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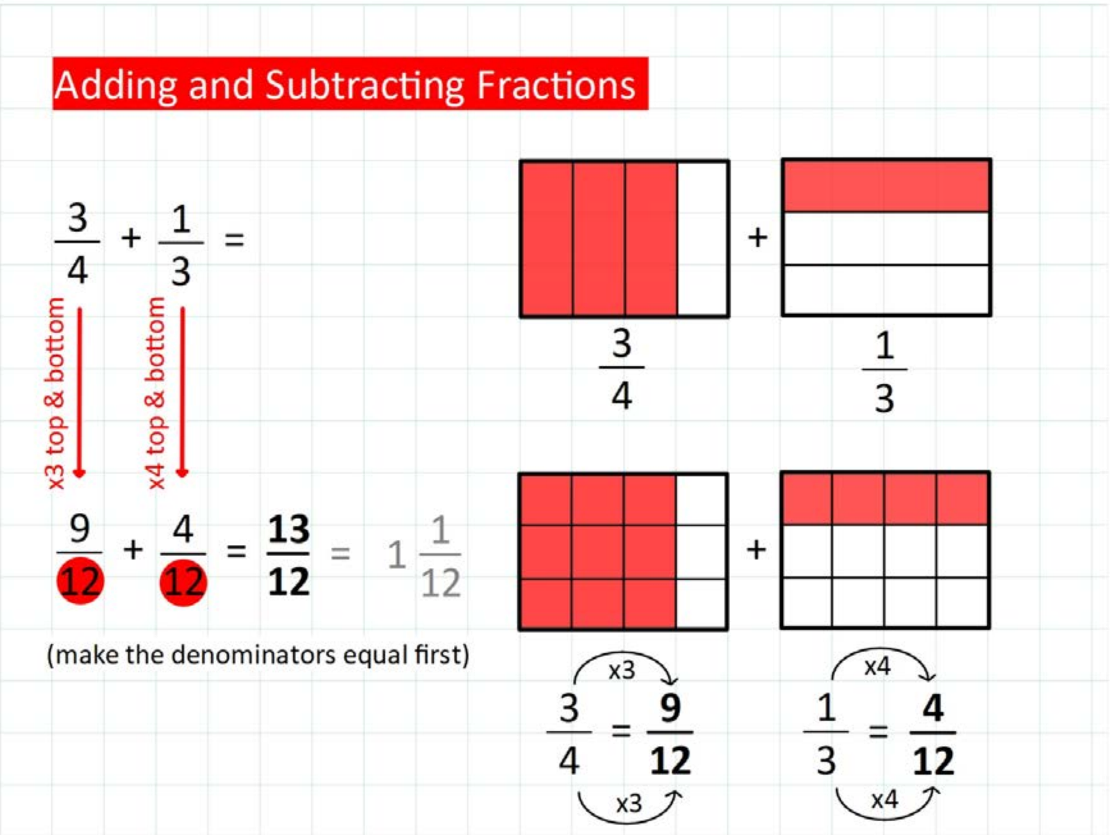
S3FR4	Recognise and show, using diagrams, equivalent fractions with small denominators.
S2FR2	Write simple fractions for example, 1/2 of 6 = 3 and recognise the equivalence of 2/4 and 1/2.

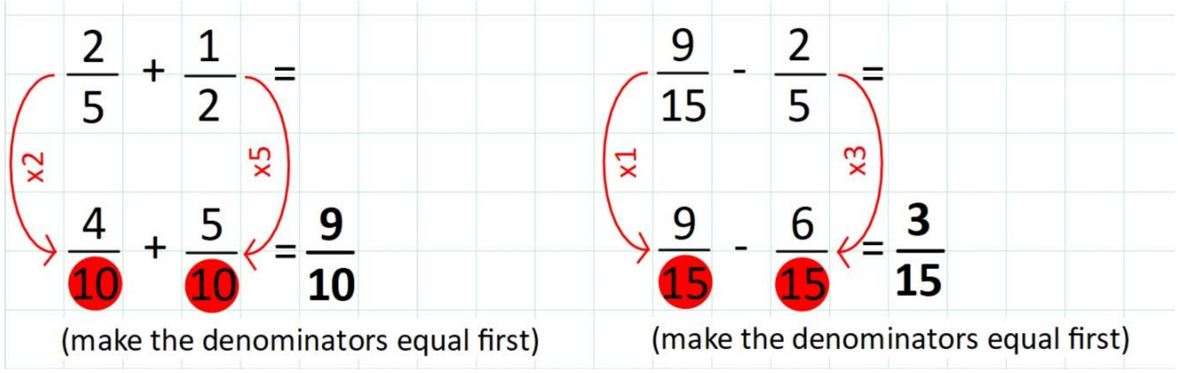
Adding / subtracting fractions

Notes:

