A Level Further Mathematics

Transition Activity

The questions below are an extension to the A Level Maths transition questions, therefore students taking Double Maths you should complete both the Maths AND Further Maths Transition activities.

Additional resources that you may find interesting:

+plus magazine (http://plus.maths.org/content/) for interesting articles on application of mathematics e.g.

- The maths of infectious diseases:
- Constructing our lives: the mathematics of engineering
- Mathematics and the nature of reality

Enriching mathematics site (http://nrich.maths.org/public/) which has a wide range of puzzles and articles

There are many interesting popular maths books, here are just a few examples:

- **'Professor Stewart's Cabinet of Mathematical Curiosities' by Ian Stewart** ISBN**-10: 1846680646**

- **'Fermat's Last Theorem: The story of a riddle that confounded the world's greatest minds for 358 years' by Simon Singh** ISBN**-10: 1841157910**

- **The Penguin Dictionary of Curious and Interesting Numbers (Penguin Press Science)** ISBN**-10: 0140261494**

1. **Indices:**

   Which of these values is the odd one out?: 2^6 4^3 $8^{5/3}$ $16^{3/2}$ $32^{6/5}$

2. **Simultaneous equations**

   a) If 5x - y = 18 and 5y - x = 12, what is the value of x - y?

   b) Suppose that $\boldsymbol{x - \frac{1}{x} = y - \frac{1}{y}}$ and $\boldsymbol{x \neq y}$. What is the value of *xy*?

3. **Equations**

   Solve the following equations

   a) $2x^2 - 3x - 5 = 0$

   b) $x^3 + 10x^2 + 21x = 0$

4. **Surds**

   $\sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} = 2^x$. What is the value of x?

5. **Sequences:**

   Complete the next two terms in these sequences

   a) 2 5 8 11 14 17

   b) 2 4 8 16 32 64

   c) 1 4 9 16 25 36

   d) 2 3 5 7 11 13

   e) 1 2 6 24 120 720

   f) 1 1 2 3 5 8

6. Miscellaneous

a) Put the results of the following questions in order from smallest to largest

1. The number of cm in a foot
2. The number of grams in an ounce
3. The mean of the prime numbers between 20 and 40
4. The median of the first 10 square numbers
5. The mode of the number of days in a month
6. The circumference of a circle with radius 5 units
7. The area of a rhombus with base 8 units and height 3.8 units
8. The perimeter of a rhombus with base 8 units and height 3.8 units
9. The surface area of a cube with side length 2.3 units
10. The volume of a cube with side length 3.1 units

b) A bag contains red, yellow, green and purple marbles. When a marble is drawn from the bag it is not replaced. At the beginning of each question there are 3 red, 3 yellow, 3 green and 3 purple marbles in the bag.

I. If 1 marble is drawn from the bag, what is the probability that it is red?

II. If 2 marbles are drawn from the bag, what is the probability that they are the same colour?

III. How many marbles should be drawn from the bag to ensure two marbles of the same colour are drawn?

IV. How many marbles should be drawn from the bag to ensure at least one marble of each colour is drawn?

V. 3 marbles are drawn from the bag. What is the probability that none of them are purple?

7. An Introduction to Complex Numbers

You may have been told that it is not possible to find the root of a negative number. If you try to find the value of $\sqrt{-3}$ using a calculator, you will usually be presented with a "Math Error".

However, it is possible to solve this problem by defining i to be equal to $\sqrt{-1}$. This number is known as an imaginary number.

Using i, an expression can be written for the square root of any negative number.

Example1: Find an expression for the square root of -4 and that of -7 in terms of i

$$\sqrt{-4} = \sqrt{4} \times \sqrt{-1} = 2i$$

$$\sqrt{-7} = \sqrt{7} \times \sqrt{-1} = \sqrt{7}i$$

A number can be made up of a real and an imaginary part, e.g. 5 + 4i. 5 is the real part of the number and 4i is the imaginary part. These numbers are known as complex numbers.

Complex numbers result from the solution of a quadratic equation where the discriminant, (b^2 - 4ac), is negative.

Example 2: Solve $x^2 + 2x + 5 = 0$.

Using the quadratic formula $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ a=1, b=2, c=5

$$x = \frac{-2 \pm \sqrt{2^2-4\times1\times5}}{2\times1}$$

$$x = \frac{-2 \pm \sqrt{-16}}{2}$$

$$x = \frac{-2 \pm 4i}{2}$$

$$x = -1 \pm 2i$$

Complex numbers can be added together by adding their real and complex parts individually.

Example 3: Add the complex numbers (4 + 5i) and (2 - i)

(4 + 5i) + (2 - i) = 6 + 4i

Complex numbers can be multiplied together. Note that since $i = \sqrt{-1}$ then i × i = -1.

Example 4: Find 5i × 6i

5i × 6i = 30 × (-1)

5i × 6i = -30

Example 5: Find (3 + 2i) × (4 + 5i)

3 × (4 + 5i) = 12 + 15i

2i × (4 + 5i) = 8i - 10

(3 + 2i) × (4 + 5i) = (12 + 15i) + (8i - 10)

(3 + 2i) × (4 + 5i) = 2 + 23i

Questions:

Write the following negative roots as an expression in terms of i

a) $\sqrt{-5}$ b) $\sqrt{-9}$ c) $\sqrt{-121}$ d) $\sqrt{-8}$

Solve the following equations

e) $x^2 + 2x + 10 = 0$ f) $x^2 - 3x + 4 = 0$ g) $2x^2 - x + 3 = 0$

Find the values of the following

h) (2 + i) + (4 + 5i)

i) (3 + 2i) + (5 - 10i)

j) 4 × 3i

k) 6i × -2

l) 3i × 5i

m) 3 × (4 + i)

n) 3i × (4 + i)

o) (2 + i) × (4 + 3i)

p) (2 - i)((4 + 3i)

q) (5 - 2i) × (2 - 3i)