Using two 4 by 3 grids for adding quarters and thirds (to show equivalence).

Subtraction works the same but taking away squares at the final stage.





S6FR3	Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.
S5FR4	Add and subtract fractions with the same denominator and denominators that are multiples of the same number.

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Multiplying fractions (1)

Notes:

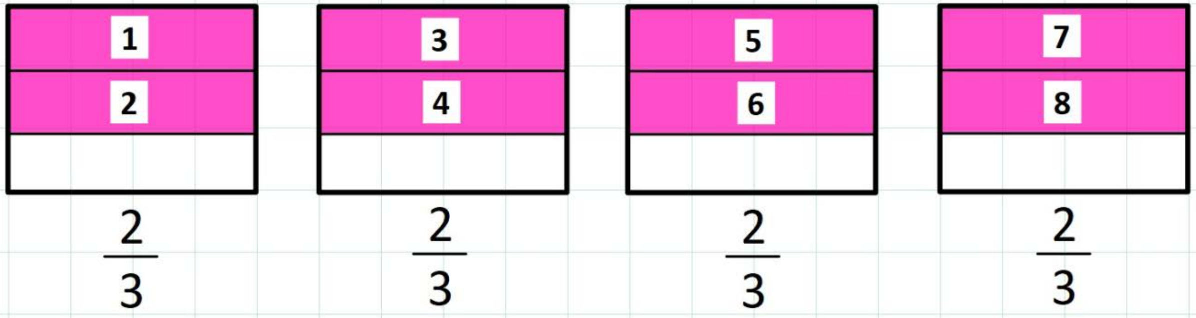
This emphasises the idea of “4 lots of 2/3”, as opposed to 2/3 lots of 4.

Keep the “wholes” separate to avoid confusion (e.g. thinking that the answer is 8/12).

Multiplying (by an integer)

$$\frac{2}{3} \times 4 = \frac{8}{3} = 2\frac{2}{3}$$

(multiply the numerator)



S7FR12	Use the four operations, with proper and improper fractions, and mixed numbers.
S7FR2	Use the four operations, including formal written methods, applied to integers and decimals.
S6FR5	Divide proper fractions by whole numbers [for example, 1/3 ÷ 2 = 1/6].
S6FR4	Multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, 1/4 × 1/2 = 1/8].

Multiplying fractions (2)

Notes:

Again, use an array whose dimensions match the two denominators.

Emphasise the fact that multiplying by a fraction is the same as finding that fraction "of" the first number.

This diagram can give pupils a clear understanding of where the "rule" comes from (multiply the numerators and the denominators).

Alternative approach:

Directly links to grid / array multiplication.

Multiplying (by a fraction)

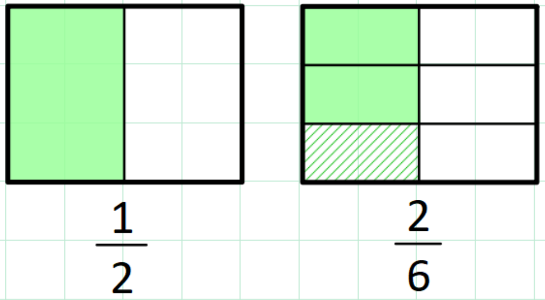
$$\frac{2}{3} \times \frac{1}{2} = \mathbf{\frac{2}{6}} = \frac{1}{3}$$

(numerator x numerator)

(denominator x denominator)

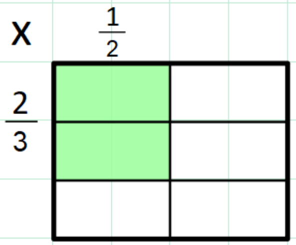
Read as: " $\frac{2}{3}$ of $\frac{1}{2}$ "

(find 2/3 of the half)



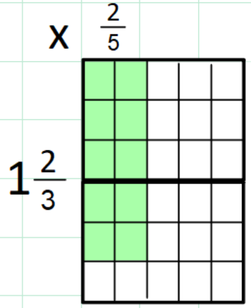
Alternative approach:

$$\frac{2}{3} \times \frac{1}{2} = \mathbf{\frac{2}{6}} = \frac{1}{3}$$



This leads well into calculations such as:

$$1\frac{2}{3} \times \frac{2}{5}$$



S7FR12	Use the four operations, with proper and improper fractions, and mixed numbers.
S7FR2	Use the four operations, including formal written methods, applied to integers and decimals.
S6FR5	Divide proper fractions by whole numbers [for example, 1/3 ÷ 2 = 1/6].
S6FR4	Multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, 1/4 × 1/2 = 1/8].

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Dividing fractions (1)

Notes:

Standard method: “flip the second fraction, then multiply”. At KS2 (Y6) this may well be fine, since understanding what is actually going on is actually very complicated.

Alt. method:

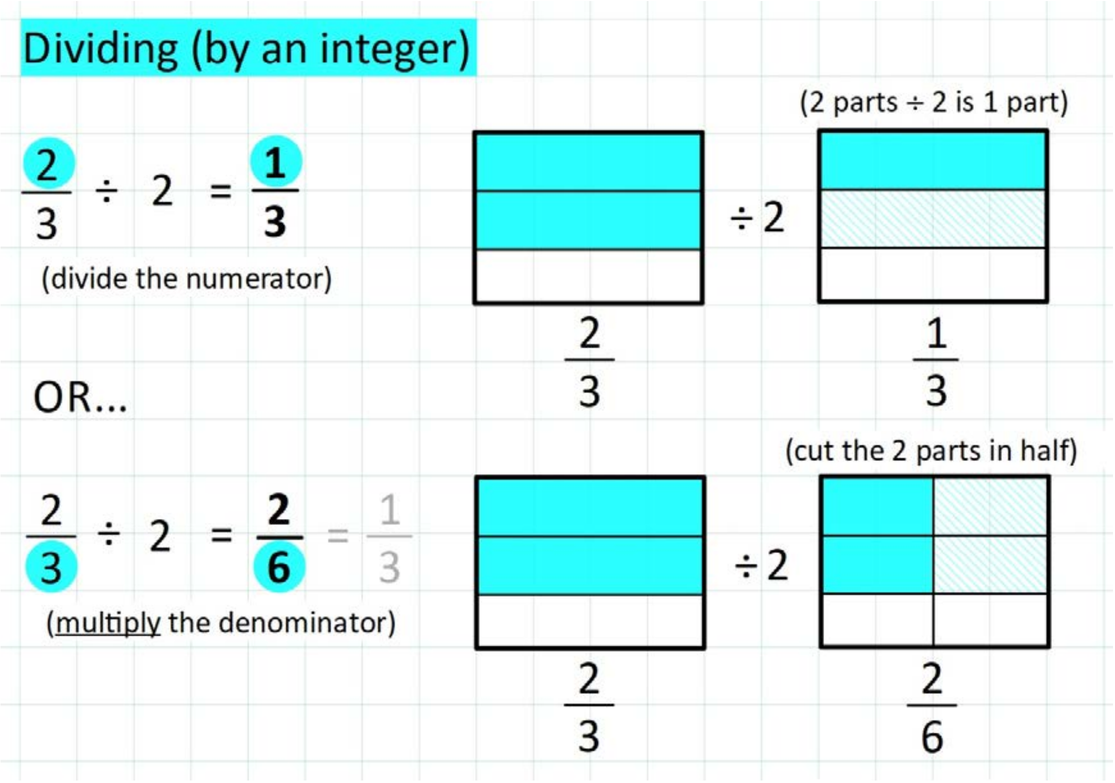
May work at KS3?

1) re-write both fractions with a common denominator:

$$\frac{\mathbf{2}}{\mathbf{3}} \div \frac{\mathbf{1}}{\mathbf{5}} = \frac{10}{15} \div \frac{3}{15}$$

2) Now divide the numerators:

$$10 \div 3 = \frac{10}{3} = \mathbf{3}\frac{\mathbf{1}}{\mathbf{3}}$$



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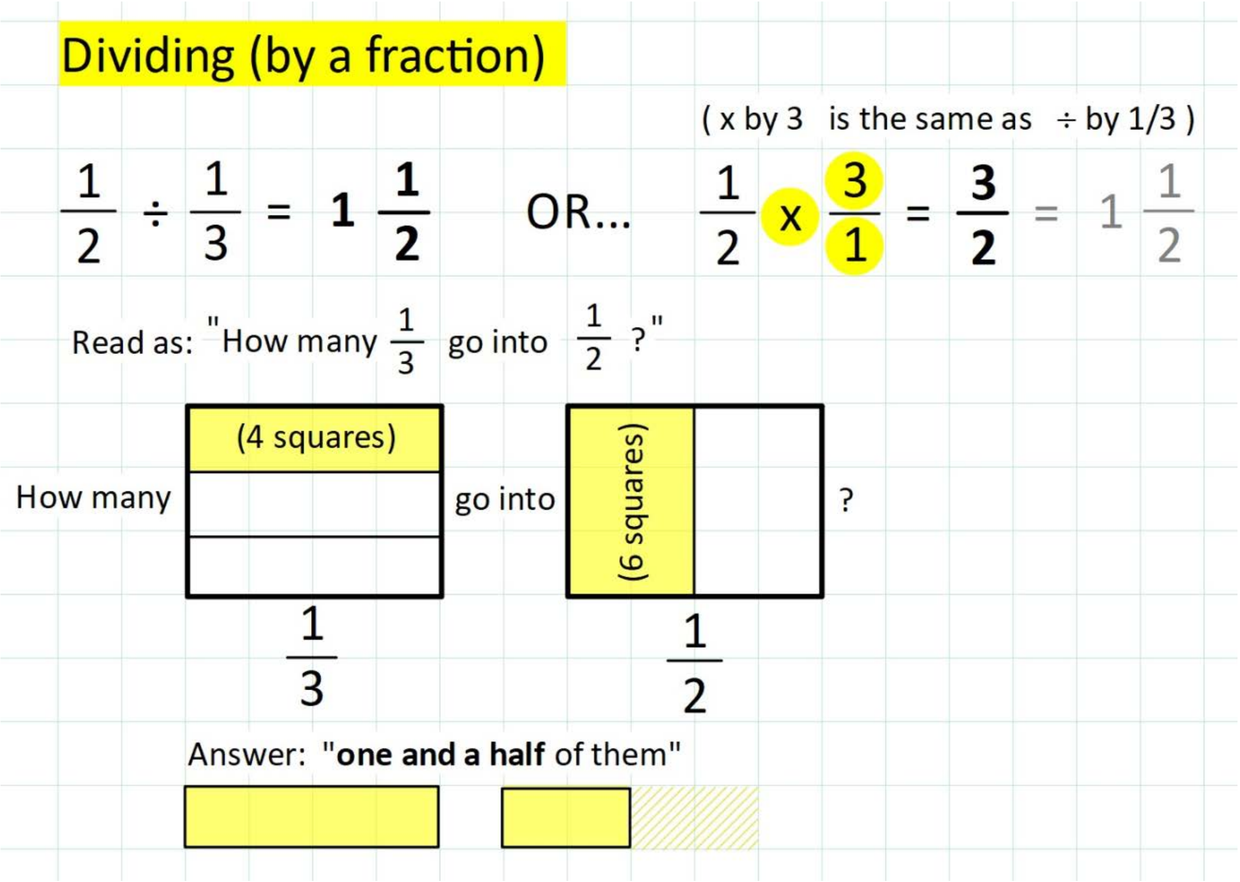
S6FR4	Multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, 1/4 × 1/2 = 1/8].

Dividing fractions (2)

Notes:

This is still a very difficult concept to understand but does at least provide opportunity to see that the answers obtained through the numerical approach are sensible.



S7FR12 Use the four operations, with proper and improper fractions, and mixed numbers.

S7FR2 Use the four operations, including formal written methods, applied to integers and decimals.

S6FR5 Divide proper fractions by whole numbers [for example, 1/3 ÷ 2 = 1/6].

S6FR4 Multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, 1/4 × 1/2 = 1/8].

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Geometry & Measures

- Converting Units of Measure

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Converting Units of Measure - metric

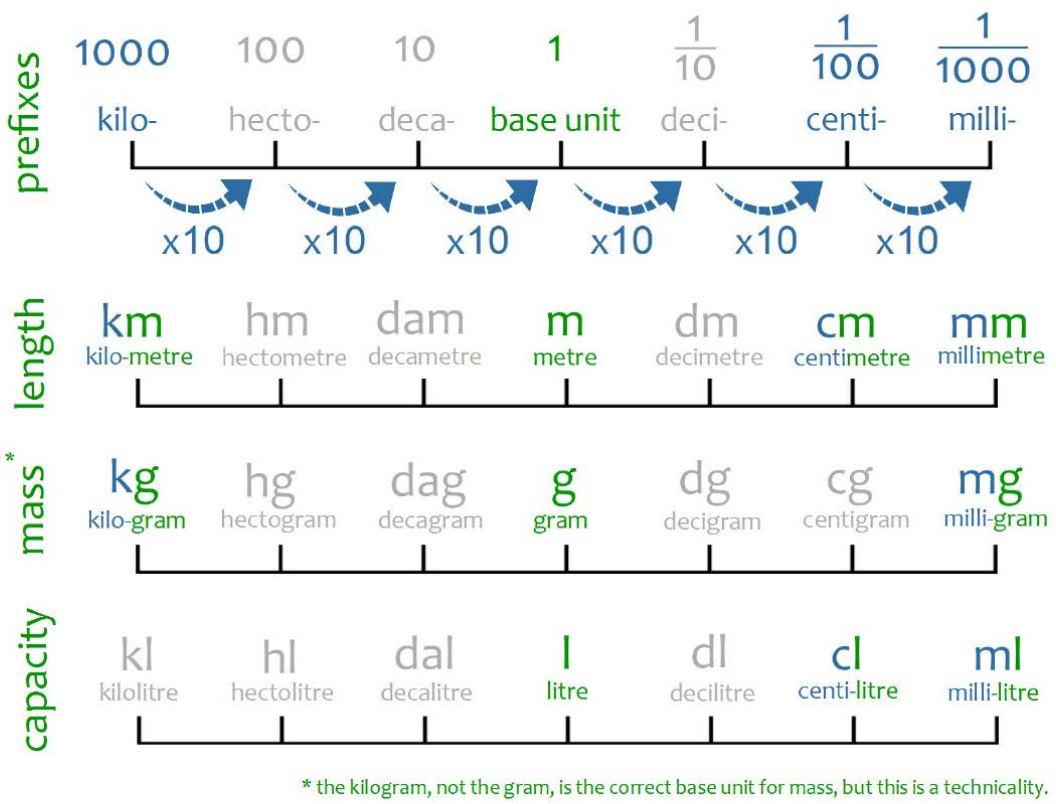
Notes:

Emphasise prefixes and their meaning.

Show the (rarely used) in-between prefixes to reveal the (base 10) structure of the metric system.

Using ratio as a basis for conversion re-enforces a key topic in KS3 (especially using a multiplier both “across” and “down” the ratio).

This topic provides an ideal opportunity for some history relating to the origin of metric measures (and their relationship to imperial measures).



÷ 100

m : cm

0.34 m : 34 cm

x 1000 x 100

km : m : cm

2.8 km : 2800 m : 280 000 cm

S6MS2	Use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places.
S6MS1	Solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate.
S5MS7	Use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling.
S5MS2	Understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints.
S5MS1	Convert between different units of metric measure (for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre).

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Algebra

- Manipulating Expressions – collecting like-terms
- Manipulating Expressions – expanding brackets
- Manipulating Expressions – factorising
-
-
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Manipulating expressions – collecting like terms

Notes:

Emphasise vocabulary: ‘term’, ‘like-term’, ‘variable’, ‘expression’.

Highlight terms using circles / or a highlighter – different colours to denote sets of like-terms.

Go back to substitution / number calculations where misconceptions arise (e.g: “Why is 3a + 5 not equal to 8a?” “It’s just like 3 x 8 + 5 is not the same as 9 x 8”)

Emphasise equivalence of simplified expressions by showing the number equivalent e.g:

$3a - 2a + 9a = 10a$

$3\times5 - 2\times5 + 9\times5 = 10\times5$

$15 - 10 + 45 = 50$

(understanding of BIDMAS is an essential prerequisite here).

S8AG4	Simplify and manipulate algebraic expressions to maintain equivalence by: taking out common factors, expanding products of two binomials.
S8AG3	Understand and use the concepts and vocabulary of inequalities and factors.
S7AG4	Simplify algebraic expressions to maintain equivalence by: collecting like terms, multiplying a single term over a bracket.
S7AG3	Understand and use the concepts and vocabulary of expressions, equations, and terms.

Manipulating expressions – expanding brackets (1)

Notes:

Use grid multiplication and the expanded form of the answer (using the distributive law of multiplication) to show the link between partitioning and expanding brackets.

$7 \times 58 = 406$

x	50	8
7	350	56

$7 \times 58 = (7 \times 50) + (7 \times 8)$

Place value partitioning;

x	40	18
7	280	126

$7 \times 58 = (7 \times 40) + (7 \times 18)$

alternative partition;

x	60	-2
7	420	-14

$7 \times 58 = (7 \times 60) - (7 \times -2)$

alternative partition

$a(7a + 9) = 7a^2 + 9a$

x	$7a$	9
a	$7a^2$	$9a$

$a \times (7a + 9) = (a \times 7a) + (a \times 9)$

$-4c(-8 + 3b - 5c) = 32c - 12bc + 20c^2$

x	-8	$3b$	$-5c$
$-4c$	$32c$	$-12bc$	$20c^2$

$-4c \times (-8 + 3b - 5c) = (-4c \times -8) + (-4c \times 3b) + (-4c \times -5c)$

S8AG4	Simplify and manipulate algebraic expressions to maintain equivalence by: taking out common factors, expanding products of two binomials.
S8AG3	Understand and use the concepts and vocabulary of inequalities and factors.
S7AG4	Simplify algebraic expressions to maintain equivalence by: collecting like terms, multiplying a single term over a bracket.
S7AG3	Understand and use the concepts and vocabulary of expressions, equations, and terms.

Manipulating expressions – expanding brackets (2)

Notes:

Again, linking to grid method when partitioning both numbers.

Vary the order / number of terms in each bracket (e.g. sometimes numbers first, sometimes different variables rather than the same one, sometimes 3 terms in a bracket etc.)

$27 \times 58 = 406$

x	50	8
20	1000	160
7	350	56

x	30	28
30	900	840
-3	-90	-84

Place value partitioning; alternative partition

$(a - 3)(4a + 7) = 4a^2 + 7a - 12a - 21$

$= 4a^2 - 5a - 21$

x	4a	7
a	$4a^2$	7a
-3	-12a	-21

S8AG4	Simplify and manipulate algebraic expressions to maintain equivalence by: taking out common factors, expanding products of two binomials.
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Manipulating expressions – factorising

Notes:

Again, linking to grid method when partitioning both numbers.

This can be simply thought of as a puzzle (exactly like missing number puzzles when learning grid multiplication). This can be a useful starter task to lead into factorising.

Puzzles such as: "two numbers sum to 20 and add to 12" work well for leading into factorising quadratics.

N.B: this lays foundations for synthetic division of polynomials in A-level.

$30a + 15 = 15\,(2a + 1)$

a)

x	?	?
?	30a	15

b)

x	?	?
15	30a	15

c)

x	2a	1
15	30a	15

a) We know the products b) Take the highest common factor

c) Find the missing multipliers

$a^2 + 7a + 10 = (a + 5)\,(a + 2)$

a)

x	a	?
a	a^2	?
?	?	10

b)

x	a	5
a	a^2	?
2	?	10

c)

x	a	5
a	a^2	5a
2	2a	10

a) a^2 must come from a x a. 10 must come two numbers multiplied.

b) & c) two numbers multiply to make 10, and add to make 7.

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